FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION

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Report

on the practical task No. 2

“Algorithms for unconstrained nonlinear optimization. Direct methods”

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**Goal**

The use of direct methods (one-dimensional methods of exhaustive search, dichotomy, golden section search; multidimensional methods of exhaustive search, Gauss (coordinate descent), Nelder-Mead) in the tasks of unconstrained nonlinear optimization.

**Formulation of the problem**

I. Use the one-dimensional methods of exhaustive search, dichotomy and golden section search to find an approximate (with precision ) solution for the following functions and domains:

1.

2.

3.

Calculate the number of -calculations and the number of iterations performed in each method and analyze the results. Explain differences (if any) in the results obtained.

II. Generate random numbers and . Furthermore, generate the noisy data , where , according to the following rule:

,

where are values of a random variable with standard normal distribution. Approximate the data by the following linear and rational functions:

1. (linear approximant),
2. (rational approximant)

by means of least squares through the numerical minimization (with precision ) of the following function:

To solve the minimization problem, use the methods of exhaustive search, Gauss and Nelder-Mead. If necessary, set the initial approximations and other parameters of the methods. Visualize the data and the approximants obtained in a plot separately for each type of approximant so that one can compare the results for the numerical methods used. Analyze the results obtained (in terms of number of iterations, precision, number of function evaluations, etc.).

**Brief theoretical part**

Unconstrained minimization is the problem of finding a vector x that is a local minimum to a scalar function :

The term unconstrained means that no restriction is placed on the range of .

**Dichotomy**

The Dichotomous Search Method computes the midpoint , and then moves slightly to either side of the midpoint to compute two test points: , where is a very small number. The objective being to place the two test points as close together as possible. The procedure continues until it gets within some small interval containing the optimal solution.

Then for :

We can determine the new interval in the following way:

* if , then the new uncertainty interval is ;
* if , then the new uncertainty interval is

It is easy to see, the length of the obtained interval is . We continue this procedure until get for some , where is a tolerance. Then if we choose the midpoint of the last interval , then the error will be less than .

**Golden Section**

The Golden Section search method is used to find the maximum or minimum of a unimodal function. To make the discussion of the method simpler, let us assume that we are trying to find the minimum of a function. The previously introduced search method is somewhat inefficient because if the interval is a small number it can take a long time to find the minimum of a function. To improve this efficiency, the Golden Section Search method is suggested.

The Golden Section Search Method chooses and such that the one of the two evaluations of the function in each step can be reused in the next step. At the end of the golden section method we get the local minimum point is in interval which length is less then . We use the approximation , because the condition fulfils.

**Results**

**Table 1**

*Number of -calculations and the number of iterations performed in each method*

|  |  |  |  |
| --- | --- | --- | --- |
| **Functions** | **Exhaustive Search** | **Dichotomy** | **Golden Section** |
|  | (1001; 1001) | (21; 10) | (17; 15) |
|  | (1001; 1001) | (21; 10) | (17; 15) |
|  | (991; 991) | (21; 10) | (17; 15) |

As we can see, number of -calculations and the number of iterations performed in exhaustive search for each function are not the same. E.g., we have 991 calculations of  
 as the interval of the function starts with 0.01. That is why it’s less than others.

As for dichotomy and golden section, the seconds one is slightly more efficient (20% less of functions’ calls, but 50% more of execution iterations).

**Conclusions**

Make conclusions on the results obtained and on the achievement of the goal of your work

**Appendix**

Source code can be found in GitHub repository: github.com/ellkrauze/algorithms2022