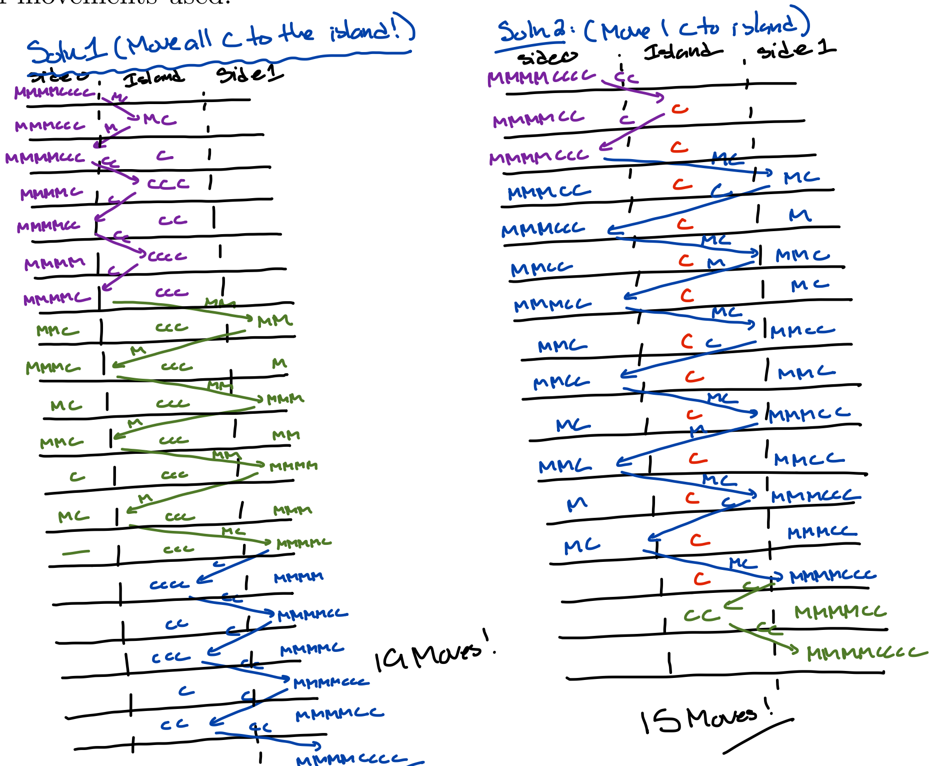
CSC 404 - Homework 1 - Name:

Problem 1 (River crossing with a middle island!)**.** On one bank of a river are **four** missionaries and **four** cannibals. There is one boat available that can hold up to **two** people and that they would like to use to cross the river. The river contains a middle island that can be used to assist the movement of missionaries and cannibals across the river. (Here we are allowed to cross from bank-to-bank without stopping at the middle island. If this were not allowed, we could not transport two cannibals across if a missionary were on the middle island.)

If the cannibals ever outnumber the missionaries on either of the river's banks, the missionaries will get eaten. How can the boat be used to safely carry all the missionaries and cannibals across the river?

a. Solve the riddle by drawing a diagram indicating the movement of missionaries and cannibals. Record the number of movements used.

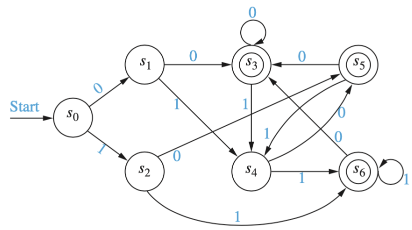


b. (Bonus) Solve the original riddle with the assumption that shore-to-shore travel is not allowed (i.e., you must stop at the middle island). Does this change the number of movements used?

c. (Bonus) Solve the riddle with $N = 5$ missionaries and $N = 5$ cannibals (without and with shore-to-shore movement)

d. (Bonus) Attempts at implementing a search algorithm (in a language of your choice) to construct a path from initial state to end state in the Missionaries and Cannibals (+ Middle Island) Riddle?

Problem 2. (Why do you hate 01?) Consider the following deterministic finite-state automaton (DFA).



a. Determine which of the following are accepted by the DFA. What state do they end at?

$w_1 =$ 101010 (42!) Yes! (s_5) $w_2 =$ 000111 Yes! (s_6) $w_3 =$ 011100 Yes! (s_3) $w_4 =$ 10100111001 (1337!) No! (s_4)

b. Identify all bit-strings of length 3 that are accepted by the DFA.

000 ~~001~~ 010 011 100 ~~101~~ 110 111

c. Identify all bit-strings of length 4 that are accepted by the DFA.

0000 ~~0001~~ 0010 0011 0100 ~~0101~~ 0110 0111

1000 ~~1001~~ 1010 1011 1100 ~~1101~~ 1110 1111

d. Identify all bit-strings of length 5 that are accepted by the DFA.

00000 ~~00001~~ 00010 00011 00100 ~~00101~~ 00110 00111

01000 ~~01001~~ 01010 01011 01100 ~~01101~~ 01110 01111

10000 ~~10001~~ 10010 10011 10100 ~~10101~~ 10110 10111

11000 ~~11001~~ 11010 11011 11100 ~~11101~~ 11110 11111

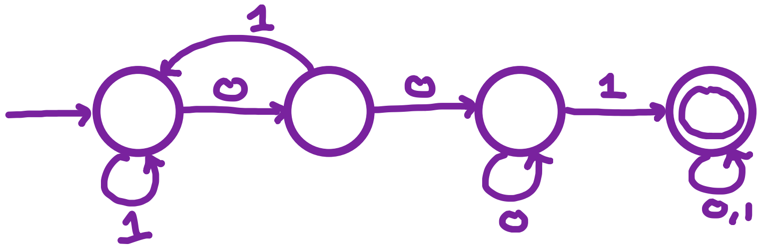
e. (Bonus) Describe (in words) the set of bit-strings recognized by this DFA. (Hopefully you see a pattern in the mess that is this machine.)

Strings of length at least 2 that do not end with 01

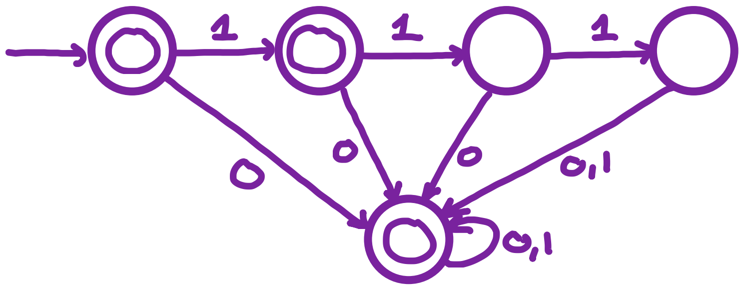
f. Blank space – add some sweet pictures – yay! Fun fact, binary numbers that end with 01 are congruent to 1 modulo 4.

Problem 3. (Yay designing DFAs - Enjoy!) For each of the following, construct a deterministic finite-state automaton (DFA) that recognizes the given set.

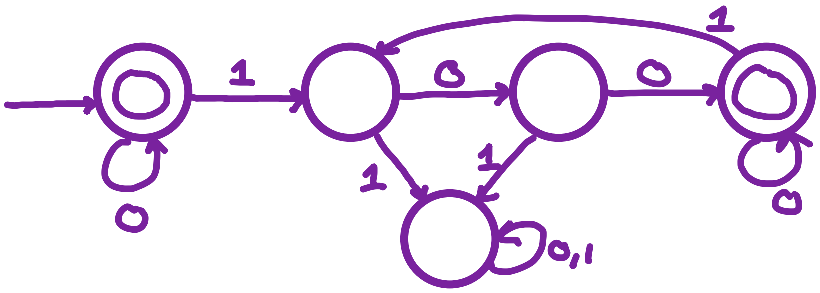
a. Set of all bit-strings that contain the substring `001`. (Use at most 4 states)



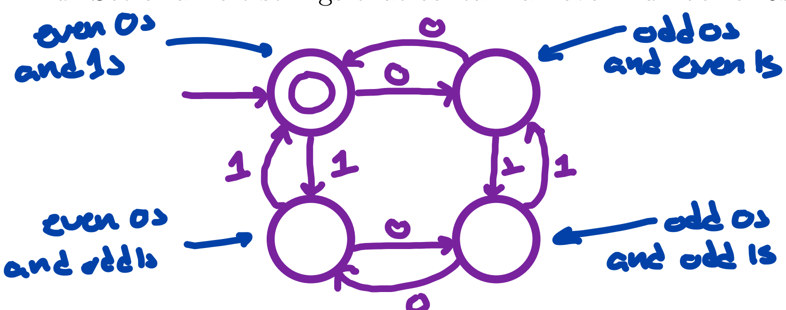
b. Set of all bit-strings except `11` and `111` (Use at most 5 states)



c. Set of all bit-strings where every `1` is followed by at least two `0`s. (e.g., `0, 100, 01000, 1001000,...`)



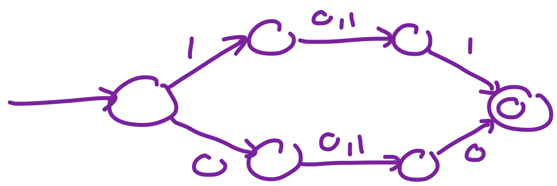
d. Set of all bit-strings that contain an even number of 0s and an even number of 1s. (Use 4 states)



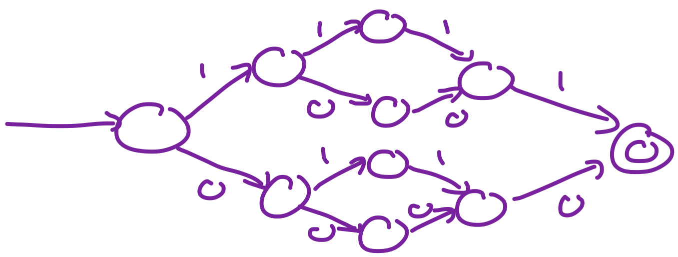
e. (Bonus) Set of bit-strings that contain an equal number of occurrences of the substrings `01` and `10`. Note `101` is accepted because `101` contains a single `10` and a single `01`, but `1010` is not because it contains two `10`s and one `01`.

Problem 4. (Palindromes and DFA are bad news :-/) A string w is palindromic if it is the same read forwards and backwards - e.g., 10101, 111, racecar, neveroddoreven (Never odd or even).

a. Construct a deterministic finite-state automaton with fixed input length $\ell = 3$ (all words/strings are of length 3) that recognizes the palindromic bit-strings of length 3 (`000,010,101,111`). (Solve with 6 states or fewer.)



b. Construct a deterministic finite-state automaton with fixed input length $\ell = 4$ that recognizes the palindromic bit-strings of length 4 (`0000,0110,1001,1111`).



c. (Bonus) Construct a deterministic finite-state automaton with fixed input length $\ell = 5$ that recognizes the palindromic bit-strings of length 5 (`00000,00100,01010,01110,10001,10101,11011,11111`).

d. (Bonus) What issues emerge when you attempt to construct a DFA that recognizes all bit string palindromes? (This is one of the examples the prompts us to look at additional models of computation)