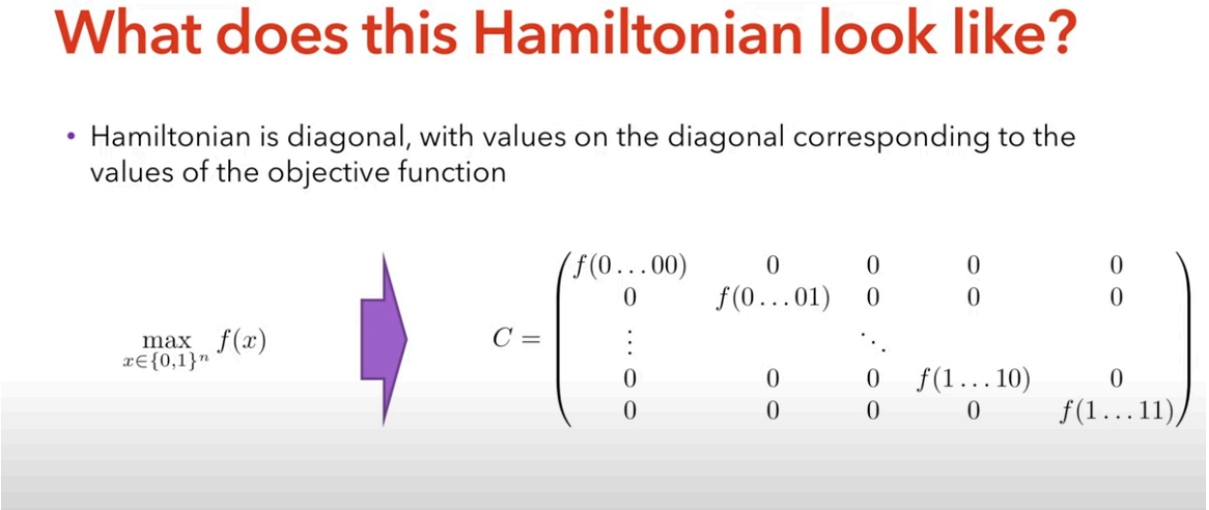
Quantum approximate optimisation algorithm (QAOA) is a combinatorial approximate optimisation algorithm. The combinatorial optimisation aspect means that it is an algorithm which seeks to find an optimal solution from a finite set of options (Schrijver 2003, p. 1.) with the approximate aspect meaning it will provide the closest to optimum solution, not the solution which is necessarily the most optimum (Bernard., Shmoys, David (2011). *The design of approximation algorithms*. Cambridge University Press. ISBN 9780521195270. OCLC 671709856.
)

One of the most common problems QAOA is used to solve is the maximum cut problem. The maximum cut problem (https://en.wikipedia.org/wiki/Maximum_cut.) aims to divide vertices in a graph by cutting through its edges, aiming to find a cut which goes through the most edges.

The max cut problem can be represented as a classical formula, this needs to be translated into quantum. This is done by converting it into a hamiltonian. A hamiltonian (https://en.wikipedia.org/wiki/Hamiltonian_(quantum_mechanics).) represents the total energy of a system, therefore our classical formula must be transformed into a system of energy. A hamiltonian is constructed via a hermitian operator which is referred to as C. Operators are mathematical representations of physical elements in a system, in our example this energy system. (https://mathforquantum.quantumtinkerer.tudelft.nl/5_operators_QM/) A hermitian operator returns a real number (not complex) and will return the same result whether it's in its complex conjugate form or not.

The solution is transformed from a binary string representation to a highest energy eigenstate (largest eigenvalue eigenvector). Because the hamiltonian is classical, the eigenstate is a computational basis state.



The classical formula, in this example an n-dimensional binary cube, is translated into a matrix with each value on the diagonal representing a binary string combination of length n. The off-diagonal values all equal 0. Each value will be a vertice of the graph of 2n with edges to each vertice being 2^n-1. This matrix represents the energy field that the classical formula is mapped to. Optimum solutions in this energy field will be dips (lower energy states) with

less optimum solutions being the peaks (higher energy states). The goal is to find the lowest energy state.

Rather than constructing the full hamiltonian, we construct little parts called Pauli Z.
https://www.youtube.com/watch?v=2MsVD9ufguk

Pauli Z matrices are hermitian.

In the MaxCut example for applying QAOA:

- MAXCUT objective:

$$\max_{\mathbf{s}} \frac{1}{2} \sum_{ij \in E} (1 - s_i s_j) \qquad s_i \in \{-1, +1\}$$

- MAXCUT Hamiltonian is constructed by **mapping binary variables s_i onto the eigenvalues of Z**

$$C = \frac{1}{2} \sum_{ij \in E} (I - Z_i Z_j)$$

- Want to show:

$$C\,|x\rangle = C(\mathbf{x})\,|x\rangle$$