Question 4.1: Basic results from automata theory tell us that the language $L = a^n b^n c^n = \varepsilon,\ abc,\ aabbcc,\ aaabbbccc,\ \ldots$ is not context free. It can be captured, however, using an attribute grammar. Give an underlying CFG and a set of attribute rules that associates a Boolean attribute ok with the root R of each parse tree, such that $R.ok = true$ if and only if the string corresponding to the fringe of the tree is in L.

Answer: Below is the context free grammar accompanied by their attribute rules

$$E \rightarrow ABC \qquad E.ok = (A.val == B.val == C.val)$$

$$\begin{aligned} A_1 &\rightarrow \alpha\ A_2 & \quad A_1.val &= 1 + A_2.val \\ &\rightarrow \varepsilon & A.val &= 0 \end{aligned}$$

$$\begin{aligned} B_1 &\rightarrow \alpha\ B_2 & \quad B_1.val &= 1 + B_2.val \\ &\rightarrow \varepsilon & B.val &= 0 \end{aligned}$$

$$\begin{aligned} C_1 &\rightarrow \alpha\ C_2 & \quad C_1.val &= 1 + C_2.val \\ &\rightarrow \varepsilon & C.val &= 0 \end{aligned}$$

Question 4.11: Consider the following CFG for floating-point constants, without exponential notation. (Note that this exercise is somewhat artificial: the language in question is regular, and would be handled by the scanner of a typical compiler.)

$$\begin{aligned} C &\rightarrow digits\ .\ digits \\ digits &\rightarrow digit\ \ more\_digits \\ more\_digits &\rightarrow digits \mid \varepsilon \\ digit &\rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{aligned}$$

Augment this grammar with attribute rules that will accumulate the value of the constant into a val attribute of the root of the parse tree. Your answer should be S-attributed.

Answer: These are the attribute rules for the above grammar. (ds = digits, md = more_digits, d = digit)

$$C \to ds_1 \ . \ ds_2 \qquad C.val = ds_1.val + \frac{ds_2.val}{10^{ds_2.mag+1}}$$

$$ds \to d \ \ md \qquad \begin{aligned} ds.mag &= md.mag \\ ds.val &= (10^{ds.mag} * d.val) + md.val \end{aligned}$$

$$md \to ds \qquad \begin{aligned} md.mag &= ds.mag + 1 \\ md.val &= ds.val \end{aligned}$$

$$md \to \varepsilon \qquad \begin{aligned} md.val &= 0 \\ md.mag &= 0 \end{aligned}$$

$$d \to 0 \mid 1 \mid 2 \ \ldots \qquad d.val = CONST$$

Postest loop:

$M_{ptl}(\text{Do } L \text{ until not } B, s)\Delta =$

$M_{stmt}(L, s) = s'$

if $M_b(B, s') = \text{false}$

s'

else

$M_{ptl}(\text{Do } L \text{ until not } B, s')$