2 Aggregated Model

In [4], the authors proposed a simplified model in which the aspect of the allocation of vehicles to the different charging stations is ignored, with a focus on an efficient online algorithm for the optimal charging of vehicles over time. The online algorithm is based on an offline model that is expressed as a convex quadratic program. In this section, we adapt the offline model to the case when there are uncertainties in the electricity costs and arrival and departure times.

2.1 Nominal model

The proposed model for N vehicles over a time horizon of T steps takes the form

$$p^* := \min_Y \sum_{t=1}^T f_t \left(y_t^0 + \sum_{i\,:\,t\in[a_i,d_i]} Y_{ti} \right) \;:\; Y \geq 0, \;\; \sum_{t=a_i}^{d_i} Y_{ti} = L_i, \;\; i = 1, \ldots, N. \tag{2}$$

In the above, f_t is are convex increasing functions that encode the cost of production at time t; y_0 is a given T-vector that corresponds to a non-flexible consumption of the charging station; variable Y is a $T \times N$ matrix, such that Y_{ti} is the power allocated to vehicle i at time t; $L \in \mathbb{R}^N$ is a (required) load vector divided by a time step parameter; finally, for each i, (a_i, d_i) are (given) arrival and departure times for vehicle i.

Let us encode the arrival and departure times in a $T \times N$ binary matrix A, where $A_{ti} = 1$ if t is between the arrival and departure times of vehicle i, and zero otherwise. With this convention, we have, for every $N \times T$ matrix Y:

$$\forall\, i \;:\; \sum_{t=a_i}^{d_i} Y_{ti} = \sum_{t=1}^T A_{ti} Y_{ti} = (Ae_i)^T Y e_i = e_i^T A^T Y e_i, \;\; \forall\, t \;:\; \sum_{i\,:\,t\in[a_i,d_i]} Y_{ti} = \sum_{i=1}^N A_{ti} Y_{ti} = e_t^T A Y^T e_t,$$

This leads to an expression of the problem in matrix form: This leads to an expression of the problem in matrix form:

$$p^* = \min_Y \; F(y^0 + \mathbf{diag}(AY^T)) \;:\; \mathbf{diag}(A^T Y) \geq L, \;\; Y \geq 0, \tag{3}$$

where $F : \mathbb{R}^T \to \mathbb{R}$ is the function with values for $z \in \mathbb{R}^T$ given by

$$F(z) := \sum_{t=1}^T f_t(z_t).$$

Note that we have replaced the equality load constraint in (2) by an inequality; this is done without loss of generality, since the costs are increasing with increasing loads. The interpretation is that it is not optimal to produce in excess of the required load.

Sohet's thesis [3] discusses several choices for the production costs, for example (constant over time) quadratic costs, where $f_t(\xi) = \xi^2$ for every t. We may also consider time-varying linear costs: $f_t(\xi) = c_t \xi$, where $c_t > 0$ for every t. In that case, the problem writes

$$p^* = \min_Y \; c^T(y^0 + \mathbf{diag}(AY^T)) \;:\; \mathbf{diag}(A^T Y) \geq L, \;\; Y \geq 0. \tag{4}$$

It turns out that in that case, the loading matrix corresponds to a somewhat trivial solution, which is to select times such that the cost is the smallest, and put all the charging load at that time step.

To illustrate, consider a toy problem where $T = 4$, $N = 3$, $y^0 = 0$, and

$$c = \begin{pmatrix} 26 \\ 25 \\ 20 \\ 29 \end{pmatrix}, \;\; L = \begin{pmatrix} 12 \\ 17 \\ 19 \end{pmatrix}, \;\; A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{5}$$

The optimal load matrix and cost are then

$$Y^*_{\rm lin} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 19 \\ 12 & 17 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

We observe that the loading matrix indeed corresponds to the simple strategy of charging "all at once" at the best (lowest cost) time step. In contrast, with a constant quadratic cost, the solution does distribute the load over the entire charging intervals:

$$Y^*_{\rm quad} = \begin{pmatrix} 0 & 0 & 12.0000 \\ 5.0000 & 0 & 7.0000 \\ 7.0000 & 5.0000 & 0 \\ 0 & 12.0000 & 0 \end{pmatrix}.$$

2.2 Robust counterparts

Uncertainty in the cost vector. As a first step, we address uncertainties in the costs. Precisely, we consider the nominal problem above, with linear, time-varying costs encoded in a cost vector $c \in \mathbb{R}^T$. Here, we assume that c is only known up to a sphere: $\|c - \hat{c}\|_2 \leq \rho$, where the "nominal" cost $\hat{c} \in \mathbb{R}^T$ and the uncertainty level $\rho \geq 0$ are both known. The robust counterpart to (4) is the SOCP

$$p^* = \min_Y \ \hat{c}^T(y^0 + \mathbf{diag}(AY^T)) + \rho\|y^0 + \mathbf{diag}(AY^T)\|_2 \ : \ \mathbf{diag}(A^TY) \geq L, \ \ Y \geq 0. \tag{6}$$

We observe that when $\rho = 0$, we recover the nominal problem with fixed costs; while for ρ large, the solution approaches that of the problem with quadratic costs. We can see that the robust model accomplishes a trade-off between the nominal cost and a term that corresponds to the case with quadratic costs. As a result, the robust model does not suffer from the drawback observed with linear costs, where charging systematically occurs at a single time step.

Returning to the toy example above, with $\hat{c}$ set as c in (5), and $\rho = 11$, we obtain indeed a better spread in charging values:

$$Y^*_{\rm rob} = \begin{pmatrix} 0.0000 & 0.0000 & 9.3003 \\ 2.3005 & 0.0000 & 9.6997 \\ 9.6995 & 15.8011 & 0.0000 \\ 0.0000 & 1.1989 & 0.0000 \end{pmatrix}.$$

As expected, when $\rho \to \infty$, the solution corresponding to quadratic costs, $Y^*_{\rm quad}$, is recovered.

Uncertainty in the arrival times. In order to cope with uncertainties in the arrival times, we assume that the matrix A is only known to be long to a finite set $\mathcal{A}$ of possible scenarios, denoted $A^{(k)}$, $k = 1, \ldots, K$. The corresponding robust counterpart is

$$p^* = \min_Y \ \max_{A \in \mathcal{A}} \ F(y^0 + \mathbf{diag}(A^{(k)}Y^T)) \ : \ \mathbf{diag}((A^{(k)})^TY) \geq L, \ \ k = 1, \ldots, K, \ \ Y \geq 0. \tag{7}$$

The problem is convex and can be solved using standard convex optimization software, for a wide variety of cost functions F.

Returning to the toy example above, let us assume that there are two scenarios ($K = 2$), with

$$A^{(1)} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \ \ A^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

We obtain

Uncertainty in the load vector. Assume for example that the load vector L is only known to satisfy $\|L - \hat{L}\|_\infty \leq \rho$, where $\hat{L}$ contains nominal values and $\rho \geq 0$ is a measure of component-wise uncertainty. The robust counterpart has a trivial expression, in the same form as the nominal model (3), but with values of L replaced with their worst-case (largest) values, $\hat{L} + \rho\mathbf{1}$.

2.3 Affine recourse

TBD

2.4 Numerical Experiments

- Use real data (as in Sohet)
- Assume uncertainty on cost, and plot worst-case objective against uncertainty level ρ.
- Next assume no uncertainty on cost, and uncertainty on A only. Work with a set of scenarios provided by Riadh; one of these is the nominal. Compute the nominal strategy and the robust strategy. Then, randomly sample among the scenarios and compute the cost for that sample, for both strategies. Average and take the worst-case. Normally the robust version does better.