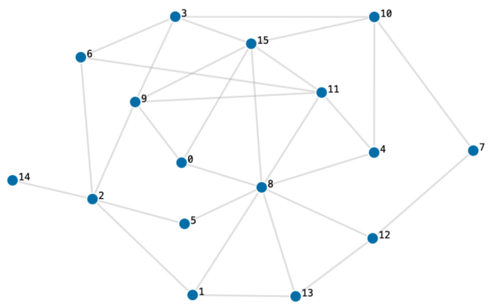
CSC 404 - Activity/Project 10 - Name:

Problem 1. Consider the following graph



a. Construct the Adjacency Matrix, A (at least the first 3 rows)

$$A = \begin{pmatrix} -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

b. Determine the number of paths of length 2 that originate from 0 and end at $0, 1, 2, ..., 15$. That is, determine the first row of A^2 (count the paths by hand and use A^2 to check your work).

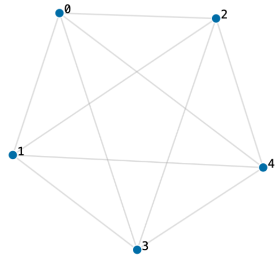
$$A^2 = \begin{pmatrix} -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

c. Determine the number of paths of length 3 that originate from 0 and end at $0, 1, 2, ..., 15$. That is, determine the first row of A^3 (count the paths by hand and use A^3 to check your work).

$$A^3 = \begin{pmatrix} -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- & -- \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

d. By using A^4, determine the number of paths of length 4 from 0 to 7. Then, list these paths :-).
(I was pretty kind here – We could have done from 0 to 11)

Definition 1. A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge. For example, the complete graph with 5 vertices, K_5 is given below (it is a pentagram!)



With adjacency matrix, A_{K_5}, given by $A_{K_5} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$

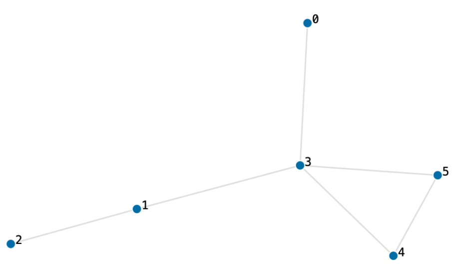
The number of edges on a complete graph K_n is $n(n-1)/2$. For example, the number of edges on K_5 is $5(4)/2 = 10$. You can see this by seeing that each vertex has $n-1$ neighbors. So, there are $n(n-1)$ '1s' in the adjacency matrix and dividing by two accounts for the double counting!

Problem 2. Draw the complete graph with 6 vertices, K_6, and confirm that it has $6(6-1)/2$ edges.

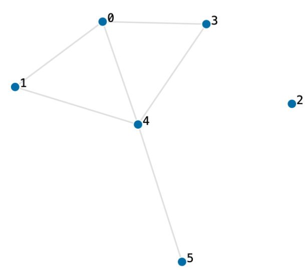
Definition 2. The density of an undirected graph G is then umber of edges of G divided by the number of possible edges in an undirected graph (i.e., the number of edges in the corresponding complete graph). That is, if $|V| = n$ (n vertices) and $|E| = m$ (m edges), then

$$D(G) = \frac{\text{edges in } G}{\text{edges in } K_n} = \frac{m}{n(n-1)/2} = \frac{2m}{n(n-1)}$$

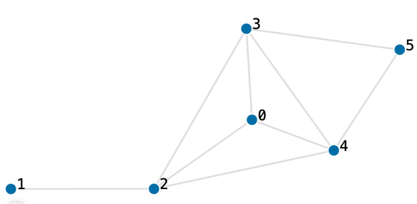
Problem 3. Determine the density of the following graphs (Here $n = 6$)



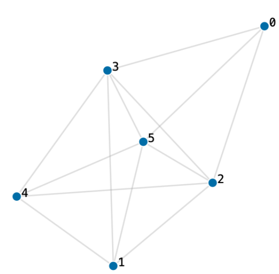
a.



b.



c.



d.

Problem 4. Determine the density of the graph in problem 1 (Here $n = 16$).