Student Information

Name : Solution

ID :

Answer 1

a)

Expected value of a die is calculated by dividing the sum of all face values to the number of faces.
For blue die; $E(B) = \dfrac{21}{6} = 3.5$

For yellow die; $E(Y) = \dfrac{24}{8} = 3$

For red die; $E(R) = \dfrac{30}{10} = 3$

b)

Using the expected values obtained in part a, $E(R) + E(Y) + E(B) = 9.5$ and $3E(B) = 10.5$. The second option (rolling three blue dice) is better, since it has a greater expected value.

c)

In this case expected value of yellow die has become 8. So $E(R) + E(Y) + E(B)$ becomes 14.5, making rolling a single die from each color a better option.

d)

Let $P(A)$ be the probability of rolling a red die and $P(B)$ be the probability of rolling 3. The given question asks the conditional probability $P(A|B)$. Using Bayes rule;

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Since all colors have an equal probability, $P(A) = \frac{1}{3}$. Among ten faces of a red die only two have the value 3, so $P(B|A) = \frac{2}{10}$. For $P(B)$, we must find the probability of obtaining a 3 for all colors. That is;

$$P(B) = (\tfrac{2}{10})(\tfrac{1}{3}) + (\tfrac{1}{6})(\tfrac{1}{3}) + (\tfrac{3}{8})(\tfrac{1}{3}) = \tfrac{89}{360}$$

This makes;

$$P(A|B) = \frac{(\frac{2}{10})(\frac{1}{3})}{(\frac{89}{360})} = \tfrac{24}{89} = 0.27$$

If the result is 3, with approximately 27% probability the rolled die is red.

e)

Let $P(a,b)$ denote the probability of obtaining a on blue die and b on yellow die. To get a total of 5, only possible options are $P(1,4), P(2,3)$ and $P(4,1)$. So;

$$P(1,4) = (\frac{1}{6})(\frac{1}{8}) = \frac{1}{48}$$
$$P(2,3) = (\frac{1}{6})(\frac{3}{8}) = \frac{3}{48}$$
$$P(4,1) = (\frac{1}{6})(\frac{3}{8}) = \frac{3}{48}$$

$P(1,4) + P(2,3) + P(4,1) = \frac{7}{48}$, with a roll of a single blue die and a single yellow die, there is approximately 15% probability that the total value will be 5.

Answer 2

a)

Discounts offered by company A follows **Poisson approximation to Binomial distribution** with $\lambda = 2$. Referring to Table $A3$, we have $F(3) = 0.857$ which indicates the probability of having less than four distributors. To calculate the answer, $1 - F(3) = 0.143$. There is approximately 14% probability that at least four distributors of company A offers a discount.

b)

In part **a**, we showed that discounts offered by company A follows **Poisson approximation to Binomial distribution** with $\lambda = 2$. Furthermore, discounts offered by company B follows **Geometric distribution**.
To calculate the probability of buying the phone of company A in two days, we must find the probability of buying zero of their phone and subtract it from 1 (Law of Total Probability). Referring to Table $A3$, there is 0.135 probability of no discounts offered by any distributor of company A. The probability of this happening twice consecutively is $(0.135)^2 = 0.018$.
Probability of not buying the phone of company B is 0.81. Since these two are independent, using Law of Total Probability $1 - (0.018) * (0.81) = 1 - 0.01458 = 0.985$.

NOTE: For this question there are more than one approach that gives an approximate result with 0.01 precision. All of these methods are accepted if backed with sufficent explanation.

Answer 3

iter = 1000;
blue = [1;2;3;4;5;6];

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yellow = [1;1;1;3;3;3;4;8];
red = [2;2;2;2;2;3;3;4;4;6];
result1 = ones(iter, 1);
result2 = ones(iter, 1);
count = 0;

for i = 1:iter
b = randsample(blue, 1, true);
y = randsample(yellow, 1, true);
r = randsample(red, 1, true);
result1(i) = b+r+y;

b1 = randsample(blue, 1, true);
b2 = randsample(blue, 1, true);
b3 = randsample(blue, 1, true);
result2(i) = b1+b2+b3;

if(result2(i) > result1(i))
count = count + 1;
endif
endfor

avg1 = mean(result1)
avg2 = mean(result2)
percentage = count / iter
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avg1 = 9.4480
avg2 = 10.385
percentage = 0.5510
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The averages are close to the expected values calculated in **Q1b**. In that question, we preferred the second option over the first one, and as it can be seen by the percentage, more than half of the time second option's total value is greater.