Time Series Analysis and Forecasting

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Chapter Outline

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III. Forecasting using Exponential Smoothing

1. What is Exponential Smoothing?
2. Types of Exponential Smoothing
3. Performing Exponential Smoothing in RStudio

Forecasting using Exponential Smoothing

What is exponential smoothing?

Motivation for exponential smoothing

Forecasts using simple moving average sma(p) gives all p time points equal weights 1/p.

Yet one can argue that the **more recent observations give more relevant information than the older ones**, hence a need for a forecasting scheme that gives decaying or decreasing weights to more distant observations.

What is exponential smoothing?

Exponential smoothing definition

Exponential smoothing methods weigh past observations using **exponentially decreasing weights** as the observations get older.

That is, they **give larger weights to more recent observations**, and the weights decrease exponentially as the observations become more distant.

What is exponential smoothing?

Exponential smoothing methods

The following are the five methods of exponential smoothing:

Method	Pars	Trend	Seas	Hyndman Taxonomy
Single Exponential Smoothing	1	✗	✗	ANN
Double Exponential Smoothing	1	✓	✗	AAdN
Holt-Winters (no seasonal)	2	✓	✗	AAN
HW (additive seas.)	3	✓	✓	AAA
HW (multiplicative seas.)	3	✓	✓	AAM

Single exponential smoothing

It is also referred to as **simple exponential smoothing** (SES) in some references. The single exponential smoothing method is used for forecasting a time series when there is no trend or seasonal pattern.

Single exponential smoothing

For any time period t, the **smoothened series** St is found by calculating

$$S_t = \alpha y_t + (1-\alpha)S_{t-1},\ 0 < \alpha \le 1.$$

The parameter a is called the smoothing or damping parameter.

Single exponential smoothing

To see how the weights decrease exponentially over time: $S_t = \alpha y_t + (1-\alpha)S_{t-1},\ 0 < \alpha \le 1.$

$$
\begin{aligned}
&= \alpha y_t + (1-\alpha)(\alpha y_{t-1} + (1-\alpha)S_{t-2}) \\
&= \alpha y_t + \alpha(1-\alpha)y_{t-1} + (1-\alpha)^2 S_{t-2} \\
&= \alpha y_t + \alpha(1-\alpha)y_{t-1} + (1-\alpha)^2(\alpha y_{t-2} + (1-\alpha)S_{t-3}) \\
&= \boxed{\alpha} y_t + \boxed{\alpha(1-\alpha)} y_{t-1} + \boxed{\alpha(1-\alpha)^2} y_{t-2} + \boxed{\alpha(1-\alpha)^3} y_{t-2} + \ldots
\end{aligned}
$$

The weight of the most recent time point y_t is the highest, followed by the weight of y_{t-1}, followed by the weight of y_{t-2} and so on.

Single exponential smoothing

Remarks about single exponential smoothing

- SES works best for time series with no trend or seasonality. It can still be applied on series with trend or seasonality, although the other methods will most likely produce better forecasts.

Single exponential smoothing

Remarks about single exponential smoothing

- The value of a is chosen such that the smoothing procedure minimizes the RMSE. Although Bowerman and O'Conell (1979) suggested values between 0.01 and 0.30.
- Don't worry though, RStudio (will do the job for us and) can estimate the optimal value of the parameter.

Double exponential smoothing

The double exponential smoothing method (DES) is applicable to time series **with linear trend but no seasonality**.

It applies ses twice in the time series. Given a time series y_t,

$$S_t = \alpha y_t + (1-\alpha)S_{t-1},\ 0 < \alpha \le 1$$
$$D_t = \alpha S_t + (1-\alpha)D_{t-1}.$$

Double exponential smoothing

Forecasts for *DES* are given by the following formula:

$$\hat{y}_{T+h} = \left(2S_T - D_T\right) + \left[\frac{\alpha}{1-\alpha}\left(S_T - D_T\right)\right]h.$$

$$= \boxed{Mean_t + Trend_t \times h}$$

The last above emphasizes that the forecast is a **linear trend model with no seasonality**.

Although the formula is given, RStudio can automatically provide forecasts using double exponential smoothing.

Holt-Winters two parameter exponential smoothing

Like DES, the Holt-Winters two-parameter smoothing (HW2) is appropriate for series with a **linear time trend and no seasonality**, but more parsimonious.

The forecast form of HW2 is

$$\hat{y}_{T+h} = Mean_t + Trend_t \times h$$

where $$mean_t = \alpha y_t + (1-\alpha)(mean_{t-1} + trend_{t-1})$$

$$trend_t = \beta(mean_t - mean_{t-1}) + (1-\beta)trend_{t-1},\ \ 0 < \alpha, \beta < 1.$$

Holt-Winters 3 parameter ES: additive seasonality

The Holt-Winters three parameter exponential smoothing with additive seasonality (HWA) is appropriate for a time series with **trend and additive seasonal variation**.

$$mean_t = \alpha\left(y_t - season_{t-s}\right) + (1-\alpha)\left(mean_{t-1} + trend_{t-1}\right)$$
$$trend_t = \beta\left(mean_t - mean_{t-1}\right) + (1-\beta)trend_{t-1}$$
$$season_t = \gamma\left(y_t - mean_{t+1}\right) + (1-\gamma)season_{t-s}, \ \ 0 < \alpha, \beta, \gamma < 1.$$

Holt-Winters 3 parameter ES: additive seasonality

The forecast form of HWA is

$$\hat{y}_{T+h} = \left(mean_T + trend_T \times h\right) + season_{T-s+h}$$

The series $season_t$ is an additive seasonal component with s being the length of seasonality.

Holt-Winters 3 parameter ES: multiplicative seasonality

In the Holt-Winters three parameter exponential smoothing with multiplicative seasonality (HWM), the three series $mean_t$, $trend_t$ and $season_t$ are obtained using the following recursive equations:

$$
\begin{aligned}
mean_t &= \alpha\left(y_t / season_{t-s}\right) + (1-\alpha)\left(mean_{t-1} + trend_{t-1}\right) \\
trend_t &= \beta\left(mean_t - mean_{t-1}\right) + (1-\beta)\,trend_{t-1} \\
season_t &= \gamma\left(y_t / mean_{t+1}\right) + (1-\gamma)\,season_{t-s}, \quad 0 < \alpha, \beta, \gamma < 1.
\end{aligned}
$$

Holt-Winters 3 parameter ES: multiplicative seasonality

The forecast form of HWM is

$$\hat{y}_{T+h} = \left(mean_T + trend_T \times h\right) \times season_{T-s+h}$$

Here, the series $season_t$ is a multiplicative seasonal component with s being the length of seasonality.

Which exponential smoothing to use?

Choosing the "best" method

- Check which components are present in the time series through decomposition. Then refer to the table in *slide 5*.
- Choose the method with the least MAPE.

Performing Exponential Smoothing in RStudio

Performing Exponential Smoothing in RStudio

```
#Load Packages
library("zoo")     #Date manipulation package
library("smooth") #For Exponential Smooting

#Set Working Directory
setwd("F:/Profession/PSAi/Datasets")

#Import Data and Manage Date
export <- read.table("export.txt", header=T)
  head(export)

export$month <- gsub("M","-",export$month)
rtime <- as.yearmon(export$month, format="%Y-%m")
export <- cbind(rtime,export["export"])
  head(export)

export.ts <- ts(export$export,frequency=12,start=c(1991,1))
  head(export.ts)
```

Performing Exponential Smoothing in RStudio

```
#Exponential Smoothing Using the smooth Package
export.ses  <- es(export.ts,model="ANN",h=12,holdout=FALSE,intervals="parametric",
                  level=0.95,cfType="MSE")
export.des  <- es(export.ts,model="AAdN",h=12,holdout=FALSE,intervals="parametric",
                  level=0.95,cfType="MSE")
export.hwn  <- es(export.ts,model="AAN",h=12,holdout=FALSE,intervals="parametric",
                  level=0.95,cfType="MSE")
export.hwa  <- es(export.ts,model="AAA",h=12,holdout=FALSE,intervals="parametric",
                  level=0.95,cfType="MSE")
export.hwm  <- es(export.ts,model="AAM",h=12,holdout=FALSE,intervals="parametric",
                  level=0.95,cfType="MSE")
export.auto <- es(export.ts,model="ZZZ",h=12,holdout=FALSE,intervals="parametric",
                  level=0.95,cfType="MSE")

#ANN  means Additive errors, No trend, No seasonality (SES)
#AAdN means Additive errors, Additive (dampled) trend, No seasonality (DES)
#AAN  means Additive errors, Additive trend, No seasonality (HW no seasonality)
#AAA  means Additive errors, Additive trend, Additive seasonality (HW add. seas.)
#AAM  means Additive errors, Additive trend, Multiplicative seasonality (HW mult. seas.)
#ZZZ  means all components are (Z) estimated
```

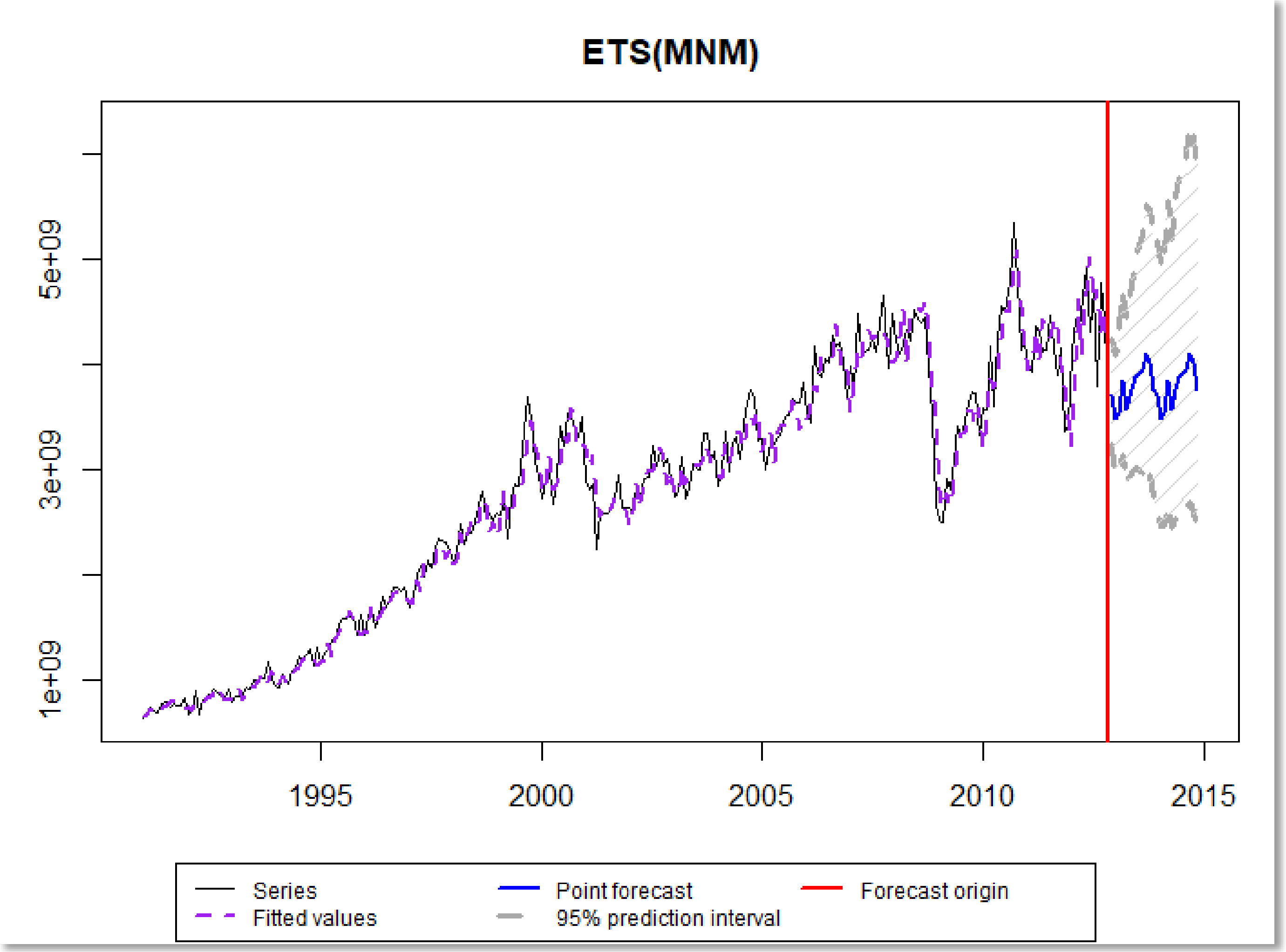
Performing Exponential Smoothing in RStudio

```
#CHOOSE LOWEST MAPE
MAPE(export.ts,export.ses$fitted)
MAPE(export.ts,export.des$fitted)
MAPE(export.ts,export.hwn$fitted)
MAPE(export.ts,export.hwa$fitted)
MAPE(export.ts,export.hwm$fitted)
MAPE(export.ts,export.auto$fitted)

#Forecast h steps using model with lowest MAPE
forecast(export.auto,h=24)
plot(forecast(export.auto,h=24))
```

Performing Exponential Smoothing in RStudio

Forecast Dates	Point Forecasts	Lower Bound	Upper Bound
Dec 2012	3.71	3.25	4.25
Jan 2013	3.49	2.97	4.09
Feb 2013	3.53	2.94	4.24
Mar 2013	3.85	3.14	4.71
Apr 2013	3.57	2.86	4.45
May 2013	3.76	2.96	4.77
Jun 2013	3.88	3.01	5.01
Jul 2013	3.92	3.00	5.13
Aug 2013	3.94	2.97	5.23
Sep 2013	4.10	3.05	5.52
Oct 2013	4.03	2.96	5.48
Nov 2013	3.75	2.72	5.17
Dec 2013	3.71	2.66	5.18
Jan 2014	3.49	2.47	4.92
Feb 2014	3.53	2.47	5.04
Mar 2014	3.85	2.67	5.55
Apr 2014	3.57	2.45	5.20
May 2014	3.76	2.55	5.53
Jun 2014	3.88	2.61	5.78
Jul 2014	3.92	2.61	5.89
Aug 2014	3.94	2.60	5.98
Sep 2014	4.10	2.68	6.28
Oct 2014	4.03	2.61	6.22
Nov 2014	3.75	2.41	5.84



Workshop

Performing Exponential Smoothing in RStudio

Workshop 3: Exponential Smoothing

Write an R code that performs exponential smoothing on the following data:

(i) RGDP and
(ii) Netflix sales (in Billion USD).

Provide 2-year (h=8) forecasts based on the most appropriate model.

Thank You