CSC 404 - Foundations of Computation

Section 1.2 – Nondeterministic Finite Automaton (NFA)

DFA: Deterministic Finite Automaton

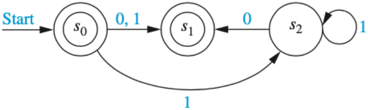
NFA: Nondeterministic Finite Automaton

LATER: NFA $\longrightarrow$ DFA

Nondeterministic Finite Automata (NFA)

Example 1.2.1.

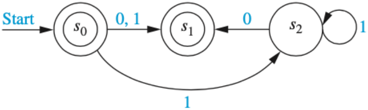
Consider the following Nondeterministic Finite Automaton (NFA)



Can leave s_0 on 1 to s_1 OR s_2.

Example 1.2.1.

Consider the following Nondeterministic Finite Automaton (NFA)



$\boxed{0}$

$s_0 \xrightarrow{0} s_1$

ACCEPT!

WHAT ABOUT $\boxed{1}$?

$\boxed{1}$

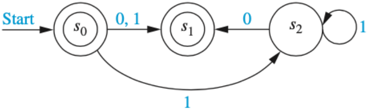
$s_0 \xrightarrow{1} s_1$

$s_0 \xrightarrow{1} s_2$

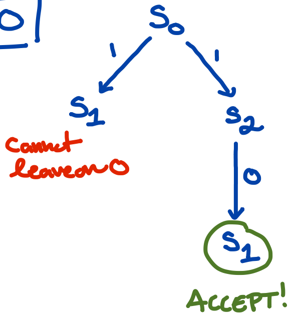
At least one ends in a final state, so we accept 1.

Example 1.2.1.

Consider the following Nondeterministic Finite Automaton (NFA)

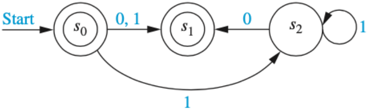


$\boxed{10}$



Example 1.2.1.

Consider the following Nondeterministic Finite Automaton (NFA)



$\boxed{00}$

REJECT!

s_0

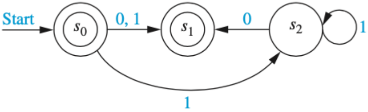
$\downarrow 0$

s_1

we cannot leave on a 0. Even though s_1 is an accept state we have not gone through the entire word/string, thus we REJECT!

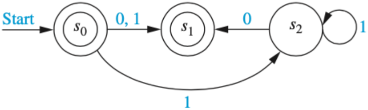
Example 1.2.2.

Find the language recognized by the NFA shown below.



Example 1.2.2.

Find the language recognized by the NFA shown below.



ACCEPTED?

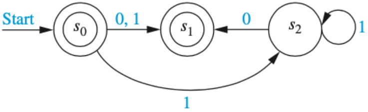
$\emptyset$, 0, 1, 10, 110, 1110, ...

Empty String

Nondeterministic Finite Automata (NFA)

Example 1.2.2.

Find the language recognized by the NFA shown below.



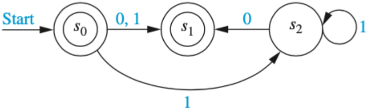
MORE LATER:

$$\underbrace{10,\ 110,\ 1110,\ \ldots}_{1\,(\text{any number of 1s})\,0}$$

$= 1\,1^*0$

Example 1.2.2.

Find the language recognized by the NFA shown below.

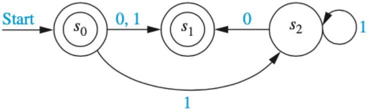


$L(M) = \phi \mid 0 \mid 1 \mid 11^*0$

NFA as DFA

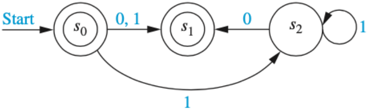
Example 1.2.3.

Find a DFA that recognizes the same language as the following NFA.

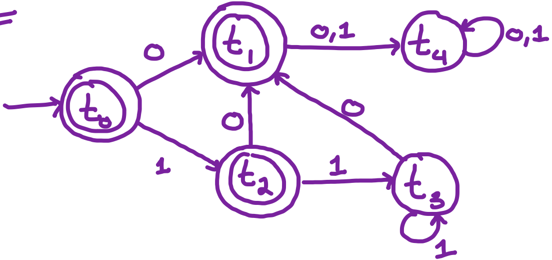


Example 1.2.3.

Find a DFA that recognizes the same language as the following NFA.

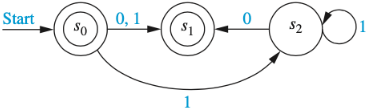


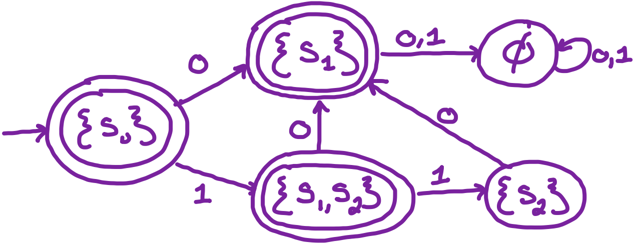
ACCEPTS: $\phi, 0, 1, \underbrace{10, 110, 1110, \dots}_{1\,\underbrace{11\dots1}_{\text{Any \# of 1s}}\,0}$



Example 1.2.3.

Find a DFA that recognizes the same language as the following NFA.

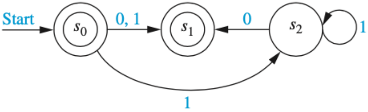


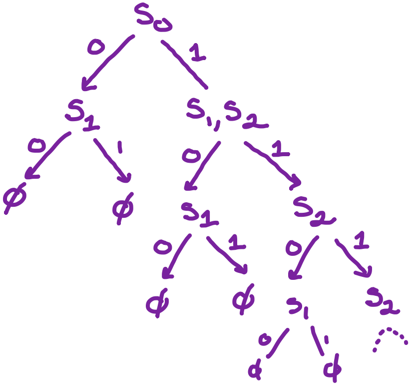


NFA as DFA

Example 1.2.3.

Find a DFA that recognizes the same language as the following NFA.





NFA as DFA

Theorem 1.2.4.

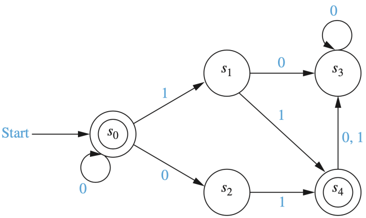
If a language L is recognized by a nondeterministic finite automaton M_0, then L is also recognized by a deterministic finite automaton M_1.

I.E. NFA $\longrightarrow$ DFA

Any NFA can be turned into a DFA!

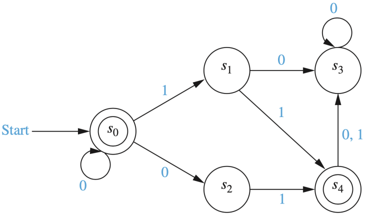
Example 1.2.5.

Consider the following Nondeterministic Finite Automaton (NFA)



Example 1.2.5.

Consider the following Nondeterministic Finite Automaton (NFA)



$\boxed{11}$

ACCEPT!

$s_0 \xrightarrow{1} s_1 \xrightarrow{1} s_4$

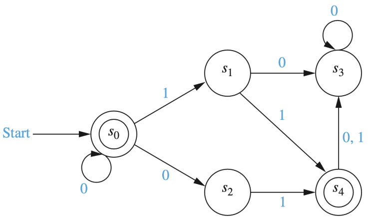
$\boxed{111}$

REJECT!

$s_0 \xrightarrow{1} s_1 \xrightarrow{1} s_4 \xrightarrow{1} s_3$

Example 1.2.5.

Consider the following Nondeterministic Finite Automaton (NFA)



$\boxed{011}$

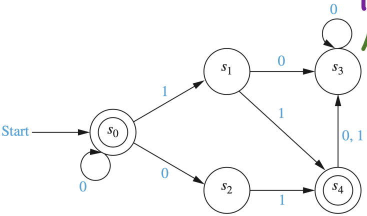
ACCEPT!

$$
\begin{tikzcd}
 & s_0 \arrow[dl, "0"'] \arrow[dr, "0"] & \\
s_0 \arrow[d, "1"'] & & s_2 \arrow[d, "1"] \\
s_1 \arrow[d, "1"'] & & s_4 \arrow[d, "1"] \\
s_4 & & s_3
\end{tikzcd}
$$

At least one ends in a final state!

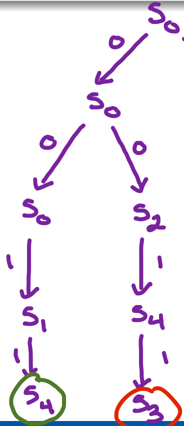
Example 1.2.5.

Consider the following Nondeterministic Finite Automaton (NFA)



$\boxed{0011}$

ACCEPT!

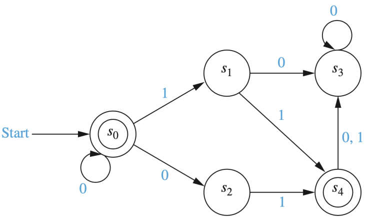


Cannot leave s_2 on 0

$\{s_0\} \xrightarrow{0} \{s_0, s_2\} \xrightarrow{0} \{s_0, s_2\} \xrightarrow{1} \{s_1, s_4\} \xrightarrow{1} \{s_4, s_3\}$

Example 1.2.5.

Consider the following Nondeterministic Finite Automaton (NFA)



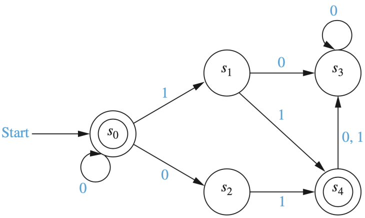
what about $\boxed{1111}$? REJECT!

$s_0 \xrightarrow{1} s_1 \xrightarrow{1} s_4 \xrightarrow{1} s_3$??

we cannot leave s_3 on 1
so we cannot proceed on this Branch.
Since this is the only branch ⇒ Reject!

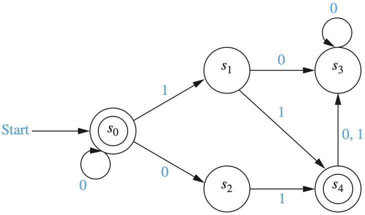
Example 1.2.6.

Find the language recognized by the NFA shown below.



Example 1.2.6.

Find the language recognized by the NFA shown below.



ACCEPT:

$\varepsilon, 0, 00, 000, \ldots = \underbrace{00\ldots0}_{\text{any \# of 0s}} = 0^*$

$=$

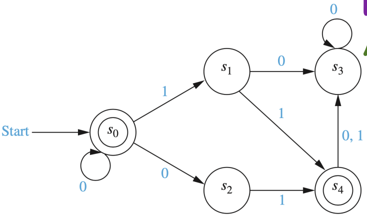
$11, 011, 0011, 00011, \ldots = \underbrace{00\ldots0}_{\text{any \# of 0s}}11 = 0^*11$

$=$

$01, 001, 0001, 00001, \ldots = \underbrace{00\ldots0}_{\text{any \# of 0s}}01 = 0^*01$

Example 1.2.7.

Find a DFA that recognizes the same language as the following NFA.



$\boxed{0011}$

ACCEPT!

$s_0 \xrightarrow{0} s_0 \xrightarrow{0} s_0 \xrightarrow{1} s_1 \xrightarrow{1} s_4$

$s_0 \xrightarrow{0} s_0 \xrightarrow{0} s_2 \xrightarrow{1} s_4 \xrightarrow{1} s_3$

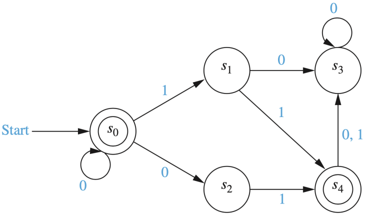
$s_0 \xrightarrow{0} s_2$

Cannot leave s_2 on 0

$\{s_0\} \xrightarrow{0} \{s_0, s_2\} \xrightarrow{0} \{s_0, s_2\} \xrightarrow{1} \{s_1, s_4\} \xrightarrow{1} \{s_4, s_3\}$

Example 1.2.7.

Find a DFA that recognizes the same language as the following NFA.

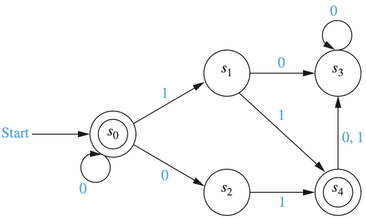


$$\{s_0\} \xrightarrow{0} \{s_0, s_2\} \xrightarrow{0} \{s_0, s_2\} \xrightarrow{1} \{s_1, s_4\} \xrightarrow{1} \{s_4, s_3\}$$

$s_0 \xrightarrow{0} s_0$, $s_0 \xrightarrow{0} s_2$ | $s_0 \xrightarrow{0} s_0$, $s_0 \xrightarrow{0} s_2$, s_2 | $s_0 \xrightarrow{1} s_1$, $s_2 \xrightarrow{1} s_4$ | $s_1 \xrightarrow{1} s_4$, $s_4 \xrightarrow{1} s_3$

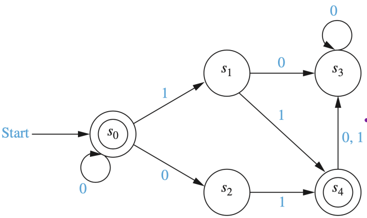
Example 1.2.7.

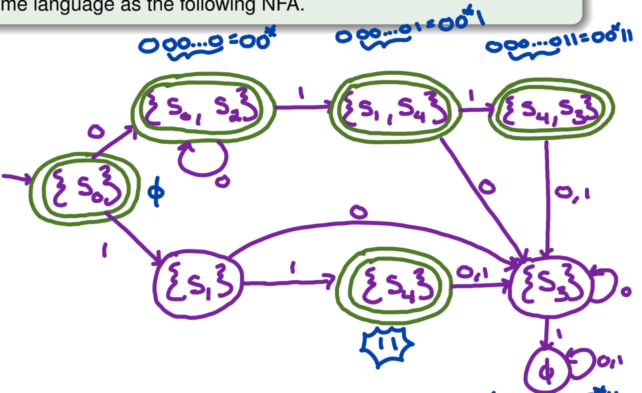
Find a DFA that recognizes the same language as the following NFA.



Example 1.2.7.

Find a DFA that recognizes the same language as the following NFA.





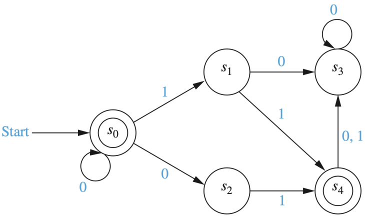
$$L(M_1) = \begin{cases} \underbrace{00\ldots0}_{\text{any \#}} = 0^* \\ 0\underbrace{00\ldots0}_{\text{any \#}}1 = 00^*1 \\ \underbrace{00\ldots0}_{\text{a \#}}11 = 0^*11 \end{cases}$$

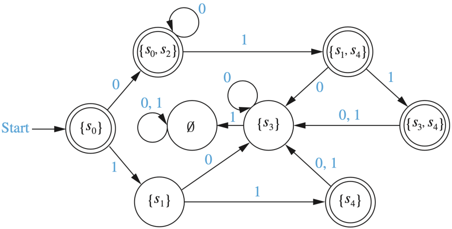
NOTE: 11 along with 00^*11 GIVES 0^*11

NFA as DFA

Example 1.2.8.

Find a DFA that recognizes the same language as the following NFA.

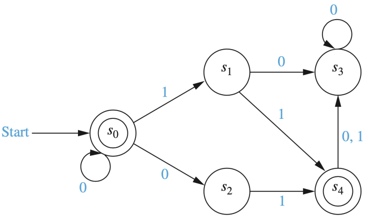


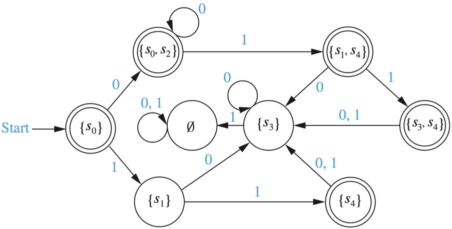


NFA as DFA

Example 1.2.8.

Find a DFA that recognizes the same language as the following NFA.





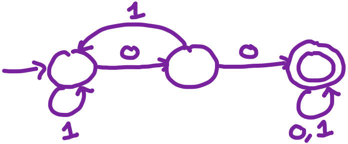
Solution 1.2.9.

The states of this deterministic automaton are subsets of the set of all states of the DFA. The next state of a subset under an input symbol is the subset containing the next states in the NFA of all elements in this subset. For example, on input of 0, $\{s_0\}$ *goes to* $\{s_0, s2\}$*, because* s_0 *has transitions to itself and to* s_2 *in the NFA. The set* $\{s_0, s_2\}$ *goes to* $\{s_1, s_2\}$ *on input of* 1 *because* s_0 *goes just to* s_1 *and* s_2 *goes just to* s_4 *on input of* 1 *in the NFA.*

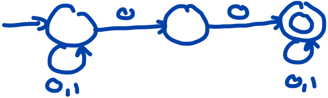
Example 1.2.10.

Construct a finite-state automaton that recognizes the set of all bit strings that contains 00.

DFA:

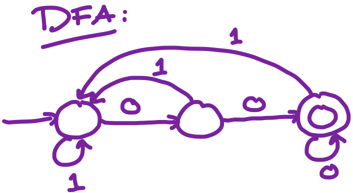


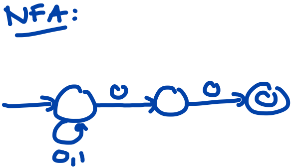
NFA:



Example 1.2.11.

Construct a finite-state automaton that recognizes the set of all bit strings that ends with two 0s.



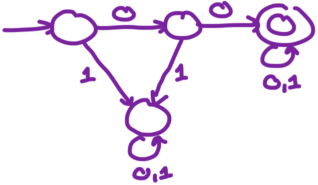


NFA and DFA

Example 1.2.12.

Construct a finite-state automaton that recognizes the set of all bit strings that begins with two 0s.

DFA:



NFA: