

Student Information

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Answer a

I have conducted a Monte Carlo study using Octave Online, after which I have used this study for estimating the probability that the total weight of all the cargo unloaded at the port in a day exceeds 300,000 tons, I can conduct a Monte Carlo simulation using the given information.

To conduct such a study I have first used Normal approximation with $\alpha = 0.01$ and $\epsilon = 0.02$, namely (since no estimator for p has been given I have directly used the following):

$$\begin{aligned} N &\geq 0.25 \left(\frac{z_{\alpha/2}}{\epsilon} \right)^2 \\ &= 0.25 \left(\frac{2.3263}{0.03} \right)^2 \\ &\approx 1503 \end{aligned}$$

I have created some variables for holding the values of distribution parameters and I have also created a vector named *TotalWeight* for keeping the total weight of ships that use the bridge for each Monte Carlo run and initialized it to 0 for all N .

Next, to find number of ships for each type, I have generated samples ($NBulks$, $NContainers$ and $NTankers$) for all ships with their corresponding Poisson parameters using sampling from Poisson.

Then, to find weights of each ship according to its type, I have used the samples that correspond to numbers for each type of ships together with their corresponding Gamma parameters. With this way I was able to generate the sample weights for all ships ($NBulks$, $NContainers$ and $NTankers$) and after summing them up at the end I have calculated the total weight for 1 Monte Carlo run and filled the corresponding place in my *TotalWeight* vector. I have repeated this study $N = 1503$ times and filled the *TotalWeight* vector accordingly.

For the answer of *part a*; after construction of *TotalWeight* vector with desired Monte Carlo runs, I have calculated the *mean* of the proportion of runs with the total weight more than 300 tons. With this way I have estimated the probability that the total weight of all the ships that pass over the bridge in a day is more than 300 tons; in other words, I have found our estimator for the desired probability.

I have simulated my solution in Octave Online a number of times and I was able to determine that my estimated probability is always in between 0.10 and 0.12 (But in general 0.12). I share a sample output (which I will refer in other parts of the answer) in below:

Estimated probability = 0.120426
Expected weight = 260617.143352
Standard deviation = 33477.635732

Answer b

For estimation of the total weight of all the ships that pass over the bridge in a day X , I have simply got the *mean* of *TotalWeight* and found the Expected weight. Expected weight for a sample simulation can be seen from the sample output shared in part a which is :

Answer c

For estimation of $Std(X)$, I have simply got the *std* of *TotalWeight* and found the Standard deviation of X . Standard deviation for a sample simulation can be seen from the sample output shared in part a.

Since initially we have created a Monte Carlo study with size N that attains our desired accuracy ($\alpha = 0.02$ and $\epsilon = 0.03$), We have guaranteed a Monte Carlo study of size N with an error not exceeding ϵ with high probability $(1 - \alpha)$ and created an estimator X with that accuracy.

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octave:75> N=1503;
lambda_Bulk = 50;
lambda_Cont = 40;
lambda_Tank = 25;
totalWeight = zeros(N,1);

for k=1:N
    U = rand; i=0;
    F = exp(-lambda_Bulk);
    while (U>=F)
        i=i+1;
        F = F + exp(-lambda_Bulk)*lambda_Bulk^i/gamma(i+1);
    end
    numberBulk = i;
    lambda_W_Bulk = 0.1;
    alphaBulk = 60;
    weightBulk = sum(-1/lambda_W_Bulk * log(rand(alphaBulk,numberBulk)));
    totalW_Bulk = sum(weightBulk);

    U = rand; i=0;
    F = exp(-lambda_Cont);
    while (U>=F)
        i=i+1;
        F = F + exp(-lambda_Cont)*lambda_Cont^i/gamma(i+1);
    end
    numberCont = i;
    lambda_W_Cont = 0.05;
    alphaCont = 100;
    weightCont = sum(-1/lambda_W_Cont * log(rand(alphaCont,numberCont)));
    totalW_Cont = sum(weightCont);

    U = rand; i=0;
    F = exp(-lambda_Tank);
    while (U>=F)
        i=i+1;
        F = F + exp(-lambda_Tank)*lambda_Tank^i/gamma(i+1);
    end
    numberTank = i;
    lambda_W_Tank = 0.02;
    alphaTank = 120;
    weightTank = sum(-1/lambda_W_Tank * log(rand(alphaTank,numberTank)));
    totalW_Tank = sum(weightTank);

    totalWeight(k) = totalW_Bulk + totalW_Cont + totalW_Tank;

end

p_est = mean(totalWeight>300000);
expectedWeight = mean(totalWeight);
stdWeight = std(totalWeight);
fprintf('Estimated probability = %f\n',p_est);
fprintf('Expected weight = %f\n',expectedWeight);
fprintf('Standard deviation = %f\n',stdWeight);
Estimated probability = 0.120426
Expected weight = 260617.143352
Standard deviation = 33477.635732

```

The octave code and the result from octave online.