Question 1: A broadcast or gather operation can be done in $\log(p)$ steps over p processes, given a suitably capable interconnection network. But an estimate of $\log(p)$ time ignores message length and the properties of the interconnect. So suppose sending messages over a particular interconnect can be done with latency α (in seconds) and bandwidth β (in bytes/second).

a. Give an estimate of the time it will take to broadcast a message of size n bytes over this network.

   Answer:
   I first start with the formula for the time it takes a single message to travel from one node to another:

$$\alpha + \frac{n}{\beta}$$

   At each step in the broadcast there are several messages being sent across the network. However, all of these messages are being sent in parallel. Therefore the time for each step in the broadcast is equal to the time for a single message. This leads to the conclusion that the total time for the broadcast is the number of steps multiplied by the time each step takes:

$$t(n, p) = \left(\alpha + \frac{n}{\beta}\right) * \log(p)$$

b. Give an estimate of the time it will take to do a gather operation over this network, where each node contributes $\frac{n}{p}$ bytes, and the final gathered result is of n bytes.

   Answer:
   The same principles from $1.a$ apply to this question. The only difference is that in a gather, each step increases the size of the message. The message size doubles at each step. The message size at step i can therefore be determined by this function:

$$m(i) = 2^i * \frac{n}{p}$$

   Step 1 corresponding to $i = 0$ and so on. Now we can calculate the total time for each step:

$$s(i) = \alpha + \frac{m(i)}{\beta}$$

   Finally to obtain the total time we just add the time for each step over the total number of steps, starting at step zero and ending at step $log(p) - 1$:

$$t(n, p) = \sum_{i=0}^{\log(p)-1} s(i)$$

   By expanding the above function we get a complete formula for the total time taken by gather:

$$t(n, p) = \sum_{i=0}^{\log(p)-1} \left(\alpha + \frac{2^i * \frac{n}{p}}{\beta}\right)$$