CSC 404 - Activity/Project 10 - Name: Chris Glunzer

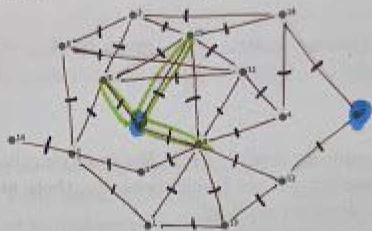
Problem 1. Consider the following graph adding the degree

vert deg sum 4

$9 = 5 + 6 + 8 = 19$

$15 = 6$

$8 = 8$



5 + 2 + 3 + 5 + 2 + 6 + 2 + 3 + 2 = 30 edges

a. Construct the Adjacency Matrix, A (at least the first 3 rows)

blank = 0

$$A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccccccccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{array} \\ \begin{pmatrix} - & - & - & - & - & - & 1 & - & 1 & 1 & - & - & - & - & - & 1 \\ - & - & 1 & - & - & - & - & - & 1 & - & - & - & - & 1 & - & - \\ - & 1 & - & - & - & 1 & 1 & - & - & 1 & - & - & - & - & 1 & - \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \end{array}$$

b. Determine the number of paths of length 2 that originate from 0 and end at $0, 1, 2, \ldots, 15$. That is, determine the first ~~row~~ of A^2 (count the paths by hand and use A^2 to check your work).

4. 19

$$A^2 = \begin{pmatrix} 3 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 3 & 1 & 1 & 0 & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

c. Determine the number of paths of length 3 that originate from 0 and end at $0, 1, 2, \ldots, 15$. That is, determine the first row of A^3 (count the paths $\boxed{\text{by hand}}$ and use A^3 to check your work). bruh.. can I just say there's 80 by hand?

80

$$A^3 = \begin{pmatrix} 4 & 3 & 3 & 4 & 5 & 2 & 6 & 2 & 13 & 11 & 3 & 5 & 2 & 3 & 1 & 11 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

d. By using A^4, determine the number of paths of length 4 from 0 to 7. Then, list these paths :-).
(I was pretty kind here – We could have done from 0 to 11)

find all intersections between: - 2 moves from 7: 7 4 13 15 8 3

and

~~triangles~~ from 0: 0 8 15 9 1 9 2 10 3 11 4 12 5 13

overlaps: 15 13 8 3 4

ways ~~[illegible]~~

7 paths

1) 0,9,13,10,7
2) 0,9,15,10,7
3) 0,15,8,12,7
4) 0,15,3,10,7
5) 0,8,4,10,7
6) 0,8,13,12,7
7) 0,8,15,10,7

c:

vert deg

9: 0 = 3, 2 = 5, 3 = 4, 11 = 5, 15 = 6 } 23

15: 0 = 3, 3 = 4, 8 = 8, 9 = 5, 10 = 4, 11 = 5 } 29

8: 0 = 3, 1 = 3, 4 = 3, 5 = 2, 4 = 5, 12 = 3, 13 = 3, 15 = 6 } 28

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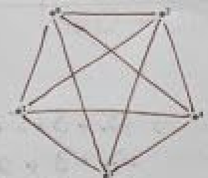
Definition 1. A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge. For example, the complete graph with 5 vertices, K_5 is given below (it is a pentagram!)

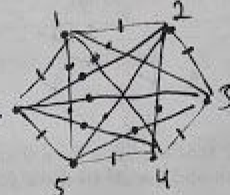


With adjacency matrix, A_{K_5}, given by $A_{K_5} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$

The number of edges on a complete graph K_n is $n(n-1)/2$. For example, the number of edges on K_5 is $5(4)/2 = 10$. You can see this by seeing that each vertex has $n-1$ neighbors. So, there are $n(n-1)$ '1s' in the adjacency matrix and dividing by two accounts for the double counting!

Problem 2. Draw the complete graph with 6 vertices, K_6, and confirm that it has $6(6-1)/2$ edges.

6
3
3
2
1
15 ✓



$\frac{30}{2}$ 15 ✓

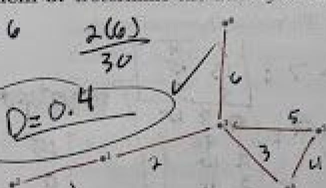
Definition 2. The density of an undirected graph G is then umber of edges of G divided by the number of possible edges in an undirected graph (i.e., the number of edges in the corresponding complete graph). That is, if $|V| = n$ (n vertices) and $|E| = m$ (m edges), then

$$D(G) = \frac{\text{edges in } G}{\text{edges in } K_n} = \frac{m}{n(n-1)/2} = \frac{2m}{n(n-1)}$$

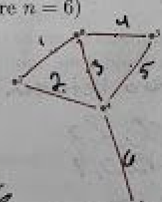
Problem 3. Determine the density of the following graphs (Here $n = 6$)

$n = 6$ $\frac{2(6)}{30}$

D = 0.4 ✓



a.

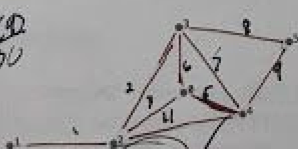


b. $n = 6$

$\frac{2(6)}{30}$

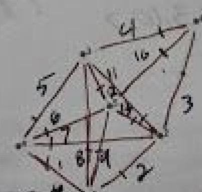
D = 0.4 ✓

$\frac{2(9)}{30}$



c. $n = 6$

D = 0.6 ✓



d. $n = 6$

$\frac{2(13)}{30}$ $\frac{26}{30}$ $\frac{13}{15}$

D = 0.86 ✓

4 + 3 + 2 + 3 + 1 = 13

Problem 4. Determine the density of the graph in problem 1 (Here $n = 16$).

$\frac{2(30)}{16 \cdot 15}$ $\frac{60}{240}$ = D = 0.25