Exploring Quarto and Latex

Orville D. Hombrebueno

1 Integration by Substitution

Theorem 1.1 (Substitution Rule)**.** *If $u = g(x)$ is a differentiable function whose range is an interval I and f is continues on I, then*

$$\int f(g(x)) \cdot g'(x)dx = \int f(u)du.$$

Example 1.1.

1. $\int (1-4x)^{\frac{1}{2}}dx$

   If we let $u = 1 - 4x$, then $du = -4dx$. We multiply the integrand by $\dfrac{-4}{-4}$. Thus,

   $$\int (1-4x)^{1/2}dx = \int (1-4x)^{1/2}.\frac{-4}{-4}dx = \int u^{1/2}\left(-\frac{du}{4}\right) = -\frac{1}{4}\int u^{1/2}du = -\frac{1}{4}.\frac{2u^{3/2}}{3}+C.$$

   We put the final answer in terms of x by substituting $u = 1 - 4x$. Therefore,

   $$\int (1-4x)^{1/2}dx = -\frac{(1-4x)^{3/2}}{6} + C$$

2. $\int x^2(x^3-1)^{10}dx$

$$\begin{aligned} 2x - 5y &= 8 \\ 3x + 9y &= -12 \end{aligned}$$