Modern Algebra

Definitions and Basic Properties of a Ring

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The most general algebraic structure with two binary operations is called a *ring*.

Definition

A **ring** $\langle R, +, \cdot \rangle$ is a set R together with two binary operations $+$ and $\cdot$, which we call *addition* and *multiplication* defined on R such that the following axioms are satisfied $\forall\ a, b \in R$:

1. $\langle R, + \rangle$ is an abelian group.
    a. $a + b \in R$. (Closure)
    b. $a + (b + c) = (a + b) + c$. (Associative)
    c. $\exists\ 0 \in R$ s.t. $a + 0 = 0 + a = a$. (Identity)
    d. $\forall\ a \in R, \exists\ -a \in R$ s.t. $a + -a = -a + a = 0$. (Inverse)
    e. $a + b = b + a$. (Commutative)
2. R is closed under $\cdot$, $a \cdot b \in R$.
3. Multiplication is associative, $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.
4. $\forall\ a, b, c \in R$, the **left distributive law**, $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ and the **right distributive law** $(a + b) \cdot c = (a \cdot c) + (b \cdot c)$ hold. ■