SOLVING THE N-PUZZLE

A Comparative Analysis of A* Heuristics

From combinatorial explosion to optimized search using Linear Conflict logic.

PROJECT: ARTIFICIAL INTELLIGENCE CSCI611
AUTHORS: FATULLA BASHIROV, ELMAR MAMMADOV
ALGORITHM: A* SEARCH

DEFINING THE MATRIX ENVIRONMENT

Problem Formulation & Constraints

START STATE (SCRAMBLED)

7	14	2	11
4	1	8	0
12	5	15	9
3	13	6	10

GOAL STATE (ORDERED)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	0

STATES

Configuration of tiles (1...N) + empty cell (0) in an n x n matrix.

ACTIONS

Valid movements of the blank space: { Left, Right, Up, Down }.
← → ↑ ↓

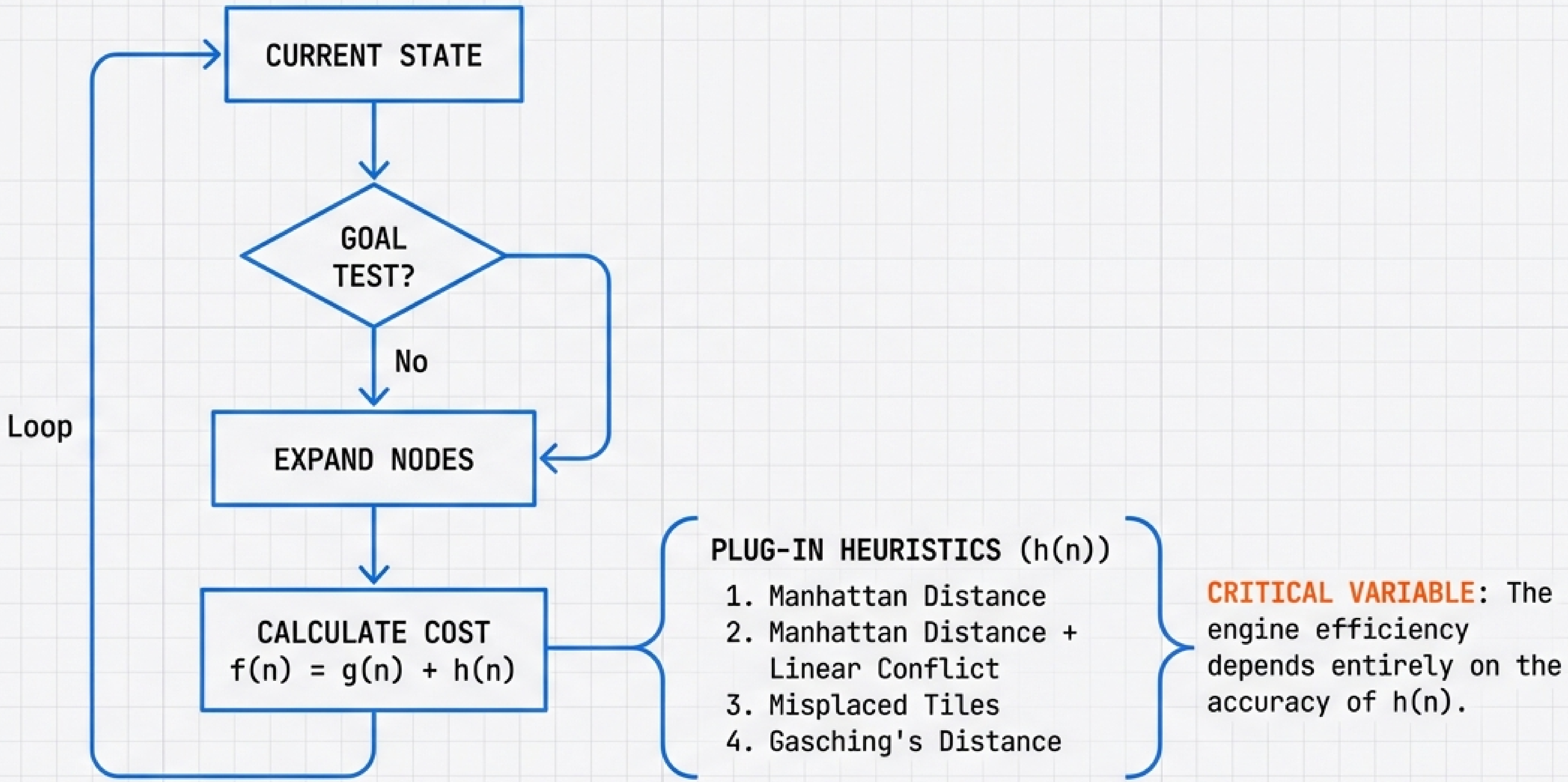
PATH COST

Uniform cost: 1 per step. 1

CONSTRAINTS

Scope: 3 <= n <= 5 (8, 15, and 24-puzzles).

A* SEARCH: THE NAVIGATION ENGINE



COMBINATORIAL EXPLOSION

Why Baseline Heuristics Fail at Scale

8-PUZZLE (3x3)

Misplaced Tiles & Gasching: EFFECTIVE.

15-PUZZLE (4x4) & 24-PUZZLE (5x5)

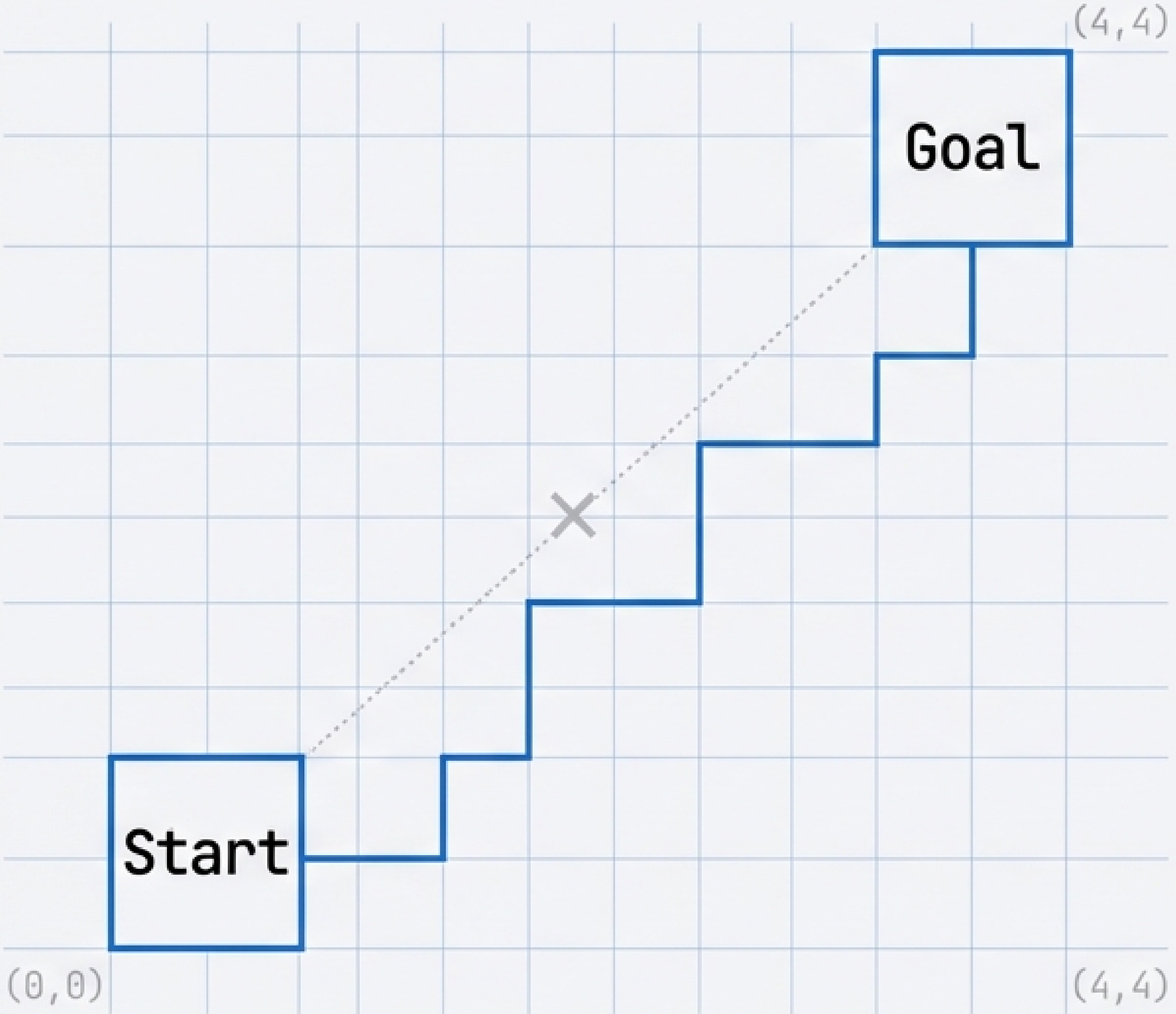
TIMEOUT

EXECUTION FAILURE
Time > 40 seconds
Search Space: Too Large
Pruning: Ineffective

Simple heuristics get lost in the branching factor as N increases. A more robust distance calculation is required.

MANHATTAN DISTANCE

Reliable but Costly (Taxicab Geometry)



DEFINITION:
The sum of the absolute differences of the Cartesian coordinates of the tiles.

PERFORMANCE:
Solves all sizes (3x3 to 5x5).
Memory Intensive.

DATA (3x3 EXAMPLE):
Nodes Expanded: 5,358
Time: 0.058s
(Slower than optimized methods)

OPTIMIZATION: LINEAR CONFLICT

Accounting for Interference

LOGIC:

1. Two tiles are in their correct goal row.
2. They are in reversed order relative to the goal.
3. They physically block each other.

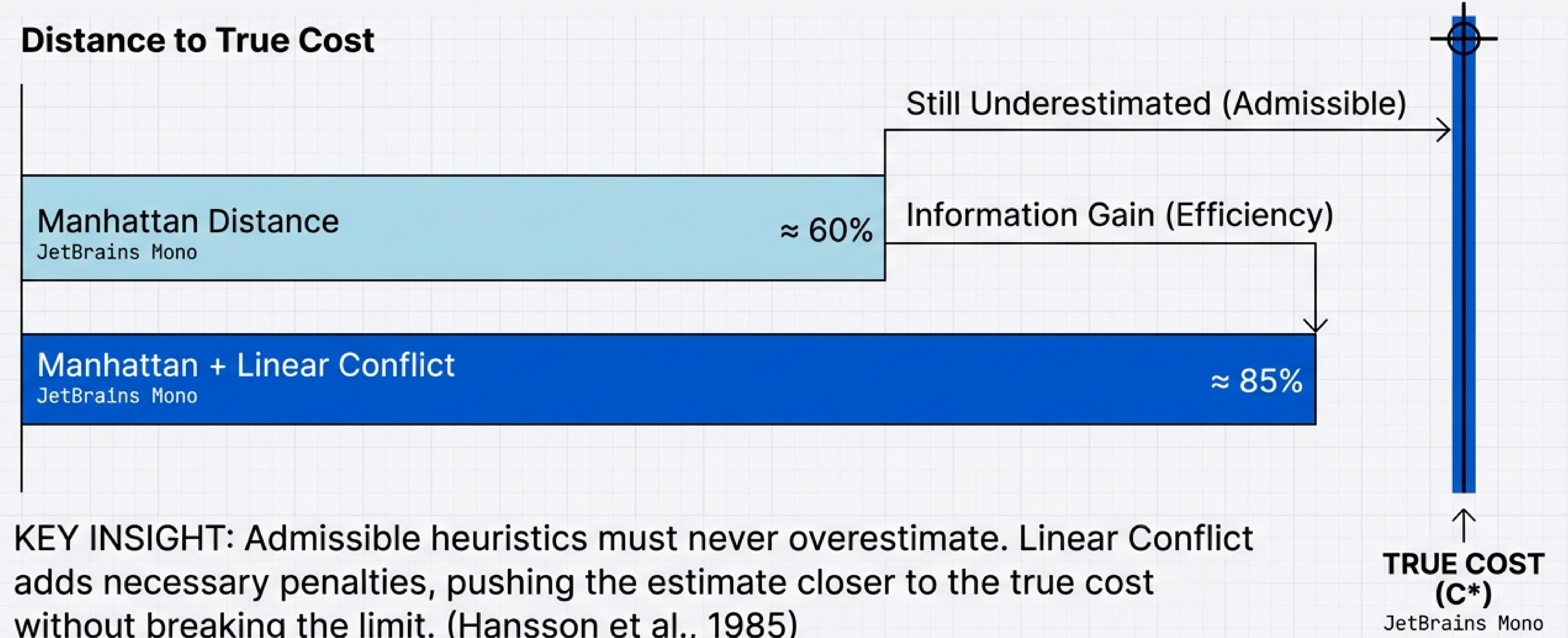
ALGORITHM:

h(n) = Manhattan Distance + 2 * (Conflict Pairs)

Penalty: +2 moves are added for every pair that must swap places.

THEORETICAL ADMISSIBILITY

Tighter Bounds = Better Pruning



KEY INSIGHT: Admissible heuristics must never overestimate. Linear Conflict adds necessary penalties, pushing the estimate closer to the true cost without breaking the limit. (Hansson et al., 1985)

PERFORMANCE BENCHMARKS

The Efficiency Gap

Heuristic	3x3 Nodes	4x4 Nodes	5x5 Nodes
Gasching	81,550 (Slowest)	TIMEOUT	TIMEOUT
Manhattan	5,358	317,594	8,848
Manhattan + Linear Conflict	**2,816 (Fastest)**	**83,056 (Dominant)**	**2,080 (Winner)**

MASSIVE REDUCTION: LC is ~3.5x more efficient than standard Manhattan for 15-puzzles.

SYSTEM BOUNDARIES

When Complexity Wins

RANDOM MATRICES

Struggles with random 4x4 and 5x5 instances due to search tree explosion.

MAX LOAD OBSERVED

8,415,013 Nodes processed in ~158 seconds.

THE HARD CEILING

All implemented heuristics fail to compute the 35-puzzle (6x6).

SUMMARY & RESOURCES

CONCLUSION:

- Manhattan Distance + Linear Conflict is the proven optimal choice for N <= 5.
  - Admissible
  - Drastically reduces node expansion
  - Superior to Gasching and Misplaced Tiles

SCAN FOR REPOSITORY

github.com/elmarMamedov485/AI_project_I

REFERENCE: Hansson, Mayer, and Yung (1985). Generating Admissible Heuristics by Criticizing Solutions to Relaxed Models.