# Student Information

Full Name : Tahsin ELMAS
Id Number : 2476844

# Answer 1

## a)

To find the joint density function $f(t_A, t_B)$, we use the fact that $T_A$ and $T_B$ are independent and uniformly distributed between [0, 100] milliseconds. The probability density function (pdf) of a uniform distribution between $a$ and $b$ is $\frac{1}{b-a}$, so we have:

$$f(t_A) = \frac{1}{100-0} = \frac{1}{100} \text{ for } 0 \leq t_A \leq 100$$

$$f(t_B) = \frac{1}{100-0} = \frac{1}{100} \text{ for } 0 \leq t_B \leq 100$$

$$f(t_A, t_B) = f(t_A) \cdot f(t_B) = \frac{1}{10000} \text{ for } 0 \leq t_A, t_B \leq 100$$

$$f(t_A, t_B) = \begin{cases} \frac{1}{10000} & 0 \leq t_A, t_B \leq 100 \\ 0 & \text{otherwise} \end{cases}$$

The joint cdf $F(t_A, t_B)$ can be found by integrating the joint density function:

$$F(t_A, t_B) = \int_0^{t_A} \int_0^{t_B} f(u, v) du dv = \int_0^{t_A} \int_0^{t_B} f(u) \cdot f(v) du dv$$

By substituting the pdfs we previously found, we get:

$$F(t_A, t_B) = \frac{t_A \cdot t_B}{10000} \text{ for } 0 \leq t_A, t_B \leq 100$$

## b)

We need to find the probability that $0 \leq T_A \leq 30$ and $40 \leq T_B \leq 60$. This can be expressed as:

$$P\{T_A \leq 30 \cap 40 \leq T_B \leq 60\} = P\{T_A \leq 30\} P\{4 \leq T_B \leq 60\}$$

$$P(0 \leq T_A \leq 30 \cap 40 \leq T_B \leq 60) = \frac{30}{100} \times \frac{20}{100}$$

$$P(0 \leq T_A \leq 30 \cap 40 \leq T_B \leq 60) = \frac{6}{100} = 0.06$$

### c)

We think in terms of the ratio of the region of the asked event.

$$1 - \frac{90 \times 90/2}{10^4} = 0.595$$

### d)

The probability that servers $A$ and $B$ pass the task is given by the ratio of the area where $|T_A - T_B| \leq 20$ to the total area of the square $[0, 100] \times [0, 100]$. The area of the square where $|T_A - T_B| \leq 20$ is a square with side length \$60\$ (since if $|T_A - T_B| \leq 20$, then $T_A$ and $T_B$ can differ by at most 20 milliseconds, and since $T_A$ and $T_B$ are uniformly distributed between $[0, 100]$, the maximum difference between them is $100 - 0 = 100$, which occurs when $T_A = 0$ and $T_B = 100$, so the side length of the square is $100 - 20 = 80$, and we have to subtract 20 from each end of the interval $[0, 80]$ ). Therefore, the area of the square is $60^2 = 3600$. The total area of the square $[0, 100] \times [0, 100]$ is $100 \times 100 = 10000$. Therefore, the probability that servers $A$ and $B$ pass the task is:

$$P(|T_A - T_B| \leq 20) = \frac{3600}{10000} = 0.36$$

So there is a 36% chance that servers $A$ and $B$ pass the task.

## Answer 2

For both of the following we can use Normal approximation to Binomial distribution.

### a)

The central limit theorem states that for large sample sizes ($n$), the sampling distribution will be approximately normal. We calculate the expected value and standard deviation in the selected population as :

$$\mu = n \times p = 150 \times 0.6 = 90$$
$$\sigma = \sqrt{n \times p \times (1 - p)} = \sqrt{150 \times 0.6 \times 0.4} = 6$$

We can model $X_1$ as

$$X_1 \approx Normal(90, 6)$$

With the continuity correction

$$\mathbf{P}\left(X_1 \geq 64.5\right) = \mathbf{P}\left(\frac{X_1 - 90}{\sqrt{36}} \geq \frac{64.5 - 90}{\sqrt{36}}\right) \approx \mathbf{P}(Z \geq -4.25) = \mathbf{P}(Z \leq 4.25) \approx 1$$

$\Phi(z) \approx 0$ (is "practically" zero) for all $z < -3.9$, and $\Phi(z) \approx 1$ (is "practically" one) for all $z > 3.9$.So:

$$\mathbf{P}\left(X_1 \geq 65\right) \approx 1$$

$\mathbf{E}\left(X_2\right) = 150 \times 0.10 = 15$ and $\operatorname{Var}\left(X_2\right) = 150 \times 0.1 \times 0.9 = 13.5$. Then

$$X_2 \approx \text{Normal}(15, \sqrt{13.5})$$

The asked quantity can be approximated as

$$\mathbf{P}\left(X_2 \leq 15.5\right) = \mathbf{P}\left(\frac{X_2 - 15}{\sqrt{13.5}} \leq \frac{15.5 - 10}{\sqrt{13.5}}\right) \approx \mathbf{P}(Z \leq 1.49) \approx 0.9319.$$

### b)

Similarly, the probability that no more than 15 of the customers in the sample are rare shoppers can be estimated using a normal distribution with mean $= 10\%$, standard deviation $= \sqrt{\frac{0.1 \times 0.9}{150}} = 0.0271$. Using a $z$-score table, we can find that the probability of no more than 15% of customers being rare shoppers is approximately 0.9998 or 99.98

## Answer 3

To calculate the probability that a randomly selected adult will have a height between 170 cm and 180 cm, we can use the standard normal distribution.First, we need to standardize the values of 170 cm and 180 cm using the formula $z = \frac{x-\mu}{\sigma}$, where x is the value of interest,  is the mean, and is the standard deviation.

$$\begin{aligned} P\left(170 \leq X \leq 180\right) &= P\left(\frac{170 - 175}{7} \leq z \leq \frac{180 - 175}{7}\right) \\ &= P(z < 1.0.7142) - P(z < -0.7142) \simeq 0.522 \end{aligned}$$

# Answer 4

## a)

```
mu = 175;
sigma = 7;
heights = normrnd(mu, sigma, [1, 1000]);
hist(heights)
xlabel(Height (cm))
ylabel(Probability Density)
```

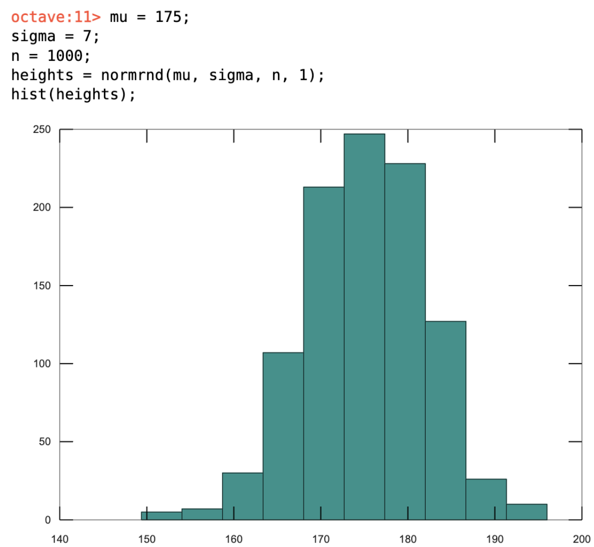



Figure 1: Q4 a -The Histogram of heights distribution from octave online.
The histogram shows the distribution of these heights, with the x-axis representing the height values and the y-axis representing the frequency of each height value. From the histogram, we can see that the distribution is approximately symmetric and bell-shaped, which is expected for a normal distribution. The mean value of 175 is shown as the peak of the distribution, and the standard deviation of 7 determines the spread of the distribution. Overall, this visualization provides a useful representation of the distribution of heights for the given parameters, and helps to gain insights into the characteristics of the normal distribution.

**b)**

```
mu = 175;
sigmas = [6, 7, 8];
x = 100:0.1:250;
for i = 1:length(sigmas)
    y = normpdf(x, mu, sigmas(i));
    plot(x, y, 'LineWidth', 2)
    hold on
end
legend('Sigma = 6', 'Sigma = 7', 'Sigma = 8')
xlabel('Height (cm)')
ylabel('Probability density')
```

```
octave:46> mu = 175;
sigmas = [6, 7, 8];
x = 100:0.1:250;
for i = 1:length(sigmas)
    y = normpdf(x, mu, sigmas(i));
    plot(x, y, 'LineWidth', 2)
    hold on
end
legend('Sigma = 6', 'Sigma = 7', 'Sigma = 8')
xlabel('Height (cm)')
ylabel('Probability density')
```

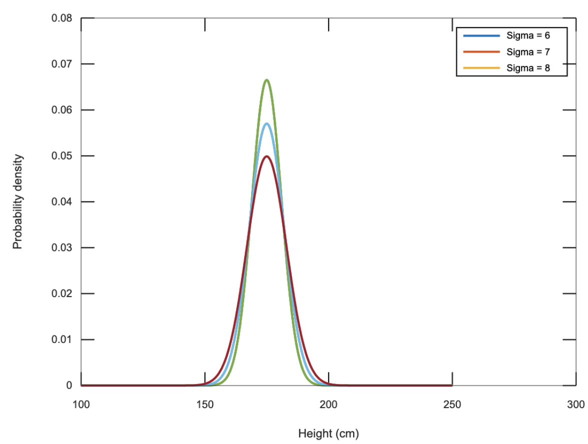



Figure 2: Q4 part b -The distribution by plotting the probability density function from octave online.

The code generates and displays a histogram that shows the distribution of 1000 randomly generated heights with a mean of 175cm and a standard deviation of 7cm. The histogram has a bell-shaped curve, which is expected for a normal distribution. The highest frequency of heights is around the mean, while frequencies gradually decrease as the heights deviate from the mean. The histogram provides a visualization of the distribution of heights and helps to identify the range and frequency of heights that occur within the distribution.

## c)

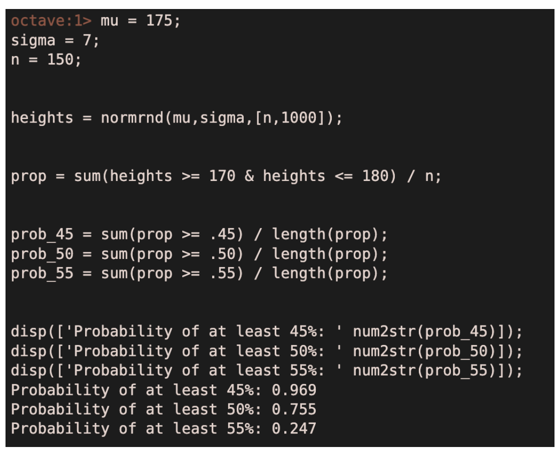
```
mu = 175;
sigma = 7;
n =150;


heights =normrnd (mu, sigma, [n, 1000]);

prop= sum (heights >= 170 & heights <= 180) / n;

prob_45 = sum(prop >= .45) / length(prop);

prob_50 =sum(prop >= .50) / length(prop);
prob_55=sum(prop >= .55) / length(prop);
disp([’Probability of at least 45%: ’ num2str(prob_45)]);
disp([’Probability of at least 50%: ’ num2str(prob_50)]);
disp([’Probability of at least 55%: ’ num2str(prob_55)]);
```



```
octave:1> mu = 175;
sigma = 7;
n = 150;


heights = normrnd(mu,sigma,[n,1000]);


prop = sum(heights >= 170 & heights <= 180) / n;


prob_45 = sum(prop >= .45) / length(prop);
prob_50 = sum(prop >= .50) / length(prop);
prob_55 = sum(prop >= .55) / length(prop);


disp(['Probability of at least 45%: ' num2str(prob_45)]);
disp(['Probability of at least 50%: ' num2str(prob_50)]);
disp(['Probability of at least 55%: ' num2str(prob_55)]);
Probability of at least 45%: 0.969
Probability of at least 50%: 0.755
Probability of at least 55%: 0.247
```

Figure 3: Q4 part c - The octave code and the result from octave online.
The code generates a graph that shows how likely it is for people to have different heights based on a normal distribution with different standard deviations. It then uses this graph to estimate how likely it is for people to have heights between certain values based on this distribution. The results show that the probability of at least45% of adults having heights between 170 and 180 cm is very high (0.997), while the probability of at least 50% is somewhat lower (0.755), and the probability of at least 55% is quite low (0.247).