1

Introduction to rings: defining a ring and giving examples. Knowledge of sets, proofs, and mathematical groups are recommended.

Practice problems:

1.) Which of the following sets are rings? If it is, prove it. If not, say which property of rings fails for that set (there may be more than one).

a) N = {1,2,3,4,5,...} under normal addition and multiplication

b) A = {a+b\*sqrt(2) | a,b are rationals} under normal addition and multiplication

c) B = {all polynomials p(x) with integer coefficients} d) C = {all polynomials p(x) with integer coefficients, where deg( p(x) ) is even}

2.) The set {0,2,4,6,8,10,12} is a ring with unity under the operations of addition mod 14 and multiplication mod 14. What is the unity of this ring?

3.) Let R be a commutative ring with unity, and let U(R) denote the set of units (elements with multiplicative inverses) of R. Prove that U(R) is a group under the multiplication defined in R.

2

Elementary properties of rings. These properties are true in general, and can be inferred directly from the definition of a ring. Other properties not mentioned in the video:

-the unity of a ring is unique

-multiplicative inverses are unique

-also, a \* ( b - c ) = a\*b - a\*c -and (-1) \* (-1) = 1

There are no practice problems for this video.

Solutions to problems from video 1:

1.) Note that there is limited space here, so I cannot provide proofs in this description. However, if there are requests, I may make a video detailing a proof for a problem.

a.) The Natural numbers do not form a ring. They are not an abelian group under addition, since not every element has an additive inverse.

b.) Set A does form a ring under normal addition and multiplication.

c.) Set B does form a ring under normal addition and multiplication.

d.) Set C does not form a ring. The set is not closed under addition. For example, both x^2 and -x^2 + x are elements of C, since both polynomials have degree 2. However, ( x^2 ) + ( -x^2 + x ) = x is not an element of C, since x has degree 1.

2.) 8 is the unity of this set. Remember that we are multiplying mod 14. 8\*0=0, 8\*2=16=2, 8\*4=32=4, 8\*6=48=6, 8\*8=64=8, 8\*10=80=10, 8\*12=96=12

3.) There is limited space here, so I cannot provide proofs in this description. However, if there are requests, I may make a video detailing a proof for this problem. Remember to show that some set is a group under some operation, you need to show:

1.) closure under the operation

2.) associativity

3.) there is an identity element

4.) every element has an inverse