CSC 404 - Activity/Project 11 - Name:

Problem 1 (Zombies!)**.** In this example we look at a simplistic model of everyones favorite pop culture staple - the zombie apocalypse (though it probably feels a bit to close to home during the current pandemic).

Consider the following system with three types of individuals, Susceptible, Infected, and Zombies. For now, we assume that the time scale is so short that (for practical purposes) there are no births and the rate of infection is constant (this really should depend on the number of zombies, but for now we keep it friendly).

i. Susceptible individuals can be infected at a rate r (and stay susceptible at a rate $1 - r$)
ii. Infected individuals can be healed at rate h or become zombies at rate z (and stay infected at rate $1 - h - z$)
iii. Zombies can stop rampaging and simply become infected at rate b (and stay zombies at rate $1 - b$).

a. Construct the state diagram for this apocalypse.

b. Construct the state transition matrix for this apocalypse.

$$P = \begin{array}{c c} & \begin{array}{ccc} S & I & Z \end{array} \\ \begin{array}{c} S \\ I \\ Z \end{array} & \begin{pmatrix} \text{------} & \text{------} & \text{------} \\ \text{------} & \text{------} & \text{------} \\ \text{------} & \text{------} & \text{------} \end{pmatrix} \end{array}$$

c. For the rates $r = 0.2$, $h = 0.1$, $z = 0.5$, and $b = 0.2$. Compute P^{100} and P^{1000} to approximate the steady state probabilities (i.e., in the long run, what percent of the population is susceptible, infected, and zombie?).

d. (Mutation!) Once infected people become zombies more quickly. Increase the z value (we must have $z \leq 1$ and $0 \leq 1 - h - z \leq 1$). Compute P^{100} and P^{1000} to approximate the steady state probabilities.

e. (Friend[ly/ier] Zombies?) We found a way to make zombies more docile. Decrease the r value (we must have $0 \leq r \leq 1$). Compute P^{100} and P^{1000} to approximate the steady state probabilities.

f. (Bonus) Fun variants/extensions to this models? (Create different rules/ideas and try to implement them) [Super zombies?] (This is a tip of a very very big iceberg looking into fun ways to explore various systems with state diagrams/state transition matrices/models and explore long term patterns - more to come!)

Problem 2. Alice and Bob are playing the following game: They have a three-card deck marked with the numbers 1, 2, and 3 and a spinner with the numbers 1, 2, and 3 on it. The game begins by dealing the cards out so that the dealer gets one card and the other person gets two. A move in the game consists of a spin of the spinner. The person having the card with the number that comes up on the spinner hands that card to the other person. The game ends when someone has all the cards.

Consider the following example. Here Alice is the dealer and deals herself a 2 (and Bob receives 1 and 3). The last column of the table denotes the state we (Alice) is in - that is `S2` denotes she has card `2`, `S23` denotes she has cards `2` and `3`, and `SW`/ `SL` denotes she wins/loses.

Spinner	Alice	Bob	State	Description
-	2	1 3	S2	Initial Deal
3	2 3	1	S23	3 is spun - Bob gives Alice a 3
2	3	1 2	S3	2 is spun - Alice gives Bob a 2
1	1 3	2	S13	1 is spun - Bob gives Alice a 1
1	3	1 2	S3	1 is spun - Alice gives Bob a 1
2	2 3	1	S23	2 is spun - Bob gives Alice a 2
1	1 2 3	-	S123 = SW	1 is spun - Bob gives Alice a 1 (Alice wins!)

a. Construct the state diagram for this game (There are 8 total states – see the matrix below for labels). Add loops on 1 at the two 'end game states' - SW and SL (i.e., make them absorbing states).

b. Construct the state transition matrix for this game.

$$P = \begin{array}{c} \\ SL \\ S1 \\ S2 \\ S3 \\ S12 \\ S13 \\ S23 \\ SW \end{array} \begin{array}{c} \begin{array}{cccccccc} SL & S1 & S2 & S3 & S12 & S13 & S23 & SW \end{array} \\ \begin{pmatrix} 1 & -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- & 1 \end{pmatrix} \end{array}$$

c. Compute P^{100} (or your favorite power of P). You should see a matrix with (essentialy) zeros in the middle and all of the data on the left and right columns. Record these values below.

$$P^{100} = \begin{array}{c} \\ SL \\ S1 \\ S2 \\ S3 \\ S12 \\ S13 \\ S23 \\ SW \end{array} \begin{array}{c} \begin{array}{cc} SL & SW \end{array} \\ \begin{pmatrix} 1 & -- \\ -- & -- \\ -- & -- \\ -- & -- \\ -- & -- \\ -- & -- \\ -- & -- \\ -- & 1 \end{pmatrix} \end{array}$$

d. From this table we can can see the (approximate) probabilities of Alice winning/losing!

i. If Alice is the dealer (i.e., she starts with one card, 1, 2, or 3) what is her probability of winning/losing?

ii. If Bob is the dealer (i.e., Alice starts with two cards, 12, 13, or 23) what is Alice's probability of winning/losing?

e. (Bonus) Cool variants of this problem? (create different rules/ideas and try to implement them)