To: Programming Group

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Subject: Trajectory Physics

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Summary

This season we may want to shoot a ball into a target. The important parameters are:

- Initial velocity
- Firing angle
- Starting position
- Target position

The central assumption is that we do not have to account for air resistance in that our speeds will be low. This assumption implies that the acceleration due to gravity is independent of mass.

Computations

The trajectory of an object with starting velocity v and direction at angle to the horizontal is well known. The equations of motion for the x and y components of the velocity are as follows:

$x(t) = x0 + v\_x * t$. x0 is the starting x-coordinate and v_x is the x component of the velocity.

$y(t) = y0 + v\_y * t - g*t^2/2.0$. g is the acceleration due to gravity, v_y is the y component of the velocity and y0 is the starting y coordinate.

Both of these equations can be solved for "t", in our case, the time of flight. If x and y now represent the target coordinates, equal time of flight for each implies that we hit the target. Note that the y equation is quadratic in "t". It is the equation of a parabola; therefore the trajectory could have the same y value at two different x locations, one before it reaches the azimuth and one after.

Time of flight for $x = (x-x0)/v\_x$

Time of flight for $y = v\_y/g \pm \mathrm{sqrt}((v\_y^2/g^2) - 2*v\_y/g)$

The computer program evaluates each expression for a range of theta and searches for the minimum difference between x and y time of flight. The minimum difference represents a possible solution (firing angle). Note that the argument within the square root must be positive. The program detects this and rejects negative arguments.

Converting Degrees to Radians

For those familiar with trig, the x-component of the velocity is v*cos(theta) where theta is firing angle from the horizontal. The y-component of the velocity is v*sin(theta). Note that in most computer languages the angle argument of trig functions is expressed in radians. Conversion from radians to degrees etc., is as follows:

Degrees = radians * 180.0/PI

Radians = degrees * PI/180.0

The functions in the program allow angle entry in degrees and the necessary conversions are performed, i.e., degrees to radians and then radians to degrees as required.