Optimal Skill Mixing Under Technological Advancements*

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Abstract

Using worker surveys and online job posting data, I document that the U.S. economy has seen a substantial increase in the mixing of skill requirements from 2005-2018, both for incumbent jobs and newly posted vacancies. American workers increasingly work in occupations that demand mixtures of analytical, computer, and interpersonal skills rather than specializing in one of them, even within granular occupations. This change occurred primarily in low- to medium-wage occupations, and workers in occupations that increasingly mix non-routine skills, or those with a broader set of these skills earn a wage premium. To understand the sources of these shifts, I build a multi-dimensional directed search and matching model with two-sided heterogeneity and endogenous choices. In this framework, firms optimally choose occupations' skill intensities before producing with a worker. Simultaneously, workers make decisions about their jobs as well as their life-time skill development trajectories. Counterfactual analysis shows that the rise in the complementarity of skills in production and in the cost of skills for occupation operation are the main drivers of skill mixing shifts and the corresponding wage and employment dynamics in this period.

Keywords: skill demand, technological changes, occupations, search and matching model, worker training
JEL Codes: J21, J23, J24, J31, E24

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I Introduction

The nature of work in the United States has seen significant changes in recent decades. A vast literature documents the decline in the demand for "routine" tasks and associated worker skills due to technological shifts (i.e., Autor, Levy, and Murnane 2003; Acemoglu and Autor 2011) and the growing importance of social skills (Cortes, Jaimovich, and Siu 2021; Deming 2017). However, as occupational skill demands adjust, it remains unclear whether employers are leaning towards specific specialized skills or seeking a broad range of skills. Additionally, if there is a trend towards a *mix* of skill demands, how does this influence workers' returns to occupation and education choices? The degree of skill mixing among occupations carries important and distinct implications: if employers seek specific skills, indicating specialization in skill demand, then workers benefit from becoming experts in those particular skill dimensions; if, however, occupations increasingly require mixtures of different skills, indicating "skill mixing," then multidisciplinary schooling and training become more advantageous.

This paper studies the phenomenon of employer skill mixing, exploring its implications for workers, and seeks to understand the underlying sources of these shifts. The analysis begins with the aggregation of suitable data and the creation of measures to assess skill mixing. For this purpose, I primarily employ the Occupational Information Network (O*NET), which surveys incumbent workers of their current jobs and details the importance of different skill requirements in occupations. By considering extended time periods and focusing on continually updated occupations, I show that O*NET allows a credible analysis of longitudinal changes in skill demand. Supplementing this, Lightcast (formerly known as "Burning Glass") provides real-time skill demand from millions of online job vacancies, enabling the measurement of the extensive margin share of jobs that require specific skills. Equipped with these datasets, I evaluate the degree of skill mixing for each occupation by calculating the cosine similarity between an occupation's skill vector and the unit vector on which skills along several domains are equally important; consequently, this "mixing index" increases as an occupation's demand for different skills gets closer to each other.

Leveraging information about skill demand for both incumbent jobs and newly posted vacancies, this paper presents evidence that from 2005 to 2018, occupations in the United States increasingly demand mixtures of different skills. Using the O*NET dataset, I show that even at the 7-digit occupation level, there is a sizable increase in the degree of skill mixing, particularly for analytical, computer, and interpersonal skills that are considered non-routine.[1] Compared to 2005, the degree of mixing of these skills in 2018 as captured by the skill mixing indexes has increased by 9.2 percentiles on average. The growth of skill mixing is even starker in higher-level 4-digit occupations, by 12.4 percentiles on average and 11 percentiles for constantly updated occupations. For example, in 2005, "Maids and Housekeeping Cleaners" valued interpersonal skill four times over analytical and twice over computer skill. By 2018, analytical skill equaled, and computer skill reached two-thirds of interpersonal skill's importance. Conversely, for "Insurance Appraisers, Auto Damage," computer skill consistently led in importance, but by 2018, analytical skill doubled and interpersonal skill tripled, both surpassing 60 percent of computer skill's importance.

I highlight two new facts about skill mixing. First, a shift-share decomposition of the rising trend in skill mixing attributes the majority of the increase to changes within occupations, rather than workers' reallocation across occupations. This pattern distinguishes skill mixing from other labor market changes for which worker reshuffling plays a key role or for which the change is mainly across-occupation.[2] Further decomposition shows that the within-occupation increases in skill mixing persist accounting for workers' gender, education, and experience, and are robust to alternative measures of skills and indexes of mixing. Second, the most pronounced rise in the mixing of the three non-routine skills appears in service and white-collar occupations, including roles like healthcare givers and housekeepers. Whereas blue-collar occupations, such as operators and machinists, have witnessed a more significant mixing of routine skill and the three non-routine skills. On the other hand, high-wage managerial and professional occupations show relatively limited skill mixing.

The phenomenon of skill mixing bears significant distributional consequences in the

[1] O*NET's occupational classification is based on the Standard Occupational Classification (SOC) system but offers more granularity. For example, in 2010, O*NET lists 1,110 unique 7-digit occupations, which correspond to 868 unique SOC 7-digit occupations. For analysis at a higher occupational level using census data, I first crosswalk O*NET occupations to the SOC. Subsequently, I employ crosswalks between SOC and census occupations from Autor and Price (2013) and developed by Deming (2017).

[2] For example, in Autor and Dorn (2013), the polarization of the labor market is attributed to the substitution of medium-skill workers in routine jobs and their flow into service jobs; in Deming (2017) across-occupation employment shift drive the rising importance of social skills. Dodini, Lovenheim, and Willen (2022) find that changes in employment concentration across existing occupations account for the skill intensity differential of unionized workers.

labor market. A notable structural shift in the U.S. labor market since the 1980s has been job polarization (Acemoglu and Autor 2011; Goos, Manning, and Salomons 2014), a trend that continues to be evident in the data from 2005 to 2018. Skill mixing emerges as a key factor in explaining these distributional dynamics. For occupations within similar wage ranks in 2005, it is observed that those who have become more skill-mixed experience greater growth in both employment shares and wages. Remarkably, the growth in employment and wages during this period is almost exclusively attributed to occupations that have become more skill-mixed. Therefore, skill mixing provides a novel and multi-dimensional lens to understand these labor market transformations.

To evaluate the impact of skill mixing on workers' labor market outcomes, I estimate the wage returns to skill mixing by combining the National Longitudinal Survey of Youth 1979 and 1997 (NLSY 79 & 97), taking advantage of the rich information on participants' abilities, employment, and educational histories. I find a significant return to skill mixing for both occupational choices and worker skills. To assess the wage premium, I estimate a regression model that incorporates multiple skills and their degrees of mixing for both occupations and individual workers, with worker and occupation fixed effects in the spirit of Abowd, Kramarz, and Margolis (1999) (hereafter AKM). My preferred specifications indicate that workers in occupations that become a standard deviation more mixed in analytical, computer, and interpersonal skills gain a 1.5 percent wage premium; meanwhile, workers who are more mixed in these skills earn 6.5 percent more. I further show some additional returns to skill mixing, both in terms of employment and college major choices.

The rich empirical findings on skill mixing pose challenges in understanding their underlying forces. I build a directed search model with several novel features to investigate the mechanisms of skill mixing. First, the model represents both firms and workers through multi-dimensional skills, laying the basis for an examination of skill mixing. Second, before producing with workers, firms of both vacant and incumbent jobs will need to design their occupations, incurring a cost payable upon operating the occupation that depends on their skill demand choices, as in Acemoglu (1999).[3] This endogenous occupation

[3]The endogenous choices of the intensity of inputs were first studied in the appropriate technology literature (Atkinson and Stiglitz 1969; Basu and Weil 1998; Acemoglu and Zilibotti 2001; Jones 2005; Caselli and Coleman 2006; León-Ledesma and Satchi 2019). Several studies in the labor literature allow firms to adjust labor usage as well as the quantity margin. In Lazear (2009), firms choose the weight on the skills workers supplied; in Eeckhout and Kircher (2018), firms trade-off between more versus higher quality workers; allows firms to choose appropriate skills given equilibrium skill prices.

design is crucial in delivering the dynamic choices of skill mixing based on the skill distribution in the labor market. Third, the model incorporates non-linear production and cost technologies, departing from the common assumption of linear production functions in standard search models. This non-linearity allows the model to capture the varying degrees of skill complementarity in production and the increasing marginal costs of combining skills in occupations.

The model provides insights into changes in skill mixing, wages, and employment that are tied closely to the empirical observations. Central to its insights is the idea that, as skills become more complementary in production or as their marginal costs increase, firms find it more profitable to mix skills than to specialize. Further, in designing the occupations, firms take into account the skills different workers bring and the likelihood of employing those workers. The model further links the production and cost technology, as well as worker skill supply adjustment to wage and employment distributions.

I then quantitatively evaluate the model to assess the relative importance of various channels' contributions to the observed skill mixing and to investigate their implications for wages and employment. Using two periods of NLSY data, I calibrate the model parameters by targeting the wage and employment distribution across different occupation and worker types, as well as the degree of skill mixing of occupations. Besides matching these targeted moments closely, the model replicates well the wage returns of skill mixing. The calibration results reveal that in a multi-dimensional matching framework, skills are substitutable in production, and firms face increasing marginal costs in operating occupations. Notably, sizable technology shifts have occurred: from the early 2000s to the late 2010s, there has been an increase in the complementarity of skills in production and also in firms' cost of skills for occupation operation. Meanwhile, the efficiency of analytical, computer, and interpersonal skills has increased but has declined substantially for routine skill.

Counterfactual analyses further illustrate that the technology shifts reflected in the increase in skill complementarity in production and in the cost of skills for occupation operation appear as the main drivers of the increase in skill mixing. Specifically, two-thirds of this adjustment in skill mixing is attributed to enhanced skill complementarity, while the remaining third is due to changes in occupational skill costs. In contrast, the changing skill efficiencies contribute negatively to skill mixing, and the shifts in worker skill supply play a negligible role.

The forces driving skill mixing also significantly influence shifts in wage and employment distributions. For the wage premium in high-wage relative to low-wage occupations, the increasing complementarity of skills and cost of skills together account for 74 percent, while the changing skill efficiencies contribute 26 percent. Conversely, in terms of employment gains in high-wage occupations, skill efficiencies play a more crucial role, accounting for 62 percent. These results indicate that while skill efficiency, a traditional focus of the task-biased technological change (TBTC) literature, is important in driving wage and employment dynamics, skill complementarity and cost are also pivotal factors. Additionally, a counterfactual training program that increases the mixing of non-routine skills compresses the wage disparities between skill specialists and non-specialists.

The rest of the paper is organized as follows. The ensuing section connects this paper to a broader set of literature and discusses the contributions. Section III presents the main empirical findings about skill mixing and many of its features. In section IV, I show the returns to mixing both at the occupation and worker levels. Section V presents a directed search model with occupation design to study the skill mixing problem and derive comparative statistics. Estimation of the model parameters and counterfactual analysis are discussed in Section VI. Section VII concludes.

II Literature Review

I study labor market dynamics emphasizing *skill mixtures* and explore new theoretical perspectives to explain them. The empirical objective aligns with the literature investigating the long-term trend of skill demand and skill-biased technological changes (i.e., Tinbergen 1974, 1975; Katz and Murphy 1992; Autor, Katz, and Krueger 1998; Autor, Levy, and Murnane 2003; Goldin and Katz 2010; Acemoglu and Autor 2011; Autor and Dorn 2013; Deming and Kahn 2018; Deming and Noray 2020).[4] My finding that the within-occupation changes drive skill mixing is consistent with other studies that find a major role played by within-occupation variation for aggregate job attributes (Autor and Handel 2013; Atalay

[4]The changes in relative efficiency of inputs is the focus of the skill-biased technological change (SBTC) literature, and has been shown to successful account for the major U.S. wage dynamics. See for example, Katz and Murphy (1992), Autor, Katz, and Krueger (1998), and Goldin and Katz (2010). This paper incorporates both changes in relative skill efficiency and changes in the skill complementarity, and show the latter's important role in determining skill mixing, wage shifts, and employment distribution post 2000s.

et al. 2020; Freeman, Ganguli, and Handel 2020; Cortes, Jaimovich, and Siu 2021).[5] Unlike these studies, this paper studies skills in their conjunction, i.e., as mixtures, and show that employers do increasingly require mixtures of skills from workers, especially non-routine ones. This paper further finds that skill mixing has important distributional implications for wage and employment and for workers' return in occupation and education choices. The evidence on skill mixing leads to unique policy implications and broadens the understanding of the influence of technological change on the labor market.

Two papers closely related to the empirical phenomenon documented in this paper are Hershbein and Kahn (2018) and Deming (2017). The former illustrates that employers in metropolitan areas hit harder by great recession were more likely to post jobs demanding cognitive and computer skills, particularly in routine-cognitive occupations. My analysis differs by demonstrating that skill mixing occurs for a broad set of skills, within a wide array of granular occupations, and is not specific to regions or economic downturns. Deming (2017) highlights that occupations requiring higher math and social skills based on O*NET 1997 have seen increased employment and wage growth from 1980 to 2012. In contrast, I use various versions of O*NET to capture longitudinal changes in skill demand and explore the wage and employment gains stemming from within-occupation skill mixing shifts.

Theoretically, I build a directed search model with multi-dimensional skills and endogenous occupation design, following the literature on directed search (i.e., Menzio and Shi 2010, 2011; Kaas and Kircher 2015; Schaal 2017; Baley, Figueiredo, and Ulbricht 2022; Braxton and Taska 2023). Two main contributions of this model are: First, I allow firms to have endogenous skill demand in the spirit of Acemoglu (1999), which delivers the comparative statics regarding skill mixing. Second, I model skills in a multi-dimensional environment with non-linearity technologies. As such, the model incorporates directed search on both the worker and firm sides with high-dimensional heterogeneity on the two sides, which departs from most search models, but allows me to analyze the changes in skill mixing and the contribution of skill complementarity and cost factors.

[5] Extracting task information from job ads, Atalay et al. (2020) reveal that the major change in job content during 1950-2000 occurred within-occupation, a pattern that is found to persist post-2000 by Freeman, Ganguli, and Handel (2020). Cortes, Jaimovich, and Siu (2021) show that from 1980 to post-2010, high-paying occupations in the United States require more social skills. Using worker-reported job tasks, Autor and Handel (2013) find that there is significant within-occupation variation in task requirements.

The foundational model for worker sorting can be traced back to the seminal work of Roy (1951). Within this framework, occupations are treated as distinct categories, each requiring a unique skill, and workers possess skills specific to particular occupations, preventing the exploration of skill mixing.[6] An earlier tradition, including theoretical work by Shi (2001) and empirical investigations such as Hagedorn, Law, and Manovskii (2017), adopt a single-dimensional index to represent worker heterogeneity. By design, these models preclude discussions on skill mixing. A burgeoning literature explores the multidimensional matching of workers and firms that features two-sided heterogeneity and skill transferability (i.e., Yamaguchi 2012; Lindenlaub 2017; Lise and Postel-Vinay 2020). While much of this literature focuses on the assortative nature of worker-firm matching and the evolution of worker skills[7], this study instead examines firms' endogenous skill demand trade-offs in response to technological advancements or shifts in skill supply.

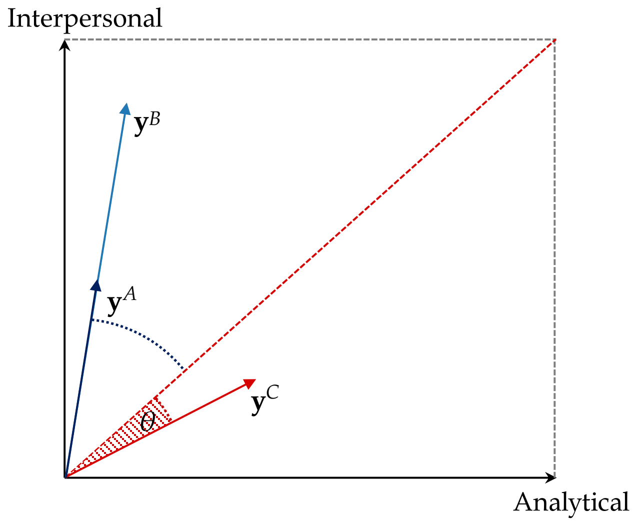
A related literature, inspired by Rosen (1983), Murphy (1986), and Heckman and Sedlacek (1985), features skill indivisibility or bundling, allowing for nonlinear wage schedules and a flexible degree of occupational specialization. Choné and Kramarz (2021) introduce a skill bundling framework featuring heterogeneous firms and using Swedish matched employer-employee data, they find that generalist workers earn more over time. In a separate study, Edmond and Mongey (2021) show that when skill are priced differently across occupations, firms tend to adopt technologies that reflect these skill prices, leading to opposing within-occupation changes in inequality. A critical aspect of these models is the need to take a stance on the aggregation of worker skills within firms, as discussed by Eeckhout and Kircher (2018). Different from this approach, I apply a matching model to address the indivisibility of skills and endogenous skill demand at the worker level, inherently delivering nonlinear wages and skill mixing.

Quantitatively, I provide model-based identification of the elasticity of substitution parameters among a number of different skills and the relevant occupation operation cost parameters under a tractable general equilibrium model of the labor market with endogenous skill intensities. These results contribute to the recent work on task-based models that has typically assumed exogeneity of the elasticity of substitution among

[6] In Roy or Ricardian type of models, workers will also specialize in a particular skill based on comparative advantages, making it harder to study skill mixing's implications for workers.

[7] A notable exception is Ocampo (2022), which introduces the optimal combination of tasks, leading to endogenous occupational heterogeneity.

Figure 1: Illustrating Skill Mixing



Notes: This figure contrasts three occupations—A, B, and C—in the two dimensional skill space of analytical and interpersonal skills. Each occupation is characterized by its skill vector ($\mathbf{y}^A$, $\mathbf{y}^B$, and $\mathbf{y}^C$), as well as by the angle (θ) between the skill vectors and the 45-degree line.

different types of skills (i.e., Autor, Levy, and Murnane 2003; Autor and Dorn 2013), and also relates to studies on the elasticity of substitution among different types of workers (Johnson 1997; Heckman, Lochner, and Taber 1998; Krusell et al. 2000).

III Evidence of Skill Mixing

In this section, I examine the shifts in the extent of skill mixing in the economy. I start by showing that an angle-based index can effectively measure the magnitude of skill mixing of occupations within a multi-dimensional skill space. Using both O*NET and Lightcast data at varying levels of granularity, I explore the growth in skill mixing, decomposing it into across- and within-occupation changes. I further explore the primary sources of this variation and the differences across occupation groups. Lastly, I underscore the significance of the mixing of different skills by relating it to the changes in employment and wage distributions.

III.A Measures and Data

The Degree of Skill Mixing: To evaluate the degree of skill mixing in an occupation, one can analyze the angular difference between the occupation's skill vector and the unit vector, on which different skill requirements are equivalent. Figure 1 illustrates this in the two-dimensional skill space of analytical and interpersonal skills, showing three occupations represented by vectors ($\mathbf{y}^A$, $\mathbf{y}^B$, and $\mathbf{y}^C$). Occupations A and B exhibit greater specialization towards interpersonal skill, as their vectors diverge from the diagonal in the direction of the interpersonal skill axis. Despite varying skill intensities — with occupation B's vector ($\mathbf{y}^B$) being notably longer than occupation A's ($\mathbf{y}^A$) — they share a similar degree of skill mixing, evident from the angle (θ) between their vectors and the diagonal line are the same. The emphasis of skill mixing is on the proportionate use of different skills (indicated by the angle) rather than the overall skill intensity (represented by the vector's length). In contrast, occupation C demonstrates a higher degree of skill mixing, evident from its smaller angle (θ) to the diagonal line.

As θ decreases, indicating a higher degree of skill mixing, $\cos(\theta)$ increases, making it an suitable measure for skill mixing. Building on this concept, I transition from the two-dimensional representation in Figure 1 to a multi-dimensional space. I accomplish this by employing the cosine similarity between an occupation's skill vector and a multi-dimensional norm vector.[8] Specifically:

Definition 1 (Degree of Skill Mixing of an occupation)**.** *The skill mixing index for an occupation j in a K-dimensional space characterized by the skill intensity vector $\mathbf{y}^j = \{y^j_1, ..., y^j_k, \ldots, y^j_K\} \in S \subset \mathbb{R}^{K+}$ is the cosine similarity between its skill vector and the norm $\hat{\mathbf{v}}$ in the skill space.*

$$Mix(\mathbf{y}^j) = \frac{\mathbf{y}^j \hat{\mathbf{v}}}{||\mathbf{y}^j|| \cdot ||\hat{\mathbf{v}}||}, where\ \hat{\mathbf{v}} = [1, 1, ..., 1]' \subseteq \mathbb{R}^{K+}. \tag{1}$$

The mixing index, as described in equation (1), captures the same intuition presented in Figure 1. It evaluates the multi-dimensional angular similarity between a skill vector $\mathbf{y}^j$ of dimension K and the multi-dimensional norm $\hat{\mathbf{v}}$. As different skills in $\mathbf{y}^j$ get closer to

[8] Cosine similarity together with other measures, such as Euclidean distance and Manhattan distance, have been used to calculate the similarity between vectors (i.e., Xia, Zhang, and Li 2015). An angle-based measure is by no means the only measure of skill mixing, though it has the clearest graphical illustration of the trade-off among skills. Online Appendix A.6 discusses two alternative skill mixing indexes: inverse Herfindahl–Hirschman Index (HHI) and normalized absolute distance.

each other, the value of $Mix(\mathbf{y}^j)$ will rise accordingly. There are three key advantages of the skill mixing index defined using cosine similarity that are worth noting. First, it easily accommodates occupations represented by multi-dimensional skills. Second, this index is independent of the length of the skill vector, and focuses on the proximity between a skill vector and the norm, which indicates the degree of skill mixing. Lastly, this measure is inherently normalized, as the cosine of an angle in the first quadrant (indicating positive skills) lies in [0,1].

Data Construction: In analyzing the extent of skill mixing within occupations over time, I primarily use the Occupational Information Network (ONET). This dataset provides detailed information about the importance of skill requirements for various occupations, offering an intensive measure of skill demand. To complement the insights from ONET, I also use data from online job postings via Lightcast. This dataset captures whether certain skills are required for the job postings, offering an extensive measure on skill demand for unfilled vacancies specifically. Below I discuss the details of data construction.

Developed by the North Carolina Department of Commerce and administered by the U.S. Department of Labor, O*NET is a successor to the Dictionary of Occupational Titles (DOT). It has become a primary resource for analyzing occupational skill requirements and work environments (i.e., see Acemoglu and Autor 2011; Yamaguchi 2012; Deming 2017). O*NET offers an comprehensive picture of occupations, covering approximately 270 descriptors categorized into nine modules.[9] While the earlier versions of O*NET include legacy ratings from DOT analysts, a shift occurred in 2003 when O*NET began sourcing responses from random samples of workers (job incumbents). To ensure consistent measurement, I choose descriptors from questionnaires updated based solely on these worker surveys.[10]

A key challenge when using O*NET comes from employing the longitudinal variation in occupation descriptors. Specifically, while each version of O*NET contains roughly 970 7-digit occupations, an average of 110 occupations undergo updates annually.[11] Such

[9]For a comprehensive overview of O*NET, refer to online Appendix A.1, and for a discussion on the descriptors employed, see online AppendixA.2.

[10]Specifically, I use descriptors from the Work Context, Work Activities, Knowledge, and Skills questionnaires. After 2003, O*NET still contain responses from job analysts for questionnaires that have small sample sizes from workers. I abstract from those questionnaires in this paper.

[11]The decision of occupation updating is based on analysts' evaluations of factors such as the size of

a pattern of updates could introduce selection bias when constructing measures of skill demand based on the descriptors. Contrasting prior research, which often explores worker reallocation across occupations using a single O*NET version, this paper's emphasis on the dynamics of skill demand requires the examination of these longitudinal changes.

To examine these longitudinal shifts in skill demand via O*NET data, I employ two approaches, following works such as Ross (2017) and Freeman, Ganguli, and Handel (2020). First, I focus on broader year intervals. For the time period from 2005 to 2018, I analyze the differences in skill requirements between the start and end of this period, during which most occupations are updated at least twice. To capture more granular time patterns, I use 4-year intervals, ensuring updates to cover over half of the occupations within these intervals. Given that each O*NET version retains data from prior years, I make a distinction between the release year and the represented year when integrating O*NET with other datasets. Online Appendix A.1 shows the specific O*NET versions used, their release dates, and the corresponding years.[12] Second, 274 7-digit occupations consistently receive updates between 2005, 2011, and 2018. While these occupations do not represent the entire economy, their trends under continual updates supplements the broader occupation analysis.

Furthermore, I utilize data from online job postings from Lightcast (previously "Burning Glass") for the years 2007 and from 2010 to 2017 that offers insights into unfilled vacancies. Lightcast is a labor market analytics firm that collects and analyzes millions of online job postings and provides detailed education requirements and thousands of codified skills extracted from the posting text. The key advantage of Lightcast data is that it provides comprehensive and up-to-date information on labor demand, and many recent studies have used this dataset to analyze trends in job skill requirements (see, i.e., Deming and Kahn 2018; Hershbein and Kahn 2018; Braxton and Taska 2023). It is essential to recognize that while O*NET gauges the level and importance of a skill (intensive margin), Lightcast identifies whether a skill is required for a vacancy (extensive margin).[13] I employ Lightcast

employment, the demand for labor, and alterations in the type of work involved. See Tippins and Hilton (2010) for more details.

[12]Specifically, O*NET versions 13.0, 18.0, 22.0, and 25.0 were released in 2008, 2013, 2017, and 2022, respectively. These versions are interpreted as representing the years 2005, 2009, 2013, and 2018, respectively.

[13]Several caveats of Lightcast data are that it may not capture jobs advertised through other channels, possibly over-represents certain sectors that tend to advertise online, and inherently might favor growing firms (Davis, Faberman, and Haltiwanger 2013).

as an additional source to complement the picture of skill mixing changes over time.

Skill Measures: Leveraging the O*NET occupation descriptors, I first derive skill measures in line with Acemoglu and Autor (2011) to focus the analysis on the degree of skill mixing. These measures are widely applied and are easily comparable with other studies. To have a feasible dimension of skills to understand their mixing, I consolidate the two routine skills (routine cognitive and manual) into one, which I call routine skill, while I keep the non-routine skills (non-routine analytical and interpersonal) separate.[14] To capture the rise of computer technology post-2000, I also construct a computer skill measure based on two components related to programming and interacting with a computer. As these work activities are not easily codifiable, computer skill is also considered to be non-routine.[15] Appendix Table A2 shows the detailed composing descriptors for each of the skill measures.

While these four skill measures including both routine and non-routine skills (hereafter RNR) serve as the core of this study's analysis, to provide a more comprehensive perspective on evolving skill demands, I also introduce two additional skills that have not been analyzed in previous studies and that are relatively non-routine—leadership and design. To enhance the reliability of these skill measures, especially for granular longitudinal time patterns, I apply principal component analysis (PCA) on the chosen descriptors following Guvenen et al. (2020) and Yamaguchi (2012). The final skill measures are linearly rescaled to lie in [0,1].[16] As a check of validity, online Appendix Table A3 shows that my constructed skills correlate highly with other similar skill measures used in the literature. In online Appendix A.2, I build "broader" skill measures that each include more relevant descriptors than Acemoglu and Autor (2011), which are also highly correlated with the benchmark ones. Along with the discussion of my empirical results, I demonstrate their robustness to using alternative measures of skills and indexes of skill mixing.

[14] Since I only use descriptors updated by job incumbents in this study, I do not use non-routine manual skill since part of the composing descriptors comes from surveys of job analysts exclusively.

[15] For subsequent references to specific non-routine skills, I use terms like analytical, interpersonal, or computer skill, excluding the prefix "non-routine."

[16] Based on Definition 1, it is crucial that skill vectors are in the positive real space for an angle-based measure to be appropriate. In that regard, normalization by standard deviation will not work unless with additional re-normalization, and linear transformation to a positive interval appears most desirable as it also retains the cardinal information that is likely to be useful for an easily interpretable skill comparison (i.e., Autor and Handel 2013; Deming 2017; Lise and Postel-Vinay 2020). Alternative measures of skills and skill mixing are discussed in online Appendixes A.6 and A.7.

Regarding the Lightcast data, I directly use the measures from Braxton and Taska (2023), which in turn are based on the methodology of Hershbein and Kahn (2018). Specifically, for the years 2007 and 2010-2017 of Lightcast data that this study uses, a vacancy is defined to use analytical skill if any of the codified job skills contain keywords such as "research", "analy", and "decision". Similarly, a vacancy is defined to require interpersonal skill if the codified job skills contain keywords such as "communication" or "teamwork".[17] Each occupation's skill measure is then determined by the proportion of vacancies demanding that specific skill, capturing the extensive margin of firm skill demand. To classify occupations within the Lightcast data, I used a 4-digit consistent census occupation code, as developed by Autor and Dorn (2013) to ensure matching with other datasets.

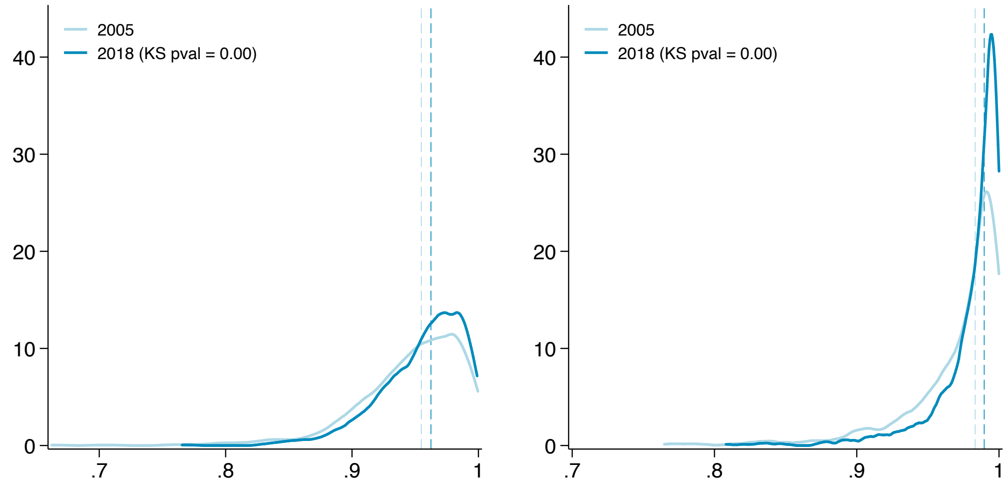
III.B Aggregate Trends

This subsection investigates the trend of skill mixing across various occupations in the U.S. economy from 2005 to 2018, highlighting a notable shift towards increased mixing, particularly in non-routine skills. The analysis begins at the most granular level using 7-digit O*NET data, and also illustrate the key skills driving this overall trend. I then examine the time patterns of skill mixing changes at the census SOC level, employing both O*NET and Lightcast data. Throughout this discussion, I examine the robustness of these findings regarding to the choice of skill mixing indexes and skill measures.

Figure 2 depicts the density and median values of two different skill mixing indexes for the years 2005 and 2018, derived from 7-digit O*NET data. The first index incorporates the four routine and non-routine (RNR) skills, while the second focuses on the three non-routine skills (Non-routine). Panel (1) first reveals a modest rightward shift in the density of skill mixing index for RNR skills during this period. The Kolmogorov-Smirnov (KS) test confirms the distributional difference as statistically significant at the 1 percent level. However, there is more noticeable shift in density for only non-routine skills (analytical, computer, and interpersonal) as shown in Panel (2). By 2018, the density of the skill mixing indexes of these non-routine skills peaks at a much higher value than 2005, and

[17] More specifically, the keywords used to capture analytical skill are: "research", "analy", "decision", "solving", "math", "statistic", and "thinking". The keywords used to capture interpersonal skill are "communication", "teamwork", "collaboration", "negotiation", and "presentation". The key words used for computer skill are "computer", or any skill flagged as software by Lightcast.

Figure 2: Density for Skill Mixing Indexes (Cosine Similarities), 2005 vs. 2018



(1) RNR Skills (2) Non-routine Skills

Notes: These figures plot the kernel density of different skill mixing indexes in 2005 (light blue line) and 2018 (dark blue line). The x-axis displays the value of skill mixing indexes with a maximum of 1 by construction. "RNR" stands for the one routine and three non-routine skills (analytical, interpersonal, computer). "Non-routine" skills only include the three non-routine skills. Specific composing descriptors of the skills are in online Appendix Table A1. These plots are created using O*NET at 7-digit occupations unweighted by employment.

the distribution's shift to the right is more pronounced, indicating a substantial growth in occupations demanding a high level of mixing of non-routine skills.[18]

This pattern of growth in skill mixing is not unique to the choice of non-routine skills and becomes even sharper when accounting for the composition of labor force across occupations. In online Appendix Figure A2 Panel (1), I show that the rightward shift in the density of the mixing index remains consistent when including other non-routine skills (leadership and design). I also combine O*NET data with detailed employment weights from the Occupational Employment and Wage Statistics (OEWS) in online Figure Appendix A2 Panel (2).[19] The rightward shift of all the skill mixing indexes becomes more pronounced when weighted by employment shares.[20]

[18] In addition to index-based evaluation of skill mixing, one can also non-parametrically examine occupation skill requirements in two-dimensional spaces. Online Appendix A.3 discusses and presents non-parametric plots for six skill pairs from both 2005 and 2018, confirming the observed increase in skill mixing, particularly for non-routine skills.

[19] The OEWS uses 6-digit SOC codes, while O*NET uses 7-digit occupation codes that are based on 6-digit SOC. I match OEWS with O*NET at a 6-digit SOC level and distribute the employment weight evenly for 7-digit O*NET occupations within a 6-digit occupation.

[20] This result implies that occupations with larger employment shares have a more significant rise in skill mixing.

Two exercises further highlights the *drivers of skill mixing* and the heterogeneity across different occupations. In online Appendix Figure A2, Panel (3), I exclude each skill individually from the three non-routine skills to compute the skill mixing index. Regardless of which skill is left out, a noticeable rightward shift in density persists, suggesting that each skill has contributed to the overall increase. In Online Appendix Table A4, I conduct a decomposition of changes in the mixing of non-routine skills using polynomial regressions.[21] The results reveal that computer skill is the most significant driver across all occupations. However, for higher-paid occupations (i.e., professionals, managerial, white-collar), interpersonal skill is more important, whereas for medium to lower-paid jobs (i.e., blue-collar, service), computer skill remains dominant.

Time Pattern: To examine the time profile of the shifts in skill mixing and understand the sources of variation, I combine the longitudinal variation in skill mixing from O*NET with worker employment and characteristics from the ACS using consistent census occupation codes from Autor and Dorn (2013).

I take three additional steps to illustrate the time pattern. First, I construct the trend at 4-year intervals so that more than half of the occupations (about 60 percent of employment) are updated between observations. Second, a limitation of the skill mixing index is that it has a long left tail and concentrates at values near 1; moreover, it cannot be comfortably treated as cardinal. To address this concern, I transform skill mixing indexes into percentile values based on their rank in the 2005 distributions. Third, I weight each skill requirement from O*NET by the total hours of work in each sex-education-industry-occupation cell in the ACS to implicitly control for changes in task inputs due to variations in gender, education, industry, and occupation compositions in the U.S. economy (see Autor, Levy, and Murnane (2003) and Deming (2017) for other examples).

Table 1 presents the 3-digit Census SOC occupations that have the highest increase in skill mixing from 2005 to 2018, in percentile units. It also details the skill compositions for

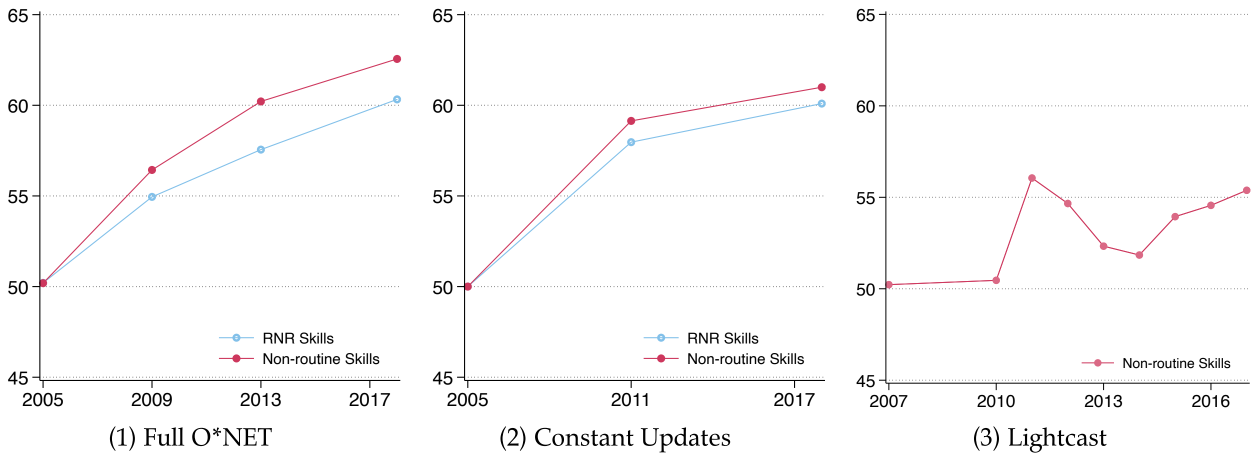
[21] Since the mixing index based on cosine similarity is a nonlinear function of the composing skills, a standard variance-decomposition can hardly partial out the variations clearly. Instead, I conduct a polynomial regressionof non-routine skills' mixing index and each composing skill's polynomials up to order N over the period from 2000 to 2020: $\text{Mix}(\mathbf{y})^{\text{percentile}}_{ijt} = \beta_1 y^1_{ijt} + \beta_2 y^2_{ijt} + ... + \beta_N y^N_{ijt}$, where $\text{Mix}(\mathbf{y})^{\text{percentile}}_{ijt}$ indicates the percentile rank of an individual's i mixing index of non-routine skills in occupation j at time t, and y_{ijt} is the measure of a specific composing skill for that individual's occupation at time t. The R-Squared values are then used to evaluate the degree to which each composing skill explains the variance in skill mixing.

Table 1: Top Occupations in Skill Mixing Growth

Top Occupations	Year	Analytical	Computer	Inter-personal	Routine	Mixing Index	Percentile
Mix of Non-routine Skills							
Packers, fillers, and wrappers	2005	0.58	0.44	0.16		0.915	1
(Operators/Fabricators/Laborers)	2018	0.52	0.40	0.42		0.994	99
Housekeepers, maids, cleaners	2005	0.00	0.10	0.24		0.753	0
(Personal Care and Services)	2018	0.28	0.20	0.25		0.990	96
Sales counter clerks	2005	0.13	0.32	0.30		0.946	7
(Sales)	2018	0.50	0.52	0.39		0.993	99
Recreation facility attendants	2005	0.24	0.18	0.39		0.947	7
(Personal Care and Services)	2018	0.38	0.40	0.35		0.998	99
Janitors	2005	0.10	0.07	0.21		0.913	1
(Food Prep/Buildings and Grounds)	2018	0.15	0.16	0.21		0.987	93
Carpenters	2005	0.50	0.14	0.44		0.915	1
(Production/Craft/Repair)	2018	0.59	0.38	0.53		0.985	90
Cashiers	2005	0.08	0.41	0.33		0.892	0
(Sales)	2018	0.31	0.41	0.49		0.984	87
Packers and packagers by hand	2005	0.16	0.16	0.30		0.951	12
(Operators/Fabricators/Laborers)	2018	0.49	0.40	0.54		0.992	99
Data entry keyers	2005	0.56	0.77	0.27		0.935	3
(Office/Admin)	2018	0.55	0.66	0.43		0.985	90
Sales supervisors and proprietors	2005	0.40	0.39	0.79		0.943	6
(Sales)	2018	0.49	0.57	0.74		0.985	92
Mix of RNR Skills							
Packers and packagers by hand	2005	0.16	0.16	0.30	0.71	0.824	0
(Operators/Fabricators/Laborers)	2018	0.49	0.40	0.54	0.70	0.979	99
Cashiers	2005	0.08	0.41	0.33	0.71	0.863	2
(Sales)	2018	0.31	0.41	0.49	0.61	0.973	99
Assemblers of electrical equipment	2005	0.35	0.25	0.34	0.82	0.894	5
(Operators/Fabricators/Laborers)	2018	0.44	0.43	0.40	0.65	0.979	99
Equipment cleaners	2005	0.23	0.24	0.26	0.63	0.896	5
(Operators/Fabricators/Laborers)	2018	0.41	0.32	0.52	0.54	0.981	99
Cooks	2005	0.24	0.16	0.34	0.59	0.899	6
(Food Prep/Buildings and Grounds)	2018	0.46	0.33	0.46	0.64	0.974	99
Painters, construction, maintenance	2005	0.29	0.12	0.28	0.56	0.892	5
(Production/Craft/Repair)	2018	0.53	0.30	0.56	0.72	0.962	94
Hairdressers and cosmetologists	2005	0.52	0.10	0.38	0.35	0.912	11
(Personal Care and Services)	2018	0.51	0.30	0.44	0.46	0.985	99
Accounting and auditing clerks	2005	0.26	0.72	0.33	0.28	0.905	7
(Office/Admin)	2018	0.40	0.69	0.33	0.44	0.960	93
Packers, fillers, and wrappers	2005	0.58	0.44	0.16	0.84	0.900	6
(Operators/Fabricators/Laborers)	2018	0.52	0.40	0.42	0.82	0.954	90
Punching and stamping operatives	2005	0.25	0.29	0.15	0.86	0.809	0
(Operators/Fabricators/Laborers)	2018	0.42	0.37	0.35	0.74	0.948	81

Notes: This table presents specific O*NET occupations at census SOC levels that have the greatest growth in skill mixing from 2005 to 2018. It provides details on compositions of skills within these occupations and the corresponding changes in skill mixing indexes. The last column translates skill mixing levels into percentiles relative to their 2005 distributions.

Figure 3: Trend of Skill Mixing in the US Economy, 2005-2018



(1) Full O*NET (2) Constant Updates (3) Lightcast

Notes: These figures plot the employment-weighted skill mixing indexes in the U.S. economy from 2005-2018. The y-axis is the percentile of skill indexes in year 2005. By construction, each index has a mean of 50 percentiles in 2005; succeeding points are employment-weighted means mapped to its percentiles in 2005. Panel (1) and (2) combine O*NET and ACS data with consistent 4-digit occupation codes from Autor and Price (2013) and developed by Deming (2017). The matching of different O*NET releases and ACS years are detailed in online Appendix Table A1. Panel (1) show the trend for the universe of occupations while Panel (2) only include 274 7-digit occupations that are constantly updated between 2005, 2011, and 2018. Panel (3) combines Lightcast job posting data and the ACS with the same occupation coding. Employment weights from ACS are the total hours of work aggregated to sex-education-industry-occupation cells.

those occupations, as well as their broader 2-digit occupational categories (in parentheses). To ease the presentation, the table includes only those occupations that constitute a minimum of 0.2% of overall employment, though all occupation codes have been used in the analysis.

The occupations with the highest increase in non-routine skill mixing include service sector, particularly in housekeeping and cleaning, sales roles, and blue-collar jobs involving operation, production, and labor tasks. In these jobs, analytical and computer skills have become increasingly important. For example, in 2005, the role of a housekeeper heavily relies on interpersonal skills; by 2018, the importance of computer skills equals three-quarters of that of interpersonal skills, and analytical skills become just as crucial. Similarly, in the case of sales clerks, while interpersonal and computer skills are predominant in 2005, by 2018 analytical skills equal the importance of computer skills and surpass interpersonal skills. In contrast, blue-collar occupations such as packers, which primarily required routine skills in 2005, have seen a marked increase in all non-routine skills, resulting in a significant rise in their RNR skill mixing index.

Figure 3 demonstrates that the degree of skill mixing has risen substantially and steadily between 2005 and 2018. By construction, each index has a mean of 50 percentiles in 2005; succeeding points are employment-weighted means of each index mapped to its percentiles in 2005. By 2018, the degree of mixing in non-routine skills for an average occupation in the US economy is 12.4 percentiles higher than its 2005 level. The degree of skill mixing in RNR skills has also increased steadily to a slightly lesser extent, averaging 10.1 percentiles higher.[22] A potential concern of using O*NET data to obtain the longitudinal variation of skill demand is that the trend could be affected by the inconsistency in occupation updating. In Panel (2) of Figure 3, I compute these trends focusing solely on the 274 7-digit occupations that are constantly updated between 2005, 2011, and 2018, thus reflecting a consistent updating of skill requirements among these occupations. The same qualitative pattern holds, that is, there has been a sharp increase in the degree of skill mixing, particularly of non-routine skills between 2005 and 2018. Nonetheless, for the constantly updated occupations, the shift is mostly pronounced before 2011.[23]

In Panel (3) of Figure 3, I complement the picture of changing degree of skill mixing using the Lightcast data through a similar pairing with O*NET data starting in 2007, the first year when the company starts to collect job postings. Overall, firms are more likely to post job requirements that contain more mixed-skill demands. By 2017, the degree of skill mixing in online posted vacancies averaged 5.2 percentiles higher compared to 2007. The time pattern of skill mixing among online job postings appears to be more volatile, first peaking in 2011, then sliding down until 2014, before rising dramatically afterwards. Despite the greater variance, the same qualitative pattern holds that occupations have a higher demand for the mixing of non-routine skills.[24]

[22]The inclusion of routine skill decreases the magnitude of the rise in skill mixing implies that the speed of mixing of routine with other skills is slower than the speed of mixing among non-routine skills. Online Appendix Figure A6 depicts the trend of skill mixing for specific skill pairs. The findings reveal a modest increase in the mixing of routine with computer skills at 2.9 percentiles from 2005 to 2018. Conversely, the degree of mixing between routine and other non-routine skills has remained stable.

[23]In online Appendix Figure A3, I show employment percentages and hourly wages across various job categories in the full sample and the sample for constantly updated occupations. The hourly wage rates across the categories are fairly consistent between the full and selected samples, with minor discrepancies: the selected sample has less presence of professionals and sales occupations.

[24]The higher degree of volatility is partly driven by the nature of the measure and the data. The measures of skills from job postings are whether firms require a particular skill in the text of job ads, which are naturally noisier than the questions on level and importance from O*NET. Moreover, firm job posting is more influenced by firm entry and exit patterns.

Robustness of the trend: One may be concerned about that the overall patterns shown so far are driven by the choice of skill measures or the choice of skill mixing index. To address this concern, in online Appendix A.6 and A.7, I demonstrate the robustness of these trends across various skill measures, alternative skill mixing indexes, as well as skill mixing indexes of distinct skill pairs. For example, using standardized (or broader) measures of skills, the increase in the degree of mixing of non-routine skills is 6 (or 13) percentiles from 2005 to 2018; using inverse Herfindahl-Hirschman Index, the increase in the mixing indexes of any given skill pair is above 10 percentiles during the same period. Across these checks, the qualitative picture remains consistent: there has been a notable rise in the degree of skill mixing, particularly for non-routine skills.

III.C Decomposing the Sources

To gain a deeper understanding of the variations underlying changes in skill mixing, I undertake three exercises. First, I decompose the longitudinal changes in skill mixing in the U.S. economy, differentiating between intensive margin skill mixing index changes and extensive margin employment shifts across occupations. This analysis reveals that within-occupation skill mixing shifts play a more influential role in driving skill mixing than across-occupation employment shifts. Second, I perform a regression analysis that include extensive controls such as various skill supply measures, as well as gender, industry, and occupation fixed effects. I find that the pronounced trend of increasing skill mixing persists.

Table 2 shows a shift-share decomposition of the changes in the employment-weighted skill mixing indexes into within-occupation index shifts and across-occupation employment changes, at both 7-digit O*NET occupation and 4-digit census occupation levels using employment weights from the OEWS and ACS respectively. I conduct the analysis both for the full O*NET data and the subset of persistently updated occupations, alongside the Lightcast data. Irrespective of the dataset or skill groupings, within-occupation variation predominantly drives the rise in skill mixing. For example, for the 12.4 percentile increase in the mixing of non-routine skills in the full O*NET data at 4-digit occupation level, within-occupation increase contribute 9.7 percentiles while only 2.7 percentiles stem from worker reallocation; for the 5.2 percentile increase in the mixing of non-routine skills in

Table 2: Shift-Share Decomposition of Skill Mixing Index Changes

	Skill Groups	7-digit Occupations			3-digit Occupations		
		total	within	across	total	within	across
Full O*NET	RNR Skills	6.78	4.93	1.85	10.12	9.46	0.66
	Non-routine Skills	9.21	5.62	3.59	12.37	9.72	2.65
Constant Updates	RNR Skills	5.59	6.73	-1.14	10.09	10.74	-0.65
	Non-routine Skills	4.05	5.33	-1.29	11.00	9.69	1.31
Lightcast	Non-routine Skills				5.16	4.37	0.78

Notes: This table shows a shift-share decomposition of changes in the average level of different mixing indexes between 2005-2018 in percentile units. Specifically, for a change in the percentile of a mixing index over two periods t and τ, its change $\Delta T_\tau = T_\tau - T_t$ which can be decomposed to $\Delta T = \sum_j \left(\Delta E_{j\tau}\alpha_j\right) + \sum_j \left(E_j \Delta\alpha_{j\tau}\right) = \Delta T^a + \Delta T^w$ where $E_{j\tau}$ is employment weight in occupation j in year τ, and $\alpha_{j\tau}$ is the level of mixing index h in occupation j in year τ, $E_j = \frac{1}{2}(E_{jt} + E_{j\tau})$ and $\alpha_j = \frac{1}{2}(\alpha_{jt} + \alpha_{j\tau})$. ΔT^a and ΔT^w then represent across-occupation and within-occupation change.

Lightcast data, within-occupation increases account for 4.4 percentiles.[25] Interestingly, for the constantly updated occupations at 7 digits, worker reallocation actually contributes negatively to the increase in skill mixing. This pattern implies that for these granular occupations under regular updates, the contribution of within-occupation variation more than accounts the increase in skill mixing. At 4-digit occupations, worker reallocation does contribute positively to these increase in the mixing of non-routine skills, but the influence is still marginal compared to within-occupation variation; for RNR skills, the contribution remains negative.

An alternative explanation of the employer-side shifts in accounting for skill mixing could be that even within occupations, the supply of labor might have changed due to for example, rising human capital or labor force participation of female workers. To further shed light on the sources, Table 3 shows results from a regression of skill mixing indexes on a linear time trend (year indicator) across combinations of O*NET and Lightcast with the ACS data. I further control for the interaction between gender and education fixed effects, and between industry and occupation fixed effects; additionally and I include flexible polynomials and interactions of years of education and experience. The table shows a universal increase in the degree of skill mixing at a magnitude of 0.65 to 0.75 percentiles per year using O*NET data and 0.33 percentiles per year using Lightcast data. This increase

[25] Online Appendix A5 shows the decomposition results using skill mixing indexes for different skill pairs and a similar result holds.

Table 3: Annual Changes in Skill Mixing Indexes (in Percentiles)

	RNR Skills		Non-routine Skills	
	(1)	(2)	(3)	(4)
*A. Full O*NET, 2005-2018*				
Year indicator	0.77***	0.70***	0.81***	0.71***
	[0.14]	[0.07]	[0.08]	[0.06]
Observations	237,885	237,885	237,885	237,885
R-squared	0.10	0.83	0.08	0.83
*B. O*NET Constant Updates, 2005-2018*				
Year indicator	0.77***	0.75***	0.68***	0.65***
	[0.12]	[0.11]	[0.11]	[0.11]
Observations	107,956	107,956	107,956	107,956
R-squared	0.29	0.81	0.15	0.82
C. Lightcast, 2007-2017				
Year indicator	—		0.42**	0.33**
	—		[0.19]	[0.15]
Observations	—		532,636	532,636
R-squared	—		0.25	0.87
Experience and edu controls	X	X	X	X
Gender × education FE	X	X	X	X
Industry × occupation FE		X		X

Notes: This table provides regression results on the relationship between the percentile values of RNR skills and Non-routine skills, based on their distributions in the year 2005, and a time trend variable (year values). The analysis incorporates data from the full O*NET, constantly updated O*NET, and Lightcast datasets combined with ACS. See online Appendix A.1 and A.6 for the data construction. The regressions include controls for gender-education fixed effects, industry-occupation fixed effects, polynomials of years of work experience up to power 4, and the interaction of experience polynomials and education fixed effects and gender. Education fixed effects include 5 categories (no high-school, high-school graduate, some college, college graduate, post-college). *** p<0.01, ** p<0.05, and * p<0.1.

persists within gender, education, industry, and occupation groups and is unaffected by controls of worker's labor supply. Despite the varying trends among different industry and occupation groups, these did not alter the overall increase in skill mixing. This finding suggests that the increase in skill mixing withstands adjustments for worker composition, highlighting the important impact of demand-side forces.

Lastly, I investigate the relationship between changes in skill mixing and two industrial-level shifts: the increase in IT capital and the adoption of industrial robots. I obtain productive capital stock for "Total information processing equipment" from the Bureau of Labor Statistics Total Multifactor Productivity tables. Additionally, following Acemoglu and Restrepo (2020), I use data on the stock of robots from the International Federation of Robotics (IFR).[26] Online Appendix Table A6 presents regressions of skill mixing indexes from 2005-2015 on IT capital levels and robot adoption changes, accounting for worker composition across gender, education, and industry groups, as well as flexibly controlling for years of experience and levels of education. The results indicate that a rise in productive IT capital stock by 10 billion is associated with a 0.1 percentile increase in non-routine skill mixing and a 0.1 percentile decrease in RNR skill mixing, driven by rising demand for computer skills and a marginal decline in the importance of interpersonal skills.[27] On the other hand, an increase of one industrial robot per thousand workers has no significant association with the mixing of non-routine skills but is associated with a 1.24 percentile decrease in mixing RNR skills.

III.D Variation in Skill Mixing Changes by Subgroups

Beneath these general trends of skill mixing are diverse patterns among occupations and worker groups. Panel (1) of Figure 4 illustrates the changes of skill mixing indexes from 2005 to 2018 across four primary occupation categories, grouped by wage levels and encompassing all non-agricultural employment in the U.S.[28] The units of changes are in percentiles of the skill mixing indexes' 2005 distributions, similar to Figure 3.

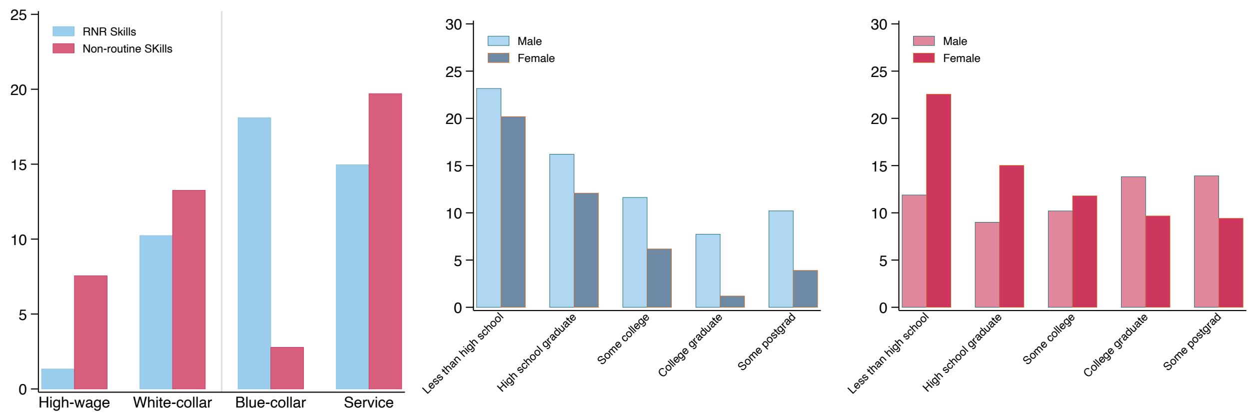
Considering the occupational trends, for the mixing of non-routine skills, service and white-collar occupations see higher increases than other occupations. For the mixing of RNR skills, on the other hand, blue-collar followed by service jobs show most pronounced rise. In contrast, high-wage occupations show the least increase in skill mixing for both skill groups. This pattern highlights that the bulk of skill mixing occurs in medium- to lower-wage professions, especially within the service sector.

[26]Specifically, I focus on the average number of industrial robots per thousand workers in five European countries to isolate the impact of global technological advancements. These 5 countries are Denmark, Finland, France, Italy, and Sweden. Germany is omitted due to its growth in robotics way above other countries.

[27]The average IT capital stock across industries in year 2005 is 70 billion. The results on the association between IT capital and skill mixing is significant at 5 percent using O*NET data and not precisely estimated using Lightcast data.

[28]The categorization into four groups is based on Acemoglu and Autor (2011), which is derived from 10 1-digit occupational groups that cover the entirety of US non-agricultural employment. Specifically, "High-wage" includes Managers, Professionals, and Technicians; "White-collar" comprises Office/Administrative and Sales roles; "Blue-collar" includes Production, as well as Operators/Laborers; and "Service" consists of Protective Services, Food/Cleaning Service, and Personal Care occupations.

Figure 4: Skill Mixing Index Change by Occupation and Gender Groups, 2005-2018



(1) Occupation groups

(2) Gender and education groups – RNR skills

(3) Gender and education groups – Non-routine skills

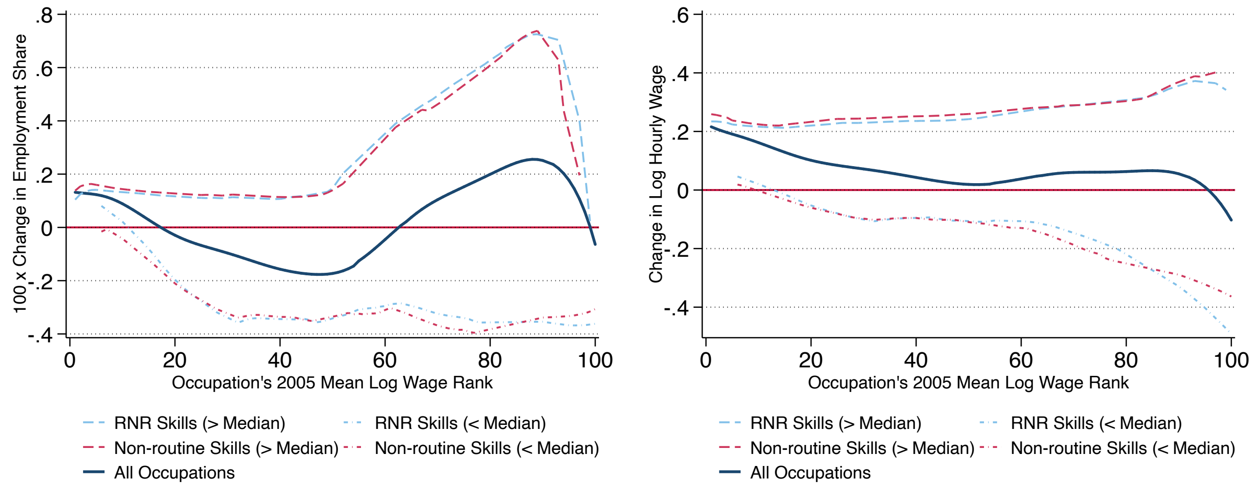
Notes: This figure plots the changes in skill mixing indexes across different occupation groups and gender-education combinations. The units of the index changes are percentiles of their distributions in 2005. The occupation groups (High-wage, White-collar, Blue-collar, Service) follow Acemoglu and Autor (2011). The education grouping aligns with the educational categories defined in the ACS. O*NET and ACS data are combined for these figures with consistent occupation codes from Autor and Price (2013) and developed by Deming (2017).

Panel (2) and (3) of Figure 4 depict changes in skill mixing for RNR skills and non-routine skills, respectively, across gender and education groups. For the mixing of RNR skills, workers with less than a college degree show a greater increase than other groups. However, gender difference appears; on average, male workers experience 5 percentiles higher skill mixing than their female counterparts, and the difference is particularly evident among highly educated workers. Regarding the mixing of non-routine skills, male workers across different education levels experience roughly similar increases in skill mixing, whereas female workers observe a slower increase in mixing as education level rises, less than their male counterparts for those with a college education or higher.

In online Appendix A.4, I show the decomposition of skill mixing by industries, and a similar pattern holds. The private service sector followed by retail trade and construction lead others in the growth of skill mixing, while public, social and professional services sectors demonstrate only modest increases, particularly for RNR skills.[29] I also show the decomposition of occupations' changing mixing of distinct skill pairs, which confirms that

[29]The sectors that have the least growth in skill mixing are public and education and social services. This result is consistent with Hershbein and Kahn (2018) that industries with locally consumed goods are more likely to change skill demand.

Figure 5: Smoothed Employment and Wage Changes by Skill Percentile, 2005-2018



Notes: These figures plot the smoothed changes of share of total hours workded (Panel A) and hourly wage (Panel B) for occupations between 2005-2018. On the x-axis, occupations are ranked into 100 percentiles by the average log wages of workers in those occupations in 2005. The changes in the share of hours worked and percent wage growth are then calculated for each percentile, which fit into smoothed lines using cubit polynomial fit. Solid lines depict the smoothed employment/wage changes for all the occupations, while dashed (or dotted) lines depict the changes for occupations with above-median (or below-median) increases in the skill mixing indexes.

non-routine skills drive skill mixing in all occupations, while routine skill is only more mixed with other skills in blue-collar and service occupations.

Distributional Implications: One of the key structural changes in the U.S. labor market post-1980 is the pronounced job polarization or hollowing out of middle-skill employment and wage growth, due potentially to the routine-biased technological change and offshoring (Acemoglu and Autor 2011; Goos, Manning, and Salomons 2014). To see how much skill mixing can relate to these distributional dynamics, Figure 5 depicts the smoothed observed changes in both the share of total hours worked and log wage in 2005-2018 for occupations ranked by their hourly wage percentiles in 2005. I reconstruct these smoothed employment/wage changes for two groups of occupations: those with above-median increases in skill mixing indexes and those below the median.

Figure 5 first confirms the inverted bell shape (polarization) of observed employment and to a lesser extent, the change of wages. Furthermore, it illustrates key differences for occupations that have become more skill mixed. For occupations within similar wage ranks in 2005, those that become more mixed in skills have a higher increase in employment share and wage growth. In fact, almost the entirety of employment and wage growth is accounted for by occupations that have become more skill mixed during this period. Therefore, relating to polarization, the differential growth in employment and wage among occupations at the top and bottom end of the 2005 wage distribution are entirely accounted for by skill-mixing occupations during this period. Besides being an important phenomenon for labor market dynamics, skill mixing also provides a unified and multi-dimensional perspective of the polarization changes.

IV Returns to Skill Mixing

In order to better understand the influence of skill mixing on workers' labor market outcomes, this section examines the wage returns associated with skill mixing in relation to occupational choices and inherent worker skills. Additionally, I discuss the return on investment for a college major with a more mixed skill set.

IV.A Data and Measurement

To assess wage returns associated with skill mixing, I use the National Longitudinal Survey of Youth (NLSY) datasets from both the 1979 and 1997 cohorts, which offer comprehensive records of the participant's employment and educational histories. I combine these two cohorts to increase the sample size, limiting to the period from 2005 to 2019 to align with the timing of my skill mixing measurements from O*NET as discussed in the previous section.[30] The NLSY data are connected with O*NET via the census occupation information in NLSY and the crosswalk formulated by Autor and Dorn (2013). My principal focus is the real log hourly wage, adjusted to 2013 dollars. As in Altonji, Bharadwaj, and Lange (2012), I trim values of the real hourly wage below 3 or above 200. The results of wage returns are robust to considering alternative sample constructions, such as excluding respondents over the age of 55 or using the unprocessed real hourly wage.

The key advantage of NLSY is that it is a worker-level panel, and also contains information on workers' pre-market abilities. This allows for the control of worker characteristics

[30]The NLSY 1979 and NLSY 1997 are nationally representative surveys of youth, capturing data from individuals aged 14 to 22 in 1979 and 12 to 16 in 1997, respectively. During my sample period, the median age is 37, and 91 percent of the sample is below 50.

in assessing occupational wage returns to skill mixing and also facilitates the evaluation of returns to the worker-level degree of skill mixing. The selected measures of worker abilities are chosen to align well with the skill measures in O*NET: the Armed Forces Qualifying Test (AFQT) scores represent analytical skill, the social skills measure developed by Deming (2017) is employed to represent interpersonal skill,[31] and routine skill is measured by the workers' Armed Services Vocational Aptitude Battery (ASVAB) mechanical orientation scores.[32] As NLSY offers scant information on workers' computer skills, I adopt the worker's occupation or college major's computer skill value in the year 2005 as a proxy for the worker's initial endowment of computer skill. Online Appendix Table B1 lists the corresponding measures.

IV.B Wage Returns

To estimate the returns to skill mixing, I regress the log wage of workers on the levels of different skills required by their employed occupations, as well as the skill mixing indexes of these skills. Conditional on skill levels, the coefficients on skill mixing indexes identify the returns to working in occupations that are more mixed among the skills. To further examine the worker level returns to skill mixing, I add to the right-hand side the levels of the skills that workers have, and their degrees of mixing. The coefficients on worker-level mixing indexes then identify the wage premium to the mixing of worker skills conditional on occupational skill requirements. To streamline the discussion, I focus on the degree of mixing of the three non-routine skills (analytical, computer, interpersonal), given its significant increase during the observation period as in Section III. Analysis of returns to mixing between routine and other skills as well as to individual skills is deferred to the online Appendix A.8.

Throughout all the specifications, I include ethnicity by gender, age, metropolitan status, individual year, years of education, census region, and urbanicity fixed effects. I also

[31] I use the AFQT scores constructed by Altonji, Bharadwaj, and Lange (2012) that are consistent across NLSY waves and account for age-at-test, test format, and other peculiarities. For interpersonal skills, I use the social skill measure developed by Deming (2017) assessing extraversion, which is constructed based on sociability in childhood and adulthood in NLSY79, and two questions from the Big 5 inventory in NLSY97 respectively.

[32] ASVAB test scores are only available for the NLSY79 survey. For NLSY97, I impute their ASVAB scores using a regression model with indicators for gender and ethnicity, and fixed effects that include age, year, census division, metropolitan area, and urbanicity.

Table 4: Return to Skill Mixing: Occupations and Workers

Dependent: ln (hourly wage)	(1)	(2)	(3)	(4)
Mix (non-routine skills): Occupation	0.017***	0.015***	0.001	0.014***
	[0.005]	[0.005]	[0.006]	[0.005]
Mix (non-routine skills): Worker		0.065***	0.070***	
		[0.017]	[0.017]	
Interaction			0.032***	
			[0.008]	
Ethnicity, gender, age/year, region, edu FE	X	X	X	X
Occupation FE	X	X	X	X
Worker FE				X
Observations	88,391	79,343	79,343	88,391
R-squared	0.41	0.43	0.43	0.76

Notes: This table reports the result of estimating wage equations using pooled NLSY79&97 data for employed workers from 2005-2019. Log hourly wages are the outcome variables and person-year is the unit of observation. The occupational skill and skill mixing measures come directly from O*NET and are merged to NLSY79&97 based on census occupation codes. The worker-level skill measures are constructed to correspond to occupation-level measures as in Table B1 and skill mixing indexes are then calculated accordingly. All measures of skill and skill mixing are normalized to have mean 0 and standard deviation 1. Ethnicity-by-gender, age, year, census region, urbanicity, and a 5-category (no high-school, high-school graduate, some college, college graduate, post-college) education fixed effects are included for all regressions, with additional fixed effects as indicated in the table. Standard errors are clustered at the individual level. *** p<0.01, ** p<0.05, * p<0.10.

include occupation fixed effect to control for time-invariant differences across occupations, which allows me to focus on how the changes in skill requirements within occupations are affecting wage returns, consistent with the empirical finding that this margin is the main driver of skill mixing. Standard errors are clustered at the individual worker level to account for within-group correlation and heteroskedasticity among repeated observations at the individual level.

Occupation and Worker Level Returns: Table 4 shows the wage returns to skill mixing at both the occupation and individual levels, indicating a positive premium for mixing non-routine skills. Column (1) reveals that workers in occupations that become one standard deviation more mixed among analytical, computer, and interpersonal skills earn a wage premium of 1.7 percent per year, significant at the 1 percent level. In column (2), I incorporate the mixing index of worker abilities, which enhances the precision of occupation-level wage premiums and estimates the return to skill mixing at the worker level. The results suggest that on the worker side, workers who are a standard deviation more mixed among the non-routine skills earn a wage premium of 6.5 percent. Meanwhile, the wage premium for the three non-routine skills remains at 1.5 percent per year at the occupational level. In column (3), I further restrict the analysis to within-worker variation by adding worker fixed effects; combined with the occupation fixed effects, this specification closely aligns with an AKM model.[33] The magnitude of the returns to skill mixing presented in column (3) is similar to that in column (2). Workers in occupations that become one standard deviation more mixed among non-routine skills experience a wage increase of 1.4 percent.

Given the positive wage premium associated with mixing non-routine skills at both occupation and worker levels, below I discuss some additional returns to skill mixing. I first examine the robustness of the wage return results. Next, I explore the returns of skill mixing in for employment and college major choices.

Discussions:

Robustness: To gain a more detailed view of the drivers of the positive premium of skill mixing, online Appendix Table A8 uses mixing indexes of skill pairs instead of a high-dimensional mixing index for non-routine skills, which indicates the positive wage returns primarily arise from the mixing of analytical with computer and analytical with interpersonal skills. Robustness checks in online Appendix A10 show that the occupational returns to skill mixing conditional on worker fixed effects are robust to alternative skill measures and indexes of mixing. Specifically, the results consistently suggest a wage premium of 1 to 2.5 percent in occupations that mix these skills. Further, in online Appendix Table A8 I show that there is also a positive employment premium for workers with a more mixed skill set: workers with a more mixed level of all the skill pairs except for routine and interpersonal are also more likely to exit unemployment.[34]

Additional Returns: Moreover, using the college education information from NLSY, I assess the returns of pursuing college majors with different degrees of skill mixing.[35] I

[33]Using within worker variation to study wage growth has been discussed and applied in i.e., Neal (1999); Gibbons et al. (2005); Lazear (2009) and Deming (2017). Choné and Kramarz (2021) found that under a worker assignment model with bundled skills, the implied wage equation also has an AKM form.

[34]Throughout my analysis, I classify a worker as employed if the worker earns a wage greater than zero and has held one or more jobs since the last NLSY interview, consistent with Altonji, Bharadwaj, and Lange (2012) and Deming (2017). Further, workers without a paying job for 24 months are considered to be out of the labor force.

[35]There are some inconsistencies in NLSY's field of study coding: NLSY79 uses its own major codes that

calculate for workers who studied a particular major, the employment weighted average of skill intensities of their occupations in O*NET to compute skill mixing index for that major.[36] Online Appendix Table A11 highlights majors based on their skill mixing levels and changes. Notably, Architecture and Environmental Design stands out in mixing the three non-routine skills, with Computer and Information Sciences, and Communications following closely. Additionally, Social Sciences and Agriculture and Natural Resources are among the top majors in mixing routine and non-routine skills. Online Appendix Table A8 column (4) quantifies workers' human capital by their majors' skill contents, and shows a positive return of around 3 percent studying a college major that is associated with a standard deviation higher mixing of non-routine skills.

V A Directed Search Model with Occupation Design

The rich empirical findings on skill mixing pose challenges in understanding their driving forces. In what follows, I attempt to provide an overarching framework to investigate the mechanisms. For this purpose, I build a directed search model with several novel features: First, both firms and workers are represented by multi-dimensional skills; Second, firms must make decisions about occupation design before producing with workers, a process that involves a cost payable upon operating the occupation as in Acemoglu (1999);[37] Third, the model incorporates non-linear production and operation cost technologies. Despite the rich setup, the model remains tractable satisfying Block Recursivity as in Menzio and Shi (2011). Under these specifications, the model offers clear insights regarding changes in skill mixing, wages, and employment that are linked to the empirical findings.

V.A Environment

Workers: Time is discrete. At each period, there is a unit measure of heterogeneous workers that lives forever. Each worker of type i is characterized by a vector of multi-dimensional

contain 25 two-digit categories, while NLSY97 uses another set codes for years leading to 2010 and transfers to National Center for Education Statistics (NCES)'s 2010 College Course Map (CM10) for years after 2010. For consistency, I map the two different types of major codes in NLSY97 to the 25 two-digit major categories in NLSY79. Online Appendix Table A12 shows the crosswalk of different types of major field of study codes.

[36] I take the first field within a year as representing a worker's major field in the case of multiple fields.

[37] As such, the model incorporates directed search on both the worker and firm sides with high-dimensional heterogeneity on the two sides.

skills $\mathbf{x}^i = \{x_1^i, ..., x_k^i, \ldots, x_K^i\} \in S \subset \mathbb{R}^{K+}$, where K is the dimension of a closed skill space S. Workers draw their initial skill vectors at the beginning of the period from an exogenous distribution $G(\mathbf{x})$. Workers are risk-neutral, have linear utilities over consumption, and discount the future with a factor β.

Firms: On the other side of the market, there is a mass of risk-neutral firms each running one vacancy. Firms pay a cost c to post their vacancies across different occupations $j = \{1, ..., J\}$, with $J \geq 2$. Each occupation is characterized in the same multi-dimensional skill space as workers' skills, $\mathbf{y}^j = \{y_1^j, ..., y_k^j, \ldots, y_K^j\} \in S \subset \mathbb{R}^{K+}$, which has the interpretation of a vector of skill requirements or skill importance for each of the worker skills. Firms share workers' discount factor β.

The production function of each worker-firm pair takes a CES form of the skill inputs of workers and skill requirements of an occupation that the firm operates:

$$f(\mathbf{x}^i, \mathbf{y}^j) = \left[\sum_{k=1}^{K} (x_k^i \alpha_k y_k^j)^{\sigma^j} \right]^{\frac{1}{\sigma^j}}, \tag{2}$$

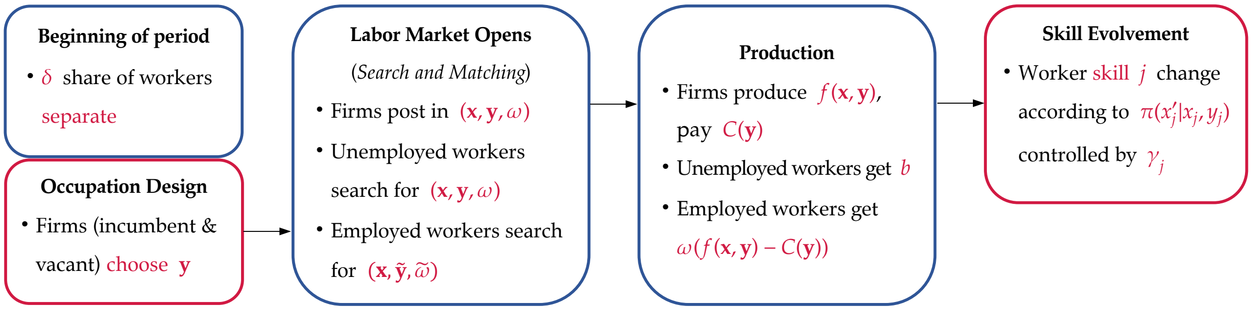
where α_k controls the efficiency between worker skill and job skill requirements for a particular skill k, and σ^j controls the elasticity of substitution among different skills for an occupation j.[38] This production technology represents an extension of the production technology used in the multi-dimensional skill matching literature (i.e., Lise and Postel-Vinay 2020; Lindenlaub 2017; Ocampo 2022), where worker and firm attributes take a multiplicative form for their output associated with efficiency α_k differing by skill, which I term as "skill efficiency". I also allow complementarity across skills regulated by σ^j. When multi-dimensional skill distributions of workers and firms are considered, such a production technology gives a clear portrayal of the interaction between skill demand and supply, as well as the interaction among different skills.[39] Due to the one-to-one matching nature of the model, I omit the superscripts for worker and occupation skill vectors i and j in the exposition below.

A unique feature of this model is that I allow firms to actively *design* the jobs before

[38]Since labor is the only input in the model, it can be understood as "equipped" labor, and occupations' skill requirement or importance $\mathbf{y}$ takes a factor augmenting form, essentially acting as demand shifters.

[39]As such, the model explores both the role of changes in relative input efficiency that is the focus of task-based literature, and changes in skill complementarity.

Figure 6: Model Timing



producing with the worker (Acemoglu 1999), delivering endogenous demand specialization and the degree of skill mixing. Specifically, firms with both filled and unfilled vacancies design their occupations by optimally choosing the occupational skill requirements $\mathbf{y}$ in each period.[40] Such an intensity choice of skills in an occupation alters the efficiency of that skill will and essentially leads to different production technologies for that occupation, capturing the overall quality of the occupation and the optimal degree of skill mixing. In designing the occupation, both worker skill profiles and skill complementary play an important role, as firms would want to exploit what skills the workers supply given the technology.[41]

Nonetheless, such a job design incurs a cost $C(\mathbf{y})$ that is payable upon producing with a worker. This cost is convex and strictly increases in the skill level that the firm chooses ($\frac{\partial C(\mathbf{y})}{\partial y_k} > 0, \frac{\partial^2 C(\mathbf{y})}{\partial y_k^2} > 0, \forall k$), and represents the necessary expenses to operate an occupation for a given skill requirement choice. This cost can be understood as an operation cost as in Hopenhayn (1992) that increases in the skill level of the occupation.[42]

Labor Market: There is a continuum of submarkets that are indexed by worker and occupation skill profiles $(\mathbf{x}, \mathbf{y})$, as well as the share of worker-firm surplus ω that firms

[40] This feature is consistent with the empirical finding that both incumbent jobs and vacancies have changing degrees of skill mixing.

[41] For example, in designing an occupation (i.e., salespersons) for lower-skill workers who might have a greater supply of interpersonal than analytical skill, firms may want interpersonal skill to be more intensively used to take advantage of the labor supply; on the contrary, if online marketing has increased the complementarity between analytical and interpersonal skills, firms may adjust accordingly to let the analytical skill to be more intensive. In equilibrium, this endogenous intensity will depend on other forces in the model.

[42] For example, to design and operate an occupation that employs high-skill workers, a firm will need to incur higher expenses in terms of better offices and equipment rentals.

promise to workers.[43] Workers with skill profile $\mathbf{x}$ direct their search towards different occupations and surplus shares, meeting one vacancy at a time. Matching between workers and firms is frictional and is regulated by a standard constant to scale matching function. Under this directed search environment, each submarket has a separate tightness (vacancy-unemployment ratio), denoted by $\theta(\mathbf{x}, \mathbf{y}, \omega)$. In each submarket, workers find job with probability $p(\theta(\mathbf{x}, \mathbf{y}, \omega))$ and firms fill the vacancy with probability $q(\theta(\mathbf{x}, \mathbf{y}, \omega)) = p(\theta(\mathbf{x}, \mathbf{y}, \omega))/\theta(\mathbf{x}, \mathbf{y}, \omega)$.[44]

The timing of the model evolves as follows. At the beginning of each period, a fraction δ of worker-firm pairs separate exogenously. Before the labor market opens and unlike standard search models, firms of both unposted vacancies and incumbent jobs will first need to design the occupations at this stage. The labor market that is comprised of different submarkets then opens, and both unemployed and employed search for unfilled vacancies and form matches with firms under the constant return to scale matching technology. The labor market then closes, firms produce the output and pay the occupation operation cost as well as the wage, which is a share of the surplus. Unemployed workers receive a transfer with a value of b. Lastly, workers are able to learn by doing, and their skills evolve according to the Markov process depending on their employment status, as described below.

Aggregate and Individual State: The aggregate state of the economy is the distribution of workers across employment status, skill profiles, occupational skill requirements, and surplus shares, denoted as $\psi \in \Psi$. I subsume aggregate state in the exposition of model equilibrium in the next section and show that in fact, the model equilibrium is independent of the aggregate state.

Nonetheless, in the model, I allow workers to learn on the job, and their subsequent skill profiles are contingent upon their current employment status, as in Lise and Postel-Vinay (2020). Specifically, considering each skill j in the worker's skill profile $\mathbf{x}$ as an element of the finite set S, the evolution of this skill follows a Markov process $\pi(x_j'|x_j, y_j)$, conditional

[43]This arrangement can be considered as an employment contract simply specifies the surplus share ω promised to the worker contingent on the state for the current period, as well as the continuation value of the match in the subsequent period (see next section). The contract is assumed to be fully committed by both the workers and firms.

[44]Functions p and q also satisfy usual regularity conditions: twice continuously differentiable; $p'(\theta) > 0, p''(\theta) < 0, p(0) = 0$; $q'(\theta) < 0, q''(\theta) > 0, q(0) = 1$.

on the worker's current skill level and employed occupation. If a worker is matched with an occupation that requires a skill level exceeding his or her own ($x_j < y_j$), the worker's skill j will adjust upward in the next period: $x'_j > x_j$, and the inverse applies for a worker whose skill is lower than the requirements of their current occupation. The probability, or the speed of skill adjustment, is contingent upon the specific skill j. For unemployed workers, they are treated such that their present occupation demands a zero level for all skills. The calibration of the skill adjustment probability is discussed in Section VI.

V.B Model Equilibrium

I will now characterize the optimal strategies for workers' job search and firms' job creation and continuation. The value functions for workers are described at the point of the production stage when the labor market comes to a close, while for firms I also consider the job design stage before the labor market opens.

Worker's Problem: Let $U(\mathbf{x})$ denote the value of being unemployed and searching for a job for worker $\mathbf{x}$. Similarly, let $W(\mathbf{x}, \mathbf{y}, \omega)$ be the total discounted returns from holding a job of skill requirements $\mathbf{y}$ and surplus share ω at time t. These values can be written as:

$$\begin{aligned} U(\mathbf{x}) &= b + \beta E\Big\{\max_{\mathbf{y}',\omega'} \; p(\theta(\mathbf{x}', \mathbf{y}', \omega'))W(\mathbf{x}', \mathbf{y}', \omega') \\ &\quad + \big[(1 - p(\theta(\mathbf{x}', \mathbf{y}', \omega'))\big]U(\mathbf{x}')\Big\} \\ W(\mathbf{x}, \mathbf{y}, \omega) &= \omega(f(\mathbf{x}, \mathbf{y}) - C(\mathbf{y})) + \delta U(\mathbf{x}') \\ &\quad + \beta(1-\delta)E\Big\{\max_{\tilde{\mathbf{y}}',\tilde{\omega}'} \; p(\theta(\mathbf{x}', \tilde{\mathbf{y}}', \tilde{\omega}'))W(\mathbf{x}', \tilde{\mathbf{y}}', \tilde{\omega}') \\ &\quad + \big[(1 - p(\theta(\mathbf{x}', \tilde{\mathbf{y}}', \tilde{\omega}'))\big]W(\mathbf{x}', \mathbf{y}', \omega)\Big\} \end{aligned} \tag{3}$$

Unemployed workers gain a utility b through the current period's transfer. In the subsequent period, their skills may transition to $\mathbf{x}'$, which are likely to depreciate due to their unemployed status. Meanwhile, within the submarket that aligns with their skill profiles, workers engage in the search for vacancies that span a variety of occupations $\mathbf{y}$ and surplus shares ω, looking for the highest continuation value. In choosing $\mathbf{y}$ and ω, workers face the tradeoff between the value of employed and the success probability of a match $p(\theta(\mathbf{x}', y', \omega')$, both of which hinge on the occupation and surplus share that the workers target. Should the match prove successful, the workers enjoy the continued value that employment offers; otherwise, their status of unemployment persists.

Workers currently employed in a firm characterized by $(\mathbf{y}, \omega)$ receive a wage equivalent to the share ω of the output from their match. When the subsequent period arrives, they face a probability δ of an exogenous separation, in which case they become unemployed with a value $U(\mathbf{x}')$ and engage in job search immediately. Employed workers perform on-the-job searches in their current match for new occupations and surplus shares $(\tilde{\mathbf{y}}', \tilde{\omega}')$, on the premise that there is a positive probability $p(\theta(\mathbf{x}', \tilde{\mathbf{y}}', \tilde{\omega}'))$ that the continuation value from the new match offers exceeds that of the original firm. In the absence of such possibilities or if the transition is not successful, the worker remains with the initial firm.

Firm's Problem: Consider a firm running occupation $\mathbf{y}$, offering surplus share ω, and employing worker $\mathbf{x}$. Let $J(\mathbf{x}, \mathbf{y}, \omega)$ denote the total discounted profits to this firm:

$$\begin{aligned} J(\mathbf{x}, \mathbf{y}, \omega) = & \max_{\mathbf{y}} (1 - \omega)(f(\mathbf{x}, \mathbf{y}) - C(\mathbf{y})) \\ & + \beta(1 - \delta)E\Big\{(1 - p(\theta(\mathbf{x}', \tilde{\mathbf{y}}', \tilde{\omega}'))J(\mathbf{x}', \mathbf{y}', \omega)\Big\} \end{aligned} \tag{4}$$

In the current period, firms receive a portion $(1 - \omega)$ of the worker-firm surplus, after paying the workers their wages. In the production process, firms also need to cover the occupation operation cost $C(\mathbf{y})$, which depends on the skill levels required by the occupation that the firms designed. The labor market operates under free entry for firms, hence, maintaining a vacancy bears no value. In the case of exogenous separation, or with a probability $p(\theta(\mathbf{x}', \tilde{\mathbf{y}}', \tilde{\omega}'))$ that the worker finds another job at an optimal occupation $\tilde{\mathbf{y}}'$ and surplus share $\tilde{\omega}'$ through on-the-job search, the firm accrues no profits. In the case where the match persists, the firm continues to acquire discounted profits from the match.

$$c = \beta E\Big\{q(\theta(\mathbf{x}, \mathbf{y}, \omega))J(\mathbf{x}, \mathbf{y}, \omega)\Big\} \tag{5}$$

The free-entry condition further highlights firms' choice of optimal degree of skill mixing and the tradeoff that agents face in the model. Prior to the opening of the labor market in each period, firms of incumbent jobs and unfilled vacancies re-design the occupation, taking into consideration the overall production technology and worker skills within their respective submarkets.[45] Given that the value of a vacancy is zero, firms will opt for an optimal skill mixing that equates the firm's anticipated discounted profits to the cost of vacancy posting as in equation (5). This condition implicitly pins down market tightness $\theta(\mathbf{x}, \mathbf{y}, \omega)$. If an occupation for a specific worker type becomes more profitable, the number of vacancies posted will increase, leading to a rise in market tightness but at the same time a reduction in the job-filling rate.[46]

The free entry condition also reflects the tradeoff faced by workers. Since workers receive the remaining surplus claimed by the firms, in markets with higher job-finding probabilities (i.e., tighter markets), the value of employment is likely to be lower. Workers' job-finding probability also feeds back to firms' discounted profits through worker on-the-job search and the chance that the firm attracts other employed workers.

Block-recursive Equilibrium: Despite the multi-dimensional skill setup, the model still achieves analytical tractability by relieving the dependence on the entire distribution of agents across aggregate states in characterizing agents' value functions and market tightness. Such a convenient feature was coined as "block-recursive" in Menzio and Shi (2010) and Menzio and Shi (2011) for a broad range of directed search models.[47] This is a result of two features of the model. First, as search is directed and workers choose optimally the occupation and surplus share, their life utility does not depend on their outside options, and workers do not need to forecast the wage depending on the entire distribution of employment. Second, there are separate markets for workers of different profiles, and workers search for jobs within their own submarket, in which firms carry different occupations. This additional degree of separability implies that the market tightness of a submarket is independent of the worker distribution in other markets, relieving the burden of workers and firms to forecast other markets in making their decisions.[48] In online Appendix B.2, I

[45] Considering that incumbent firms and new entrants utilize identical production technologies and confront the same worker skills within each submarket, their choices align.

[46] As in other directed search models, only a portion of submarkets may open in equilibrium, depending on firm's value and corresponding market tightness in different markets

[47] Block recursivity allows not only analytical tractability but also enables standard numerical techniques to solve the model. The framework considered in this paper involves more heterogeneity and requires an additional degree of directness, as discussed.

[48] Such additional directness implies that, i.e., computer scientists only confront other computer scientists in job search, while sales clerks only compete with other sales clerks. In reality, the degree of separability will

formally define a Block-recursive equilibrium for the economy and show its existence and uniqueness.

Skill Mixing, Wages and Employment: The model yields several predictions regarding changes in skill mixing, wages, and employment that align closely with the empirical findings detailed in Sections III and IV. These predictions emphasize the role of skill complementarity within a production framework that features indivisible skills. The formal propositions and proofs of these outcomes can be found in the online Appendix B.1, and a concise discussion is provided here.

Under the production technology described in equation (2) and the occupational design cost $C(\mathbf{y})$ that is strictly increasing and convex in skill requirements, increased complementarity in production or a higher degree of increasing marginal costs leads firms to find it more profitable to employ a mixture of different skills rather than specializing in one, leading to increased skill mixing. Additionally, the supply of skills by workers influences these outcomes: as workers supply a more diverse set of skills, it becomes more efficient to design jobs that require this mix of skills. In terms of wages and employment, if skills become more complementary in production or less costly to combine, the output of the worker-firm match rises, leading to wage increases. Through the free entry condition specified in equation (5), this increased joint worker-firm value results in a tighter labor market and an elevated job-finding probability for unemployed workers. I quantitatively calibrate the model and test these predictions in the next section.

VI Model Quantification

I will now calibrate the model to evaluate the quantitative significance of various channels contributing to skill mixing and examine their implications for wages and employment. First, I outline the data construction and measurement, followed by a discussion on calibration strategy and estimated parameters. Next, I analyze worker sorting and job ladder under the baseline calibration. In Section VII, I will perform counterfactual analyses to decompose the shifts in skill mixing as well as employment and wage distributions.

depend on specific occupations and the overall economic condition. As reported by Osberg (1993), search directedness is procyclical and is higher when the market is tight. In bringing the model to the data, I use economic recovery periods and more coarse occupations to be consistent with the model.

VI.A Measurement and Calibration

I apply the same combination of NLSY 79 & 97 along with O*NET data as in Section IV to calibrate the model. The datasets provide counterparts to the model variables: the worker abilities correspond to worker skills ($\mathbf{x}$) as discussed in Section IV, while O*NET provides occupational skill requirements ($\mathbf{y}$). NLSY also provides information on employment distribution and wage levels. The model is calibrated to two periods of data from the early 2000s to late 2010s separately, which coincides with a substantial shift in skill mixing and abstracts from the great recession. Specifically, the steady state of the model is fitted to the data from 2005–2006 and 2016–2019 to ensure comparability of sample sizes across these two periods, and I restrict to those workers with information on their skills.[49] Finally, for both worker and job skill profiles, I consider the same set of skills (analytical, computer, interpersonal, routine) as in Section III and IV, only that I combine analytical and computer skills to have a three-dimensionality feasible for quantitative analysis ($K = 3, k = \{$analytical/computer (a), interpersonal (p), routine (r)$\}$).[50]

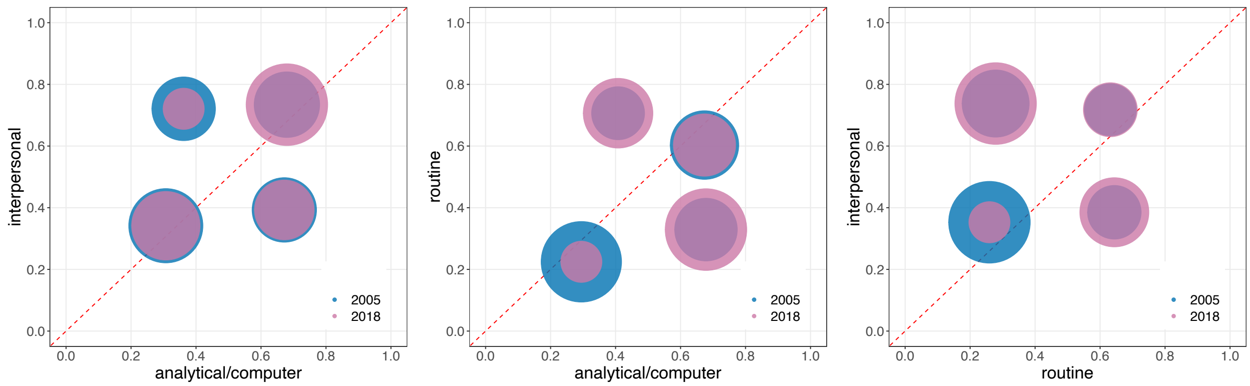
Considering the potential influence of skill supply variation on skill mixing, I calibrate two key aspects of it. First, the distribution of worker skills $G(\mathbf{x})$ varies *across* the two data periods to align with the workers' choice of occupations and college majors (if attended) as in the NLSY data, following Lise and Postel-Vinay (2020). Specifically, a worker accumulates γ_j times the gap between the worker's endowment and an occupation's or college major's requirement of skill j in each year, with γ_j depending on upward or downward accumulation. Table 6 panels B shows the calibrated γ_j using the estimates from Lise and Postel-Vinay (2020).[51] Second, the Markov process $\pi(x_j'|x_j, y_j)$ for worker skill adjustment *within* a model steady state is calibrated to make a worker's skill level adjust upward or downward in the next model period if the worker's occupation requires a higher or lower skill level than the worker has. The Markov adjustment probability is equal to the annual skill adjustment rates $gamma_j$ scaled by the gap between the worker's

[49] NLSY 1997 was conducted annually during 2005-2006, but only biannually in 2016-2019, so does NLSY 1979 for the later period. The same sizes for the two selected periods are 30,654 and 43,340 respectively.

[50] As I merge analytical and computer skills into one for calibration using their average values, I denote this combined skill as "analytical/computer".

[51] Workers' skills can be lost when not employed but cannot be lower than their initial endowments. For skill changes while in school, I specify that workers spend on average 3 years learning the skills of their majors.

Figure 7: Worker Skill Distribution Shifts



Notes: This figure illustrates the evolution of the skill distribution for different types of workers over the years 2005 (shown in blue) and 2018 (shown in cranberry), across three distinct two-dimensional skill spaces. These worker skills are measured using data from NLSY79&97, with the specific skill measure discussed in the online Appendix Table B1. Skill variations of these worker types are calibrated based on the skill accumulation and depreciation rates associated with different occupations and college majors, using the estimates of by Lise and Postel-Vinay (2020).

skill set and the occupation's demands.[52] Online Appendix B.4 provides further details of the skill supply calibration.

Figure 7 presents the calibrated variation in skill distributions from 2005 (in blue) to 2018 (in cranberry) for four worker types within each two-dimensional skill space. The circle sizes represent the probability of corresponding skill combinations. In the analytical/computer and interpersonal skill space, which are non-routine skills, there is a noticeable shift towards greater skill mixing. This is evidenced by an increase in workers possessing both high analytical/computer and interpersonal skill, indicated by greater areas representing these skill combinations. This shift towards more mixed skill sets is due to the rising mixing of skill demand and the worker learning by doing. In contrast, in skill spaces involving routine skill, there is a clear trend towards specialization, indicated by an increased area in off-diagonal skill combinations. This shift is similarly driven by a

[52]Specifically, the Markov probability of upward adjustment is determined by: $\frac{x_j^{up}-x_j}{y_j-x_j}\mathbf{1}(x_j^{up} < y_j) \times \gamma_j^{up}$, and of downward adjustment is given by: $\frac{x_j^{down}-x_j}{y_j-x_j}\mathbf{1}(y_j < x_j^{down}) \times \gamma_j^{down}$. Here, x_j represents the current grid value of worker skill j, while x_j^{up} or x_j^{down} denotes the value of worker skill j up or down a grid, respectively. The indicator variables $\mathbf{1}(y_j < x_j^{down})$ or $\mathbf{1}(x_j^{up} < y_j)$ evaluates whether the skill j grid value of the worker's current employed occupation is greater or smaller than the value of the worker's skill j grid. This means that a worker will only adjust up or down a grid if the occupation's skill is larger or smaller than the corresponding up or down grid value for the worker's skill.

growing demand for specialized routine skill relative to others. The implications of these changes in skill supply on labor market dynamics are explored in the next section.

To map occupations and workers in the model to the data, I set grid points as follows. I classify occupations into high- and low-wage, as in Section IV, with the former group including managerial, professional, and white-collar occupations, and the latter blue-collar and service occupations. The grid point for an occupation's requirement of a skill y_j is set such that moving up one grid corresponds to 50 percent of the average observed value of y_j for that occupation.[53] On the worker side, workers are classified based on their skill level x_j: those with skills above the average are deemed high type and assigned the mean of the above-average values; those below the average are considered low type and assigned the mean of the below-average values.[54]

Functional Forms: The model is parameterized as follows. The multi-dimensional skill production function is defined as in equation (2), which accommodates cross-skill complementarity controlled by σ and enables a sensible interaction between skill demand and supply, in line with the multi-dimensional matching literature (i.e.,Lise and Postel-Vinay 2020; Lindenlaub 2017; Ocampo 2022). I allow a flexible form of occupation operation cost $C(\mathbf{y})$ as in equation (6), where ρ regulates the convexity of the occupation operation cost function with respect to skill requirements, and τ governs the scale of the cost.[55]

$$C(\mathbf{y}) = \tau[\sum_{k=1}^{K}(y^k)^\rho] \tag{6}$$

The matching function assumes a standard Cobb-Douglas form, $M(s,v) = \mu s^\eta v^{1-\eta}$, indicating that η is the elasticity of matches concerning total search effort, and μ is the matching efficiency. This function form leads to the job finding rate being $p(\theta) = \mu\theta^{1-\eta}$ and the vacancy filling rate being $q(\theta) = \mu\theta^{-\eta}$.

[53]As the model calibration uses data of two periods with a consistent grid, I determine grid points by averaging the occupation's median values across both periods.

[54]With three chosen skills, there are 8 worker types in the model.

[55]Besides technical convenience, the functional form (6) also implies that for a given cost, firms need to trade off the choice of altering different skill intensities.

Table 5: Moments and Model Match

	First Period		Second Period	
	Data	Model	Data	Model
A. Worker moments				
Relative wage of high type				
Analytical/computer	1.46	1.62	1.60	1.78
Interpersonal	1.05	1.09	1.20	1.25
Routine	1.12	1.23	0.92	1.21
Wage return of skill mixing (untargeted)	0.07	0.04	0.07	0.04
Unemployment Rate	0.05	0.03	0.04	0.04
B. Occupation moments				
Relative wage of high skill	1.30	1.07	1.56	1.38
Corr. wage & abilities (low-wage)	0.23	0.23	0.49	0.49
Corr. wage & abilities (high-wage)	0.35	0.32	0.60	0.71
Employ. share (low-wage)	0.43	0.31	0.37	0.09
Employ. share (high-wage)	0.57	0.69	0.63	0.91
$100 \times$ Skill mixing (low-wage)	97.54	95.11	98.96	98.82
$100\times$ Skill mixing (high-wage)	95.74	96.03	94.12	94.60

Notes: This table reports the average values of the targeted moments both in the data and through model simulation. The data used for the moment calculation and for SMM estimation are two periods of pooled NLSY79&97 for employed workers: period 1 from 2005–2006 and period 2 from 2016–2019. Two types of moments are included. The worker moments include the relative wage of high type workers as well as the unemployment rate. The occupation moments include the relative wage of high skill occupations, the employment share and the skill mixing index of RNR skills in low and high skill occupations.

VI.B Calibration Strategy

The calibration of parameters falls into three categories. For parameters that regulate the search environment, I follow closely the conventions of the search and matching literature. I rely on estimates from the multi-dimensional matching literature for the skill adjustment and skill efficiency parameters. Lastly, for the parameter regulating elasticity of substitution across skills and relating to costs, I estimate them internally through Simulated Methods of Moments (SMM).

External Calibration: The model period is a year. Given that all agents are risk-neutral, the discount rate β is assigned a value of 0.96, corresponding to an annual interest rate of 4 percent. The job separation rate δ is set at 10 percent as in Shimer (2005). For employed

Table 6: Parameter Estimates

Parameter	Description	Value	
	A. Externally calibrated - search		
β	Discount Rate	0.96	
δ	Job separation rate	0.10	
ω	Worker share of surplus	0.60	
b	Unemployment benefit as a share of output	0.42	
η	Elasticity of the matching function	0.50	
μ	Matching efficiency	0.65	
	B. Externally calibrated - skill adjustment	*Up*	*Down*
γ_a	Annual adjustment speed of analytical/computer skill	0.36	0.10
γ_p	Annual adjustment speed of interpersonal skill	0.05	0.00
γ_r	Annual adjustment speed of routine skill	1.00	0.36
	C. Externally calibrated - skill efficiency	*Period 1*	*Period 2*
α_a	Skill efficiency of analytical/computer skill	0.63	0.95
α_p	Skill efficiency of interpersonal skill	0.05	0.08
α_r	Skill efficiency of routine skill	0.14	0.06
	D. Internally estimated	*Period 1*	*Period 2*
σ^{low}	Elasticity parameter of skills in production (low-wage)	0.64	0.41
σ^{high}	Elasticity parameter of skills in production (high-wage)	0.60	0.36
τ	Scaler of occupation operation cost	0.74	0.53
ϕ	Convexity of occupation operation cost	3.63	4.90
c	Vacancy posting cost as a share of output	0.56	0.82

Notes: This table shows the exogenously calibrated as well as internally estimated parameters. The data used for the internal estimation are two periods of pooled NLSY79&97 data for workers with information on their pre-market abilities. Period 1 is from 2005–2006 and period 2 from 2016–2019.

workers, their share of output ω is set at 0.6, mirroring the labor share of GDP in 2005. For unemployed workers, the unemployment benefits b is set at 41.5 percent of the earning loss of lowest-paid occupations, following the estimates of Braxton, Herkenhoff, and Phillips (2020). The elasticity of the matching function η is set at 0.5 as is standard, and the matching efficiency μ is set to 0.65, as in Mercan and Schoefer (2020). Table 6 panel A summarizes these externally calibrated parameters.

I calibrate the speed of skill adjustment (γ_j) and the skill efficiencies (α_k) following Lise and Postel-Vinay (2020) and Lindenlaub (2017), as detailed in Table 6 panels B and C.

The calibration aligns the adjustment of analytical/computer, interpersonal, and routine skills with the cognitive, interpersonal, and manual skills detailed in Lise and Postel-Vinay (2020).[56] Analytical/computer skill adjusts upward two times faster than it depreciates, while interpersonal skill changes slowly in both directions. Routine skill adjusts most rapidly in either direction. I linearly interpolate Lindenlaub (2017)'s estimates of skill efficiencies for my period of analysis.[57] Between 2005 and 2018, the productivity of analytical/computer and interpersonal skill in matching worker abilities with job skill requirements increased by about 60 percent. In contrast, the productivity of routine skill saw a decrease of more than 50 percent.

Internal Estimation: For the internal estimation, the SMM procedure initiates by determining the agents' steady-state policies based on the model's parameters, simulating a cohort of workers. Each simulation results in a distribution of employment statuses and corresponding labor market outcomes. The parameters are then estimated minimizing the distance between simulated and empirical moments.[58] The estimation targets 11 moments as shown in Table 5 for both periods of data that include: i) the relative wage of the high-type worker for each skill; ii) the unemployment rate; iii) the relative wage of high-skill occupation; iv) the within-occupation correlation between wages and worker abilities; v) the share of employment across occupations; and vi) the skill mixing index of RNR skills of occupations.[59] The model does a decent job of matching all the moments.

The model parameters are jointly identified from the moments, for which a concise summary of the key information for identification is given below with a more detailed discussion in online Appendix B.3. I first identify the complementarity parameter of skills in production σ targeting the correlation of within-occupation relative wages and worker skills. The cost parameter ρ is then estimated by leveraging the firm's optimization conditions in skill mixing. Conditional on parameters estimated at the production side,

[56] Lise and Postel-Vinay (2020)'s estimates are presented on a monthly basis, which I have adjusted to an annual scale.

[57] Lindenlaub (2017)'s estimates span from 1990 to 2010.

[58] Online Appendix B.5 provides further details on the numerical implementation.

[59] All moments are directly computed from the two periods of data from NLSY, except for unemployment, for which I use the statistics from the Bureau of Economic Analysis (BEA) to avoid the age composition effects present in NLSY. For example, by the late 2010s, a larger segment of the NLSY 79 cohort was above age 50, making them more likely to be out of the labor force. Additionally, the unemployment rate from NLSY, derived from the number of jobs held since the last survey, averages 9 percent, notably higher than BEA data. However, this decision primarily affects vacancy posting cost parameters.

the employment distribution and relative wages further aid in estimating τ. Lastly, the unemployment rate disciplines the vacancy posting cost.

Table 6 panel D presents the internally estimated parameters, which indicate considerable technological shifts between the two periods. For the initial period, the estimated σ is 0.64 and 0.6 for low- and high-wage occupations respectively, suggesting that skills are substitutable in production. In the late 2010s, there was a significant rise in skill complementarity in production, reflected in the reduction of σ to 0.4 for both types of occupations. Firms also encounter rising costs of skills in occupation operation, as reflected in the increase of both the scale and the convexity of the cost function (τ and ρ). As discussed in Section VI, this increased complementarity as well as the cost of skills intensifies firms' incentives to mix skills. Lastly, the cost of posting vacancies has risen slightly post-2010s.

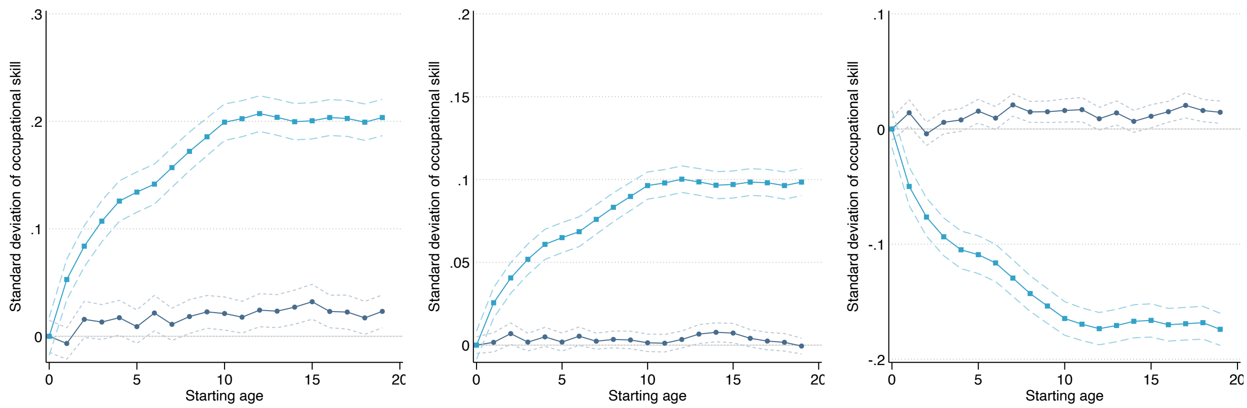
VI.C Job skills Over the Life Cycle

I now turn to discuss the paths of workers' job skill requirements or job ladder as simulated by the model, and compare them with empirical data. The model initiates with a job match determined by $G(\mathbf{x})$ and incorporates two main mechanisms that influence workers' movement across jobs overtime: active job searching, both while employed and unemployed, and the evolution of workers' skills, which is governed by a Markov process. These mechanisms provide workers with different occupational choices over time, guided by the parameters calibrated in Table 6. For empirical comparison, I apply the 2005 O*NET values to align with observations from the NLSY 79 and the year 2018 to align with observations from NLSY 97. I also impose age restrictions on the datasets to compare similar life stages, limiting the age range to 41 to 60 for NLSY 79 and 21 to 40 for NLSY 97, similarly constraining the model-simulated paths.

Figure 8 illustrates the average occupational job skills over the life cycle of workers, as predicted by the model and observed empirically, depicting the years 2005 (represented with circles) and 2018 (shown with squares) across three different skills. In 2005, the job ladder for all three skills appears flat for workers aged 41 and above, as shown by both the model simulations and empirical data. However, by 2018, there is a notable increase in the job skill requirements for analytical/computer and interpersonal skills as workers age in the labor market, although the model tends to overestimate the depth of these job ladders.

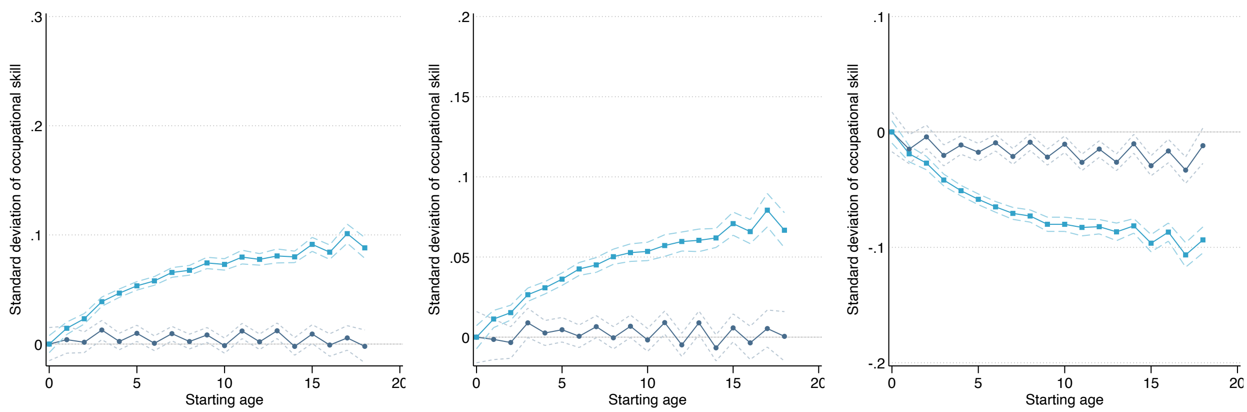
Figure 8: Predicted and Observed Occupational Skills Across Age Groups in 2005 vs. 2018

Panel A. Model Simulation



(1) Analytical/Computer (2) Interpersonal (3) Routine

Panel B. NLSY



(1) Analytical/Computer (2) Interpersonal (3) Routine

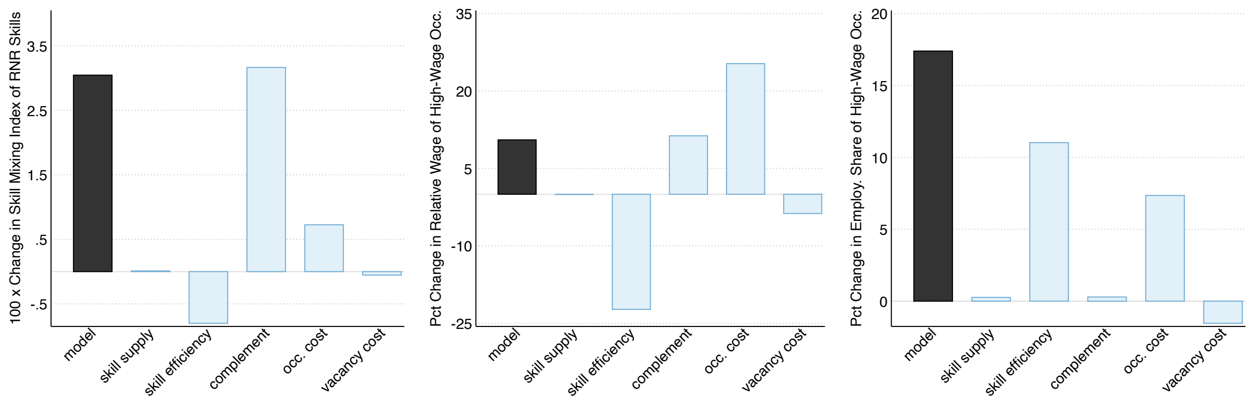
Notes: These figures display the average occupational job skills over the life cycle of workers, for both model predictions and empirical data. Model simulated or emprical data points for the year 2005 are represented by circles, and for the year 2018 are depicted as squares. The emprical observations are drawn from the NLSY 79 cohort for year 2005 and NLSY 97 cohort for year 2018

In contrast, both the model and empirical data indicate a decline in routine skills over time for workers in 2018. Thus, the model provides a reconciliation of the changing job ladder across these two periods throught the lens of underlying technology.

VII Counterfactual Analysis

What drives the observed increase in skill mixing, and what are their implications for labor market outcomes? In this section, I employ the model to perform a series of counterfactual

Figure 9: Counterfactual Decomposition



(1) Skill mixing (2) Wages (3) Employment

Notes: These figures plot the model generated changes in skill mixing in low-skill occupations (panel 1), changes in relative wage of high-wage occupation (panel 2), changes in employment share of high-wage occupation (panel 3), and different individual skills' contributions to relative wage and employment share changes (panel 4). Different model channels are shut down individually by eliminating the changes in calibrated values to highlight the contribution of each channel. The full model has all the model features. The values of skill complementarity in production, cost of skills in occupation operation, efficiency differential, and vacancy posting cost across the two periods are shown in Table 6. Worker skill supply distribution variation across the periods are calibrated according to Table B1.

experiments to assess the relative significance of each model channel in explaining the shifts in the degree of skill mixing. I then evaluate the influence of these channels on the changes in earnings and employment distributions.

For the counterfactual analysis, I take the 2018 economy and then sequentially remove shifts in calibrated parameters representing different channels, setting their values to that in the 2005 economy. Specifically, I examine the roles of changes in skill supply ($G(\mathbf{x})$), skill efficiencies (α_k), skill complementarity in production (σ), and occupation operation cost (τ, ϕ) in generating moment variations. Given the non-linear interplay of these forces, I remove these elements in different sequences and calculate the effect of each channel by averaging across those sequences.

Counterfactual Skill Mixing: I begin by assessing how different channels contribute to the growth in skill mixing within low-wage occupations, which has been noticeably observed in the data.[60] The first panel of Figure 9 illustrates that the full model predicts a rise in skill mixing within low-wage occupations over the two periods consistent with the

[60]Online appendix B.6 shows the results for high-wage occupations.

Table 7: Returns to Skill Specialization Decomposition

Decomposition	Analytical/ Computer	Interpersonal	Routine
Full model	15.45	15.16	-3.72
Skill supply	-2.60	-0.52	-3.13
Skill efficiency	26.59	1.60	-11.82
Complementarity	-23.86	11.01	12.33
Occ. cost	10.82	0.80	-7.42

Notes: This table shows the model-generated changes in relative wages of high-type workers for the three skills. The first row shows the changes with all model channels, corresponding to the first three rows of Table 5. The following rows then show the variation attributable to different model channels. See the footnote of Figure 9 for details.

observed data. Changes in the supply of worker skills and vacancy posting cost during these periods did not play a significant role in this rise. On the other hand, shifts in skill efficiency have had a negative contribution to the change in the degree of skill mixing. The latter result arises because, as the efficiency of routine skill declines and of interpersonal and analytical/computer skills improves drastically, firms are incentivized to redesign occupations to shift towards either analytical/computer or interpersonal skill away from routine skill, to a degree that it leads to a slight increase in specialization towards the skills that become more efficient.

The subsequent counterfactual results indicate that the rise in the complementarity of skills in production and occupational operation costs account for the increase in skill mixing. The increase in skill complementarity contributes to three-quarters of the increase, while changes in occupational operation cost account for another quarter. These results are consistent with the predictions in Section V and highlight the importance of skill complementarity and their cost in occupation operation in driving firms' endogenous skill demand specialization.[61]

Wage and Employment Effects: I proceed to investigate how the same model channels that influence skill mixing also impact wage and employment distributions. Column (2) of Figure 9 illustrates the changes in the relative wage between high-wage and low-wage

[61] Further analysis of the the implications of τ and ϕ individually for skill mixing changes shows that ϕ plays a bigger role.

occupations from 2005 to 2018, with the model predicting a 10 percentage point increase. As observed with skill mixing, changes in worker skill supply and vacancy posting costs have negligible impacts. In contrast, changes in skill efficiencies significantly reduce the relative wage gap, decreasing it by more than the net increase. However, the growing complementarity and changing costs of skills notably increase wage disparities, contributing to 40 and 60 percent, respectively, of the overall rise in wage premiums for high-wage occupations.

In Table 7, I analyze the drivers of wage returns for specializing in different skills. The full model indicates a 15 percent increase in wage returns for specializing in analytical/computer and interpersonal skill, while returns for specializing in routine skill decrease by 3 percent. Variations in skill supply have negatively impacted these returns. In terms of remaining forces, for analytical/computer skills, the significant rise in its efficiency is the primary factor, leading to a 30 percent increase in returns, followed by changes in skill costs; the increase in skill complementarity has reduced these returns. In contrast, for interpersonal and routine skill, the increase in complementarity is the principal driver, leading to an 11 and 12 percent rise in returns, respectively. For routine skill, the drastic decrease in efficiency and the increase in skill costs have led to a reduction in its returns.

I further decompose the influence of various model factors on the increase in employment in high-wage occupations over two periods, as depicted in column (3) of Figure 9. The full model shows a 17 percentage points rise in the employment share of high-wage occupations. The skill supply has only a marginal impact, similar to its effect on relative wages. The most significant factor is the change in skill efficiencies, accounting for 62 percent of the overall increase. Changes in skill cost are also important, contributing to 37 percent of the increase. However, the rise in complementarity plays only a marginal role in this growth. In the online Appendix B.6, I conduct a further decomposition of the contributions of individual skills to changes in wages and employment. The results demonstrate that the rise in analytical/computer skills is the most significant factor influencing employment, whereas the decline in routine skills drives the wage trends.

In summary, the counterfactual analysis demonstrates the significance of the growing complementarity of skills in production, the changing skill cost for occupation operation, and shifts in skill efficiency as primary drivers behind the observed changes in skill mixing and the increases in wage and employment gains in high-wage occupations.

Discussions:

Task-biased vs. Skill Complementarity & Cost: Biased technological change, especially task-biased technological change (TBTC), is shown to be a key driver of the recent trends in wage inequality in developed countries. Studies by Costinot and Vogel (2010) and Acemoglu and Zilibotti (2001) employ one-dimensional assignment models, while Lindenlaub (2017) uses a multi-dimensional assignment model to examine this phenomenon. This change is characterized by increasing complementarities and efficiency in cognitive tasks and a decline in routine tasks. Such a shift leads to the replacement of workers in medium-wage occupations and an increase in wages and employment in high-wage occupations.

In my model, I incorporate both changes in skill efficiency representing TBTC, as well as variations in skill complementarity and cost. My counterfactual analysis first confirms the significance of TBTC, which accounts for 62 percent of the employment gains in high-wage occupations, whereas skill complementarity and cost account for the rest. However, for the wage premium in high-wage occupations, skill complementarity and cost are more crucial, contributing three-quarters of the change, whereas TBTC accounts for only a quarter. For the wage return to skill specialization, TBTC emerges as the primary driver for analytical/computer skills; however, for interpersonal skill, skill complementarity plays a more significant role, and it also increases the return to specialization in routine skill, even though TBTC has a reduced it. The increase in skill mixing is entirely attributed to skill complementarity and cost. Overall, the results indicate that while TBTC is crucial for employment distribution, skill complementarity is more influential for wage distribution and also in shaping firms' endogenous skill specialization.

The Role of Education: In my model, although direct education investment is not explicitly included, the changing skill supply ($G(\mathbf{x})$) implies at the potential role of education in shaping labor market outcomes under skill mixing. As depicted in Figure 7, the calibrated skill supply variation based on occupational and major choices indicates an increase in the mixing of analytical/computer and interpersonal skill, while a rise in specialization in routine skill. While skill supply has not played a significant role in wage and employment distributions, it has reduced the returns to specialization in different skills by 0.5 to 3 percent. These results suggest that while education may have marginal effects on overall distributions, it significantly impacts the wage disparity between experts in specific skills and those who are not.

VIII Conclusion

Skills are inevitably embedded in workers and understanding the demand for skill *mixtures* is important in studying the dynamics of the labor market. I present a rich set of empirical findings on the phenomenon of "skill mixing", and show that between 2005-2018, there has been a sizable growth in the degree of mixing, particularly for non-routine skills such as analytical, computer, and interpersonal skills. To understand the heterogeneous within-occupation variation in skill mixing of different occupations, I provide an integrated framework incorporating multi-dimensional skill directed search and endogenous occupation design. Bringing the model to the data, I show that technological change as reflected in the increased skill complementarity and costs are the main drivers of skill mixing, as well as wage and employment distribution changes in this period.

The phenomenon of skill mixing brings forth very different policy implications for worker training and college education. Using NLSY 79 and 97 combined with O*NET data, I show that workers in occupations that become more mixed in analytical, computer, and interpersonal skills earn a positive wage premium. Further, workers who possess a more mixed set of these non-routine skills, or those who study a college more mixed in these skills earn 3 to 6 percent more. In sum, this paper's results suggest that in a world with an increasing trend of skill mixing with positive wage premiums related to technological advancements, educators and policymakers should consider providing more "mixed" skills to workers rather than focusing solely on nurturing expertise and specialization.

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Appendix for Online Publication

Table of Contents

A ADDITIONAL EMPIRICAL RESULTS

A.1 Data Construction

In this section, I give more details on data construction for the two primary datasets on job skill demand employed in Section III and IV, namely O*NET (Occupation Information Network) and Lightcast (previously known as "Burning Glass"). Specifically, I discuss strategies for leveraging the longitudinal information in these datasets with higher precision. I also present an overview of their inherent characteristics, advantages and disadvantages, and how they are cross-walked with other datasets used in the analysis.

O*NET: Administered by the U.S. Department of Labor, O*NET is a replacement for the Dictionary of Occupational Titles (DOT). It is more comprehensive and more frequently updated and has been used widely to analyze occupation skill requirements and work settings (i.e., Acemoglu and Autor 2011; Yamaguchi 2012; Autor and Price 2013).

Nonetheless, to use the longitudinal variation from O*NET, the key challenge concerns partial updating – each new version of O*NET only updates an average of 110 targeted occupations among the 970 7-digit occupations. Online Appendix Table A1 lists different versions of O*NET, the release year, and the year composition for 3 of the modules. Specifically, for each release of O*NET, I assign a "Considered Year" such that at least 55% to 60% of occupations are updated after that year.

Moreover, I use 4-year intervals. The last column of online Appendix Table A1 shows the percent of occupations that are updated from the last considered year of data included in the analysis. On average, more than 50 percent of the occupations are updated across the succeeding years included in the analysis.

O*NET contains around 270 descriptors about occupations that are grouped into 9 modules: abilities, knowledge, skills, work context, work activities, experience/education requirement, job interest, work values, and work styles. For my main analysis, I only use descriptors from 3 modules: work context, work activities, and knowledge that are more interpretable as the skill requirements and are consistently evaluated by incumbent workers for each new release. These descriptors come as importance, level, extent, and relevance. To interpret the skill measures as gauging the intensity, I use the importance information, similar to i.e., Acemoglu and Autor (2011) and Guvenen et al. (2020), but the level and importance pieces of information are highly correlated and do not affect the qualitative patterns of skill mixing shown in the paper.

In Section III, I show the longitudinal changes in skill mixing by combining O*NET and ACS datasets. O*NET uses SOC 2000 occupation classification for releases between 2000 and 2010 and SOC 2010 for years after 2010. To link O*NET and ACS, I first bridge SOC codes to the census' OCC 2000 and OCC 2010 codes respectively using crosswalks provided by the Analyst Resource Center and the Bureau of Labor Statistics. Then different years of OCC codes are homogenized using a balanced and consistent panel of occupation codes developed by Autor and Dorn (2013) and updated by Deming (2017). The same code is also used for combining all years of ACS and O*NET data.

Lightcast: Lightcast (formerly "Burning Glass Technologies") is an analytics software company that has developed a comprehensive and detailed dataset derived from online job postings, capturing real-time labor market information, and reflecting the current demand for skills and occupations. One of the key advantages of Lightcast data is its extensive coverage and high-frequency updates. By examining over 40000 online job boards and company websites, it provides a near universe of online posted vacancies; moreover, it provides a level of detail that is rarely matched by other sources of labor market data, such as job titles, employer information, and specific skill requirements. This allows for a very granular analysis of job skill requirements and labor market dynamics across different industries and regions.

The information that Lightcast collected is then parsed and deduplicated into a systematic list of thousands of codified skills. Similar to Hershbein and Kahn (2018) and Braxton and Taska (2023), the dataset that this study uses defines different skills if the codified skills from Lightcast contain relevant keywords. Specifically, the keywords used to capture analytical skill are: "research", "analy", "decision", "solving", "math", "statistic", and "thinking". The keywords used to capture interpersonal skills are "communication", "teamwork", "collaboration", "negotiation", and "presentation". For each occupation, the share of posted vacancies that require a particular skill is then the measure of skill for that occupation, capturing the extensive margin of firm skill demand.

However, like any data source, Lightcast data also has its limitations. For instance, it only covers online job postings, which may not represent the entire labor market, especially for low-skilled jobs or jobs in small firms that do not typically advertise online. It may also have a bias towards certain types of jobs or industries that use online job advertisements more frequently, and online vacancies by nature overrepresent growing firms (Davis, Faberman, and Haltiwanger 2013). One note of Lightcast data is that the measure of skill as introduced above focuses on the extensive margin – whether a job uses a skill or not – this is very different than the level and importance information that O*NET contains.

Table A1: O*NET Versions and Corresponding Years

	Released Year	Division	Work Context	Work Activities	Knowledge	Skills	Abilities	Considered Year
O*NET 13.0	2008	Post 2005	73.79%	73.79%	73.79%	73.79%	73.79%	2005
		Before 2005	26.21%	26.21%	26.21%	26.21%	26.21%	
O*NET 18.0	2013	Post 2009	57.15%	57.21%	57.21%	99.89%	57.21%	2009
		Before 2009	42.85%	42.79%	42.79%	0.11%	42.79%	
O*NET 22.0	2017	Post 2013	57.84%	57.67%	57.67%	57.67%	57.67%	2013
		Before 2013	42.16%	42.33%	42.33%	42.33%	42.33%	
O*NET 25.0	2022	Post 2018	54.52%	54.52%	54.52%	54.52%	54.52%	2018
		Before 2018	45.48%	45.48%	45.48%	45.48%	45.48%	

*Notes: The table summarizes different versions of the O*NET (Occupational Information Network) database, along with their released year, year division for the 5 modules (work context, work activities, knowledge, skills, abilities), and the considered year for each version. The "Post" and "Before" rows indicate whether the data in each version was collected post or before a particular year. The "Considered Year" column represents the year considered to be corresponding to each release of O*NET based on the year division of data.*

A.2 Details of Skill Measures

In this section, I discuss the choice of skill measures used in the main analysis. Specifically, I show the composition of descriptors of each skill used in the main analysis. I also discuss the composite skill measures' validity and correlation with other measures used in the literature.

Table A2 lists the O*NET descriptors for each of the constructed composite skill measures. The analytical measure corresponds to "non-routine cognitive analytic" and the interpersonal measure corresponds to "non-routine interpersonal" from Acemoglu and Autor (2011). I collapse Acemoglu and Autor (2011)'s "routine cognitive" (the first three items under Routine) and "routine manual" (the last three items under Routine) into a big routine skill, as occupations using these skills have been shown to have had similar labor market dynamics (Autor, Levy, and Murnane 2003; Acemoglu and Autor 2011). I didn't include the "non-routine manual" from Acemoglu and Autor (2011), since it includes descriptors from the "Abilities" module of O*NET that is evaluated solely by job analysts, and for consistency purposes I focus on occupation descriptors that are evaluated incumbents workers.

Further, I include two additional composite skills that are considered to be non-routine. First, I include a "leadership" composite skill that is comprised of descriptors of problem-solving, strategic thinking, teamwork, and communication. They all demand an ability to guide and manage teams, strategize and plan, solve problems, coordinate activities, and communicate effectively within a team or organizational context. Second, I include a "design" composite skill measure centering around technical proficiency and creativity. The composing descriptors entail a strong understanding of design principles, and the ability to draft and layout specifications for technical devices.

Table A2: O*NET Skill Measures and Composing Descriptors

Non-routine Analytical

- Analyzing data/information
- Thinking creatively
- Interpreting information for others

Non-routine Interpersonal

- Establishing and maintaining personal relationships
- Guiding, directing and motivating subordinates
- Coaching/developing others

Computer

- Interacting With Computers
- Programming
- Computers and Electronics

Design

- Design
- Drafting, Laying Out, and Specifying Technical Devices, Parts, and Equipment

Routine

- Importance of repeating the same tasks
- Importance of being exact or accurate
- Structured v. Unstructured work (reverse)
- Pace determined by speed of equipment
- Controlling machines and processes
- Spend time making repetitive motions

Leadership

- Making Decisions and Solving Problems
- Developing Objectives and Strategies
- Organizing, Planning, and Prioritizing Work
- Coordinating the Work and Activities of Others
- Developing and Building Teams
- Guiding, Directing, and Motivating Subordinates
- Provide Consultation and Advice to Others

*Notes: This table shows the detailed O*NET descriptors for skill measures. The Non-routine Analytical and Non-routine Interpersonal skills align with Acemoglu and Autor (2011)'s "non-routine cognitive analytic" and "non-routine interpersonal" skills. A unified Routine skill measure combines Acemoglu and Autor (2011)'s "routine cognitive" and "routine manual" skills, reflecting their similar market trends. The study omits "non-routine manual" to maintain consistency with incumbent worker-evaluated descriptors. Two additional skills, 'leadership' and 'design', are included to capture managerial and creative competencies.*

Table A3 shows the correlation among the chosen skills used in the main analysis, as well as math skill and social skill, which are constructed based on Deming (2017), and broader skill measures skills constructed using factor analysis as discussed in online Appendix A.6. It reveals the analytical skill (row 1), exhibits a strong positive correlation with computer skills (0.92) and a moderate correlation with math skills (0.50). This pattern suggests that positions requiring analytical skills frequently necessitate computer and mathematical proficiency. Interpersonal skills (row 3) indicate a moderate-to-strong positive correlation with social skills (0.61) and broader interpersonal skills (0.73). This correlation suggests that occupations demanding interpersonal skill also emphasize social abilities. These results validate the interpretation of the analytical and interpersonal skills with a strong positive correlation with math and social skills used in other studies.

On the other, a strong negative correlation exists between routine and interpersonal skills (-0.49) and between routine and interpersonal skills (-0.45), indicating that these skill sets rarely overlap in job requirements. The broader skill categories (rows 7 to 9) align well with their narrower counterparts, reinforcing the validity of these categorizations. In sum, there exist specific, identifiable skills in the labor market, some of which are more aligned with each other, but they tend not to overlap, reflecting distinct competencies.

Table A3: Correlations Among Skill Measures

Skill Measures	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1) analytical	1.00								
(2) routine	-0.45	1.00							
(3) interpersonal	0.44	-0.49	1.00						
(4) computer	0.92	-0.27	0.25	1.00					
(5) math skill	0.50	-0.11	0.12	0.46	1.00				
(6) social skill	0.34	-0.54	0.61	0.24	0.09	1.00			
(7) analytical (broader)	0.84	-0.59	0.55	0.68	0.63	0.57	1.00		
(8) mechanical (broader)	-0.43	0.58	-0.24	-0.38	-0.11	-0.38	-0.49	1.00	
(9) interpersonal (broader)	0.10	-0.35	0.73	0.02	-0.09	0.70	0.28	-0.22	1.00

*Notes: This table reports the correlation among different skill measures constructed using O*NET data from 2000-2020. The first four skills measures in rows (1) to (4) are the ones used in the main text and are constructed using the O*NET descriptors shown in Table A1. The next two measures in rows (5) to (6), math skill and social skill are constructed based on Deming (2017). Math skill is the average of 1) mathematical reasoning ability, 2) mathematics knowledge, and 3) mathematics skill. Social skill consists of the average of four variables, 1) social perceptiveness, 2) coordination, 3) persuasion, and 4) negotiation. Rows (7) to (9) contain the broader analytical, mechanical, and interpersonal skills constructed using factor analysis as discussed in online Appendix A.6 with their specific component variables.*

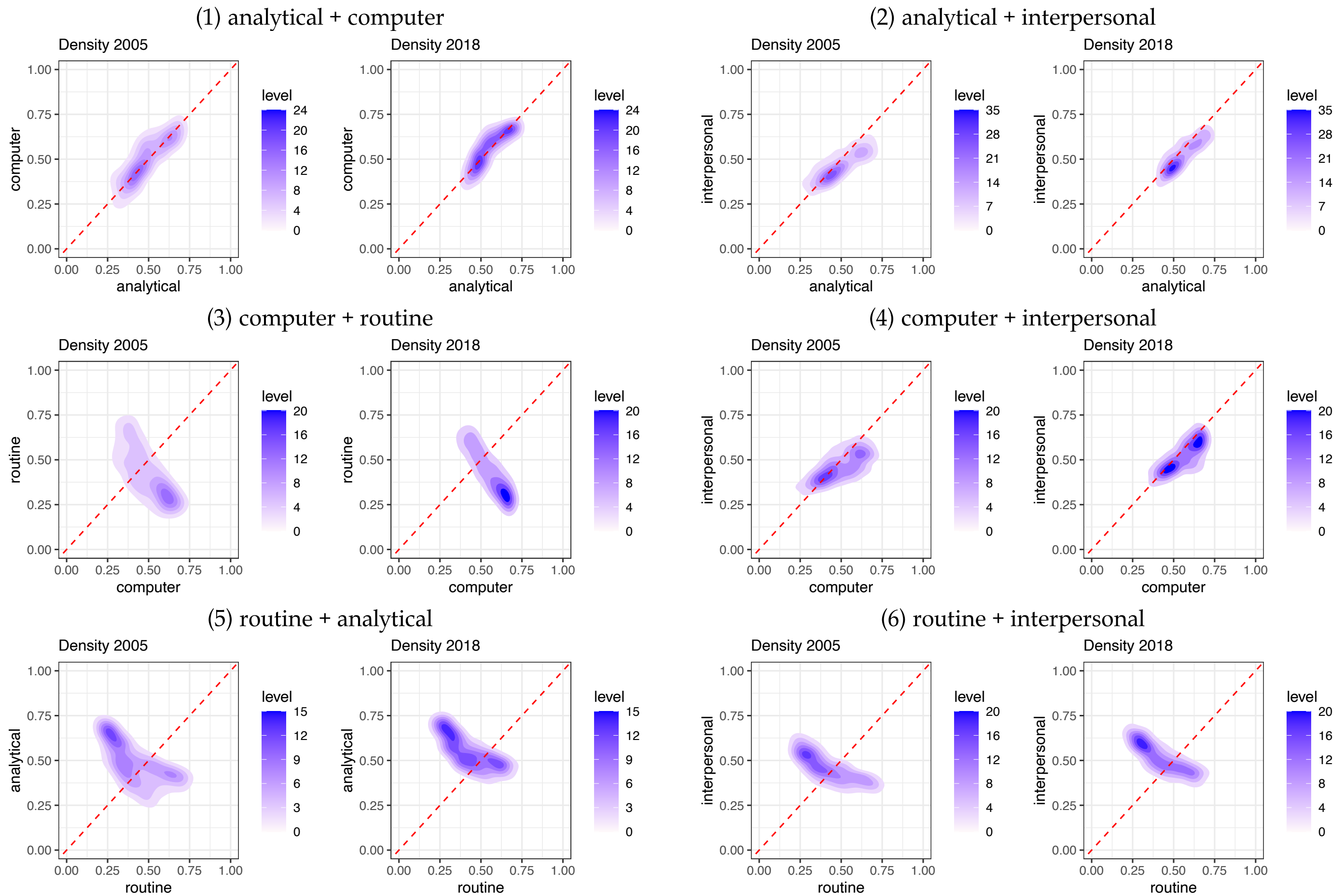
A.3 Alternative Non-parametric Examination of Trend

An intuitive alternative check of the changes in the degree of skill mixing across periods is to non-parametrically plot the density of skill intensities in different skill dimensions. Figure A1 depicts the density of skill requirements of six skill pairs out of the four constructed skills in 2005 and 2018 respectively using O*NET data combined with ACS. As in previous studies of job attributes, I aggregate the ACS to sex-education-industry cells that implicitly control for changes in task inputs due to variations in skill and industry mixes in the U.S. economy. Employment weights are obtained as the total hours of work aggregated to each cell. The ACS then supplies the O*NET data with employment across worker types to present an overarching picture of skill intensities in the economy.

From the figure, there is a clear shift towards mixed skill requirements in panel (1) pertaining to analytical and computer skills where these skills are positively correlated. Two salient changes happened in this period: first, the entire distribution of skill intensities moves near the 45-degree line; second, there is a significant increase in density around the 45-degree line. Both of these changes will lead to an increased degree of skill mixing, according to how it is defined based on the position of skill vectors relative to the 45-degree line. Such a change is also salient for other non-routine skill combinations: in the analytical and interpersonal skills space (panel 2), as well as in the computer and interpersonal skills space (panel 4).

On the other hand, one can scarcely observe changes in mixing in the routine skill spaces, as shown in panels (3),(5), and (6). From these three plots, there is an increase in density towards the non-routine direction, losing density in routine skill, and the resulting change in relationship with the diagonal does not indicate a strengthening of mixing.

Figure A1: Non-parametric Depiction of Skill Intensities, 2005 vs. 2018



*Notes: These density plots show the intensity of occupation skill requirements across the U.S. economy in 2005 (column 1) and 2018 (column 2) in six two-dimensional skill spaces, as illustrated in the six panels. Darker colors indicate higher density and the 45-degree line is also plotted. O*NET and ACS data are combined for the construction of these plots. The two datasets are merged using consistent occupation codes constructed by Autor and Price (2013) and further developed by Deming (2017). Skill measures are constructed using the O*NET descriptors shown in Table A1. All measures are normalized to [0,1].*

A.4 Robustness of Trend Results to Different Weights and Groupings

In this section, I discuss the robustness of the trend results in different weighting, granularities, and groupings. In particular, I show the density results using weighted skill mixing indexes instead of unweighted ones in the main analysis, as well as at a higher occupations level; the trend of skill mixing using indexes for different skill pairs, instead of high-dimensional indexes; the heterogeneity of skill mixing increases across occupations using indexes for different skill pairs; and the differential changes in skill mixing across industries.

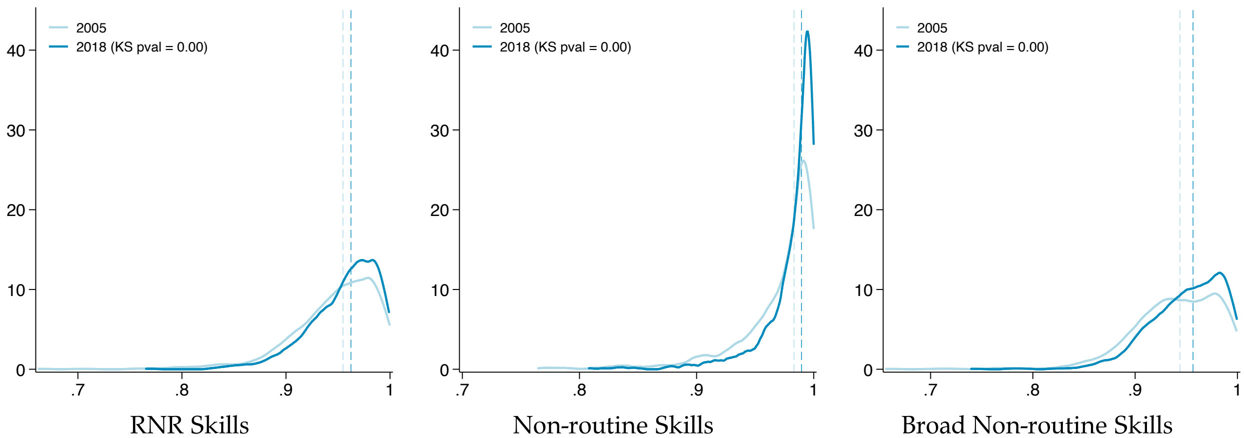
One concern of the analysis of skill mixing shown in Figure 2 is that as it shows the changes in the density of skill mixing indexes without weighting, it might not accurately represent the overall picture of mixing in the whole economy. In Figure A2 panel B, I weigh the skill mixing indexes using employment weight from the OEWS. The results show a similar message that there is a sizable increase in skill mixing particularly for non-routine skills. The only difference is that with employment weighting, the increase in the skill mixing of RNR skills is more discernable. This implies a relatively higher weight of occupations intensive in RNR skills that also increase skill mixing in these skills. In Figure A2 panel B, I show the density results at a higher 4-digit occupation level, and a similar trend holds.

Next, I discuss the changes in skill mixing using indexes of different skill pairs instead of high-dimensional indexes. Figure A6 panel (1) shows the results. The figure shows similar results as the main analysis: there is a stronger increase in skill mixing among non-routine skills. For the skill combinations involving routine skills, the change in skill mixing is negligible.

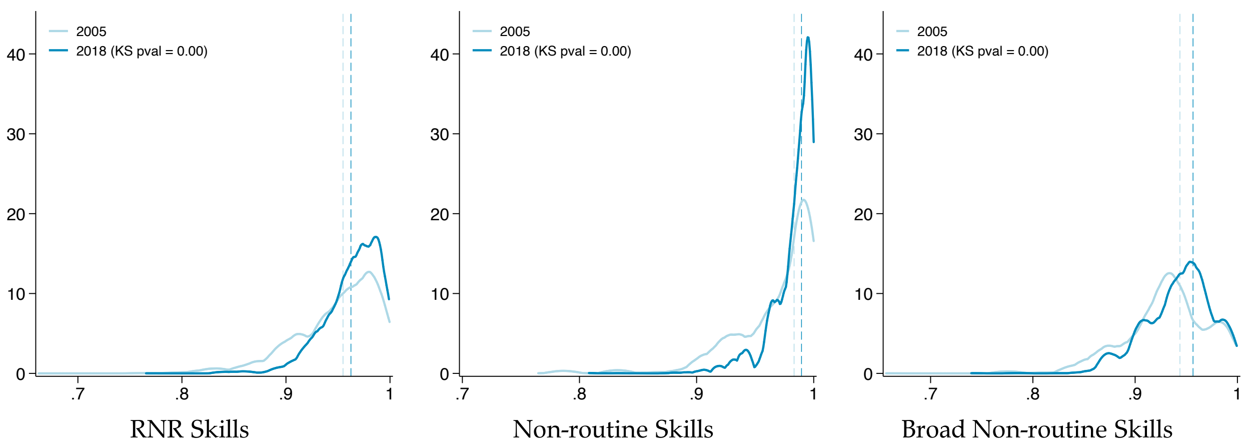
In Table A5, I show the decomposition results of the changes in the skill mixing indexes for different skill pairs across different datasets. A similar pattern as the main analysis in 2, that is within-occupation variation surpassed across-occupation variation in accounting for the increase in skill mixing. This is particularly true using constantly updated occupations at 6-digit occupation level for non-routine skill pairs, and also quite apparent in the Lightcast data. The only slight difference is that for full O*NET data at the 7-digit level, across-occupation variation does contribute to a comparable amount to the change in skill mixing for skill pairs with routine skill.

Figure A2: Density for Skill Mixing Indexes (Weighted Cosine Distances), 2005 vs. 2018

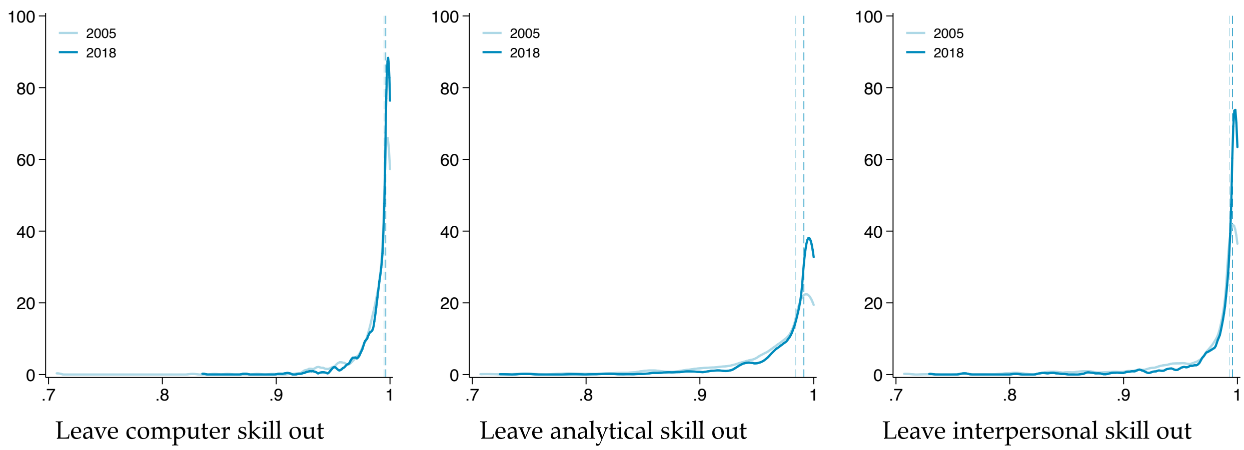
(1) 7-Digit Occupations



(2) 7-Digit Occupations Weighted by OEWS

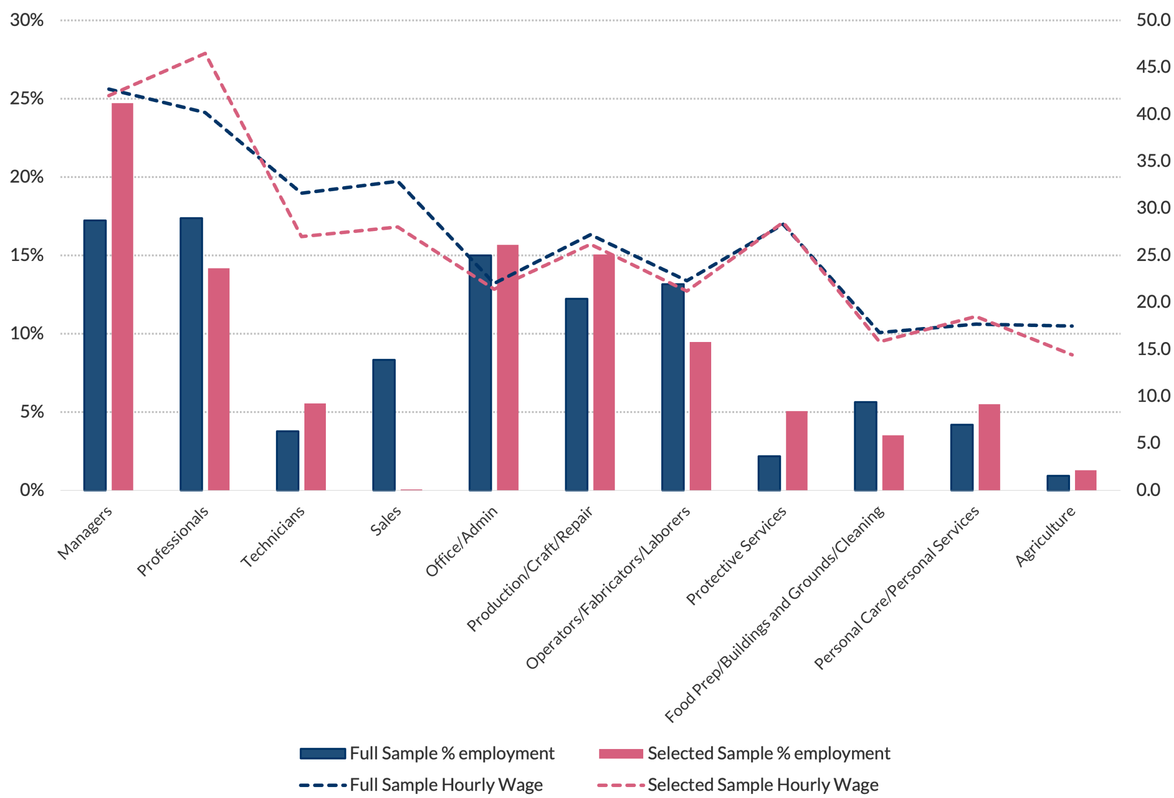


(3) Leave One Skill Out from Non-routine Skill Set



*Notes: These figures plot the PDF of different mixing indexes in 2005 (light blue line) and 2018 (dark blue line). The x-axis displays the value of mixing indexes with a maximum of 1 by construction. These plots are created using O*NET and ACS data merged with occupation codes constructed by Autor and Price (2013) and developed by Deming (2017).*

Figure A3: Employment Share and Hourly Wage of Full and Updated O*NET



Additionally, in Figure A3, I show employment percentages and hourly wages across various job categories in the full and the sample for constantly updated occupations. This information gives the occupational structure and returns for these two samples. It can be seen that while professionals make up a smaller percentage in the selected sample, they exhibit a higher average wage, suggesting a focus on higher-earning professionals in the selected sample. Conversely, the sales category shows a drastic reduction in the selected sample, indicating its limited representation. The hourly wage rates across the categories seem fairly consistent between the full and selected samples, with minor discrepancies.

Table A4: R-Squared Values for Non-Routine Skills' Mixing Index by Polynomial Order

	Analytical	Computer	Interpersonal
	1st Order Polynomial		
All occupations	0.11	0.12	0.15
High-wage	0.00	0.02	0.25
White-collar	0.17	0.00	0.38
Blue-collar	0.02	0.22	0.03
Service	0.22	0.37	0.18
	3rd Order Polynomial		
All occupations	0.15	0.48	0.21
High-wage	0.03	0.45	0.55
White-collar	0.21	0.20	0.52
Blue-collar	0.05	0.56	0.15
Service	0.30	0.62	0.20
	5th Order Polynomial		
All occupations	0.18	0.50	0.22
High-wage	0.04	0.46	0.55
White-collar	0.22	0.21	0.53
Blue-collar	0.07	0.57	0.16
Service	0.38	0.73	0.26

The table presents the R-Squared values from a polynomial regression analysis, assessing the relationship between non-routine skills' mixing index and each composing skill's polynomials up to order N over the period from 2000 to 2020. The regression formula used is $Mix(\mathbf{y})_{ijt}^{percentile} = \beta_1 y_{ijt}^1 + \beta_2 y_{ijt}^2 + ... + \beta_N y_{ijt}^N$*, where* $Mix(\mathbf{y})_{ijt}^{percentile}$ *indicates the percentile rank of an individual's* i *mixing index of non-routine skills in occupation* j *at time* t*, and* y_{ijt} *is the measure of a specific composing skill for the same individual and occupation at time* t*. The R-Squared values for polynomial orders* $N = 1, 3, 5$ *are provided, illustrating the degree to which each composing skill explains the variance in skill mixing.*

Table A5: Decomposition of Mixing Indexes' Changes by Skill Pairs

	6-digit Occupations			3-digit Occupations		
Skill Groups	total	within	across	total	within	across
*Full O*NET*						
Analytical + Computer	10.52	6.40	4.12	8.13	6.71	1.42
Analytical + Interpersonal	5.36	2.90	2.46	6.42	4.21	2.21
Computer + Routine	4.38	2.41	1.97	2.65	3.03	-0.37
Computer + Interpersonal	7.23	3.60	3.63	10.28	7.67	2.60
Routine + Analytical	4.00	2.29	1.71	1.52	3.26	-1.75
Routine + Interpersonal	1.93	0.12	1.81	-1.25	1.13	-2.38
Constant Updates						
Analytical + Computer	5.59	6.03	-0.44	6.75	5.96	0.79
Analytical + Interpersonal	3.53	4.58	-1.05	4.24	3.15	1.09
Computer + Routine	2.88	3.69	-0.81	0.77	1.97	-1.20
Computer + Interpersonal	0.78	1.86	-1.09	7.24	6.06	1.17
Routine + Analytical	2.04	2.13	-0.09	1.72	3.69	-1.96
Routine + Interpersonal	0.81	0.82	-0.01	-0.08	1.53	-1.61
Lightcast						
Analytical + Computer		—		13.20	11.74	0.90
Analytical + Interpersonal		—		2.73	2.20	0.31
Computer + Interpersonal		—		-3.90	-3.79	-0.39

Notes: This table shows the shift-share decomposition of changes in the average level of different mixing indexes between 2005-2018 in percentile units. Specifically, for a change in the percentile of a mixing index h over two periods t and τ, its change $\Delta T_{h\tau} = T_\tau - T_t$ which can be decomposed to $\Delta T_h = \sum_j \left(\Delta E_{j\tau}\alpha_{jh}\right) + \sum_j \left(E_j \Delta\alpha_{jh\tau}\right) = \Delta T_h^a + \Delta T_h^w$ where $E_{j\tau}$ is employment weight in occupation j in year τ, and $\alpha_{jh\tau}$ is the level of mixing index h in occupation j in year τ, $E_j = \frac{1}{2}(E_{jt} + E_{j\tau})$ and $\alpha_{jh} = \frac{1}{2}(\alpha_{jht} + \alpha_{jh\tau})$. ΔT_h^a and ΔT_h^w then represent across-occupation and within-occupation change.

Table A6: Relationship between Robotics, IT Capital, and Skill Mixing Shifts

	Non-routine Skills		RNR Skills	
	(1)	(2)	(3)	(4)
*A. Skill Mixing Index, 2005-2018 (O*NET)*				
IT capital stock	-0.00	0.09***	0.12**	-0.09**
	[0.02]	[0.03]	[0.04]	[0.03]
Observations	821,030	821,030	821,030	821,030
R-squared	0.07	0.19	0.11	0.24
B. Skill Mixing Index, 2007-2017 (Lightcast)				
IT capital stock	0.02	0.06		
	[0.07]	[0.07]		
Observations	518,520	518,520		
R-squared	0.09	0.26		
*C. Change in Skill Mixing Index, 2005-2010 and 2010-2015 (O*NET)*				
Δ industrial robots	-1.38***	-0.41	-1.77**	-1.24***
	[0.36]	[0.38]	[0.54]	[0.31]
Observations	97,650	97,650	97,650	97,650
R-squared	0.02	0.12	0.04	0.14
Year FE	X	X	X	X
Experience and education controls	X	X	X	X
Gender $\times$ education FE	X		X	
Gender $\times$ education $\times$ industry FE		X		X

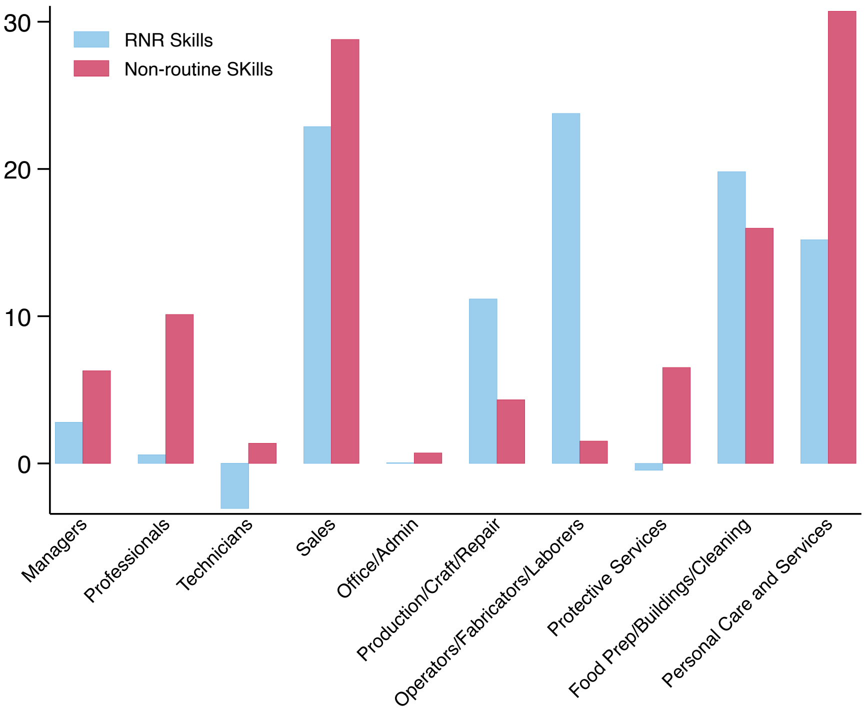
*Notes: This table provides regression results on the changes in skill mixing indexes for non-routine and RNR skills between 2005-2018, measured in percentile units based on their distributions in 2005. The analysis integrates data from O*NET and Lightcast to derive skill intensities for calculating skill mixing, which are then merged with ACS data. The data on IT capital stock is sourced from the Bureau of Labor Statistics Total Multifactor Productivity tables. It reflects the productive capital stock for "Total information processing equipment" in billions of 2017 dollars, which is then converted into logarithmic values. The information on the number of industrial robots per thousand workers is sourced from the International Federation of Robotics (IFR), which covers seven industries including Manufacturing, Agriculture, Mining, Utilities, Construction, Education, and Services, and covers the periods 2004-2010 and 2010-2014. Following the methodology in Acemoglu and Restrepo (2020), I use IFR data from Denmark, Finland, France, Italy, and Sweden to assess the influence of global technological progress. Robust standard errors are reported in brackets. *** p<0.01, ** p<0.05, and * p<0.1.*

A.5 Additional Results on Trend Heterogeneity

Now, I turn to discuss the robustness of the occupation heterogeneity in skill mixing changes. Figure A5 provides a detailed view of the changes in the skill mixing of different skill pairs across various occupations. Overall, the increase in the degree of mixing of non-routine skill pairs is higher than the increase in the mixing of skill pairs that include routine skills. Service and blue-collar occupations experience the highest increases in skill mixing of different skills, surpassing white-collar and high-wage occupations. When it comes to routine skills, blue-collar jobs lead other occupations in terms of increase in mixing.

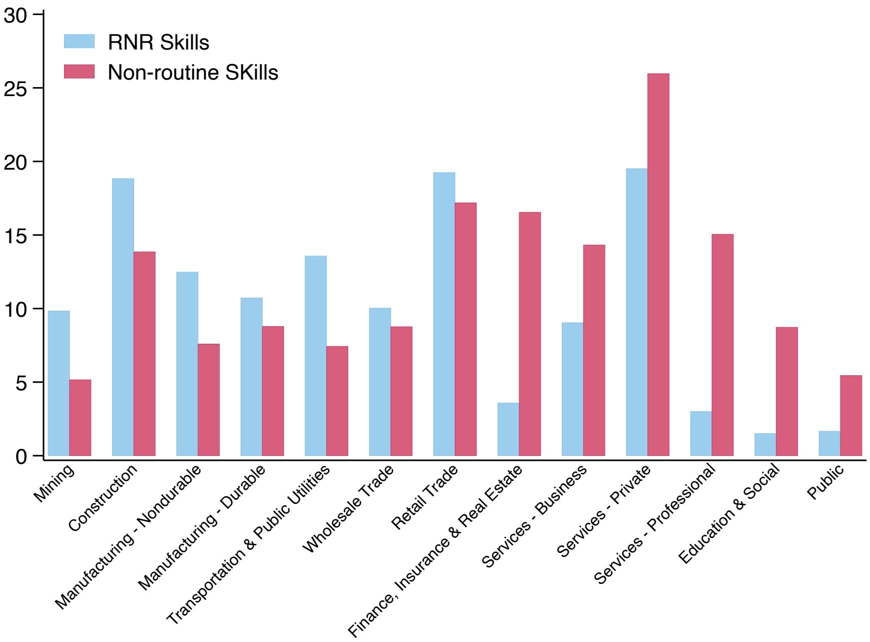
Figure A5 also provides a detailed view of the changes in the skill mixing of RNR and non-routine skills across various industries. The main patterns indicate that the private service sector, followed by retail trade and construction, leads others in the growth of skill mixing, while public, education, social, and professional services experience the least increases in skill mixing. There is also noticeable heterogeneity across industries in terms of the skills that are mixed. For instance, in finance, real estate, and professional services, there is much higher mixing in non-routine skills relative to RNR skills; conversely, in industries like mining, transportation, public utilities, and construction, RNR skills are mixed in a higher degree.

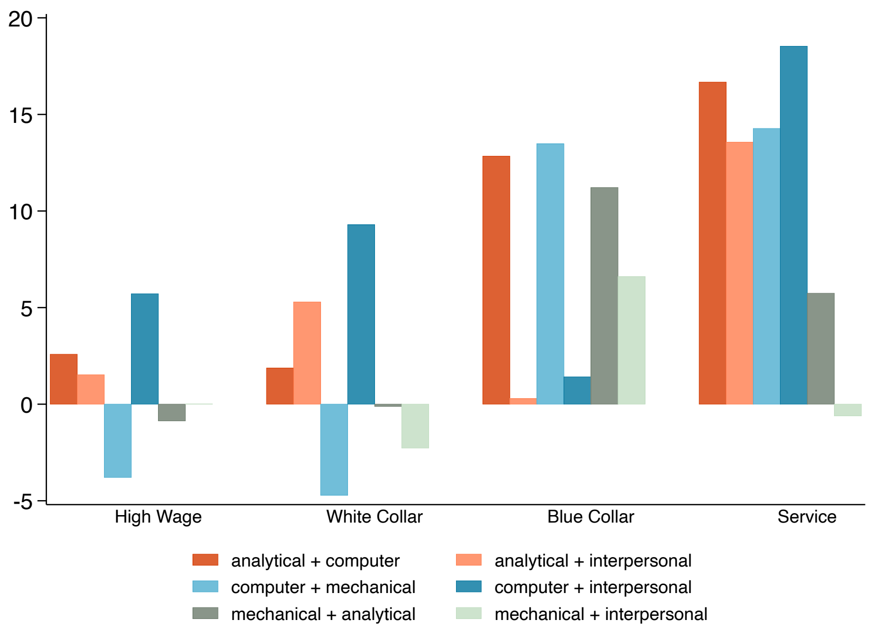
Figure A4: Mixing Index Change by Occupation Groups, 2005-2018



*Notes: This figure plots the changes in skill mixing indexes across different occupation groups. The units of the index changes are percentiles of their distributions in 2005. Workers are categorized into 10 1-digit occupational groups that cover the entirety of US non-agricultural employment following Acemoglu and Autor (2011). O*NET and ACS data are combined for these figures with consistent occupation codes from Autor and Price (2013) and developed by Deming (2017).*

Figure A5: Mixing Index Change by Industry and Occupation Groups, 2005-2018





*Notes: These two figures plot the changes in mixing indexes across different industry and occupation groups. The units of the index changes are percentiles of their distributions in 2005. The industry grouping is based on the industrial classification from the 1990 census. The occupation groups (High-wage, White-collar, Blue-collar, Service) follow Acemoglu and Autor (2011). O*NET and ACS data are combined for these figures with consistent occupation codes from Autor and Price (2013) and developed by Deming (2017).*

A.6 Robustness of Trend Results to Measures of Skills

In this section, I discuss the robustness of the trend results to using alternative measures of skills. Specifically, I present alternative trend results using different ways of processing skill descriptors from O*NET, such as not using PCA, and standardizing rather than rescaling. I also show the robustness using broader skill measures than those applied in the main analysis.

Alternative Construction of Skills: Since O*NET contains a large number of descriptors, many of which capture the same dimensions of skill requirements, it becomes standard practice to first abstract useful information from the descriptors to construct lower-dimensional measures of skills. The first approach, as in Autor, Levy, and Murnane (2003), Acemoglu and Autor (2011) and Deming (2017), takes the average of a subset of variables and assumes that such average represents a particular broader skill intensity and not others. The other approach, as in Lise and Postel-Vinay (2020), applies PCA to the entire set of variables, which assumes that each of the variables contains information about underlying components that are orthogonally distributed. Both approaches impose different assumptions, with the first one giving more easily interpretable skill groups while the second being more data-driven. A third approach, as in Yamaguchi (2012), first picks descriptors that are ex-ante most easily interpretable with respect to each skill dimension, and then conducts PCA on those descriptors to abstract the most relevant variation. The main body of the paper adopts the third approach; here I show robustness checks using alternative skill measures.

Online Appendix Figure A6 presents the trend results using skill measures constructed by taking an average of the descriptors without imposing PCA (panel 2) and using skill measures normalized by standard deviation rather than linearly scaled to $[0, 1]$ (panel 3). Normalizing by standard deviation necessarily creates negative values for the skills; since the mixing index is defined based on positive real values, having these negative values invalidates the mixing index in measuring skill mixing. One solution is to add a positive number to the skill measures. As any number chosen is essentially arbitrary, here I added the negative of the smallest value such that the re-scaled measure lies exactly above 0. For both of these robustness exercises, the main message is similar to the main text: there is a significant increase in mixing for non-routine skills, and less so for RNR skills.

Skill Measures: Another concern is that by using skill measures from Acemoglu and Autor (2011), each of which is constructed from a few descriptors, the resulting skill measures could be relatively "narrow" and do not provide a comprehensive depiction of the skill spaces. To alleviate this concern, I construct skill measures using a broader set of descriptors, similar to that of Lise and Postel-Vinay (2020). I first select descriptors from abilities, knowledge, skills, and work activities files that are more relevant for job skill demand, leaving me with around 163 descriptors. I then combine each year's O*NET data with ACS and conduct PCA on the merged data from the years 2005-2018.

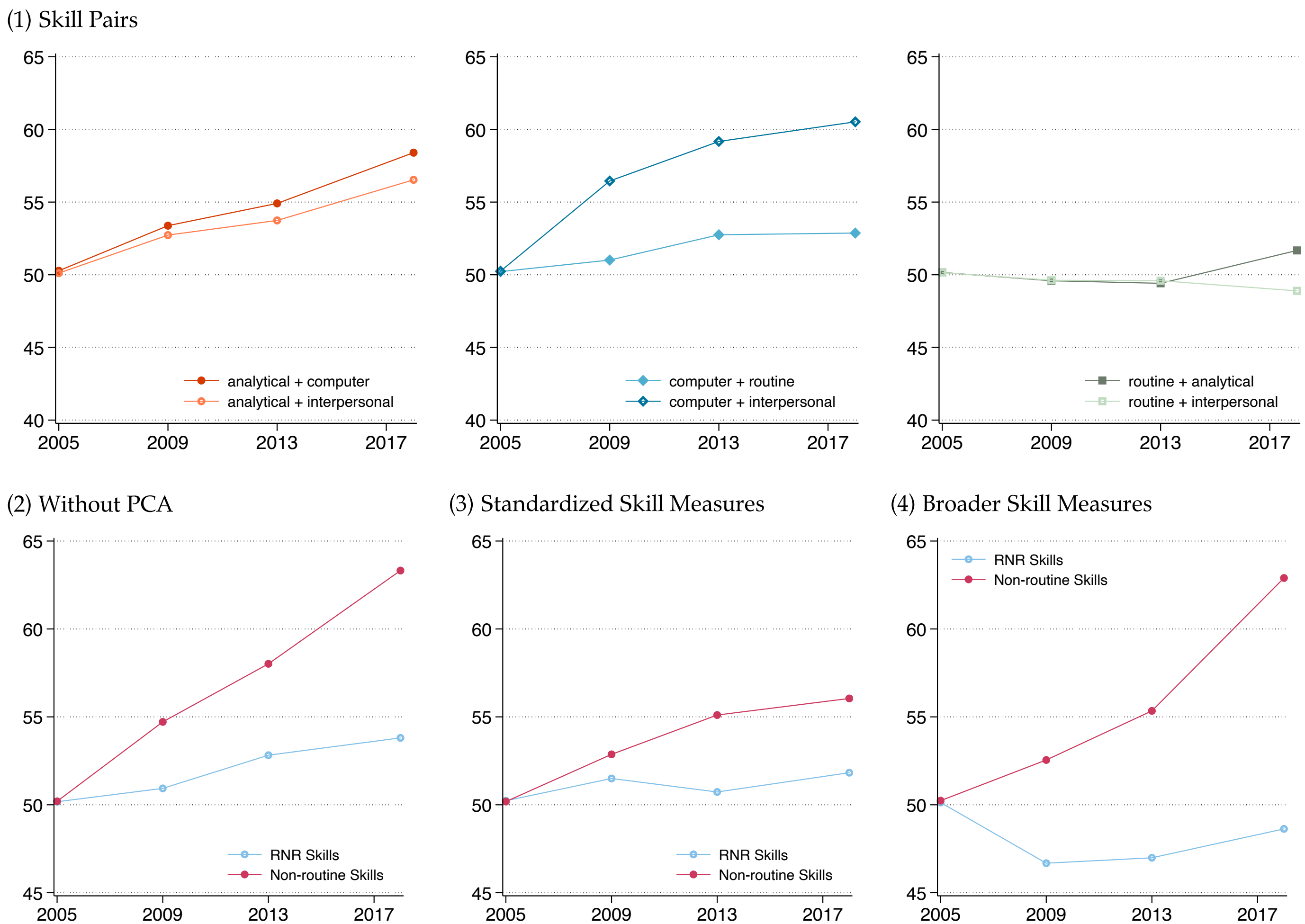
The result from this approach supports the choice of analytical, routine, and interpersonal skills in the main text The first three factors out of PCA explain around 60% of the variation across all the descriptors for years. The first factor has a strong positive association with reason and math skills, such as "Deductive Reasoning", "Inductive Reasoning" and "Mathematics", while the second factor relates more to motor coordination and mechanical work, such as "Multi-limb Coordination", "Mechanical" and "Equipment Maintenance". The third factor is clearly more associated with interacting with other people, such as "Selling or Influencing Others" and "Resolving Conflicts and Negotiating with Others". I interpret the second factor as "mechanical" rather than routine for the broader skill measures.

After conducting PCA, one could directly extract the factors imposing the assumptions that these factors are orthogonal to each other. While this is obviously quite convenient, it nevertheless creates the challenge of interpretability, since each of the factors has been constructed such that it is positively or negatively correlated with all of the 163 descriptors, and the assumption of orthogonality appears strong if the underlying skills are complementary in production across occupations. To take a fine balance between comprehensibility and interpretability, I adopt an approach similar to the measurement validation literature (Costello and Osborne 2005; Thompson and Daniel 1996), where I first conduct PCA/factor analysis to reveal the underlying dimensionality and structure of the measure (as has been done in the previous step). Guided by the factor loadings, I then hand-pick the skill descriptors into three broad groups "analytical", "mechanical" and "interpersonal" without imposing the orthogonality assumption.

Online Appendix Table A7 illustrates the selected descriptors for each of the composite skill measures. These descriptors are broadly in line with Acemoglu and Autor (2011) but have several distinctions. First, the descriptors coming from factor analysis lean more toward reasoning, comprehension, and expression. Second, the mechanical skill used in the main text is the average of two ASVAB test scores that are constructed by the weighted average of 26 O*NET descriptors. The ASVAB "Mechanical Comprehension" tests contestants' "understanding of the principles of mechanical devices, structural support, and properties of materials" and the ASVAB Electronics Information tests contestants' "understanding of electrical current, circuits, devices, and systems", both stressing one's knowledge basis. On the other hand, the descriptors chosen by conducting PCA relate more to physical control, coordination, and machine operation aspects rather than mental perception. Third, the descriptor choices for interpersonal skill from factor analysis also emphasize interactions with others as in Acemoglu and Autor (2011) but are more comprehensive.

Online Appendix Figure A6 panel (4) illustrates the trend results using these broader skill measures. The message on the growth of skill mixing remains the same as the main text, that is there is strong growth of skill mixing for non-routine skills. Nonetheless, for RNR skills, the degree of skill mixing has decreased using the broader measures.

Figure A6: Trend of Skill Mixing with Alternative Skill Measures



*Notes: These three panels plot the employment-weighted mixing indexes of different skills in the U.S. economy from 2005-2018 using O*NET and ACS data. Panel (1) shows the changes in skill mixing indexes of 6 distinct skill pairs of the 4 skills. In panel (2), skill mixing indexes are calculated using skill measures without using PCA, and in panel (3), skill measures are normalized to have mean 0 and standard deviation 1. Panel (4) shows the changes in mixing indexes using broader skill measures as described in online Appendix A.6.*

Table A7: Components of Broader Skill Measures

Analytical	Mechanical	Interpersonal
• Deductive Reasoning	• Multilimb Coordination	• Assisting and Caring for Others
• Inductive Reasoning	• Speed of Limb Movement	• Selling or Influencing Others
• Mathematical Reasoning	• Mechanical	• Resolving Conflicts and Negotiating
• Number Facility	• Performing General Physical Activities	• Coaching and Developing Others
• Mathematics	• Handling and Moving Objects	• Staffing Organizational Units
• Economics and Accounting	• Controlling Machines and Processes	• Service Orientation
• Reading Comprehension	• Operate Vehicles, Mechanized Devices or Equipmnt	• Administration and Management
• Writing	• Repairing and Maintaining Mechanical Equipment	• Customer and Personal Service
• Speaking	• Repairing and Maintaining Electronic Equipment	
• Oral Comprehension	• Installation	
• Written Comprehension	• Equipment Maintenance	
• Oral Expression	• Repairing	
• Written Expression	• Production and Processing	

*Notes: This table lists the O*NET descriptor components for each of the constructed (broader) composite skill groups as discussed in online Appendix A.6.*

A.7 Robustness of Trend Results to Measures of Skill Mixing

I introduce two additional measures and show the robustness of the trend results using these alternative mixing measures.

A first commonly used measure for concentration or specialization based on the share of a total quantity is the Herfindahl–Hirschman Index (HHI).[62] Equation (7) shows how to use inverse HHI to measure skill mixing for an occupation represented by $(\alpha_{ja}, \alpha_{js})$. Observe that this index is maximized when $\alpha_{ja} = \alpha_{js}$, exactly corresponding to the case when the skill vector lies on the unit vector and becomes most mixed. If one skill's intensity is greater than the other, the occupation becomes less mixed and this index becomes smaller. Similar to an angle-based mixing index, this measure is insensitive to the length of a skill vector, since each skill is normalized by the total quantity of skills in that occupation.

$$\left[\left(\frac{\alpha_{ja}}{\alpha_{ja}+\alpha_{js}}\right)^2 + \left(\frac{\alpha_{js}}{\alpha_{ja}+\alpha_{js}}\right)^2\right]^{-1} \tag{7}$$

$$-\frac{|\alpha_{ja}-\alpha_{js}|}{\alpha_{ja}+\alpha_{js}} \tag{8}$$

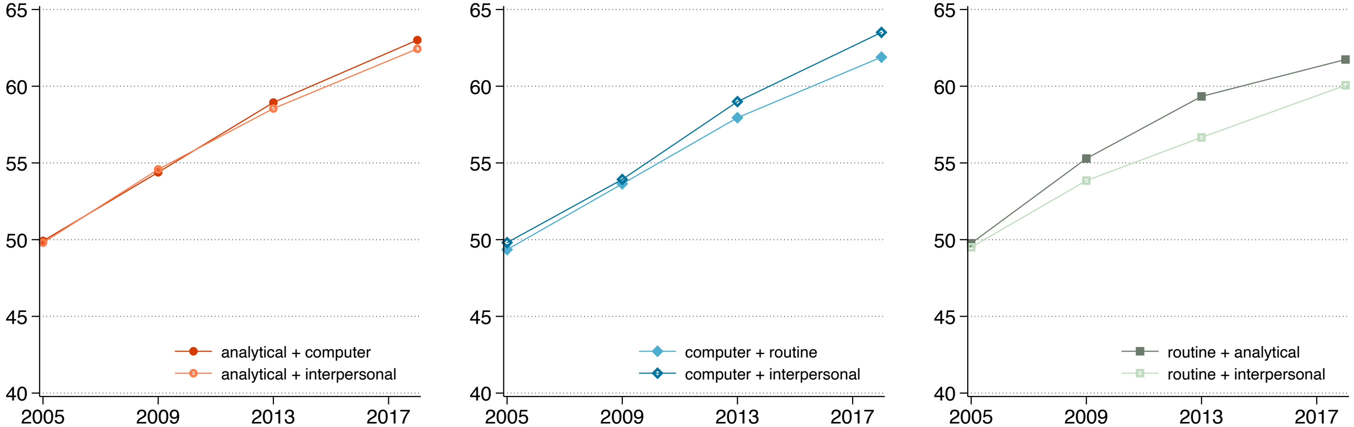
Under a similar vein, the degree of skill mixing could also be measured by normalizing the absolute distance between skill intensities for a skill vector: as this distance decreases, the overall skill portfolio becomes more balanced; normalization then eliminates the effect of the length of the skill vector. Equation (8) gives a particular specification of such a measure. As can be seen from this construction, as the absolute distance between skill intensities decreases and the degree of mixing increases, this measure also increases, though from the direction of $(-\infty, 0)$.

In Online Appendix Figure A7, I show the robustness of the trend results using these alternative measures in panels (1) and (2). Both measures deliver the same message as the cosine mixing index in the main text, that is, there is a sizable increase in skill mixing, particularly for non-routine skills. The only difference is that for the HHI skill index, there is also a comparable increase in skill mixing for RNR skills.

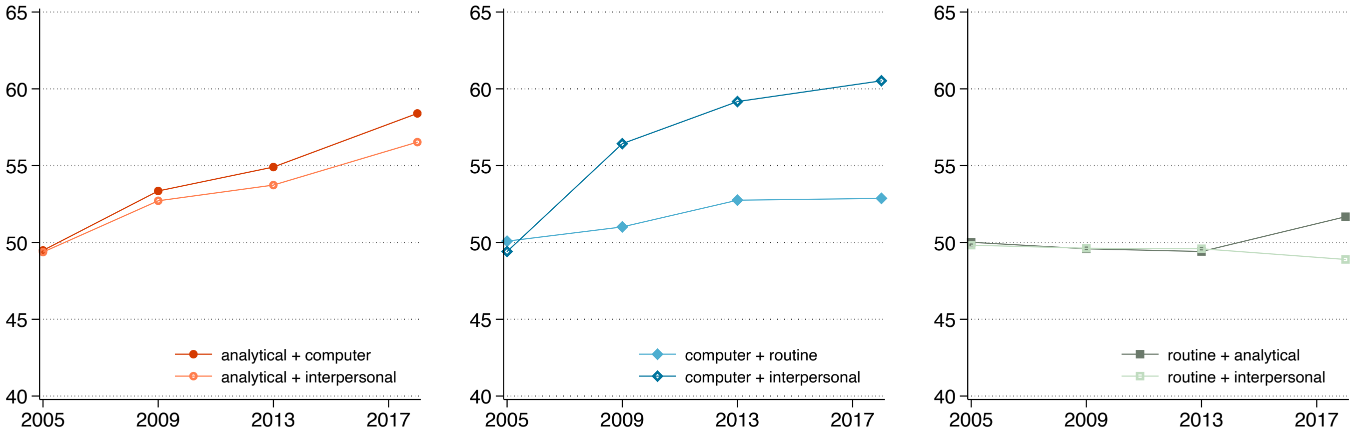
[62]For applications in the labor literature, Ransom and Phipps (2017) and Jin (2017) use the inverse of HHI as the "variety index" to examine how diverse the jobs held for students who graduated from a certain major. Similar logic can be applied to the measurement of skill mixing: in the context of 2-dimensional skill space, the more "varied" skills an occupation uses essentially means that skills are more mixed.

Figure A7: Trend of Skill Mixing with Alternative Indexes

(1) Inverse Herfindahl



(2) Absolute Distance



*Notes: These three panels plot the employment-weighted mixing indexes of different skills in the U.S. economy from 2005-2018 using O*NET and ACS data. In panels (1) and (2), mixing indexes are calculated using the Inverse Herfindahl index and Absolute Distance as discussed in online Appendix A.7.*

A.8 Additional Results on Wage and Employment Returns

In this section, I provide more detailed results on wage returns, and results relating employment to occupation skill mixing. I also provide robustness results to the analysis of the returns to skill mixing in the main paper.

Detailed Results on Wage and Employment Returns: I first check the returns to individual skills and how they interact with the returns to skill mixing. Table A8 column (1) shows that in a cross-sectional regression analytical and computer skills both have significant positive returns. Workers employed in occupations requiring a higher degree of these two skills earn more. Nonetheless, workers in occupations that require a higher level of interpersonal skills have a wage reduction.

Column (2) of Table A8 shows that by restricting to within-occupation variation and including skill mixing measures an important pattern appears: the coefficients for most individual skills become slightly more negative (except for routine skill),[63] while the skill mixing indexes of analytical paired with interpersonal skills, as well as routine and interpersonal skills show significant positive returns. Such a pattern persists in columns (3) and (4) including worker skills and fixed effects, only that the skill mixing of analytical and computer skills is more precisely estimated to have a positive return. This indicates that the mixing of skills earns separate and additional rewards beyond those predicted by individual skills.

Turning to employment, there is also a positive employment premium for workers with a more mixed skill set. Column (6) of Table A8 shows that workers with a more mixed level of computer and interpersonal routine skills, or computer and interpersonal skills, or routine and interpersonal skills, are more likely to move from unemployment to employment. Workers with a more mixed level of analytical and computer, or analytical and interpersonal skills, are also more likely to exit unemployment, but the results are not precisely estimated. On the other hand, workers with a more mixed level of routine and interpersonal skills are less likely to find employment.

[63] The insignificant or even negative return to analytical skill over time also finds support from the literature. Lise and Postel-Vinay (2020) shows a strong negative 14.4 percent return on cognitive skill using NLSY data with 3-digit occupation fixed effects. Deming (2017) found that the return to cognitive skills has declined across the NLSY79 and NLSY97 cohorts, similar to Castex and Kogan Dechter (2014).

Robustness Checks: Table A10 shows the robustness checks to the results in Table 4. Specifically, Columns (1) and (2) utilize the Absolute Distance and Inverse Herfindahl measures to formulate mixing indexes (refer to online Appendix A.7 for details), while Columns (3) and (4) employ standardized and broader measures of skills (refer to online Appendix A.6 for details).

The findings presented in Table A10 clearly indicate a consistent trend: workers experience a positive return when they are employed in occupations that are more mixed with analytical with computer skills, analytical with interpersonal skills, and routine with interpersonal skills. Specifically, a notable increase in wage is observed with workers in occupations more mixed of analytical and computer skills, especially when applying standardized and broad skill measures; similarly, occupations becoming more mixed of analytical and interpersonal skills, when assessed using the Absolute Distance and Inverse Herfindahl measures, also show a significant positive return. The mixing of routine and interpersonal skills exhibits a positive return as well across the different measures.

On the other hand, the mixing of computer and routine skills, computer and interpersonal skills, and routine and analytical skills all exhibit significant negative wage returns at the occupational level. These negative coefficients may indicate that the combination of these particular skills is less beneficial or leads to inefficiency

Table A8: Return to Skill Mixing Full Table

Dependent Variable:	ln(hourly wage)				Employed
	(1)	(2)	(3)	(4)	(5)
Occupation Skills					
Analytical	-0.023**	-0.023**	-0.022**	-0.015*	
	[0.009]	[0.010]	[0.010]	[0.008]	
Computer	-0.008	-0.014	-0.015	-0.009	
	[0.010]	[0.011]	[0.011]	[0.009]	
Interpersonal	-0.009	-0.014	-0.015*	-0.013*	
	[0.009]	[0.009]	[0.009]	[0.008]	
Mechanical	0.021**	0.029***	0.028***	0.019**	
	[0.010]	[0.011]	[0.011]	[0.009]	
Mix (analytical + computer + interpersonal)	**0.017*** **	**0.015*** **	**0.001**	**0.014*** **	
	[0.005]	**[0.005]**	**[0.006]**	**[0.005]**	
Mix (routine + computer)	-0.035***	-0.045***	-0.044***	-0.037***	
	[0.008]	[0.008]	[0.008]	[0.007]	
Mix (routine + analytical)	-0.041***	-0.045***	-0.042***	-0.039***	
	[0.007]	[0.008]	[0.008]	[0.007]	
Mix (routine + interpersonal)	0.029***	0.035***	0.033***	0.025***	
	[0.009]	[0.009]	[0.009]	[0.008]	
Worker Skills					
AFQT (analytical)		0.074***	0.073***		-0.009**
		[0.011]	[0.011]		[0.004]
Computer		0.045***	0.044***		0.056***
		[0.006]	[0.006]		[0.002]
Social (interpersonal)		0.016***	0.015***		-0.001
		[0.005]	[0.005]		[0.002]
ASVAB mechanical (routine)		-0.015	-0.014		-0.002
		[0.015]	[0.015]		[0.005]
Mix (AFQT + computer + social)		**0.065*** **	**0.070*** **		**0.135*** **
		[0.017]	**[0.017]**		**[0.009]**
Mix (ASVAB mechanical + computer)		0.029*	0.024		0.038***
		[0.017]	[0.017]		[0.010]
Mix (ASVAB mechanical + AFQT)		0.006	0.007		0.000
		[0.008]	[0.008]		[0.004]
Mix (ASVAB mechanical + social)		-0.039***	-0.038***		0.030***
		[0.008]	[0.008]		[0.004]
Interaction			**0.032*** **		
			[0.008]		
Ethnicity*Gender, Age/Year, Region, Edu FE	X	X	X	X	X
Occupation FE	X	X	X	X	X
Worker FE				X	
Observations	88,391	79,343	79,343	88,391	94,062
R-squared	0.416	0.430	0.431	0.756	0.136

Notes: See Table 4 notes.

Table A9: Return to Skill Mixing of Skill Pairs

Dependent Variable:	ln(hourly wage)				Employed
	(1)	(2)	(3)	(4)	(5)
Occupation Skills					
Analytical	-0.019**	-0.019**	-0.012	-0.033***	
	[0.009]	[0.009]	[0.008]	[0.011]	
Computer	-0.002	-0.008	-0.003	-0.016	
	[0.010]	[0.011]	[0.009]	[0.013]	
Interpersonal	-0.019**	-0.022**	-0.021***	-0.025**	
	[0.009]	[0.009]	[0.008]	[0.011]	
Routine	0.027***	0.035***	0.025***	0.048***	
	[0.010]	[0.011]	[0.009]	[0.015]	
Mix (analytical + computer)	0.007	0.011**	0.013***	0.012	
	[0.005]	[0.005]	[0.005]	[0.008]	
Mix (analytical + interpersonal)	0.016***	0.016***	0.015***	0.027***	
	[0.005]	[0.005]	[0.004]	[0.007]	
Mix (computer + routine)	-0.022**	-0.029***	-0.021***	-0.029**	
	[0.009]	[0.009]	[0.008]	[0.012]	
Mix (computer + interpersonal)	-0.008	-0.012**	-0.014***	-0.010	
	[0.006]	[0.006]	[0.005]	[0.009]	
Mix (routine + analytical)	-0.050***	-0.056***	-0.050***	-0.055***	
	[0.008]	[0.009]	[0.008]	[0.012]	
Mix (routine + interpersonal)	0.023***	0.029***	0.019**	0.021*	
	[0.008]	[0.009]	[0.008]	[0.012]	
Worker Skills					
AFQT (analytical)		0.065***		-0.010	0.016***
		[0.012]		[0.025]	[0.005]
Computer		0.045***		0.024	0.052***
		[0.006]		[0.021]	[0.002]
Social (interpersonal)		0.015***		-0.044*	-0.002
		[0.005]		[0.026]	[0.002]
ASVAB mechanical (routine)		-0.008		-0.018	-0.008
		[0.016]		[0.020]	[0.006]
Mix (AFQT + computer)		0.044*		-0.006	0.007
		[0.023]		[0.010]	[0.013]
Mix (AFQT + social)		0.028*		0.070***	0.003
		[0.015]		[0.015]	[0.007]
Mix (computer + ASVAB mechanical)		0.013		0.019	0.048***
		[0.025]		[0.014]	[0.014]
Mix (computer + social)		0.008		-0.061***	0.098***
		[0.013]		[0.015]	[0.007]
Mix (ASVAB mechanical + AFQT)		0.001		-0.056***	0.013***
		[0.009]		[0.019]	[0.004]
Mix (ASVAB mechanical + social)		-0.040***		0.005	-0.021***
		[0.011]		[0.014]	[0.005]
Ethnicity*Gender, Age/Year, Region, Edu FE	X	X	X	X	X
Occupation FE	X	X	X	X	
Worker FE				X	
Observations	87,655	78,719	87,655	50,529	95,440
R-squared	0.426	0.439	0.431	0.758	0.101

Notes: See Table 4 notes.

Table A10: Robustness Checks of Return to Skill Mixing

Dependent Variable:	ln(hourly wage)		
	(1)	(2)	(3)
Occupation Skills			
Analytical	-0.014*	-0.008	-0.013
	[0.008]	[0.033]	[0.008]
Computer	-0.002	0.069**	-0.038***
	[0.009]	[0.027]	[0.010]
Interpersonal	-0.019**	-0.118***	-0.014*
	[0.008]	[0.030]	[0.008]
Routine	0.026***	0.091***	0.010
	[0.009]	[0.017]	[0.008]
Mix (analytical + computer)	0.007	-0.040	0.020***
	[0.005]	[0.036]	[0.007]
Mix (analytical + interpersonal)	0.010**	0.156***	0.025***
	[0.004]	[0.042]	[0.005]
Mix (computer + routine)	-0.028***	-0.045***	-0.087***
	[0.007]	[0.015]	[0.013]
Mix (computer + interpersonal)	-0.011**	-0.019	-0.021***
	[0.005]	[0.033]	[0.008]
Mix (routine + analytical)	-0.033***	-0.080***	-0.041**
	[0.007]	[0.015]	[0.018]
Mix (routine + interpersonal)	0.010	0.033**	0.026**
	[0.007]	[0.016]	[0.012]
Ethnicity*Gender, Age/Year, Region, Edu FE	X	X	X
Occupation FE	X	X	X
Worker FE	X	X	X
Observations	87,655	87,655	87,655
R-squared	0.757	0.757	0.758

*Notes: This table reports the robustness checks to the results in Table 4. Columns (1) and (2) use Absolute Distance and Inverse Herfindahl measures to construct mixing indexes (see online Appendix A.7 for details) and Columns (3) and (4) use standardized and broad measures of skills (see online Appendix A.6 for details). Log hourly wages are the outcome variables and person-year is the unit of observation. The occupational skill and skill mixing measures come directly from O*NET and are merged to NLSY79&97 based on census occupation codes. All measures of skill and skill mixing are normalized to have mean 0 and standard deviation 1. Ethnicity-by-gender, age, year, census region, urbanicity, and a 5-category (no high-school, high-school graduate, some college, college graduate, post-college) education fixed effects are included for all regressions, with additional fixed effects as indicated in the table. Standard errors are clustered at the individual level. *** p<0.01, ** p<0.05, * p<0.10.*

A.9 Additional Results on College Major's Skill Mixing

In online Appendix Table A11, I list the top majors both in terms of the levels and changes in the degree of skill mixing for different skill pairs. Architecture and Environmental Design stands out as the highest major in mixing the three non-routine skills, followed by Computer and Information Sciences, and Communications. Two other majors: Social Sciences and Agriculture and Natural Resources are among the top majors in mixing routine and non-routine skills.

In Table A8 column (4), I represent a worker's human capital by the skill content of a worker's accumulated education experience.[64] Such a designation necessarily restricts the analysis to those who have entered college and brings up selection concerns; however, controlling for worker fixed effects and fixed and time-varying occupation attributes, the estimates show whether it is rewarding to studying a more skill-mixed major conditional on one's job choices. The result in column (4) shows a positive return of around 3 percent studying a college major that is associated with a standard deviation higher mixing of non-routine skills. Interestingly, when taking into account the skill mixing of a worker's college major, the wage premium to occupational skill mixing becomes insignificant. This is due to the correlation between the skill mixing of college majors and subsequent occupational choices, and shows that the former plays a more significant role in driving the wage returns.

[64] This is determined using rolling averages of skill and mixing measures from the worker's entire educational history, since workers may have studied multiple majors.

Table A11: Top College Majors in Skill Mixing

Hybrid Index – Level	Hybrid Index – Change
analytical + computer + interpersonal	
Physical Sciences	Architecture and Environmental Design
Engineering	Computer and Information Sciences
Letters	Communications
analytical + computer	
Physical Sciences	Interdisciplinary Studies
Engineering	Area Studies
Letters	Computer and Information Sciences
analytical + interpersonal	
Public Affairs and Services	Architecture and Environmental Design
Business and Management	Computer and Information Sciences
Social Sciences	Communications
computer + interpersonal	
Social Sciences	Architecture and Environmental Design
None, General Studies	Computer and Information Sciences
Public Affairs and Services	Engineering
routine + computer	
Transportation	Social Sciences
Fine and Applied Arts	Agriculture and Natural Resources
Engineering	Foreign Languages
routine + analytical	
Transportation	Agriculture and Natural Resources
Health Professions	Social Sciences
Computer and Information Sciences	Foreign Languages
routine + interpersonal	
Transportation	Agriculture and Natural Resources
Health Professions	Architecture and Environmental Design
Military Sciences	Social Sciences

*Notes: This table lists the top 3 college majors for each mixing index both in terms of levels and in terms of changes from 2005 to 2019. To calculate the degree of skill mixing for college majors, I first map the occupation level degree of skill mixing contained in the O*NET data to NLSY, and then calculate for each college major's students, the employment weighted average of skill intensities and mixing indexes of their occupations. I use both NLSY79&97 to get the employment weight on occupations.*

Table A12: Return to Skill Mixing Full Table with Individual Skills

NLSY97 Code (before 2010)	Major Field of Study	NLSY79 Code
0	None, no major yet (didn't/don't) have to declare yet;	.
1	Agriculture/Natural resources	1
2	Anthropology	22
3	Archaeology	22
4	Architecture/Environmental design	2
5	Area studies	3
6	Biological sciences	4
7	Business management	5
8	Communications	6
9	Computer/Information science	7
10	Criminology	22
11	Economics	22
12	Education	8
13	Engineering	9
14	English	15
15	Ethnic studies	3
16	Fine and applied arts	10
17	Foreign languages	11
18	History	22
19	Home economics	13
20	Interdisciplinary studies	49
21	Mathematics	17
22	Nursing	12
23	Other health professions	12
24	Philosophy	15
25	Physical sciences	19
26	Political science and government	21
27	Pre-dental	4
28	Pre-law	14
29	Pre-med	4
30	Pre-vet	4
31	Psychology	20
32	Sociology	22
33	Theology/religious studies	23
36	Nutrition/Dietetics	4
37	Hotel/Hospitality management	5
38	Other - Recoded to Liberal Arts and Sciences	49
39	Other - Recoded to Automobile/Automotive Mechanics Technology/Technician	24
40	Other - Recoded to Human Services, General	21
41	Other - Recoded to Social Work	21
42	Other - Recoded to Electrical/Electronics Maintenance and Repair Technology	24
43	Other - Recoded to Geography	22
44	Other - Recoded to International Relations & Affairs	21
45	Other - Recoded to transportation & materials moving	25
46	Other - Recoded to security and protective services	21
47	Other - Recoded to legal support services	14
48	Other - Recoded to other sciences/applied sciences	49
99	UNCODABLE	99

NLSY97 Code (CM10)	Major Field of Study	NLSY79 Code
1	Agriculture, agriculture operations, & related sciences	1
3	Natural resources and conservation	1
4	Architecture and related services	2
5	Area, ethnic, cultural, gender, and group studies	3
9	Communications, journalism, and related programs	6
10	Communications technologies/technicians & support services	6
11	Computer & information sciences & support services	7
12	Personal and culinary services	49
13	Education	8
14	Engineering	9
15	Engineering technologies & engineering-related fields	9
16	Foreign languages, literatures, and linguistics	11
19	Family and consumer sciences/human sciences	13
22	Legal professions and studies	14
23	English language and literature/letters	15
24	Liberal arts and sciences, general studies & humanities	49
25	Library science	16
26	Biological and biomedical sciences	4
27	Mathematics and statistics	17
28	Military science, leadership, and operational art	18
29	Military technologies and applied sciences	18
30	Multi/interdisciplinary studies	49
31	Parks, recreation, leisure, and fitness studies	21
32	Basic skills development/remedial education	8
33	Citizenship activities	21
34	Health-related knowledge and skills	12
35	Interpersonal and social skills	6
36	Leisure and recreational activities	49
37	Personal awareness and self-improvement	8
38	Philosophy and religious studies	15
39	Theology and religious vocations	23
40	Physical sciences	19
41	Science technologies/technicians	24
42	Psychology	20
43	Homeland security, law enforcement, firefighting, and related protective services	18
44	Public administration and social service professions	21
45	Social sciences	22
46	Construction trades	24
47	Mechanic and repair technologies/technicians	24
48	Precision production	24
49	Transportation and materials moving	25
50	Visual and performing arts	10
51	Health professions and related programs	12
52	Business, management, marketing, & related support services	5
53	High school/secondary programs and certificates	8
54	History	22
60	Residency programs	12
999	Uncodable	99

NLSY79 Code	Major Field of Study
0	None, General Studies
1	Agriculture and Natural Resources
2	Architecture and Environmental Design
3	Area Studies
4	Biological Sciences
5	Business and Management
6	Communications
7	Computer and Information Sciences
8	Education
9	Engineering
10	Fine and Applied Arts
11	Foreign Languages
12	Health Professions
13	Home Economics
14	Law
15	Letters
16	Library Science
17	Mathematics
18	Military Sciences
19	Physical Sciences
20	Psychology
21	Public Affairs and Services
22	Social Sciences
23	Theology
24	Mechanics
25	Transportation
49	Interdisciplinary Studies
99	Other

B THOERY AND QUANTITATIVE

B.1 Propositions and Proofs

Lemma 1. *An occupation $\mathbf{y}^j = \{y_1^j, ..., y_k^j, \ldots, y_K^j\} \in S \subset \mathbb{R}^{K+}$ within a closed skill space S of dimension K is more mixed in skills based on Definition 1 if for any pair of skills (h, k), the ratio of $\frac{y_h}{y_k}$ becomes closer to 1.*

Proof of Lemma 1: For the occupation $\mathbf{y}$ we want to establish how the degree of skill mixing changes if the skill dimensions for j and k are to vary. The lemma can be simply proved by considering the skill mixing index for this occupation. Let $y_k = ry_h$ and denote y_h by y, the mixing index for $\mathbf{y}$ is:

$$\frac{y + ry + A}{\sqrt{K}\sqrt{y^2 + r^2y^2 + B}},$$

where A and B are two constants that do not depend on y_k and y_h. The above equation is maximized at $r = 1$. Therefore, for any y_h, the occupation is more skill-mixed if the ratio r is close to 1. This completes the proof. *Q.E.D.*

Proposition 1 (Changes in Skill Mixing)**.** *Consider an occupation $\mathbf{y}^j = \{y_1^j, ..., y_k^j, \ldots, y_K^j\} \in S \subset \mathbb{R}^{K+}$ within a closed skill space S of dimension K. Assume that firms operate the occupation with a production technology as described by equation (2) and under an occupation operation cost defined by equation (6). Under these conditions, occupation $\mathbf{y}^j$ will show an increased degree of skill mixing given the following conditions:*

(i) The skills within the vector $\mathbf{y}^j$ demonstrate a rise in complementarity in production (a decrease in σ), provided that σ does not undergo a change in sign.

(ii) The skills within the vector $\mathbf{y}^j$ exhibit an higher increasing marginal cost (an increase in ρ), under the condition that $\rho > \sigma^j$.

(iii) Additionally, occupation $\mathbf{y}^j$ will exhibit a increased degree of skill mixing in the (y^j_k, y^j_h) dimension if the ratio between (x_k, x_h) approaches unity.

Proof of Proposition 1: Lemma 1 posits that an occupation $\mathbf{y}^j$ exhibits greater skill mixing if the ratio across all skill dimensions approximates 1. Therefore, establishing the influence of the ratio on the degree of skill mixing suffices. The initial step concentrates on any two skills within the vector (y^j_k, y^j_h). I subsume occupation superscript j in the proof below.

The firm value function indicates that the firm re-optimizes the choice of $\mathbf{y}$ in each period. Consequently, within a given submarket at a particular time instance $(\mathbf{x}, \mathbf{y}, \omega)$, the firms' choices of $\mathbf{y}$ remain uninfluenced by the continuation value, rendering it a static problem. Time subscript is subsumed in the subsequent proof.

By deriving the first-order condition of firms' optimization problems in the submarket $(\mathbf{x}, \mathbf{y})$ and taking ratios, one obtains the following condition: $\frac{y_h}{y_k} = (\frac{x_h}{x_k})^{\frac{\sigma}{\rho-\sigma}}(\frac{\alpha_h}{\alpha_k})^{\frac{\sigma}{\rho-\sigma}}$. Therefore, the ratio of firms' optimal skill requirement choices for any two skills (y_h, y_k) is influenced by four variables: the elasticity parameter of substitution in production σ, the degree of increasing marginal occupation operation cost ρ, the ratio of worker skills in the submarket (x_h, x_k), and the ratio of skill efficiencies (α_h, α_k).

From the equation, it is evident that as σ decreases, indicating an increase in skill complementarity in production, $\frac{y_h}{y_k}$ will converge to 1 for any two skills (y_h, y_k), under the assumption that σ does not change sign. Similarly, as ρ increases, $\frac{y_h}{y_k}$ will approximate 1 for any two skills (y_h, y_k), given that $\rho - \sigma$ does not change sign.

The influence of worker skill bias on the degree of skill mixing of $\mathbf{y}$ presents a more complex scenario, as a change in the ratio $\frac{x_h}{x_k}$ does not directly imply a change in the ratio of other skill pairs. Consequently, to gauge its impact on the degree of skill mixing, the focus must remain on the (y_h, y_k) dimension. For this specific dimension, if $\frac{x_h}{x_k}$ converges to 1, then $\frac{y_h}{y_k}$ also approaches 1. *Q.E.D.*

Proposition 2 (Changes in Wage and Job Finding)**.** *Consider an occupation $\mathbf{y} \in S \subset \mathbb{R}^K$ within a closed skill space S of dimension K. Assume that firms operate the occupation with a* *production technology as described by equation (2) and under an occupation operation cost defined by equation (6). Also, these firms offer an output share ω to workers and have value functions described by equation (4). Further, let worker value be described by equation (3). Under these conditions, workers in occupation $\mathbf{y}$ will earn a higher wage, and unemployed workers will have a higher job-finding probability under conditions (i) and (ii) of Proposition 1*

Proof of Proposition 2:

Wages: To establish the change in wages, one needs to show that the output of the worker-firm match increases as the elasticity parameter σ decreases and approaches 0 from 1, or if σ decreases in the negative range, consistent with skills becoming more complementary in production. At a particular output share rate ω, such value changes of σ will lead to higher wages.

Now, let us obtain the first derivative of σ for the production function 2. WLOG, let's consider the case of two skills, and express y_1x_1 and y_2x_2 as m and n. The output of a worker-firm match can be expressed as $q = (m^\sigma + n^\sigma)^{1/\sigma}$. We can take log of the production function $\ln(q) = \frac{1}{\sigma}\ln(m^\sigma + n^\sigma)$ and then take logarithmic differentiation that gives the following:

$$\frac{1}{q}\frac{\partial q}{\partial \sigma} = -\frac{1}{\sigma^2}\ln(m^\sigma + n^\sigma) + \frac{1}{\sigma}\frac{1}{m^\sigma + n^\sigma}(m^\sigma \ln(m) + n^\sigma \ln(n))$$

Solving for $\frac{\partial q}{\partial \sigma}$ gives:

$$\frac{\partial q}{\partial \sigma} = q[-\frac{1}{\sigma^2}\ln(m^\sigma + n^\sigma) + \frac{1}{\sigma}\frac{1}{m^\sigma + n^\sigma}(m^\sigma \ln(m) + n^\sigma \ln(n))]$$
$$\frac{\partial q}{\partial \sigma} = q[-\frac{1}{\sigma}\ln(q) + \frac{1}{\sigma}q^{-\sigma}(m^\sigma \ln(m) + n^\sigma \ln(n))]$$

In the case of the calibration of the model, since m, n, and y are all in the range of $[0, 1]$, one can show that the above derivative is negative when $0 < \sigma < 1$ or when $\sigma < 0$.

With respect to (ii) of Proposition 1, it is easy to see that since for the analysis of this paper, both $(\mathbf{x}, \mathbf{y})$ are in the range $[0, 1]$, therefore the occupation operation cost is decreasing in ρ, so wage should increase as marginal cost increases.

Employment: For job finding probability, it suffices to show that $p(\theta_t(\mathbf{x}, \mathbf{y}, \omega))$ is increasing in σ and ρ. This becomes simpler since the above proof establishes that worker-firm output is increasing in both σ and ρ, and so does the firm's value $J(\mathbf{x}, \mathbf{y}, \omega)$. By the free entry condition in equation (5), at a fixed vacancy posting cost, an increase in $J(\mathbf{x}, \mathbf{y}, \omega)$ implies a decrease in $q(\theta_t(\mathbf{x}, \mathbf{y}, \omega))$ and therefore implies an increase in $p(\theta_t(\mathbf{x}, \mathbf{y}, \omega))$ under constant return to scale matching technology. *Q.E.D.*

B.2 Equilibrium Definition and Block Recursivity

In this section, I define a block-recursive equilibrium (BRE) for the economy following Menzio and Shi (2011). I further show that the equilibrium of the economy is unique and is block-recursive.

Definition 2 (Block-recursive Equilibrium)**.** *Let $\psi \in \Psi$ be the aggregate state of the economy, which is a distribution of agents across employment status $e = U, W$, skill profiles $\mathbf{x}$, occupational skill requirements $\mathbf{y}$, and output shares ω.*

A block-recursive equilibrium for this economy consists of value functions for both unemployed and employed workers $U(\mathbf{x}) : S \to \mathbb{R}$, $W(\mathbf{x}, \mathbf{y}, \omega) : S \times S \times [0, 1] \to \mathbb{R}$, and their respective policy functions $y'_U(\mathbf{x}) : S \to S \times [0, 1]$, $y'_W(\mathbf{x}, \mathbf{y}, \omega) : S \times S \times [0, 1] \to S \times S \times [0, 1]$; firms' policy function $J(\mathbf{x}, \mathbf{y}, \omega) : S \times S \times [0, 1] \to \mathbb{R}$ and corresponding policy function $y'_J(\mathbf{x}, \mathbf{y}, \omega) : S \times S \times [0, 1] \to S \times S \times [0, 1]$; labor market tightness $\theta(\mathbf{x}, \mathbf{y}, \omega) : S \times S \times [0, 1] \to \mathbb{R}_+$; and aggregate state $\psi \in \Psi$ such that:

1. *The worker's value functions $U(\mathbf{x})$ and $W(\mathbf{x}, \mathbf{y}$ satisfy (3) for all states $\psi \in \Psi$ and $y'_U(\mathbf{x})$, $y'_W(\mathbf{x}, \mathbf{y}, \omega)$ are the associated policy functions respectively*

2. *Firms' value function $J(\mathbf{x}, \mathbf{y}, \omega)$ satisfy (4) for all states $\psi \in \Psi$ and $y'_J(\mathbf{x}, \mathbf{y}, \omega)$ is the associated policy function*

3. *The labor market tightness $\theta(\mathbf{x}, \mathbf{y}, \omega)$ in each submarket $(\mathbf{x}, \mathbf{y}, \omega)$ for all states $\psi \in \Psi$ is consistent with free-entry condition in equation (5)*

From the above definition of block-recursive equilibrium agents' value functions and policy functions, as well as the market tightness are independent of the aggregate state, only requiring that they are consistent with the aggregate state distribution of agents. Such an equilibrium is easier to characterize analytically and solve numerically. Note a key difference between the above definite of BRE and the one defined in Menzio and Shi (2011). In the economy studied in this paper, because I use the model to study the steady-state equilibrium, the value functions, policy functions, and market tightness are entirely independent of the aggregate state. Whereas Menzio and Shi (2011) studies out-of-steady-state dynamics, the value functions, policy functions, and market tightness still depend on the aggregate productivity shocks but are independent of the distribution of agents across employment status and match-specific shocks.

Now, I show that a block-recursive equilibrium exists and is unique.

Proposition 3 (Existence and Uniqueness of BRE)**.** *Under the model specification of linear utility and invertible and constant returns to scale matching function, also assume that the support for worker and occupation skill profiles S has bounded, then: i) all equilibria are block recurve as defined in definition 2; ii) there exists a unique block-recursive equilibrium.*

Proof of Proposition 3:

The proof first establishes the uniqueness of value functions (U, W, J), as well as policy functions and market tightness $(y'_U, \omega'_U, y'_W, \omega'_W, y'_J, \theta)$; then, the proof establishes their independence from the aggregate state.

Uniqueness: I first show that the value functions for workers and firms as defined in equation (3) and (4) are contractions. Let $\Theta = S \times S \times [0, 1]$, which is bounded based on the assumption that S is bounded. Let $B(\Theta)$ the space of bounded functions $V : \Theta \to \mathbb{R}$ and the operator associated with the worker or firm value functions denoted by $T : B(\Theta) \to B(\Theta)$. It is straightforward to verify that T satisfies monotonicity and discounting properties:

1. (monotonicity) For $V, V' \in B(\Theta)$, $V \leq V'$ implies $T(V) \leq T(V')$
2. (discounting) For $V \in B(\Theta)$ and $\epsilon > 0$, $T(V + \epsilon) =$

The above conditions establish that the operator T associated with either firm or worker values functions is a contraction bunder Blackwell's sufficient conditions. Therefore, the optimal values workers and firms obtain through dynamic optimization problems are unique.

Next, I show that the policy functions and market tightness are also unique. Since the optimal values firms and workers obtain for their dynamic optimization problems (3) and (4) is unique, the associated policy functions $(y'_U, \omega'_U, y'_W, \omega'_W, y'_J)$ are also unique due to concavity of the production function defined in equation (2) and workers have linear utility over consumption. To show the uniqueness of market tightness, first note that since it is assumed that the matching function is invertible, one may directly obtain market tightness through the market clearing condition (5) with $\theta > 0$. The uniqueness of θ then follows from the uniqueness of firms' value function.

Independence of Aggregate State: In the model economy, workers with different skill profiles $\mathbf{x}$ search in their own market, and firms with different skill requirements $\mathbf{y}$ post jobs in these separated markets, therefore, one can establish that the value functions of firms, workers and the market tightness are all independent of the aggregate state ψ. I establish this argument more rigorously through a backward induction argument as in Braxton and Taska (2021). For this purpose, I introduce back time subscript in the notation.

At the terminal period $t = T$, for an employed worker, the continuation value is zero for $T + 1$ onward, so the worker's dynamic programming problem does not depend on the aggregate distribution across states, and is equal to the worker's share of output $W_T(\mathbf{x}, \mathbf{y}, \omega) = \omega f(\mathbf{x}, \mathbf{y})$.

Similarly, the firm's value function also remains independent of the aggregate distribution $J_T(\mathbf{x}, \mathbf{y}, \omega) = (1 - \omega) f(\mathbf{x}, \mathbf{y})$. As a result, through the free entry condition in equation (5), the market tightness $\theta_T(\mathbf{x}, \mathbf{y}, \omega)$ is also independent of the aggregate distribution.

Firms at $T - 1$ make occupation design choices $\mathbf{y}$ to solve the firm dynamic programming problem in equation (4); workers at $T - 1$ make labor market search choices over occupations $\mathbf{y}$ to solve the worker dynamic programming problem in equation (3); As long as $\mathbf{y}$ is within a bounded interval, the extreme value theorem assures at least one solution to this problem. This process is repeated stepping back from $t = T - 1, ..., 1$, which completes the proof. *Q.E.D.*

B.3 Identification of Parameters

I begin by estimating the elasticity parameters in production and occupation operation cost, denoted by σ and ρ. As highlighted by Caselli and Coleman (2006), the challenge arises when allowing for the endogenous choice of the efficiency of inputs under constraints, as the elasticity parameters cannot be separately identified. To overcome this challenge, I estimate σ using the *relative wage within occupation* instead of relying on absolute wage levels.

Specifically, based on the model, the wage that workers receive per period is given by the share ω of the output of the worker-firm match, reduced by the occupation design cost, formulated as $w(\mathbf{x}, \mathbf{y}) = \omega f(\mathbf{x}, \mathbf{y}) - C(\mathbf{y})$. Consequently, within each occupation, the difference in wage relative to a base worker type $\Delta w(\mathbf{x}, \mathbf{y})$ can be articulated as follows:

$$\Delta w(\mathbf{x}, \mathbf{y}) = \omega \left[\sum_{k=1}^{K} (x^k y^k)^{\sigma} \right]^{\frac{1}{\sigma}} - A, \tag{9}$$

where A is occupation-specific and does not depend on the cost parameter τ or ρ. This formulation enables the identification of σ independent of the cost parameters. To carry out the estimation equation (9), I first adjust the wage for occupation fixed effects in order to account for A and ω. Next, I compute the within-occupation difference of the adjusted wage relative to the lowest skill type worker.[65] Last, I target the correlation between this adjusted within-occupation relative wage and worker abilities $\mathbf{x}$.[66]

I now turn to the identification of the cost parameters ρ and τ. To begin with, note that the first-order condition of firms' optimization problems in the submarket $(\mathbf{x}, \mathbf{y})$ can be simplified in ratios to $\frac{y_h}{y_k} = \left(\frac{x_h}{x_k}\right)^{\frac{\sigma}{\rho-\sigma}}$, a relationship that exclusively depends on the parameters σ and ρ. With σ already estimated, I then target the skill ratio y_j/y_k, which aligns with the moment of the degree of hybridization of occupations. Further, for employed workers, the distribution of employment across various occupations is governed by wages $w(\mathbf{x}, \mathbf{y})$. Given the parameters described above, this functional relationship allows the estimation of τ.

[65] Refer to Section VI for an in-depth discussion on how worker skill types are calibrated.

[66] According to equation (9), σ can be identified from the correlation of any skill with the adjusted wage, which is what I use as the target.

Lastly, given the calibrated unemployment benefits b, the parameters of the matching, production and cost functions, equation (3) reveals that the probability of exiting unemployment only depends on the vacancy posting cost. By targeting unemployment level, c is identified.

B.4 Calibration of Skill Supply

I carry out the calibration of two key aspects of skill supply variation: the Markov probability of worker skill adjustment in a steady state equilibrium and the variation in worker skill supply spanning two data periods that the model aims to align with two steady states. I will first delve into the details of the skill variation between data periods and then explore the skill evolution within a model period as guided by the Markov process, following the approach of Lise and Postel-Vinay (2020).

Across-period Skill Supply Variation: Considering the potential influence of skill supply variation on skill mixing, I calibrate the model to reflect workers' choices in occupation, college major (if attended), and employment status, in line with the approach of Lise and Postel-Vinay (2020). This calibration introduces variation in worker skill supply across two periods. Worker skills are adjusted based on the requirements of an occupation or a college major; they increase if the requirements exceed the original skills and decrease if the requirements are lower or if the worker is unemployed. The speed of this adjustment is asymmetric and skill-specific.

Specifically, following the estimates from Lise and Postel-Vinay (2020), as presented in online Appendix Table B1, a worker's skills accumulate at a rate of γ_j times the gap between the worker's skill j and the occupation's requirement for that skill each year. The value of γ_j depends on whether it relates to learning or depreciation (upward or downward accumulation). Additionally, workers can lose skills when not employed, with unemployment treated as requiring a zero level for all skills. However, I specify such that a worker's skill level cannot fall below their initial endowments. For changes in skills while in school, I specify that workers spend an average of three years learning the skills of their majors.

I incorporate two modifications into this framework. First, since Lise and Postel-Vinay

(2020)'s estimates are based on weekly data, I adjust them by multiplying by the number of working weeks, set at 47. Second, I align Lise and Postel-Vinay (2020)'s estimates of cognitive, interpersonal, and manual skills with my analysis's categories of analytical, interpersonal, and routine skills.[67] Since Lise and Postel-Vinay (2020)'s estimates do not include computer skills, I use their cognitive skill estimates as a proxy.

In calculating the skill adjustment, I first standardize both worker skills and occupation skill requirements. Then, for example, if a worker is employed in an occupation that requires a standard deviation higher in analytical skill compared to the worker's analytical skill, the worker will accumulate 0.36 standard deviations of analytical skill in a year due to learning on the job. Conversely, if a worker's interpersonal skill is higher than required, it will decrease by only 3×10^4 standard deviations, almost remaining unchanged, as interpersonal skills are estimated to be very hard to lose.

Markov Skill Supply Adjustment: I now discuss the Markov process of skill adjustment. Specifically, considering each skill j in the worker's skill profile $\mathbf{x}$ as an element of the finite set S, the evolution of this skill follows a Markov process $\pi(x_j'|x_j, y_j)$, conditional on the worker's current skill level and employed occupation. If a worker is matched with an occupation that requires a skill level exceeding his or her own ($x_j < y_j$), the worker's skill j will adjust upward in the next period: $x_j' > x_j$, and the inverse applies for a worker whose skill is lower than the requirements of their current occupation.

The calibration of the Markov adjustment probability is conducted in a similar manner to that of the across-period skill supply variation. The annual adjustment rates for different skills *gamma* represent the rate at which worker skills approach occupation skill requirements, and it is regarded as the probability that a worker's skill j will adjust to the corresponding value.

The key challenge in this calibration process arises when quantifying the model: both worker skill and occupation skill requirements are discretized as grid values. To accommodate this discretization, the probability that a worker moves up or down a grid for skill j based on the occupation is scaled as below.

[67] Their exclusion restriction imposes that (i) the ASVAB mathematics knowledge score only reflects cognitive skills; (2) the ASVAB automotive and shop information score only reflects manual skills; (3) the Rosenberg self-esteem score only reflects interpersonal skills.

The Markov probability of upward adjustment is determined by:

$$\frac{x_j^{up} - x_j}{y_j - x_j}\mathbf{1}(x_j^{up} < y_j) \times \gamma_j^{up}$$

Similarly, the Markov probability of downward adjustment is given by:

$$\frac{x_j^{down} - x_j}{y_j - x_j}\mathbf{1}(y_j < x_j^{down}) \times \gamma_j^{down}$$

Here, x_j represents the current grid value of worker skill j, while x_j^{up} or x_j^{down} denotes the value of worker skill j up or down a grid, respectively. The indicator variables $\mathbf{1}(y_j < x_j^{down})$ or $\mathbf{1}(x_j^{up} < y_j)$ evaluates whether the skill j grid value of the worker's current employed occupation is greater or smaller than the value of the worker's skill j grid. This means that a worker will only adjust up or down a grid if the occupation's skill is larger or smaller than the corresponding up or down grid value for the worker's skill. This process specifies the interplay between skill adjustment and occupation requirements and allows for a precise calibration within the model's framework.

Table B1: Annual Skill Learning and Depreciation Rate

O*NET Measure	NLSY Measure	γ_{school}^{learn}	γ_j^{up}	γ_j^{down}
analytical	AFQT score	0.33	0.36	0.10
interpersonal	Deming (2017) social skill	0.33	0.05	0.00003
routine	ASVAB	0.33	1	0.36
computer	OCC/Major's 2005 Value	0.33	0.36	0.10

*Notes: This table illustrates for each O*NET skill measure, its corresponding skill measure using NLSY79&97 data, and the learning and depreciation rate for these different skills. The AFQT is the same as the one used by Altonji, Bharadwaj, and Lange (2012) followed by Deming (2017), which controls for age-at-test, test format, and other idiosyncrasies. Deming (2017)'s social skill measure consists of sociability in childhood and sociability in adulthood in NLSY79, and two questions from the Big 5 inventory gauging the extraversion in NLSY97. The average of workers' ASVAB mechanical orientation and electronics test scores are used for mechanical skill. Since ASVAB scores are not available for the NLSY97 survey, they are imputed based on predictive regression using the NLSY79 survey. Workers' occupations' or college majors' O*NET computer skill scores in the year 2000 are used as their endowed computer skill. The skill accumulation/depreciation rate is directly from Lise and Postel-Vinay (2020)'s estimates based on monthly data converted to annual values. Skill learning/depreciating while attending college is specified to be 33% per year.*

B.5 Algorithm and Solution Method

The quantitative method used for estimation is SMM. Given the parameters in the model that are internally estimated $\Theta = \{\sigma, \rho, \tau, c, \alpha_k\}$, each iteration of SMM first solves the steady state firm and worker policy function, after which a panel of worker is simulated to obtain the equilibrium distribution of labor market outcomes.

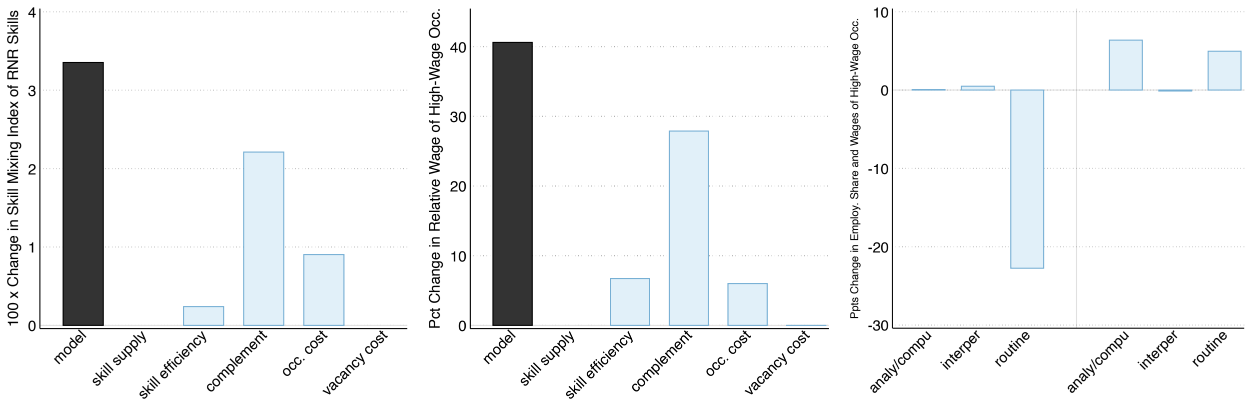
Specifically, to find the steady state policy of agents, I use value function iteration:

1. Fix the number of periods T
2. Starting from the terminal period T, solve the firm problem as in equation (4)
3. Use the free entry condition in equation (5) to obtain the market tightness $\theta_T(\mathbf{x}, \mathbf{y}, \omega)$
4. With the market tightness, solve the worker dynamic programming problem in equation (3)
5. Repeated stepping back from $t = T - 1, ..., 1$
6. Check if the difference in worker value $U_{t+1} - U_t$, $W_{t+1} - W_t$ and the firm value $J_{t+1} - J_t$ is less than a predetermined tolerance level. If yes stop, if not increase T and go back to the first step

Next, I simulate 10,000 workers to obtain a distribution of labor market outcomes across different occupations and worker types. Finally, the SMM procedure minimizes the Euclidean distance between the model-implied moments and the data moments.

B.6 Additional Counterfactual Results

Figure B1: Model Counterfactual



(1) Skill mixing

(2) Wages – unweighted

(3) Wages (left) and employment (right) by individual skills

Notes: These figures plot the model generated changes in skill mixing in high-skill occupations (column 1), changes in wages unweighted by employment (column 2), and changes in wages and employment from individual skills (column 3). Different model channels are shut down individually by eliminating the relative calibrated values to highlight the contribution of each channel. The full model has all the model features. The values of skill complementarity in production, cost of skills in occupation operation, efficiency differential, and vacancy posting cost across the two periods are shown in Table 6. Worker skill supply distribution variation across the periods are calibrated according to Table B1. Panel (3) and (4) depict the model generated changes in skill mixing in low-skill occupation and the relative wage of high-skill occupations by shutting down the skill efficiency differential for analytical/computer, interpersonal, and routine skills individually; also by shutting down τ and ϕ individually.