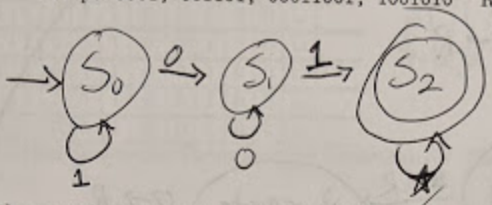
CSC 404 - Exam 1 - Name: Chris Glanzer

Problem 1 (01 and 001 - weee!). For each of the following, construct a finite automaton that recognizes the given language. Then, write the language via regular expressions, implement, and test against the given sets.

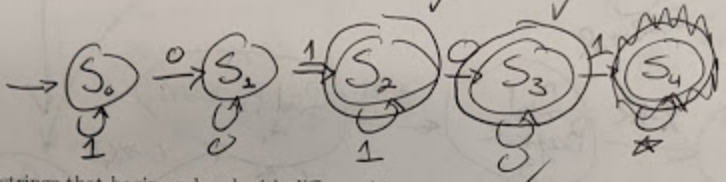
a. Bit-strings that contain the substring 01.

Accept: 0001, 001101, 00011001, 1001010 Reject: 0000, 1000, 111110



b. Bit-strings that contain exactly one copy of the substring 01.

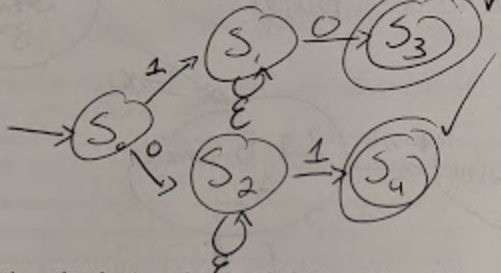
Accept: 01, 0011, 0001110, 1000111000, 11110100 Reject: 0101, 0111011, 1101101001



Note: Mistakenly circled S_4 twice... S_2 & S_3 Should be only Accept state

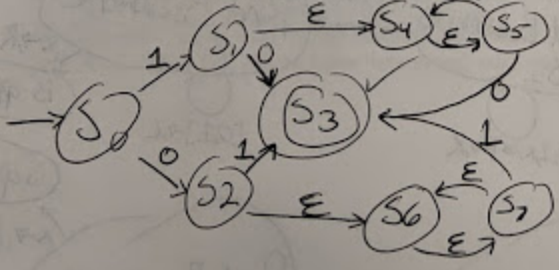
c. Bit-strings that begin and end with different bits.

Accept: 101010, 000101, 01111 Reject: 10100111001, 01010, 00



d. Bit-strings that begin and end with different bits and have even length if it starts with a 1 and odd length if it starts with a 0.

Accept: 101010, 0001101, 00111 Reject: 10100111001, 01010, 010101, 100



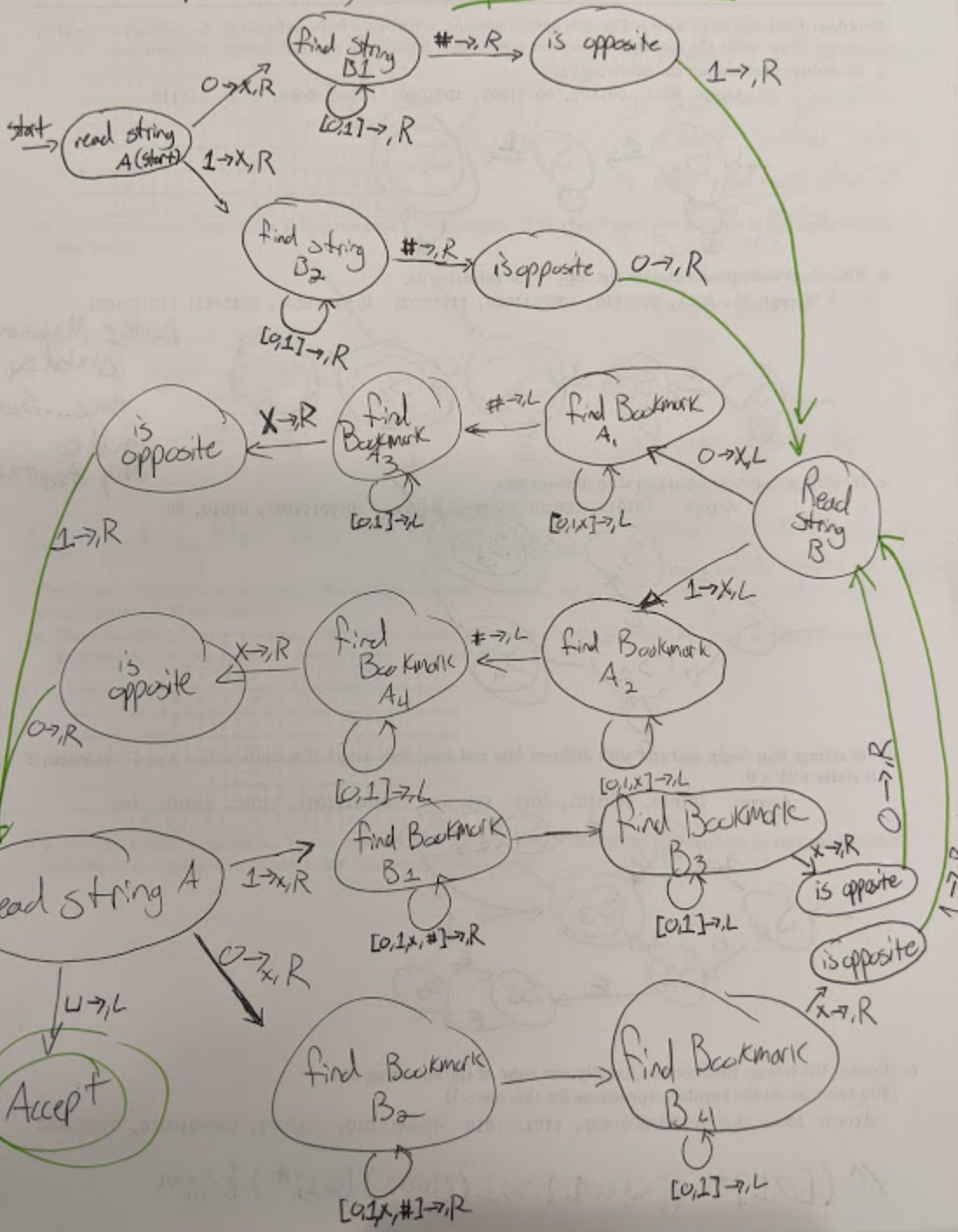
e. (Bonus) Bit-strings that contain exactly one copy of the substring 001.
(You can just do the regular expressions for this one :-))

Accept: 10001,000110,1000001101, 1101001010 Reject: 010, 0010011, 0001010010, 001001001

/^([01]* (?<!001)001(?!001)[01]*)$/gm

(No bookmarks on first read, so special branch)

Problem #2

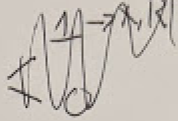


Problem 2 (Complements – You are awesome!). In the first part of this problem, we look at the question of seeing if two strings are bitwise complements of each other (i.e., 1s and 0s are swapped).

a. Describe how a Turing machine, COMPLEMENT, would accept the string 1011#0100. Give a rough idea what your machine will do (you will design a state diagram below).

String A   String B

···	⊔	1	0	1	1	#	0	1	0	0	⊔	···
···	⊔	1	0	1	1	#	0	1	0	0	⊔	···
···	⊔	1	0	1	1	#	0	1	0	0	⊔	···
···	⊔	1	0	1	1	#	0	1	0	0	⊔	···
···	⊔	1	0	1	1	#	0	1	0	0	⊔	···
···	⊔	1	0	1	1	#	0	1	0	0	⊔	···
···	⊔	1	0	1	1	#	0	1	0	0	⊔	···



1. read left string & branch
2. Bookmark position
3. check "other" string
4. read next char of "other" string
5. Bookmark
6. check string
7. move bookmark 2

etc.

b. Construct a state diagram for Turing machine COMPLEMENT. Then, implement the machine in turingmachine.io and test it :-).

tm.IO

see separate sheet (Back of sheet 1)

Also nothing that this has a thousand states bruh! — you can rebuild the strings from the remaining parity information but I didn't include that function

Problem 3 (Different = awesome). Are strings different? That is, our goal is to design a Turing Machine that accepts strings that are different.

a. Describe how a Turing machine, DIFFERENT, would accept the string 1011#10101. Give a rough idea what your machine will do (you will design a state diagram below).

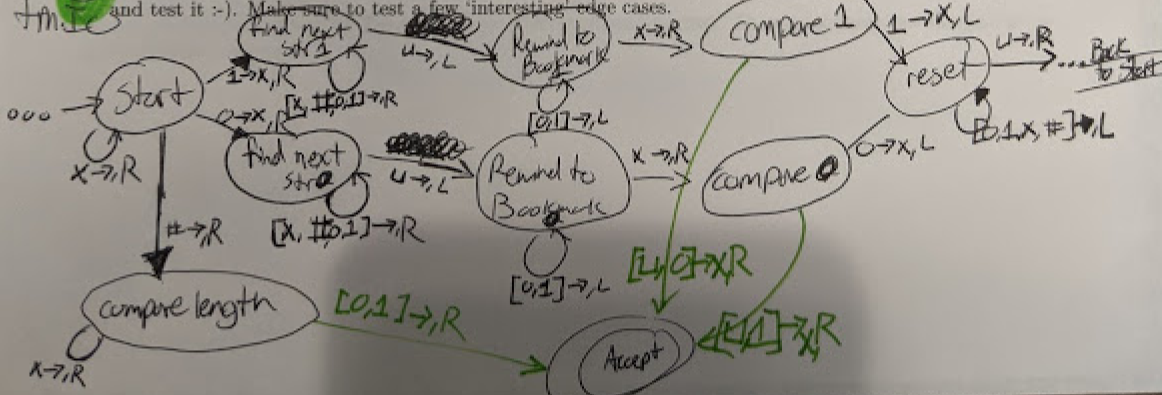
···	⊔	1	0	1	1	#	1	0	1	0	1	⊔	···
···	⊔	1	0	1	1	#	1	0	1	0	1	⊔	···
···	⊔	1	0	1	1	#	1	0	1	0	1	⊔	···
···	⊔	1	0	1	1	#	1	0	1	0	1	⊔	···
···	⊔	1	0	1	1	#	1	0	1	0	1	⊔	···
···	⊔	1	0	1	1	#	1	0	1	0	1	⊔	···
···	⊔	1	0	1	1	#	1	0	1	0	1	⊔	···

Here we Destroy All Data to minimize states... maximize bookmarking

screw efficiency. hail simplicity

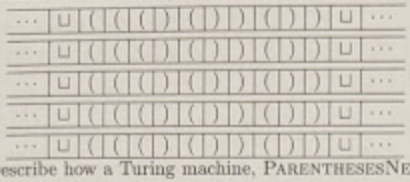
b. Construct a state diagram for Turing machine DIFFERENT. Then, implement the machine in turingmachine.io and test it :-). ~~Make sure to test a few 'interesting' edge cases.~~

tm.IO

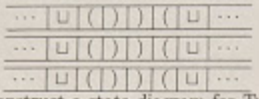


Problem 4 (Parentheses Nesting – Ah the struggle of finding a missing (or)). In this problem, we explore a classic issue – matching left and right parentheses :-/. (The ability to match parentheses is something finite automaton and regular expressions cannot handle!).

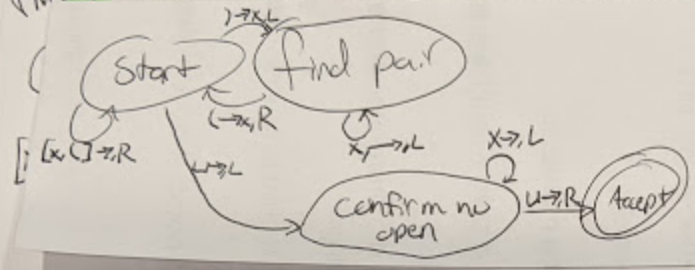
a. Describe how a Turing machine, PARENTHESESNESTING, would accept the string ((()())())



b. Describe how a Turing machine, PARENTHESESNESTING, would reject the string ())(



c. Construct a state diagram for Turing machine PARENTHESESNESTING. Then, implement the machine in turingmachine.io and test it.

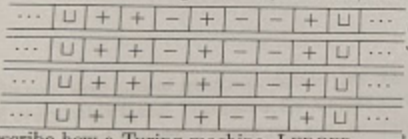


lollar for each + and loses one dollar
ie negative. For example,

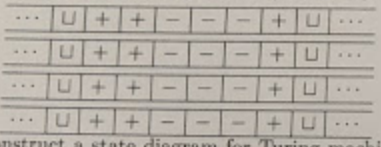
Accept: ++-+, +-++, ++-+--+, +-++-+-+-- Reject: +--, -++, ++----+, ++-+---+++

(Note: there is a reason this problem is right after parentheses nesting – they are very similar problems!)

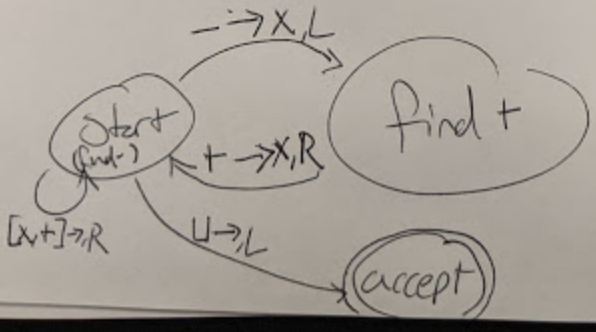
a. Describe how a Turing machine, LEDGER, would accept the string ++-+---+.



b. Describe how a Turing machine, LEDGER, would reject the string ++----+ (negative after the 5th input).



c. Construct a state diagram for Turing machine PARENTHESESNESTING. Then, implement the machine in turingmachine.io and test it.



Problem 6 (Three Equal Lengths – Weee!). Consider the (non-regular) language of all strings of 0s followed by an equal number of 1s and then an equal number of 2s,

$$L = \{012, 001122, 000111222, 000011112222, ...\} = \{0^k 1^k 2^k \mid k = 0, 1, 2, ...\}$$

a. Describe how a Turing machine would accept the string 000001111122222

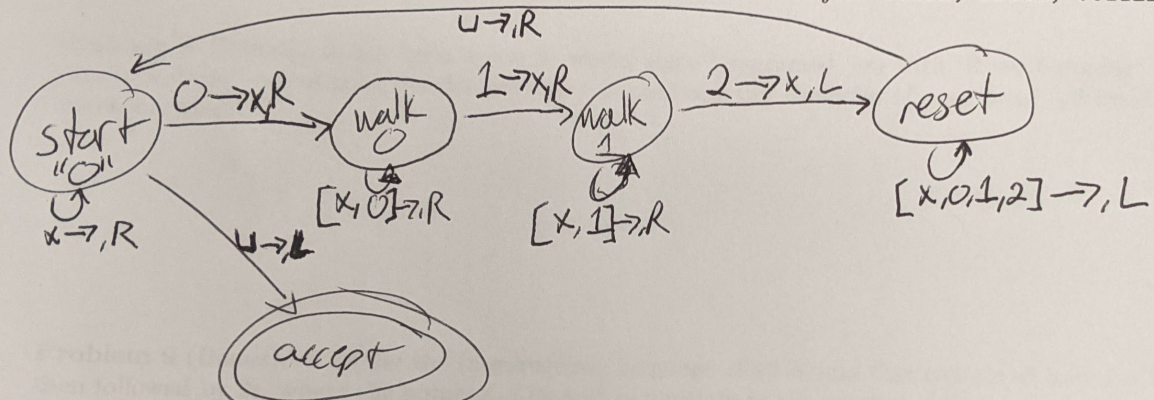
···	⊔	0	0	0	0	0	1	1	1	1	1	2	2	2	2	2	⊔	···
···	⊔	0	0	0	0	0	1	1	1	1	1	2	2	2	2	2	⊔	···
···	⊔	0	0	0	0	0	1	1	1	1	1	2	2	2	2	2	⊔	···
···	⊔	0	0	0	0	0	1	1	1	1	1	2	2	2	2	2	⊔	···
···	⊔	0	0	0	0	0	1	1	1	1	1	2	2	2	2	2	⊔	···
···	⊔	0	0	0	0	0	1	1	1	1	1	2	2	2	2	2	⊔	···

0,x → 1,x → 2,x.

if we don't hit end of string on same pass as 0|1 & 1|2 then fail

b. Construct a state diagram for Turing machine, ThreeEqualLengths. Then, implement the machine in turingmachine.io and test it. (Make sure it rejects the last example in the Reject List - this is the one most often missed :-))

Accept: 000011112222, 000001111122222 Reject: 00112, 00122, 001122012



c. (Bonus) Most *initial* machines decide this language in $\mathcal{O}(n^2)$. Extending the ideas from Section 2.3 (crossing off every other 0 and 1) construct a machine that decides this language in $\mathcal{O}(n \log_2(n))$. If you already did this in parts a and b – yay, I guess you can use this space to draw a sweet picture!

we gonna have a ton of states the

like 8 states just tracking even-odd symmetries

and that doesn't even include the "every-other" mechanisms or the position reset mechanisms.

bruh nuh-uh noway. not handdrawing all that jazz

Problem 7 (Bonus)**.**

a. How are things going in the course? Provide some comments about how the course is going so far (e.g., Activities/Project, Homework, Notes, Videos, D2L).

smooth so far.

b. As we move into the Algorithms/Optimization and Information Theory/Cryptography stage of the course, we move away from the cute machine world and begin 'actual' implementation. Here you are free to work in any programing language. I will typically work in Python and C – are there other languages you would like to explore?

I'll probably just follow suit

Problem 8 (Bonus)**.** It has been a couple weeks since I spammed you with 'River Crossing' Problems – with your new found love of river puzzles have you created any fun scenarios of your own? (Wired/Strange rule sets are encouraged.)

nope.

Problem 9 (Bonus)**.** Consider the (non-regular) language of all strings that contain at least one 0 followed by 1s then followed by 2s, where the number of 0s and 1s multiply to the number of 2s.i.e.,

$$L = \{012, 01122, 00122, 00112222, 00111222222, ...\} = \{0^{\mathtt{i}}1^{\mathtt{j}}2^{\mathtt{k}} \mid 1 \leq \mathtt{i} \text{ and } \mathtt{ij} = \mathtt{k}\}$$

A Turing machine that decides L is

M = "On input string w :

1. Scan the input from left to right to determine whether it is a member of $0^*1^*2^*$ and reject if it is not
2. Return the head to the left-hand end of the tape
3. Cross of a 0 and scan to the right until a 1 occurs. Bounce between 1s and 2s, crossing off one of each until all 1s are gone. If all 2s have been crossed off and some 1s remain, reject
4. Restore the crossed off 1s and repeat stage 3 if there is another 0 to cross off. If all 0s have been crossed off, determine whether all 2s also have been crossed off. If yes, accept; otherwise, reject

a. Describe how a Turing machine, M, would accept the string 00111222222 ($i = 2, j = 3, ij = 2 \cdot 3 = 6 = k$)

···	⊔	⊔	0	0	1	1	1	2	2	2	2	2	2	⊔	⊔	···

b. Describe how a Turing machine, M, would reject the string 0011222.

···	⊔	⊔	0	0	1	1	2	2	2	⊔	⊔	···

c. Construct a state diagram for Turing machine M.