Middle East Technical University | Department of Computer Engineering

CENG 222

Statistical Methods for Computer Engineering

Spring 2022-2023

Homework II

Due date: 30.04.2023, Sunday, 23:59

Introduction

In this assignment, there are three classical questions and one programming question related to the 4^{th} chapter of your text book. While answering the questions, please **show your work** and the steps of your calculations. Give an explanation about what numbers mean in those steps. Otherwise, you may not get any point. Also, include the code for the programming question in your report.

Questions

Q1. (**30 pts.**) In a computer network simulation, servers A and B are located in different data centres. Each server receives a packet simultaneously. The servers are programmed to process the packet and send a response. The time taken to process and send the response, T_A and T_B, are independent and uniformly distributed between [0, 100] milliseconds.

- **a)** What are the joint density function f(t_A, t_B) and the joint cdf F(t_A, t_B)? *(7.5 pts)*
- **b)** What is the probability that server A processes the packet and sends a response within the first 30 milliseconds, while server B takes between 40 and 60 milliseconds to process the packet and send a response? *(7.5 pts)*
- **c)** What is the probability that server A processes the packet and sends a response no later than 10 milliseconds after server B?*(7.5 pts)*
- **d)** Servers are considered to have failed the task if their response times differ by more than 20 milliseconds. What is the probability that they pass the task?*(7.5 pts)*

(Hint: You may find it helpful to think in terms of the ratio of the regions corresponding to the asked events to the [0, 100] $\times$ [0, 100] square)

Q2. (**20 pts.**) Assume that an e-commerce store has 100,000 registered customers. The store classifies its customers into three categories based on their purchase frequency: Frequent shoppers (60%), Occasional shoppers (30%) and Rare shoppers (10%). A random sample of 150 customers from the store is taken to analyze their purchase habits. Estimate the following using the central limit theorem:

- **a)** The probability that at least 65% of the customers in the sample are frequent shoppers. *(10 pts)*

- **b)** The probability that no more than 15% of the customers in the sample are rare shoppers. *(10 pts)*

Q3. (**15 pts.**) Suppose the height of adults in a certain population follows a normal distribution with a mean height (μ) of 175 cm and a standard deviation (σ) of 7 cm. Calculate the probability that a randomly selected adult will have a height between 170 cm and 180 cm.

Q4. (**35 pts.**) For this question, use Octave or MATLAB to simulate the specifications given in **Q3** over 1000 iterations and

- **a)** visualise the distribution of heights(assuming a normal distribution with a mean of 175 cm and standard deviation of 7 cm). *(10 pts)*

- **b)** visualize the effect of sigma on the distribution by plotting the probability density function (PDF) of the normal distribution for different values of sigma: 6, 7, and 8. *(10 pts)*

- **c)** In a group of 150 randomly selected adults, estimate the probability of having at least 45%, 50%, and 55% of adults with heights between 170 cm and 180 cm. In your estimations use the parameters defined in **Q3** and calculate the probabilities over 1000 iterations. *(15 pts)*

Add your code to your report. Also, add a screenshot of the output of Octave/MATLAB where the items listed above is clearly visible. **Briefly comment on the results**.

Specifications

- You are expected to write your answers in LaTeX format. You can use the given template.
- Please do not skip the calculation steps. Show every step of your work.
- You have a total of 2 late days for this homework. For each day you have submitted late, you will lose 25 points. If you submit your homework at least 2 days later than the deadline, you will get zero.
- Cheating is forbidden. The violators will be punished according to the department's regulations.
- Follow the course page on ODTUClass for any updates and clarifications. Please ask your questions on ODTUClass instead of e-mailing if they do not contain some part of the solution. If they contain, you can send an email to "cetinkaya@ceng.metu.edu.tr".

Submission

Submissions will be done via ODTUClass. If you do not have access to ODTUClass for some reason, please e-mail assistants about that. You are expected to submit a PDF file named "hw2.pdf" including your solutions along with the source codes and visuals required in Q4. You should also submit your source codes in a separate text file.