Optimal Skill Mixing Under Technological Advancements*

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Abstract

I document that the U.S. economy experienced a significant increase in skill mixing from 2005 to 2024, across existing jobs and new vacancies. Employers increasingly demand mixed non-routine skills instead of specialization, particularly for low- to medium-wage occupations. Meanwhile, workers in occupations that mix non-routine skills or those with a broader set of these skills earn a wage premium. To understand these shifts, I build a multi-dimensional directed search model with endogenous occupation design and skill investment. Counterfactual analysis indicates rising skill complementarity and operational costs as key drivers of skill mixing, as well as aggregate wage and employment shifts.

Keywords: skill demand, technological changes, occupations, multi-dimensional skill,

search and matching model, worker training

JEL Codes: J21, J23, J24, J31, E24

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The nature of work in the United States has undergone significant changes in recent decades. A vast literature documents the decline in the demand for "routine" tasks and associated worker skills due to technological shifts (e.g., Autor, Levy, and Murnane 2003; Acemoglu and Autor 2011) and the growing importance of social skills (Cortes, Jaimovich, and Siu 2021; Deming 2017). However, as occupational skill demands shift, it remains unclear whether employers are favoring specific specialized skills or seeking a broader range of skills—what I refer to as "skill mixing." The degree of skill mixing among occupations carries important and distinct implications: if employers seek specific skills, indicating specialization in skill demand, then workers benefit from becoming experts in those particular skill dimensions; if, however, occupations increasingly require mixtures of different skills, indicating skill mixing, then multidisciplinary schooling and training become more advantageous.

This paper examines the phenomenon of employer skill mixing, its implications for workers, and the underlying drivers of these shifts. I begin by gathering relevant data and creating effective metrics to assess skill mixing. Using the Occupational Information Network (O*NET) that surveys workers about the importance of occupational skill requirements, I analyze longitudinal changes in skill demand by considering extended periods and consistently updated occupations. I complement this with Lightcast (formerly "Burning Glass"), which provides real-time online job posting data, capturing extensive margin skill demands. I then introduce a novel "mixing index" by calculating the cosine similarity between an occupation's skill vector and a unit vector where skills are equally important, with a higher index indicating greater skill balance. This index simplifies longitudinal analysis of multi-skill mixing and remains unaffected by proportional measurement errors.

Leveraging data about skill demand for both existing jobs and new vacancies, this paper demonstrates that from 2005 to 2018, U.S. occupations increasingly require mixtures of different skills. Analysis of the O*NET dataset indicates a significant rise in skill mixing even at the 7-digit occupation level, particularly for non-routine skills like analytical, computer, and interpersonal. Compared to 2005, the average degree of skill mixing of occupations has increased by 9.2 percentiles in 2018. The rise is more pronounced in 3-digit occupations, averaging 12.4 percentiles, and 11 percentiles for constantly updated occupations. For instance, in 2005, a sales clerk mostly needs interpersonal and computer skills, with analytical less than half as important. By 2018, analytical matches the importance of the other two skills. For packaging and data entry jobs, analytical and computer skills dominate in 2005, but by 2018,

¹O*NET's occupation classification is based on the Standard Occupational Classification(SOC) system but offers more granularity. For example, in 2010, O*NET lists 1,110 unique 7-digit occupations, compared to 868 unique SOC 6-digit occupations. For higher-level analysis using census data, I first map O*NET occupations to SOC, then use crosswalks from Autor and Price (2013) and Deming (2017).

interpersonal skill rises to 65 to 80 percent of their importance.

Further, I use detailed job posting data to decompose current skill composition into its historical sources. In 2024, 47.6 percent of the skills observed across 4-digit ISCO occupations did not appear in the same occupation in 2010; of these, 45.4 percent were drawn from other occupations. This pattern is consistent across all major occupational groups, highlighting skill mixing as a key channel of labor demand change. To examine its underlying drivers, I analyze the co-occurrence structure of major skill groups. Between 2010 and 2024, the share of postings requiring joint use of analytical and interpersonal skills rose from 17.3 to 24.4 percent; analytical and computer skills from 16.5 to 20.8 percent; and interpersonal and computer skills from 18.5 to 23.7 percent. In contrast, routine skill pairings remain sparse. Regression estimates show that the rise in mixed skill composition is most closely associated with growth in non-routine skill intensity—particularly interpersonal skills—rather than routine tasks. This suggests that modern occupations increasingly require mixed sets of non-routine skills.

I highlight two new facts about skill mixing. First, a shift-share decomposition attributes most of the increase in skill mixing to changes within occupations, rather than worker reallocation across occupations. This contrasts with other labor market changes for which worker movement is key and changes primarily occur across occupations.² Further decomposition shows that within-occupation increases in skill mixing persist even after accounting for workers' gender, education, and experience, and are robust to alternative skill measures and mixing indexes. Second, the most significant rise in mixing non-routine skills appears in service and white-collar occupations, such as sales clerks, cleaners, and data entry keyers, while blue-collar jobs like operators and machinists show a greater mixing of routine and non-routine skills. High-wage managerial and professional occupations, however, exhibit a relatively limited increase in skill mixing.

To explore the implications of skill mixing for workers' labor market outcomes, I estimate the returns to skill mixing by combining O*NET and the National Longitudinal Survey of Youth 1979 and 1997 (NLSY 79 & 97), leveraging detailed information on workers' abilities, employment, and education. I estimate a regression model that accounts for multiple skills and their degrees of mixing, controlling worker characteristics and fixed effects for workers and occupations, akin to Abowd, Kramarz, and Margolis (1999). My preferred estimates indicate that workers in occupations with a standard deviation increase in the mixing of analytical, computer, and interpersonal skills earn a 1.5 percent wage premium, while workers who are

²For example, Autor and Dorn (2013) link labor market polarization to medium-skill workers moving from routine to service jobs, while Deming (2017) attribute the growing importance of social skills to across-occupation employment shifts. Dodini, Lovenheim, and Willén (2023) find that changes in employment concentration across occupations explain the skill intensity differences in unionized workers.

more mixed in these skills earn 6.5 percent more. Further, the interaction between occupationand worker-level skill mixing shows positive returns, which indicates their complementarity and motivates a bilateral matching model.

The rich findings on skill mixing pose challenges in understanding their underlying forces. To make progress, I develop a multi-dimensional directed search model with endogenous production technology and skill investment. Unlike most search models, this model incorporates multi-dimensional skills and non-linear production and cost technologies to capture skill mixing. Specifically, The output from worker-firm matches depends on both the efficiency of individual skills and complementarity of different skills, as well as the operational costs of their use. Moreover, the model includes endogenous choices from both firms and workers: firms of vacant and incumbent jobs must design occupations before meeting with workers, taking into account production technology, vacancy filling probability, also subject to a cost, as in Acemoglu (1999). Workers, in turn, seek jobs that maximize their surplus and invest in their skills through learning-by-doing.

Central to the model's insights is the idea that as skills become more complementary or as their marginal costs rise, firms benefit more from mixing skills than specializing. General equilibrium forces further shape this dynamic: firms must balance creating high-surplus jobs with operating in tighter markets, taking into account worker skill profiles. Since workers capture the remaining surplus, higher job-finding probabilities in tighter markets lower their value of employment. Additionally, workers' job-finding affects firms' profits through on-the-job search and firms' probability of attracting other employed workers.

I then quantitatively evaluate the model to understand the relative importance of channels driving skill mixing and their effects on wages and employment. I calibrate the model by aligning its production and cost parameters with NLSY data on within occupation wage elasticity, overall wage and employment distributions, and occupations' degrees of skill mixing.⁶ The model not only matches these targeted moments but also replicates wage returns

³This cost accounts for all non-wage expenses that rise with skill levels, such as better offices and equipment, comparable to operation costs in Hopenhayn (1992). Structurally, any changes in employment distribution and skill demand not explained by wages are attributed to this cost.

⁴Endogenous input intensity choices were first studied in the appropriate technology literature (Atkinson and Stiglitz 1969; Basu and Weil 1998; Acemoglu and Zilibotti 2001; Jones 2005; Caselli and Coleman 2006). Several labor studies allow firms to adjust labor besides the quantity margin: Lazear (2009) lets firms choose the weight of worker skills; Eeckhout and Kircher (2018) balances workers' quantity against quality; Edmond and Mongey (2021) let firms select skill intensity based on equilibrium prices. My model differs by incorporating high-dimensional heterogeneity and endogenous choices in a directed search framework.

⁵Specifically, workers' skills evolve via a Markov process that adjusts their future skill levels based on current job requirements. If a job demands a higher skill level than a worker has, their skill increases; if less, it decreases. The rate of skill adjustment varies by skill and direction. Since workers choose their occupations optimally, they also choose their skill development trajectories accordingly.

⁶I also use estimates from multi-dimensional matching literature for skill efficiency and adjustment, and

from skill mixing and changes in job skill intensities over workers' lifecycles. The calibration results indicate that skills are substitutable in production, and firms face rising marginal costs of skills in occupation operation. Notably, sizable technology shifts have occurred: from the early 2000s to the late 2010s, there has been an increase in the complementarity and operational costs of skills; meanwhile, the efficiency of analytical, computer, and interpersonal skills has increased and of routine skill has declined.

Counterfactual analyses illustrate that the primary drivers of increased skill mixing are the shifts in technology, specifically the rise in skill complementarity in production and the increased costs of skills in occupation operation. Higher skill complementarity accounts for 86 percent of the increase in skill mixing, while changes in operational skill costs contribute 12 percent. In contrast, changes in skill efficiencies lead to more specialization and reduce skill mixing, while shifts in worker skill supply have a limited impact.⁷

How have the forces driving skill mixing influenced shifts in wage and employment distributions? To answer this, I quantify their aggregate effects on the labor market. I find that skill complementarity accounts for 88 percent of the changing wage gap between high- and low-wage occupations, while changes in skill efficiencies contribute 9 percent. In contrast, skill efficiencies are the main driver of employment gains in high-wage occupations, accounting for 73 percent. Rising skill costs significantly narrow both wage and employment gaps. These findings suggest that while skill efficiency—a traditional focus in the biased technological change literature—remains crucial for employment shifts, skill complementarity and costs are more important in shaping wage distribution and firms' skill specialization. Additionally, a counterfactual training program that increases non-routine skill mixing reduces wage disparities between specialists and non-specialists.

The rest of the paper is organized as follows. The next section reviews the literature and outlines contributions. Section II presents the main empirical findings about skill mixing. In section III, I show the wage returns to skill mixing. Section IV presents a multi-dimensional directed search model to study the skill mixing problem. Model quantification and counterfactual analysis are discussed in Sections V and VI. Section VII concludes.

calibrate changing worker skill supply in the NLSY data to reflect occupation and college major choices based on Lise and Postel-Vinay (2020).

⁷The limited role of skill supply in the model counterfactual stems from two factors: first, the directed search model under perfect information contains submarkets for different worker types, allowing firms to tailor occupations and reduce the influence of worker distribution on labor outcomes. Second, workers' skills adjust to occupational requirements, minimizing the influence of initial skill levels.

I Literature Review

I study labor market dynamics focusing on *skill mixtures* and explore new theoretical perspectives. This aligns with the research on long-term skill demand trends and technological changes (e.g., Tinbergen 1974, 1975; Katz and Murphy 1992; Autor, Levy, and Murnane 2003; Goldin and Katz 2010; Acemoglu and Autor 2011; Autor and Dorn 2013; Deming and Kahn 2018). My findings that within-occupation changes drive skill mixing aligns with studies highlighting the role of within-occupation variation in job attributes (Autor and Handel 2013; Atalay et al. 2020; Freeman, Ganguli, and Handel 2020; Cortes, Jaimovich, and Siu 2021). Unlike these studies, this paper examines skills as mixtures and reveals that employers increasingly demand mixed non-routine skills. It also explores how skill mixing impacts aggregate distributions, as well as workers' returns from occupations and education, deepening the understanding of labor market's transformation.

Two closely related studies are Hershbein and Kahn (2018) and Deming (2017). Hershbein and Kahn (2018) find that employers in areas hardest hit by the Great Recession increased their demand for both cognitive and computer skills, especially in routine-cognitive jobs. Using an angle-based index, my analysis shows that skill mixing occurs broadly across various skills and occupations, beyond specific regions or downturns. Deming (2017) shows that occupations requiring higher math and social skills saw increased employment and wages from 1980 to 2012. In contrast, I use multiple versions of O*NET and Lightcast data to track longitudinal changes in skill demand and show how skill mixing within occupations drives wage and employment changes empirically and quantitatively.

Theoretically, I build a multi-dimensional directed search model with endogenous occupation design and skill investment, extending the directed search literature (e.g., Menzio and Shi 2010, 2011; Kaas and Kircher 2015; Schaal 2017; Baley, Figueiredo, and Ulbricht 2022; Braxton and Taska 2023). This model makes two main contributions: First, it incorporates multi-dimensional skills with non-linear technologies. Second, it allows for endogenous skill demand by firms following Acemoglu (1999), ¹⁰ and worker learning-by-doing as in Lise and

⁸The skill-biased technological change (SBTC) literature, focusing on changes in the relative input efficiency, effectively explains major U.S. wage dynamics, as shown by Katz and Murphy (1992), Autor, Katz, and Krueger (1998), and Goldin and Katz (2010). This paper incorporates both skill efficiency and complementarity and highlights the latter's crucial role in determining skill mixing, wage, and employment shifts post-2000s.

⁹Extracting task information from job ads, Atalay et al. (2020) reveal that major job content changes from 1950 to 2000 occurred within occupations, a trend that Freeman, Ganguli, and Handel (2020) find continues post-2000. Cortes, Jaimovich, and Siu (2021) show that high-paying U.S. occupations increasingly require social skills from 1980 to post-2010. Autor and Handel (2013) find significant within-occupation variation in task requirements based on worker-reported job tasks.

¹⁰Acemoglu (1999) introduces a 1-1 search and matching model where firms "design occupations" by choosing

Postel-Vinay (2020).¹¹ This approach integrates endogenous choices on both worker and firm sides within a framework of high-dimensional heterogeneity.

The foundational model for worker sorting, originating from Roy (1951), treats occupations as distinct categories, limiting the exploration of skill mixing. ¹² Earlier approaches, such as those by Shi (2001) and Hagedorn, Law, and Manovskii (2017), use single-dimensional indices to represent worker heterogeneity, which inherently excludes skill mixing. Recent literature on the multidimensional matching of workers and firms featuring two-sided heterogeneity (e.g., Yamaguchi 2012; Lindenlaub 2017; Lise and Postel-Vinay 2020) explores the assortativeness of matching as well as the evolution of worker skills. Ocampo (2022) studies optimal task combinations leading to endogenous occupational heterogeneity. This study differs by examining the firms' endogenous skill demand trade-offs and workers' occupation choices in a general equilibrium search framework.

A related literature, inspired by Rosen (1983), Murphy (1986), and Heckman and Sedlacek (1985), examines skill indivisibility and nonlinear wage schedules. For example, Choné and Kramarz (2021) introduce a skill bundling framework with heterogeneous firms and empirically find that generalist workers earn more over time; Edmond and Mongey (2021) show that varying occupational skill prices affected by worker supply lead firms to adopt technologies that reflect these prices. Unlike models that aggregate skills within firms, my approach uses a matching model to handle skill indivisibility and endogenous demand at the worker level, aligning with individual-level returns to skill mixing. In this model, each worker receives a unique skill price due to 1-1 matching, resulting in nonlinear price differentials, while workers adjust skill supply through search and learning-by-doing.

II Evidence of Skill Mixing

I begin by showing that the U.S. labor market from 2010 to 2024 has featured significant skill mixing across posted jobs, based on Lightcast data. I then examine which types of skills are most frequently mixed with others, identifying the dimension where mixing is most

labor-augmenting input (capital) before meeting workers, incurring a cost that reduces their surplus. Firms adjust occupation design based on the worker types and efficiency. My contribution extends this to a multi-dimensional framework with continuous workers and firms, unlike the discrete setup in Acemoglu (1999), and implements it in a directed search environment to handle the added complexity.

¹¹I follow Lise and Postel-Vinay (2020) for quantitative tractability. Earlier learning-by-doing models (Heckman 1976; Weiss 1972; Killingsworth 1982; Shaw 1989; Altug and Miller 1990; Heckman, Lochner, and Taber 1998) propose that skills are acquired as a by-product of work, where skill acquisition does not compete with work activities. In these models, each hour of work produces general skills applicable across all sectors.

¹²In Roy or Ricardian type of models, workers will always specialize in a particular skill based on comparative advantages, making it harder to study skill mixing's implications for workers.

prevalent. Combining Lightcast with O*NET data at varying levels of granularity, I then explore the growth in skill mixing and identify the dimensions where it is most significant. Further, I decompose these shifts into across- and within-occupation changes, and also adjust for variations in worker composition and skill supply. Lastly, I highlight the differences in changes in skill mixing across occupation and worker groups.

II.A Data and Measures

Data Construction: To analyze skill mixing trends in occupations over time, I mainly use the Occupational Information Network (O*NET) data with detailed skill requirements, providing an intensive measure of skill demand. Additionally, I incorporate Lightcast data to capture extensive margin skill demand for unfilled online job postings. Below I discuss the details of data construction.

Developed by the North Carolina Department of Commerce and as a successor to the Dictionary of Occupational Titles (DOT), O*NET has become a primary resource for analyzing occupational skill requirements and work environments (e.g., see Acemoglu and Autor 2011; Yamaguchi 2012; Deming 2017). O*NET covers around 270 occupational task descriptors divided into nine modules for 800 to 970 seven-digit occupations each year. While the earlier versions of O*NET include legacy ratings from DOT analysts, a shift occurred in 2003 when O*NET began sourcing responses from random samples of workers (job incumbents). I use descriptors from O*NET releases and questionnaires updated *solely* based on worker surveys to ensure consistent measurement.

A key challenge with O*NET is that only about 110 occupations are updated annually. ¹⁵ I address O*NET's updating issue using two methods akin to Ross (2017) and Freeman, Ganguli, and Handel (2020). First, I analyze skill requirement changes over broader intervals from 2005 to 2018 when all occupations are updated at least twice. For granular time patterns, I use 4-year intervals so that updates cover over half of the occupations. Given that O*NET retains data from prior years, I make a distinction between the release year and the represented year. ¹⁶ Second, I examine 274 7-digit occupations that consistently receive updates between

¹³For a comprehensive overview of O*NET, refer to online Appendix A.1, and for a discussion on the descriptors employed, see online Appendix A.2.

¹⁴Specifically, I use descriptors from the Work Context, Work Activities, Knowledge, and Skills questionnaires. After 2003, O*NET still contain responses from job analysts for questionnaires that have small sample sizes of workers. I abstract from those questionnaires in this paper.

¹⁵The decision of occupation updating is based on analysts' evaluations of factors such as the size of employment, the demand for labor, and alterations in the type of work involved. See Tippins and Hilton (2010) for more details.

 $^{^{16}}$ Online Appendix $\mathrm{A.1}$ shows the specific O*NET versions used, their release dates, and the corresponding

2005, 2011, and 2018 to complement the broader analysis. 17

Moreover, I use Lightcast data (formerly "Burning Glass") from 2007 and 2010 to 2017, which analyzes millions of online job postings and provides detailed codified skills and education requirements. Studies have employed this up-to-date information to examine job skill trends (Deming and Kahn 2018; Hershbein and Kahn 2018; Braxton and Taska 2023). Lightcast complements O*NET by identifying explicit skill requirements for vacancies (extensive margin), whereas O*NET assesses skill importance (intensive margin). Combining these datasets offers a more comprehensive view of skill demand changes.

Skill Measures: Using O*NET occupation task descriptors, I derive skill measures following Acemoglu and Autor (2011) as these measures are widely used and easily comparable with other studies. To simplify the skill dimensions, I use the two non-routine skills (analytical and interpersonal) and combine routine cognitive and manual skills into one.¹⁹ To address the rise of computer technology post-2000, I create a computer skill measure based on descriptors of programming and computer interaction, considering it as non-routine due to its complexity.²⁰ Detailed skill descriptors are in Appendix Table A2.

These four skill measures including both routine and non-routine skills (hereafter "RNR") are the core of this study's analysis. To enhance reliability, I normalize these skills using principal component analysis (PCA) following Guvenen et al. (2020) and Yamaguchi (2012), and linearly rescale them to [0,1] in the main analysis, with results on normalization by standard deviation detailed in the online Appendix A.6.²¹ I further construct alternative skill measures for robustness checks: I build two additional non-routine skills—leadership and design—whose descriptors are detailed in Appendix Table A2. In online Appendix A.2, I develop "broader" skill measures including much more descriptors than the Acemoglu and

years. Specifically, O*NET versions 13.0, 18.0, 22.0, and 25.0 were released in 2008, 2013, 2017, and 2022, respectively. These versions are interpreted as representing the years 2005, 2009, 2013, and 2018, respectively.

¹⁷Figure A3 compares employment percentages and hourly wages across job categories in the full and selected samples of constant updates. Although the selected sample overrepresents managerial roles and underrepresents sales roles, both employment share and wages remain consistent across the samples.

¹⁸Limitations of Lightcast data include potential underrepresentation of non-digitally advertised jobs, overrepresentation of sectors prone to online advertising, and possible bias toward growing firms (Davis, Faberman, and Haltiwanger 2013).

¹⁹Since I only use descriptors updated by job incumbents in this study, I do not use non-routine manual skill since part of the composing descriptors comes from surveys of job analysts exclusively.

²⁰For subsequent references to specific non-routine skills, I use terms like analytical, interpersonal, or computer skill, excluding the prefix "non-routine."

²¹Based on Definition 1, skill vectors must be in the positive real space for angle-based measures to be appropriate. Normalization by standard deviation is insufficient without additional re-normalization; therefore, linear transformation to a positive interval is preferable as it preserves cardinal information useful for clear skill comparisons (see Autor and Handel 2013; Deming 2017; Lise and Postel-Vinay 2020). Alternative methods for measuring skills and skill mixing are detailed in online Appendices A.6 and A.7.

Autor (2011) benchmark.

For the Lightcast data, I adopt the skill measures from Braxton and Taska (2023) based on the methodology of Hershbein and Kahn (2018). A vacancy is classified as requiring analytical skill if its job description includes keywords such as "research" or "analy", and interpersonal skills if it contains keywords "communication" or "teamwork". The skill measure for each occupation is then computed as the percentage of job postings that demand these skills, capturing the extensive margin of firm skill demand.²² I use a 3-digit consistent census occupation code developed by Autor and Dorn (2013) to ensure compatibility with other datasets.

Skill Mixing Measures: In a multi-dimensional skill space, an occupation can be represented by its skill requirement vector, characterized by both an angle and a length. While the length reflects overall skill intensity, skill mixing focuses on the proportion of different skills, captured by the angle. Therefore, to measure skill mixing, one can compare the angle of the occupation's skill vector with that of a unit vector, where all skill requirements are equal.

I define skill mixing in a multi-dimensional space using the cosine similarity between an occupation's skill vector and a multi-dimensional norm vector.²³ Specifically:

Definition 1 (Degree of Skill Mixing of an occupation). The skill mixing index for an occupation j in a K-dimensional space characterized by the skill intensity vector $\mathbf{y}^j = \{y_1^j, ..., y_k^j, ..., y_K^j\} \in S \subset \mathbb{R}^{K+}$ is the cosine similarity between its skill vector and the norm $\hat{\mathbf{v}}$ in the skill space.

$$Mix(\mathbf{y}^j) = \frac{\mathbf{y}^j \hat{\mathbf{v}}}{||\mathbf{v}^j|| \cdot ||\hat{\mathbf{v}}||}, where \, \hat{\mathbf{v}} = [1, 1, ..., 1]' \subseteq \mathbb{R}^{K+}.$$
 (1)

The mixing index in equation (1) captures the multi-dimensional angular similarity between a skill vector \mathbf{y}^j of dimension K and a multi-dimensional norm $\hat{\mathbf{v}}$. As different skill requirements in \mathbf{y}^j get closer, the value of $Mix(\mathbf{y}^j)$ increases. This skill mixing index defined using cosine similarity has three main advantages. First, it easily accommodates multi-dimensional skill space. Second, it abstracts from the skill vector's magnitude and focuses solely on the closeness between the skill vector and the norm, thereby capturing skill mixing. This approach simplifies the analysis of longitudinal changes in skill demand: if measurement errors affect all skills proportionally in a given year, the angle similarity remains

²²More specifically, the keywords used to capture analytical skill are: "research", "analy", "decision", "solving", "math", "statistic", and "thinking". The keywords used to capture interpersonal skill are "communication", "teamwork", "collaboration", "negotiation", and "presentation". The keywords used for computer skill are "computer", or any skill flagged as software by Lightcast.

²³Cosine similarity together with other measures, such as Euclidean distance and Manhattan distance, have been used to calculate the similarity between vectors (e.g., Xia, Zhang, and Li 2015).

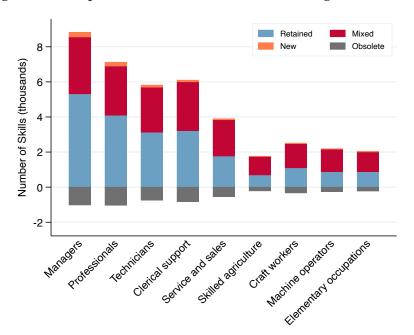


Figure 1: Composition of Skills in US Job Postings, 2010 to 2024

Notes: This figure presents the distribution of skill types across major occupational groups in the United States, distinguishing four categories of skills based on their emergence and persistence over time. The occupation classification follows International Standard Classification of Occupations (ISCO), with calculations first conducted at the 4-digit level and then averaged at the 1-digit level. Values are expressed in thousands, and declining skills are plotted below the horizontal axis for visual clarity.

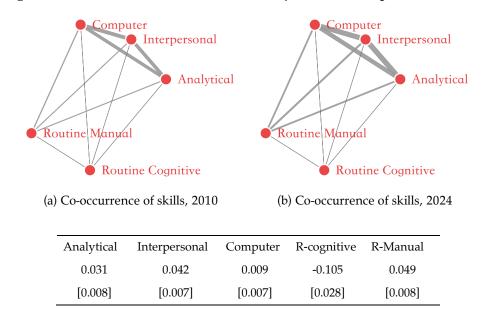
unaffected. Lastly, it is inherently normalized to [0,1] for positive skill requirements.

An angle-based measure is by no means the only measure of skill mixing, though it has the clearest graphical illustration of the trade-off among skills. Online Appendix A.6 discusses two alternative skill mixing indexes: inverse Herfindahl–Hirschman Index (HHI) and normalized absolute distance.

II.B The Evolution of Skill Mixing

What drives the evolution of occupational skill demands over time? To answer this question, I decompose the current skill composition into its historical sources using detailed job posting data. Figure 1 shows the composition of skills in U.S. job postings from 2010 to 2024, classified at the 4-digit level of the International Standard Classification of Occupations (ISCO) and then averaged at the 1-digit level. Skills are grouped into four categories: retained, new, obsolete, and mixed. Among these, mixed skills—those that originated in other occupations—account for a substantial share. Specifically, across occupations, an average of 47.6 percent of the skills observed in 2024 did not appear in 2010; of these, 45.4 percent came from other occupations. This pattern is consistent across all major occupational groups. This result underscores skill

Figure 2: Co-Occurrence Network of Major Skill Groups, 2010 vs. 2024



Notes: This figure visualizes the co-occurrence structure of five major skill categories within U.S. job postings using force-directed network graphs for the years 2010 and 2024. Each node represents a skill group, while edges reflect the share of postings where two skills appear together, with thickness of each edge is proportional to the co-occurrence share. The table shows how changes in the prevalence of mixed skills within 4-digit ISCO occupations relate to shifts in the intensity of major skill categories. Skill intensity is defined as the share of job postings within each 4-digit ISCO occupation that include the relevant keywords. All variables are expressed as year-on-year differences, and the regression includes year and occupation fixed effects.

mixing as a key channel of labor market demand change.

To analyze what drives the growing presence of skills mixed from other occupations, Figure 2 visually capture the co-occurrence of different skills in a network graph, where thicker edges indicate higher co-occurrence among these skill types in 2024. Between 2010 and 2024, postings show a marked rise in the joint appearance of non-routine skills, especially analytical, interpersonal, and computer skills. For instance, the share of postings requiring both analytical and interpersonal skills increased from 17.3 percent to 24.4 percent; analytical and computer skills rose from 16.5 percent to 20.8 percent; and interpersonal and computer skills from 18.5 percent to 23.7 percent. These upward shifts contrast with the persistently low co-occurrence of routine skill pairings, such as routine cognitive and routine manual, which together remain below 1 percent. Regression estimates further confirm that rising mixed skill composition within 4-digit occupations is strongly associated with increases in non-routine skill intensity—particularly interpersonal skill—while routine cognitive skill shows no such relationship. These patterns suggest that modern jobs increasingly require workers to integrate diverse non-routine skills.

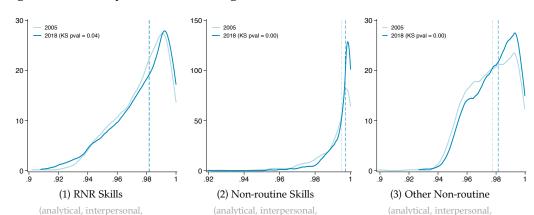


Figure 3: Density for Skill Mixing Indexes (Cosine Similarities), 2005 vs. 2018

Notes: These figures plot the kernel density of different skill mixing indexes in 2005 (light line) and 2018 (dark line). The x-axis displays the value of skill mixing indexes, with a maximum of 1 by construction. "RNR" stands for one routine skill and three non-routine skills (analytical, interpersonal, computer). "Non-routine" includes only the three non-routine skills. Specific descriptors of the skills are provided in online Appendix Table A2. These plots are created using O*NET at 7-digit occupation, unweighted by employment.

computer)

computer, leadership, design)

II.C Measuring Skill Mixing

computer, routine)

Next, I apply an angle-based index tomeasure the magnitude of skill mixing in a multidimensional skill space, among other measures. I begin by using granular 7-digit level data spanning from 2005 to 2018. Then, I examine the detailed time profile at the 3-digit level and pinpoint occupations where skill mixing is most evident. Throughout, I discuss the robustness of these findings to alternative measures and weighting methods.

Broad Pattern: Figure 3 shows the density and median values of two skill mixing indexes for 2005 and 2018 using 7-digit O*NET data. The first index covers four routine and non-routine (RNR) skills, while the second focuses on three non-routine skills (analytical, computer, and interpersonal). Panel A shows a modest rightward shift in the RNR skill mixing index, with the Kolmogorov-Smirnov test confirming this shift as statistically significant at the 1 percent level. Panel B reveals a more pronounced rightward shift in the non-routine skill mixing index, with a higher peak in 2018 compared to 2005, indicating a greater growth of non-routine skill mixing in occupations.²⁴

The growth in skill mixing is not unique to the choice of non-routine skills and becomes more pronounced when considering employment shares. Online Appendix Figure A2 panel A shows that the rightward shift in the mixing index persists with additional non-routine

²⁴In addition to index-based evaluations, one can non-parametrically analyze skill requirements in two-dimensional spaces. Online Appendix A.3 provides non-parametric plots for six skill pairs from 2005 and 2018, confirming an increase in skill mixing especially for non-routine skills.

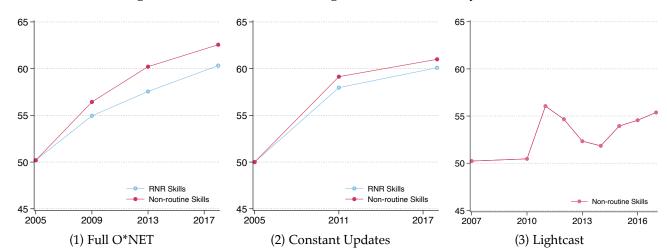


Figure 4: Trend of Skill Mixing in the US Economy, 2005-2018

Notes: These figures plot the employment-weighted skill mixing indexes in the U.S. economy from 2005 to 2018. The y-axis represents the percentiles of skill indexes in 2005. By construction, each index has a mean of 50th percentile in 2005; subsequent points are employment-weighted means mapped to their percentiles in 2005. Panels A and B combine O*NET and ACS data with consistent 3-digit occupation codes from Autor and Price (2013) and developed by Deming (2017). The matching of different O*NET releases and ACS years is detailed in online Appendix Table A1. Panel A shows the trend for the universe of occupations, while Panel B includes only 274 7-digit occupations that are consistently updated in 2005, 2011, and 2018. Panel C combines Lightcast job posting data and ACS using the same occupation coding. Employment weights from ACS represent the total hours of work aggregated by sex-education-industry-occupation cells.

skills (leadership and design). Panel B combines O*NET data with employment weights from the Occupational Employment and Wage Statistics (OEWS), showing a more pronounced rightward shift in skill mixing indexes when weighted by employment.²⁵ ²⁶

Time Trend: To examine the detailed time trend of skill mixing, I merge O*NET with ACS across various years using consistent 3-digit census occupation codes from Autor and Dorn (2013). I improve the analysis's reliability in two steps: First, I construct the trend at four-year intervals so that over half of the occupations (about 60 percent of employment) are updated between observations. Second, I follow the methods used by Autor, Levy, and Murnane (2003) and Deming (2017) to transform the skill mixing indexes into percentile values based on their initial 2005 rankings. I also weight these indices by the work hours in each sex-education-industry-occupation cell in the ACS, which helps control for changes in task inputs due to shifts in worker composition across these groups.

Figure 4 shows a substantial and steady rise in skill mixing between 2005 and 2018.

²⁵The OEWS uses 6-digit SOC codes, while O*NET uses 7-digit occupation codes that are based on 6-digit SOC. I match OEWS with O*NET at a 6-digit SOC level and distribute the employment weight evenly for 7-digit O*NET occupations within a 6-digit occupation.

 $^{^{26}}$ This result implies that occupations with larger employment shares have a higher rise in skill mixing.

By construction, each index has a mean of 50 percentiles in 2005; succeeding points are employment-weighted means of each index mapped to its percentiles in 2005. By 2018, the degree of non-routine skill mixing for the average US occupation is 12.4 percentiles higher than in 2005, while RNR skill mixing has increased by 10.1 percentiles.²⁷ Figure 4 panel B confirms this trend among constantly updated occupations, with the most significant increase occurring before 2011.

In panel C of Figure 4, I complement the analysis of skill mixing by combining Lightcast data with ACS, starting from 2007 when Lightcast first collects job postings. Firms increasingly post job requirements with mixed-skill demands, with skill mixing in online job postings averaging 5.2 percentiles higher in 2017 compared to 2007. Skill mixing trends in job postings show more volatility, first peaking in 2011, declining until 2014, and then rising substantially. Despite this variance, the trend confirms a higher demand for non-routine skill mixing.²⁸

To clarify the actual magnitude of the change, Table 1 details the occupations with the highest increases in skill mixing and their 2-digit occupation categories (in parentheses).²⁹ The largest increases in non-routine skill mixing appear in the service sector, particularly for sales and cleaning. For example, in 2005, sales counter clerks mainly need interpersonal and computer skills, with analytical skill being only 40 percent as important. By 2018, analytical skill become as crucial as computer skill and slightly more important than interpersonal skill. In contrast, the growth of interpersonal skill drives the skill mixing of occupations like packers and data entry keyers. For these jobs, analytical and computer skill are two to three times more important than interpersonal skill in 2005; by 2018, interpersonal skill rises to 65 to 80 percent of the importance of the other two skills. For routine-heavy occupations such as packers, assemblers, and operators, there is an increasing demand for all non-routine skills, raising their RNR skill mixing levels.

Robustness of the Trend: To address concerns that the observed trends might be influenced by the choice of skill measures or mixing indexes, I show the robustness of these trends in online Appendix A.6 and A.7 using various skill measures, alternative skill mixing indexes, as well as analyzing specific skill pairs. For example, using standardized or broader measures,

²⁷The inclusion of routine skill reducing the increase in skill mixing suggests that routine skills mix with other skills more slowly than non-routine skills do. Online Appendix Figure A7 shows the trend of skill mixing for specific skill pairs, indicating a rise of 2.9 percentiles in the mixing of routine with computer skills from 2005 to 2018. However, the mixing of routine with other non-routine skills has remained stable.

²⁸The volatility is partly due to the nature of the measure and data. Job postings are inherently noisier than O*NET's importance metrics and are influenced by firm entry and exit patterns.

²⁹To ease the presentation, the table includes only those occupations that constitute a minimum of 0.2 percent of overall employment, though all occupation codes have been used in the analysis. Online Appendix Table 1 shows the full table.

Table 1: Top Occupations in Skill Mixing Growth

Top Occupations	Year	Analytical	Computer	Inter- personal	Routine	Mixing Index	Percentile
	Panel A. Mix of Non-routine Skills						
Packers, fillers, and wrappers	2005	0.58	0.44	0.16		0.915	1
(Operators/Fabricators/Laborers)	2018	0.52	0.40	0.42		0.994	99
Housekeepers, maids, cleaners	2005	0.00	0.10	0.24		0.753	0
(Personal Care and Services)	2018	0.28	0.20	0.25		0.990	96
Sales counter clerks	2005	0.13	0.32	0.30		0.946	7
(Sales)	2018	0.50	0.52	0.39		0.993	99
Recreation facility attendants	2005	0.24	0.18	0.39		0.947	7
(Personal Care and Services)	2018	0.38	0.40	0.35		0.998	99
Data entry keyers	2005	0.56	0.77	0.27		0.935	3
(Office/Admin)	2018	0.55	0.66	0.43		0.985	90
	Panel B. Mix of RNR Skills						
Packers and packagers by hand	2005	0.16	0.16	0.30	0.71	0.824	0
(Operators/Fabricators/Laborers)	2018	0.49	0.40	0.54	0.70	0.979	99
Cashiers	2005	0.08	0.41	0.33	0.71	0.863	2
(Sales)	2018	0.31	0.41	0.49	0.61	0.973	99
Assemblers of electrical equipment	2005	0.35	0.25	0.34	0.82	0.894	5
(Operators/Fabricators/Laborers)	2018	0.44	0.43	0.40	0.65	0.979	99
Equipment cleaners	2005	0.23	0.24	0.26	0.63	0.896	5
(Operators/Fabricators/Laborers)	2018	0.41	0.32	0.52	0.54	0.981	99
Cooks	2005	0.24	0.16	0.34	0.59	0.899	6
(Food Prep/Buildings and Grounds)	2018	0.46	0.33	0.46	0.64	0.974	99

Notes: This table presents specific O*NET occupations at census SOC levels that have the greatest growth in skill mixing from 2005 to 2018. It provides details on compositions of skills within these occupations and the corresponding changes in skill mixing indexes. The last column translates skill mixing levels into percentiles relative to their 2005 distributions.

the increase in non-routine skill mixing ranges from 6 to 13 percentiles from 2005 to 2018. With the inverse Herfindahl-Hirschman Index, the increase for any skill pair exceeds 10 percentiles during the same period. Across all checks, the consistent finding is a significant rise in the degree of skill mixing, particularly for non-routine skills.

II.D Decomposing the Sources

To better understand the underlying variations in skill mixing and how it differs from other labor market trends, I conduct three exercises. First, I examine the contributions of different skills to the overall trend in skill mixing. Second, I decompose the longitudinal changes in skill mixing into intensive margin skill mixing changes and extensive margin employment shifts. Third, I perform regression analysis with extensive controls, including various measures of

Table 2: Shift-Share Decomposition of Skill Mixing Index Changes

	Skill Groups	7-digit Occupations			3-digit Occupations		
	Skiii Groups	total	within	across	total	within	across
Full O*NET	RNR Skills Non-routine Skills	6.78 9.21	4.93 5.62	1.85 3.59	10.12 12.37	9.46 9.72	0.66 2.65
Constant Updates	RNR Skills Non-routine Skills	5.59 4.05	6.73 5.33	-1.14 -1.29	10.09 11.00	10.74 9.69	-0.65 1.31
Lightcast	Non-routine Skills				5.16	4.37	0.78

Notes: This table shows a shift-share decomposition of changes in the average level of different mixing indexes between 2005-2018 in percentile units. Specifically, for a change in the percentile of a mixing index over two periods t and τ , its change $\Delta T_{\tau} = T_{\tau} - T_{t}$ can be decomposed to $\Delta T = \sum_{j} \left(\Delta E_{j\tau} \alpha_{j}\right) + \sum_{j} \left(E_{j} \Delta \alpha_{j\tau}\right) = \Delta T^{a} + \Delta T^{w}$ where $E_{j\tau}$ is employment weight in occupation j in year τ , and $\alpha_{j\tau}$ is the level of mixing index h in occupation j in year τ , $E_{j} = \frac{1}{2}(E_{jt} + E_{j\tau})$ and $\alpha_{j} = \frac{1}{2}(\alpha_{jt} + \alpha_{j\tau})$. ΔT^{a} and ΔT^{w} then represent across-occupation and within-occupation change.

skill supply and fixed effects for worker composition to see if the trend of increasing skill mixing persists.

Driving Skill: I examine the driving skill of the mixing trends using two approaches. In the online Appendix Figure *A*2 panel C, I exclude each non-routine skill individually to compute the skill mixing index. A consistent rightward shift in density was observed regardless of the omitted skill, indicating that all skills contribute to the overall increase. Further, online Appendix Table *A*5 shows a decomposition of changes in non-routine skill mixing using polynomial regressions.³⁰ The findings confirm that each skill influences the trend, with computer skill being the most significant overall. However, interpersonal skill is more critical in higher-paid managerial and white-collar occupations, while computer skill remains dominant in medium to lower-paid blue-collar and service jobs.

Within vs. Across Occupations: Table 2 presents a shift-share decomposition of changes in employment-weighted skill mixing indexes at both 7-digit O*NET and 3-digit census occupation levels, using employment weights from the OEWS and ACS, respectively. Across all datasets and skill groups, the rise in skill mixing is primarily driven by within-occupation variations, contrasting with across-occupation changes reported in Autor and Price (2013)

 $^{^{30}}$ Since the mixing index from cosine similarity is nonlinear in the skills, standard variance decomposition proves ineffective. Instead, I employ polynomial regression for the non-routine skill mixing index and each composing skill's polynomials up to order N: $\text{Mix}(\mathbf{y})^{\text{percentile}}_{ijt} = \beta_1 y^1_{ijt} + \beta_2 y^2_{ijt} + ... + \beta_N y^N_{ijt}$. Here, $\text{Mix}(\mathbf{y})^{\text{percentile}}_{ijt}$ represents the percentile rank of an individual's mixing index of non-routine skills in occupation j at time t, and y_{ijt} is the measure of a specific skill for that occupation. The R^2 evaluates the extent to which each skill explains the variance in skill mixing.

Table 3: Annual Changes in Skill Mixing Indexes (in Percentiles)

	RNR Skills (1)	Non-routine Skills (2)				
	Panel A. Full O*NET, 2005-2018					
Year indicator	0.70***	0.71*** [0.06]				
Observations	237,885	237,885				
R-squared	0.83	0.83				
	Panel B. Constant Updates, 2005					
Variation that is	0.75***	0.65***				
Year indicator	[0.11]	[0.11]				
Observations	107,956	107,956				
R-squared	0.81	0.82				
	Panel C.	Lightcast, 2007-2017				
N/ 11 /		0.33**				
Year indicator		[0.15]				
Observations		532,636				
R-squared		0.87				
Experience and edu controls	X	X				
Gender \times edu \times ind \times occ FE	X	X				

Notes: This table provides regression results on the relationship between the percentile values of RNR skills and Non-routine skills, based on their 2005 distributions, and a time trend variable (year values). The analysis includes data from the full O*NET, constantly updated O*NET, and Lightcast datasets combined with ACS. See online Appendix A.1 and A.6 for data construction details. The regressions control for gender-education-industry-occupation fixed effects, polynomials of work experience up to the fourth power, and the interaction of experience polynomials with education fixed effects and gender. Education fixed effects include five categories (no high school, high school graduate, some college, college graduate, post-college). *** p<0.01, ** p<0.05, and * p<0.1.

and Deming (2017). For example, for the 12.4 percentile increase in non-routine skill mixing in O*NET data at the 3-digit level, 9.7 percentiles come from within-occupation changes; for Lightcast data, the 5.2 percentile increase in non-routine skill mixing is driven by 4.4 percentiles of within-occupation changes.³¹ Interestingly, for consistently updated 7-digit occupations, within-occupation variation more than accounts for skill mixing growth in these regularly updated jobs.³²

Worker Composition and Supply: An alternative explanation for employer-side shifts in skill mixing might involve changes in labor supply, such as increased human capital or female

³¹Online Appendix A6 provides further decomposition results using skill mixing indexes for various skill pairs, showing consistent findings.

³²At 3-digit occupations, worker reallocation does contribute positively to these increases in the mixing of non-routine skills, but the influence is still marginal compared to within-occupation variation; for RNR skills, the contribution remains negative.

workforce participation. Table 3 provides regression results of skill mixing indexes against a linear time trend using O*NET and Lightcast combined with ACS data, adjusting worker composition across gender, education, industry, and occupation groups, along with flexible polynomials of education and experience years as well as their interactions. The results indicate a consistent annual increase in skill mixing of 0.65 to 0.75 percentiles using O*NET data and 0.33 percentiles with Lightcast data. This trend persists across all demographic and sectoral groups and is robust to labor supply controls, highlighting the important impact of demand-side forces.

II.E Heterogeneity in Skill Mixing Changes

To illustrate which occupations and worker groups have seen the highest growth in skill mixing, panel A of Figure 5 shows the changes in skill mixing indexes from 2005 to 2018 across four major occupational groups by wage levels, covering all non-agricultural employment.³³ Changes are measured in percentiles based on the 2005 distributions of skill mixing indexes, similar to Figure 4. For non-routine skill mixing, service and white-collar occupations experience higher increases than other groups. In contrast, blue-collar and service occupations show the largest rises in mixing RNR skills, whereas high-wage occupations exhibit the least growth in mixing RNR skills and only modest increases in non-routine skills. This trend indicates that most skill mixing occurs in medium- to lower-wage occupations, particularly within the service sector.³⁴

Turning to skill mixing across demographic groups, panels B and C of Figure 5 detail these shifts for RNR skills and non-routine skills categorized by gender and education levels. Workers without a college degree see more increases in RNR skill mixing. Gender differences appear, with male workers experiencing a 5 percentile higher increase in skill mixing than female workers on average, particularly among the highly educated. For non-routine skill mixing, increases are relatively consistent across education levels for male workers, while female workers show slower increases as education levels rise, remaining below those of their male counterparts with college education or higher.

I present additional heterogeneity in skill mixing changes in online Appendix A.4. For

³³The categorization into four groups is based on Acemoglu and Autor (2011), which is derived from 10 1-digit occupational groups that cover the entirety of US non-agricultural employment. Specifically, "High-wage" includes Managers, Professionals, and Technicians; "White-collar" comprises Office/Administrative and Sales roles; "Blue-collar" includes Production/Craft/Repair, as well as Operators/Fabricators/Laborers; and "Service" consists of Protective Services, Food Preparation/Cleaning, and Personal Care Services occupations.

³⁴For more detailed data at the 2-digit occupation level, see online Appendix A.4. Notably, Personal Care and Sales occupations show substantial increases in non-routine skill mixing, while Operators/Fabricators/Laborers, Sales, and Food Prep/Buildings/Cleaning occupations exhibit significant growth in RNR skill mixing.

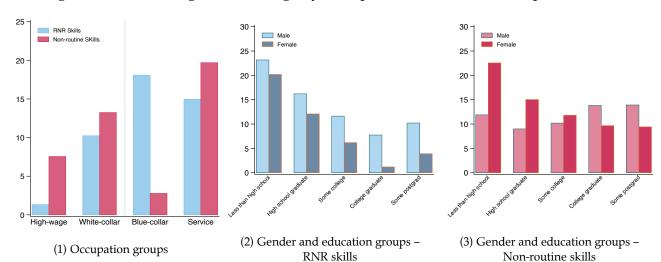


Figure 5: Skill Mixing Index Change by Occupation and Gender Groups, 2005-2018

Notes: This figure plots the changes in skill mixing indexes across different occupation groups and gender-education combinations. The units of the index changes are percentiles of their distributions in 2005. The occupation groups (High-wage, White-collar, Blue-collar, Service) follow Acemoglu and Autor (2011). The education grouping aligns with the educational categories defined in the ACS. O*NET and ACS data are combined for these figures using consistent occupation codes from Autor and Price (2013) and developed by Deming (2017).

different industries, the private service sector followed by retail trade and construction leads others in the growth of skill mixing, while public, social, and professional services sectors demonstrate only modest increases, particularly for RNR skills.³⁵ I also show the decomposition of occupations' changing mixing of distinct skill pairs, which indicates that non-routine skills drive skill mixing in all occupations, while routine skill is only more mixed with other skills in blue-collar and service occupations.

III Returns to Skill Mixing

In order to better understand the connection between skill mixing and workers' labor market outcomes, this section examines the returns associated with skill mixing linked to occupation choices and inherent worker abilities. I show that there is a positive wage premium associated with working in an occupation that becomes more mixed of the non-routine skills, or possessing a more mixed set of these skills. Furthermore, I demonstrate the complementarity between occupation-level and worker-level skill mixing, as their interaction yields positive

³⁵The sectors that have the least growth in skill mixing are public and education and social services. This result is consistent with Hershbein and Kahn (2018) that industries with locally consumed goods are more likely to change skill demand.

returns. This motivates the development of a bilateral matching model in the next section, which replicates the wage returns presented here. Additionally, I provide indicative evidence of an employment premium for mixing non-routine skills and wage returns for studying college majors with a greater mix of non-routine skills.

III.A Data and Measurement

To assess wage returns related to skill mixing, I use National Longitudinal Survey of Youth (NLSY) data from the 1979 and 1997 cohorts, which provide detailed records of workers' employment and education histories. I combine these cohorts to increase the sample size and focus on the period from 2005 to 2019 to align with my O*NET skill mixing data.³⁶ I link the NLSY data to O*NET using NLSY's census occupation information and the crosswalk from Autor and Dorn (2013). My primary measure is the real log hourly wage, adjusted to 2013 dollars, with values below \$3 or above \$200 trimmed following Altonji, Bharadwaj, and Lange (2012). The results are robust to excluding respondents over age 55 or using unprocessed real hourly wages.

The key advantage of the NLSY data is that it is a worker-level panel with detailed workers' pre-market abilities. This allows for controlling worker characteristics when assessing occupational wage returns and evaluating returns to the worker-level skill mixing. The chosen measures of worker abilities align closely with the skill categories in O*NET: AFQT scores represent analytical skill, the social skill measure by Deming (2017) captures interpersonal skills,³⁷ and routine skill is measured using ASVAB mechanical orientation scores.³⁸ Since NLSY has limited data on workers' computer skill, I use the computer skill value of the worker's occupation or college major in 2005 as a proxy for initial computer skill level. Online Appendix Table B1 lists these measures.

³⁶The NLSY 1979 and NLSY 1997 are nationally representative surveys of youth, capturing data from individuals aged 14 to 22 in 1979 and 12 to 16 in 1997. During my sample period, the median age is 37, with 91 percent of the sample under 50.

³⁷I use AFQT scores from Altonji, Bharadwaj, and Lange (2012), which are consistent across NLSY waves and account for factors such as age-at-test and test format. For interpersonal skill, I use the social skill measure developed by Deming (2017), based on childhood and adulthood sociability in NLSY79, and two questions from the Big 5 inventory in NLSY97.

³⁸The ASVAB scores are only available in NLSY79. For NLSY97, I impute ASVAB scores using a regression model including gender, ethnicity, age, year, census division, metropolitan area, and urbanicity. This is less critical since the main returns to skill mixing are associated with non-routine skills.

III.B Wage Returns

To examine the wage returns to skill mixing, I regress workers' real log wages on the skill levels required by their occupations and the mixing indexes for these skills. The coefficients on the skill mixing indexes, conditioned on skill levels, capture the wage returns to working in more skill-mixed occupations. Additionally, I include workers' skill levels and their degrees of mixing to determine the wage returns to worker skill mixing, conditioned on occupational skill requirements. This focus remains on the mixing of three non-routine skills (analytical, computer, interpersonal) due to its notable increase over the study period, as discussed in Section II. A detailed examination of the returns to mixing routine skill with other skills and to individual skills is presented in the online Appendix A.8.

In all specifications, I include fixed effects for ethnicity by gender, age, metropolitan status, individual year, years of education, census region, and urbanicity. I also add occupation fixed effects to control for time-invariant differences across occupations, allowing me to focus on how changes in skill requirements within occupations relate to wage changes, as this margin is the primary source of skill mixing. I cluster standard errors at the worker level to address within-group correlation and heteroskedasticity among repeated observations at the individual level.

Occupation and Worker Level Returns: Table 4 presents the wage returns to skill mixing at both the occupation and individual levels. Column (1) shows that workers in occupations that become one standard deviation more mixed among analytical, computer, and interpersonal skills earn a wage premium of 1.7 percent per year. In column (2), I include the mixing index of worker abilities, which suggests that conditional on the occupational-level returns, workers who are one standard deviation more mixed in non-routine skills earn 6.5 percent more. Meanwhile, the wage premium at the occupational level for mixing the three non-routine skills remains at 1.5 percent per year.

I further analyze the complementarity between occupation-level and worker-level skill mixing by including an interaction term between the two in column (3). The results show a 3.2 percent wage premium, indicating positive complementarity between skill mixing at both levels. While the worker-level skill mixing premium is accurately measured at 7 percent, the occupational-level returns are not statistically significant. This suggests that the economic benefits of increased skill mixing within an occupation mainly accrue to workers who possess a mixed set of non-routine skills to begin with. This observed complementarity provides the rationale for developing a bilateral matching model in which the match output depends on both worker and occupation skill profiles.

Table 4: Return to Skill Mixing: Occupations and Workers

Dependent: In (hourly wage)	(1)	(2)	(3)	(4)
Mix (non-routine skills): Occupation	0.017***	0.015***	0.001	0.014***
Mix (non-routine skills): Worker	[0.005]	[0.005] 0.065*** [0.017]	[0.006] 0.070*** [0.017]	[0.005]
Interaction			0.032*** [0.008]	
Ethnicity, gender, age/year, region, edu FE	X	X	X	Χ
Occupation FE	X	X	X	X
Worker FE				X
Observations	88,391	79,343	79,343	88,391
R-squared	0.41	0.43	0.43	0.76

Notes: This table reports the result of estimating wage equations using pooled NLSY79&97 data for employed workers from 2005-2019. Log hourly wages are the outcome variables and person-year is the unit of observation. The occupational skill and skill mixing measures come directly from O*NET and are merged to NLSY79&97 based on census occupation codes. The worker-level skill measures are constructed to correspond to occupation-level measures as in Table B1 and skill mixing indexes are then calculated accordingly. All measures of skill and skill mixing are normalized to have mean 0 and standard deviation 1. Ethnicity-by-gender, age, year, census region, urbanicity, and a 5-category (no high-school, high-school graduate, some college, college graduate, post-college) education fixed effects are included for all regressions, with additional fixed effects as indicated in the table. Standard errors are clustered at the individual level. *** p<0.01, ** p<0.05, * p<0.10.

Lastly, in column (4), I restrict the analysis to within-worker variation by adding worker fixed effects; combined with the occupation fixed effects, this specification closely aligns with an AKM model.³⁹ Despite further limiting the variations to estimate wage returns, the magnitude of the returns to skill mixing shown in column (4) is comparable to those presented in column (2). Workers in occupations that become one standard deviation more mixed among non-routine skills experience a wage increase of 1.4 percent.

Robustness and Additional Returns: To better understand the drivers of the positive wage returns to skill mixing, online Appendix Table ?? uses skill pair mixing indexes rather than high-dimensional ones. The results show that wage returns mainly come from mixing analytical with computer skills and analytical with interpersonal skills. Robustness checks in online Appendix A10 confirm that occupational returns to skill mixing, adjusted for worker fixed effects, remain robust to alternative skill measures and indexes of mixing, with a wage premium of 1 to 2.5 percent in occupations that mix these skills. Additionally, online Appendix Table A9 provides indicative evidence of a positive employment premium for unemployed

³⁹Using within-worker variation to study wage growth has been discussed and applied in e.g., Neal (1999); Gibbons et al. (2005); Lazear (2009) and Deming (2017). Choné and Kramarz (2021) found that under a worker assignment model with bundled skills, the implied wage equation also has an AKM form.

workers with a more mixed set of non-routine skills.⁴⁰

Using college education data from NLSY, I further assess the returns to different college majors' skill mixing. ⁴¹ I compute the skill mixing index for each major by calculating the employment-weighted average of skill intensities in O*NET of occupations employing workers from that major. ⁴² Online Appendix Table A11 shows the skill mixing levels and changes for different majors. Architecture and Environmental Design, followed by Computer and Information Sciences, exhibit high levels of non-routine skill mixing. Social Sciences and Agriculture and Natural Resources stand out at mixing routine and non-routine skills. Notably, online Appendix Table A9 column (5) indicates a positive wage return of about 3 percent for workers who studied majors that are a standard deviation higher in non-routine skill mixing.

IV A Multi-Dimensional Directed Search Model with Occupation Design

The rich empirical findings on skill mixing present challenges in understanding their underlying forces. To address this, I develop a multi-dimensional skill directed search model with several novel features: First, firms and workers operate in a multi-dimensional skill space with non-linear production and cost technologies. Second, firms decide their occupations' skill demands before meeting with workers, incurring a cost payable upon operation as in Acemoglu (1999). Third, workers indirectly invest in their skills alongside their occupation choices. The model thus allows for endogenous decisions by both firms and workers under high-dimensional heterogeneity. Despite the richness, the model remains tractable satisfying Block Recursivity as in Menzio and Shi (2011).

IV.A Environment

Workers: Time is discrete. At each period, there is a unit measure of heterogeneous workers that lives forever. Each worker of type i is characterized by a vector of multi-dimensional

⁴⁰Throughout my analysis, I classify a worker as employed if the worker earns a wage greater than zero and has held one or more jobs since the last NLSY interview, consistent with Altonji, Bharadwaj, and Lange (2012) and Deming (2017). Additionally, workers without a paying job for 24 months are considered to be out of the labor force.

⁴¹There are some inconsistencies in NLSY's field of study coding: NLSY79 uses its own major codes that contain 25 two-digit categories, while NLSY97 uses another set codes for years leading to 2010 and transfers to National Center for Education Statistics (NCES)'s 2010 College Course Map (CM10) for years after 2010. For consistency, I map the two different types of major codes in NLSY97 to the 25 two-digit major categories in NLSY79. Online Appendix Table A12 shows the crosswalk of different types of major field of study codes.

⁴²I take the first field within a year as representing a worker's major field in the case of multiple fields.

skills $\mathbf{x}^i = \{x_1^i, ..., x_k^i, ..., x_K^i\} \in S \subset \mathbb{R}^{K+}$, where K is the dimension of a closed skill space S. Workers draw their initial skill vectors at the beginning of the period from an exogenous distribution $G(\mathbf{x})$. Workers are risk-neutral, have linear utilities over consumption, and discount the future with a factor β .

Firms: On the other side of the market, there is a mass of risk-neutral firms each running one vacancy. Firms pay a cost c to post their vacancies across different occupations $j = \{1, ..., J\}$, with $J \ge 2$. Each occupation is characterized in the same multi-dimensional skill space as workers' skills, $\mathbf{y}^j = \{y_1^j, ..., y_k^j, ..., y_K^j\} \in S \subset \mathbb{R}^{K+}$, which has the interpretation of a vector of skill requirements or skill importance for each of the worker skills. Firms share workers' discount factor β.

The production function of each worker-firm pair takes a CES form of the skill inputs of workers and skill requirements of an occupation that the firm operates:

$$f(\mathbf{x}^i, \mathbf{y}^j) = \left[\sum_{k=1}^K (x_k^i \alpha_k y_k^j)^{\sigma^j}\right]^{\frac{1}{\sigma^j}},$$
(2)

where α_k controls the efficiency between worker skill and job skill requirement for a specific skill k, while σ^j governs the elasticity of substitution among different skills for an occupation j.⁴³ This production technology extends prior multi-dimensional skill matching models (i.e., Lise and Postel-Vinay 2020; Lindenlaub 2017; Ocampo 2022) by incorporating across-skill complementarity controlled by σ^j , in addition to the multiplicative combination of worker and firm attributes intervened by skill efficiencies. Considering multi-dimensional skill distributions of workers and firms, this technology allows a clear portrayal of the interaction between skill demand and supply and among different skills.⁴⁴ Due to one-to-one matching, I omit the superscripts for worker and occupation skill vectors i and j below.

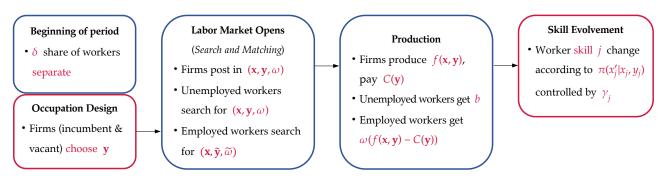
A unique feature of this model is that firms actively *design* the jobs before meeting with workers (Acemoglu 1999), leading to endogenous demand specialization and degrees of skill mixing. Firms with both incumbent jobs and unfilled vacancies set the occupational skill requirements **y** each period, changing the efficiency of skills and essentially altering the production technologies that reflect the quality and optimal skill demands of the occupation.⁴⁵ In designing occupations, firms consider worker skill supply to optimize the use of available

⁴³Since labor is the only input in the model, it can be understood as "equipped" labor, and occupations' skill requirement or importance **y** takes a factor augmenting form, essentially acting as demand shifters.

⁴⁴As such, the model explores both the role of changes in relative input efficiency that is the focus of task-based literature and changes in skill complementarity.

⁴⁵This reflects empirical observations of changing skill mixing in both incumbent roles and vacancies.

Figure 6: Model Timing



skills given the production technology.⁴⁶ The general equilibrium intensity of skills will also determined by the search forces.

Nonetheless, such a job design incurs a cost $C(\mathbf{y})$ that is payable upon producing with a worker. This cost is strictly convex and increases with the skill level chosen by the firm $(\frac{\partial C(\mathbf{y})}{\partial y_k} > 0, \frac{\partial^2 C(\mathbf{y})}{\partial y_k^2} > 0, \forall k)$, representing all non-wage expenses necessary to operate an occupation that increase in skill level, such as operation costs as in Hopenhayn (1992).⁴⁷

Labor Market: There is a continuum of submarkets that are indexed by worker and occupation skill profiles (\mathbf{x}, \mathbf{y}) , as well as the share of worker-firm surplus ω that firms promise to workers. Workers with skill profile \mathbf{x} direct their search towards different occupations and surplus shares, meeting one vacancy at a time. Matching between workers and firms is frictional and is regulated by a standard constant to scale matching function. Under this directed search environment, each submarket has a separate tightness (vacancy-unemployment ratio), denoted by $\theta(\mathbf{x}, \mathbf{y}, \omega)$. In each submarket, workers find job with probability $p(\theta(\mathbf{x}, \mathbf{y}, \omega))$ and firms fill the vacancy with probability $p(\theta(\mathbf{x}, \mathbf{y}, \omega)) = p(\theta(\mathbf{x}, \mathbf{y}, \omega))/\theta(\mathbf{x}, \mathbf{y}, \omega)$.

The timing of the model evolves as follows. At each period's beginning, a fraction δ of worker-firm pairs separate exogenously. Before the labor market opens, unlike standard

⁴⁶For instance, in designing roles like salespersons for lower-skill workers with stronger interpersonal skills, firms might prioritize these skills to leverage the available labor supply. Conversely, if technological changes enhance the complementarity between analytical and interpersonal skills, firms might adjust to intensify the use of analytical skills.

⁴⁷For example, to design and operate an occupation that employs high-skill workers, a firm will need to incur higher expenses in terms of better offices and equipment rentals. Structurally, any variation in employment distribution and skill demand not explained by wages is explained by this cost.

 $^{^{48}}$ This arrangement can be considered as an employment contract simply specifies the surplus share ω promised to the worker contingent on the state for the current period, as well as the continuation value of the match in the subsequent period (see next section). The contract is assumed to be fully committed by both the workers and firms

⁴⁹Functions p and q also satisfy usual regularity conditions: twice continuously differentiable; $p'(\theta) > 0$, $p''(\theta) < 0$, p(0) = 0; $q'(\theta) < 0$, $q''(\theta) > 0$, q(0) = 1.

search models, firms design occupations for both unfilled vacancies and incumbent jobs. The labor market, comprising various submarkets, then opens; both unemployed and employed workers search for vacancies and form matches using a constant return to scale matching technology. Once the labor market closes, firms produce, incur operation costs, and pay wages that are a constant share of the surplus. Unemployed workers receive a transfer valued at *b*. Lastly, worker skills evolve through learning by doing, governed by a Markov process based on their employment status, as described below.

Aggregate State and Worker Skill Investment: The aggregate state of the economy is the distribution of workers across employment status, skill profiles, occupational skill requirements, and surplus shares, denoted as $\psi \in \Psi$. I subsume aggregate state in the exposition of model equilibrium in the next section and show that in fact, the model equilibrium is independent of the aggregate state.

$$\pi(x_j'|x_j, y_j) = \frac{y_j - x_j}{x_j' - x_j} \mathbf{1}(x_j < y_j) \times \gamma_j^{up} + \frac{y_j - x_j}{x_j' - x_j} \mathbf{1}(y_j < x_j) \times \gamma_j^{down}.$$
 (3)

Nonetheless, in the model, I allow workers to indirectly invest in their skills through on-the-job learning, with their skill profiles evolving based on their current employment status as in Lise and Postel-Vinay (2020). Specifically, Each skill j within a worker's profile \mathbf{x} follows a Markov process $\pi(x_j'|x_j,y_j)$ as in equation 3. A worker's skill level x_j' in the next period depends on how their current skill level x_j compares to the skill level y_j of their current occupation. If y_j exceeds x_j , the worker's skill j will increase: $x_j' > x_j$, and vice versa. The speed of skill adjustment γ_j is specific to each skill and the adjustment direction. Since occupation choices are directed in the model, workers effectively determine their optimal skill paths along with their chosen occupations. Unemployed workers are considered to be in an occupation that demands a zero level for all skills.

IV.B Model Equilibrium

I will now characterize the optimal strategies for workers' job search and firms' job creation and continuation. The value functions for workers are described at the point of the production stage when the labor market comes to a close, while for firms I also consider the job design stage before the labor market opens.

Worker's Problem: Let $U(\mathbf{x})$ denote the value of being unemployed and searching for a job for worker \mathbf{x} . Similarly, let $W(\mathbf{x}, \mathbf{y}, \omega)$ be the total discounted returns from holding a job of

skill requirements y and surplus share ω at time t. These values can be written as:

$$U(\mathbf{x}) = b + \beta E \left\{ \max_{\mathbf{y}',\omega'} p(\theta(\mathbf{x}',\mathbf{y}',\omega')) W(\mathbf{x}',\mathbf{y}',\omega') + \left[(1 - p(\theta(\mathbf{x}',\mathbf{y}',\omega'))] U(\mathbf{x}') \right] \right\}$$

$$W(\mathbf{x},\mathbf{y},\omega) = \omega (f(\mathbf{x},\mathbf{y}) - C(\mathbf{y})) + \delta U(\mathbf{x}')$$

$$+ \beta (1 - \delta) E \left\{ \max_{\tilde{\mathbf{y}}',\tilde{\omega}'} p(\theta(\mathbf{x}',\tilde{\mathbf{y}}',\tilde{\omega}')) W(\mathbf{x}',\tilde{\mathbf{y}}',\tilde{\omega}') + \left[(1 - p(\theta(\mathbf{x}',\tilde{\mathbf{y}}',\tilde{\omega}'))] W(\mathbf{x}',\mathbf{y}',\omega) \right] \right\}$$

$$(4)$$

Unemployed workers receive a utility b from the current period's transfer. In the next period, their skills transition to \mathbf{x}' , which may depreciate due to unemployment. During this time, within the submarket of their skill profiles, they search for vacancies across various occupations \mathbf{y} and surplus shares ω to maximize their continuation value. In selecting \mathbf{y} and ω , workers weigh the potential value of employment against the probability of successfully matching, $p(\theta(\mathbf{x}', \mathbf{y}', \omega'))$, both influenced by their targeted occupation and surplus share. If a match is successful, they gain the continued benefits of employment.

Employed workers at a firm defined by (\mathbf{y}, ω) earn wages that are a share ω of the output after operation costs. As the next period starts, they face a separation probability δ , potentially leading to unemployment with a subsequent value $U(\mathbf{x}')$ and an immediate job search. These workers also search on-the-job for new job opportunities $(\tilde{\mathbf{y}}', \tilde{\omega}')$, with a probability $p(\theta(\mathbf{x}', \tilde{\mathbf{y}}', \tilde{\omega}'))$ that a new of transition to the new job. If no better opportunities arise or if attempts to transition fail, they remain with their current employer.

Firm's Problem: Consider a firm running occupation \mathbf{y} , offering surplus share ω , and employing worker \mathbf{x} . Let $J(\mathbf{x}, \mathbf{y}, \omega)$ denote the total discounted profits to this firm:

$$J(\mathbf{x}, \mathbf{y}, \omega) = \max_{\mathbf{y}} (1 - \omega) (f(\mathbf{x}, \mathbf{y}) - C(\mathbf{y}))$$

$$+ \beta (1 - \delta) E \left\{ (1 - p(\theta(\mathbf{x}', \tilde{\mathbf{y}}', \tilde{\omega}')) J(\mathbf{x}', \mathbf{y}', \omega) \right\}$$
(5)

In the current period, firms receive a portion $(1 - \omega)$ of the worker-firm surplus after production, during which they must cover the occupation operation cost $C(\mathbf{y})$ that depends on the skill levels firms choose. The labor market operates under free entry, so maintaining a vacancy holds no value. If there is exogenous separation or the worker finds another job characterized by $\tilde{\mathbf{y}}'$ and $\tilde{\omega}'$ through on-the-job search with probability $p(\theta(\mathbf{x}', \tilde{\mathbf{y}}', \tilde{\omega}'))$, the firm earns no profits. However, if the match persists, the firm continues to gain discounted profits

from the match.

$$c = \beta E \Big\{ q(\theta(\mathbf{x}, \mathbf{y}, \omega)) J(\mathbf{x}, \mathbf{y}, \omega) \Big\}$$
 (6)

The free-entry condition determines market tightness $\theta(\mathbf{x}, \mathbf{y}, \omega)$ and illustrates firms' tradeoffs in skill demand. With the value of a vacancy at zero, firms will seek an optimal skill mix that equates anticipated discounted profits to the cost of posting vacancies, as shown in equation (6). Firms aim to design high-value occupations to retain workers, but if a particular occupation becomes more profitable, the number of posted vacancies will increase, leading to higher market tightness and a lower job-filling rate.⁵⁰ Since firms with incumbent jobs and unfilled vacancies use the same production technologies and confront the same worker skills within each submarket, their choices align.

The free entry condition also reflects the trade-offs faced by workers. Since workers receive the remaining surplus claimed by the firms, in markets with higher job-finding probabilities (i.e., tighter markets), the value of employment is lower. Workers' job-finding probability also feeds back to firms' discounted profits through worker on-the-job search and the chance that the firm attracts other employed workers.

Block-recursive Equilibrium: Despite the multi-dimensional skill setup, the model remains tractable by avoiding reliance on the full distribution of agents when characterizing value functions and market tightness, a feature termed "block-recursive" in Menzio and Shi (2010) and Menzio and Shi (2011) for various directed search models. This results from two model features. First, since the search is directed and workers optimally choose their occupation and surplus share, their life utility does not depend on outside options, eliminating the need to forecast wages based on the full employment distribution. Second, separate markets exist for different worker profiles, where firms offer various occupations. This separability means submarket tightness is independent of other markets so workers and firms need not predict conditions elsewhere. In online Appendix B.2, I formally define a Block-recursive equilibrium and prove its existence and uniqueness.

⁵⁰As in other directed search models, only a portion of submarkets may open in equilibrium, depending on firm's value and corresponding market tightness in different markets.

⁵¹Block recursivity allows not only analytical tractability but also enables standard numerical techniques to solve the model. The framework considered in this paper involves more heterogeneity and requires an additional degree of directness, as discussed.

⁵²This directness means, for example, that computer scientists only compete with other computer scientists, while sales clerks compete only with other sales clerks. In reality, separability varies by occupation and economic conditions. As noted by Osberg (1993), search directedness is procyclical and increases when the market is tight. In bringing the model to the data, I focus on economic recovery periods and use broader occupational groups to be consistent with the model.

Thanks to its block-recursiveness and analytical tractability, the model predicts changes in skill mixing, wages, and job finding, highlighting the influence of production technology, skill complementarity, associated costs, and worker skill heterogeneity. Formal propositions and proofs of these outcomes are detailed in the online Appendix B.1.

V Model Quantification

I parameterize the model equilibrium to two periods of data from the early 2000s to the late 2010s, which captures a substantial shift in skill mixing and abstracts from the Great Recession. First, I outline the data construction, measurement, as well as functional form assumptions. Second, I discuss the calibration strategy, the estimated results of key parameters, and the implied technological changes from the estimation results. Lastly, I present worker sorting and the job ladder under the baseline calibration.

V.A Measurement and Functional Forms

I use the same combination of NLSY 79 & 97 and ONET data as in Section III to quantify the model with three skills. NLSY data informs us about worker abilities (x) and captures changes in employment and wages, while ONET provides occupational skill requirements (y). For consistency, the sample is restricted to data from 2005–2006 and 2016–2019 and includes only workers with available skill information.⁵³ Finally, for both worker and job skill profiles, I consider the same set of skills (analytical, computer, interpersonal, routine) as in Sections II and III, only that I combine analytical and computer skills to have a three-dimensionality feasible for quantitative analysis (K = 3, $k = \{$ analytical/computer (a), interpersonal (p), routine (r) $\}$).⁵⁴

Considering the potential influence of skill supply on skill mixing, I calibrate worker skill distribution $G(\mathbf{x})$ across two periods to reflect choices of occupations and college majors, aligning with the Markov adjustment described in equation 3. Workers accumulate skills at a rate γ_j based on the gap between their current skills and the requirements of their occupation or college major, using NLSY's occupational/major data. The adjustment rate γ_j varies by skill and the direction of adjustment using estimates from Lise and Postel-Vinay (2020). Figure 7

⁵³NLSY 1997 was conducted annually during 2005-2006 but biannually in 2016-2019, as was NLSY 1979 for the later period. The sample sizes for these two periods are 30,654 and 43,340 respectively.

⁵⁴As I merge analytical and computer skills into one for calibration using their average values, I denote this combined skill as "analytical/computer".

⁵⁵Details of this calibration are available in online Appendix B.4. Workers' skills adjust downward when unemployed but cannot be lower than their initial endowments. For skill changes while in college, I specify that workers spend on average 3 years learning the skills of their majors.

Figure 7: Worker Skill Distribution Shifts

Notes: This figure illustrates the evolution of the skill distribution for different types of workers over the years 2005 (shown in blue) and 2018 (shown in cranberry), across three distinct two-dimensional skill spaces. These worker skills are measured using data from NLSY79&97, with the specific skill measure discussed in the online Appendix Table B1. Skill variations of these worker types are calibrated based on the skill accumulation and depreciation rates associated with different occupations and college majors, using the estimates of by Lise and Postel-Vinay (2020).

illustrates changes in $G(\mathbf{x})$ for four worker types in each two-dimensional skill space, with circle sizes reflecting the probability of skill combinations. In spaces for analytical/computer and interpersonal skills, there is a clear increase in the mixing of worker skill supply. In contrast, for routine skills, there is a trend toward specialization, shown by larger areas in off-diagonal skill combinations.

To map occupations and workers in the model to the data, I set grid points as follows. I classify occupations into high- and low-wage, as in Section III, with the former group including managerial, professional, and white-collar occupations, and the latter blue-collar and service occupations. The grid point for an occupation's requirement of a skill y_j is set such that moving up one grid corresponds to 50 percent of the average observed value of y_j for that occupation.⁵⁶ On the worker side, workers are classified based on their skill level x_j : those with skills above the average are deemed high type and assigned the mean of the above-average values; those below the average are considered low type and assigned the mean of the below-average values.⁵⁷

Functional Forms: The functional forms are chosen as follows. The multi-dimensional skill production function is defined in equation (2), which extends the multi-dimensional matching literature (i.e., Lise and Postel-Vinay 2020; Lindenlaub 2017; Ocampo 2022) by

⁵⁶As the model calibration uses data of two periods with a consistent grid, I determine grid points by averaging the occupation's median values across both periods.

⁵⁷With three chosen skills, there are 8 worker types in the model.

Table 5: Moments and Model Match

	First Period		Second	d Period	
	Data	Model	Data	Model	
	Panel A. Worker moments				
Relative wage of high type					
Analytical/computer	1.46	1.83	1.60	1.61	
Interpersonal	1.05	1.13	1.20	1.22	
Routine	1.12	1.45	0.92	1.47	
Wage return of skill mixing (untargeted)	0.07	0.04	0.07	0.04	
Unemployment Rate	0.05	0.07	0.04	0.07	
	Panel B. Occupation moments				
Relative wage of high skill	1.30	1.20	1.56	1.50	
Corr. wage & abilities (low-wage)	0.69	0.64	0.49	0.47	
Corr. wage & abilities (high-wage)	0.58	0.62	0.60	0.64	
Employ. share (low-wage)	0.43	0.33	0.37	0.28	
Employ. share (high-wage)	0.57	0.67	0.63	0.72	
100× Skill mixing (low-wage)	97.54	98.58	98.96	99.46	
100× Skill mixing (high-wage)	95.74	95.41	94.12	95.78	

Notes: This table reports the average values of the targeted moments both in the data and through model simulation. The data used for the moment calculation and for SMM estimation are two periods of pooled NLSY79&97 for employed workers: period 1 from 2005–2006 and period 2 from 2016–2019. Two types of moments are included. The worker moments include the relative wage of high type workers as well as the unemployment rate. The occupation moments include the relative wage of high skill occupations, the employment share and the skill mixing index of RNR skills in low and high skill occupations.

incorporating both skill-specific efficiency of matching α_k and cross-skill complementarity σ^j . For the occupation operation cost function, I apply a simple and flexible formulation, $C(\mathbf{y}) = \tau[\sum_{k=1}^K (y^k)^\rho]$, which is uniform across all occupations.⁵⁸ Here, ρ determines the convexity of the cost function relative to skill levels, and τ sets the cost scale. The matching function adopts a standard Cobb-Douglas format, $M(s,v) = \mu s^{\eta} v^{1-\eta}$, where η measures the elasticity of matches in relation to total search effort and μ reflects matching efficiency, leading to a job finding rate of $p(\theta) = \mu \theta^{1-\eta}$ and a vacancy filling rate of $q(\theta) = \mu \theta^{-\eta}$.

V.B Calibration Strategy

The calibration of parameters falls into three categories. For parameters that regulate the search environment, I follow closely the conventions of the search literature. For skill adjustment and efficiency parameters, I draw on estimates from the multi-dimensional matching literature.

⁵⁸Besides technical convenience, the functional form also implies that for a given cost, firms need to trade off the choice of altering different skill intensities.

Finally, I internally estimate the production technology parameters, which govern the elasticity of substitution across skills and the scale and convexity of the operational costs of skills, using Simulated Methods of Moments (SMM).

External Calibration: The model period is a year. Given that all agents are risk-neutral, the discount rate β is assigned a value of 0.96, corresponding to an annual interest rate of 4 percent. The job separation rate δ is set at 10 percent as in Shimer (2005). For employed workers, their share of output ω is set at 0.6, mirroring the labor share of GDP in 2005. For unemployed workers, the unemployment benefits b is set at 41.5 percent of the earning loss of lowest-paid occupations, following the estimates of Braxton, Herkenhoff, and Phillips (2020). The elasticity of the matching function η is set at 0.5 as is standard, and the matching efficiency μ is set to 0.65, as in Mercan and Schoefer (2020). Table 6 panel A summarizes these externally calibrated parameters.

I calibrate the speed of skill adjustment (γ_j) and the skill efficiencies (α_k) following Lise and Postel-Vinay (2020) and Lindenlaub (2017), as detailed in Table 6 panels B and C. The calibration aligns the adjustment of analytical/computer, interpersonal, and routine skills with the cognitive, interpersonal, and manual skills detailed in Lise and Postel-Vinay (2020).⁵⁹ Analytical/computer skill adjusts upward two times faster than it depreciates, while interpersonal skill changes slowly in both directions. Routine skill adjusts most rapidly in either direction. I linearly interpolate Lindenlaub (2017)'s estimates of skill efficiencies for my period of analysis.⁶⁰ Between 2005 and 2018, the productivity of analytical/computer and interpersonal skill in matching worker abilities with job skill requirements increased by about 60 percent. In contrast, the productivity of routine skill saw a decrease of more than 50 percent.

Internal Estimation: For the internal estimation, the SMM procedure entails solving the agents' steady-state policies and simulating a cohort of workers for T(T>80) periods, resulting in a distribution of labor market outcomes. The parameters are then estimated minimizing the distance between simulated and empirical moments.⁶¹ The estimation targets 11 moments as shown in Table 5 for both periods of data that include: i) the relative wage of the high-type worker for each skill; ii) the unemployment rate; iii) the relative wage of high-skill occupation; iv) the within-occupation correlation between wages and worker abilities; v) the share of employment across occupations; and vi) the skill mixing index of

⁵⁹Lise and Postel-Vinay (2020)'s estimates are presented on a monthly basis, which I have adjusted to an annual scale.

⁶⁰Lindenlaub (2017)'s estimates span from 1990 to 2010.

⁶¹Online Appendix B.5 provides further details on the numerical implementation.

Table 6: Parameter Estimates

Parameter	meter Description			
	Panel A. Externally calibrated - search			
β	Discount Rate	0	.96	
	Job separation rate	0	.10	
	Worker share of surplus	0	.60	
b	Unemployment benefit as a share of output	0	.42	
η	Elasticity of the matching function	0	.50	
μ	Matching efficiency	0.65		
	Panel B. Externally calibrated - skill adjustment	Up	Down	
γ_a	Annual adjustment speed of analytical/computer skill	0.36	0.10	
	Annual adjustment speed of interpersonal skill	0.05	0.00	
	Annual adjustment speed of routine skill	1.00	0.36	
	Panel C. Externally calibrated - skill efficiency	2005	2018	
α_a	Skill efficiency of analytical/computer skill	0.63	0.95	
α_p	Skill efficiency of interpersonal skill	0.05	0.08	
α_r	Skill efficiency of routine skill	0.14	0.06	
	Panel D. Internally estimated	2005	2018	
σ^{low}	Elasticity parameter of skills in production (low-wage)	0.62	0.30	
	Elasticity parameter of skills in production (high-wage)	0.61	0.29	
	Scaler of occupation operation cost	0.22	0.76	
	Convexity of occupation operation cost	3.92	4.99	
,	Vacancy posting cost as a share of output	0.93	0.90	

Notes: This table shows the exogenously calibrated as well as internally estimated parameters. The data used for the internal estimation are two periods of pooled NLSY79&97 data for workers with information on their pre-market abilities. Period 1 is from 2005–2006 and period 2 from 2016–2019.

RNR skills of occupations.⁶² The model does a decent job of matching all the moments, and replicates the non-targeted wage returns of skill mixing in Section III.

The model parameters are jointly identified from the moments, for which a concise summary of the key information for identification is given below with a more detailed discussion in online Appendix B.3. I first identify the complementarity parameter of skills in production σ targeting the correlation of within-occupation relative wages and worker skills. The cost parameter ρ is then estimated by leveraging the firm's optimization conditions in skill mixing. Conditional on parameters estimated at the production side, the employment

⁶²All moments are directly computed from the two periods of data from NLSY, except for unemployment, for which I use the statistics from the Bureau of Economic Analysis (BEA) to avoid the age composition effects present in NLSY. For example, by the late 2010s, a larger segment of the NLSY 79 cohort was above age 50, making them more likely to be out of the labor force. Additionally, the unemployment rate from NLSY, derived from the number of jobs held since the last survey, averages 9 percent, notably higher than BEA data. However, this decision primarily affects vacancy posting cost parameters.

distribution and relative wages further aid in estimating τ . Lastly, the unemployment rate disciplines the vacancy posting cost c.

Table 6 panel D presents the internally estimated parameters, which indicate considerable technological shifts between the two periods. For the initial period, the estimated σ is 0.6 for low- and high-wage occupations, suggesting that skills are substitutable in production. In the late 2010s, there was a significant rise in skill complementarity in production, reflected in the reduction of σ to 0.3 for both types of occupations. Firms also encounter rising costs of skills in occupation operation, as reflected in the increase of both the scale and the convexity of the cost function (τ and ρ). As discussed in Section V, this increased complementarity as well as the operational costs of skills intensifies firms' incentives to mix skills. Lastly, the cost of posting vacancies remains relatively constant.

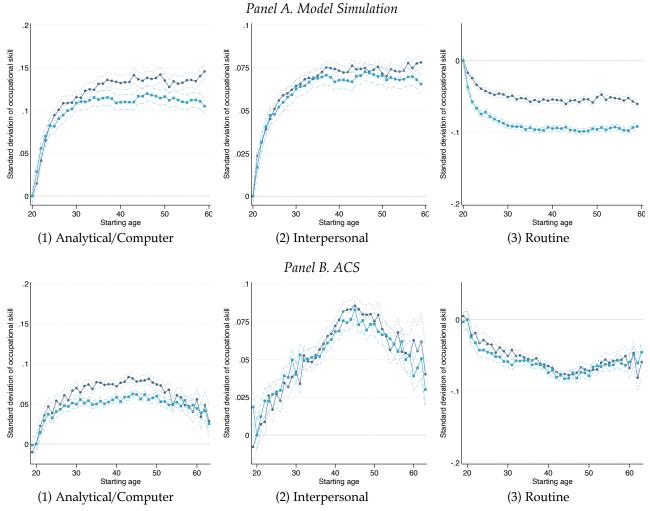
V.C Job Skills Over the Life Cycle

Before analyzing the quantitative impact of various model channels, I first present the simulated paths of workers' job skill intensities ("job ladder") from the model and compare these with empirical data. The model incorporates two main mechanisms affecting workers' job transitions over time: active job searching (both employed and unemployed) and skill investment by workers via learning by doing. These mechanisms shape workers' job trajectories. For empirical validation, I use the 2005 and 2018 ACS datasets to assess changes in workers' job skills.⁶³ I limit the age range to 20 to 60 years to accurately reflect the work experience from labor market entry to near retirement.

Figure 8 shows the average job skills across workers' lifecycles for 2005 (circles) and 2018 (squares) using both model simulations and ACS data. The standard deviation of these skills is plotted against worker ages. For analytical/computer and interpersonal skills, workers shift to jobs with higher skill requirements as they age. Notably, the job ladder for analytical/computer skills is much flatter in 2018, according to both model simulations and empirical data. However, there is no significant change in the steepness of the job ladder for interpersonal skills between the two periods. In contrast, both the model and empirical data show a decline in routine skills over time, with the model slightly over-predicting this decline in 2018. Overall, the model aligns well with empirical data and sheds light on how

⁶³The ACS is preferred over NLSY for assessing empirical worker job ladders because it covers a cross-section of workers across all age ranges, whereas NLSY is limited to specific age ranges depending on the dataset year. As a robustness check, I align the 2005 O*NET values with NLSY 79 and the 2018 values with NLSY 97, using age restrictions of 41 to 60 for NLSY 79 and 21 to 40 for NLSY 97. As shown in online Appendix B1, the model-simulated paths match the empirical job paths from NLSY, though cross-age and cross-time comparisons are necessary.

Figure 8: Predicted and Observed Occupational Skills Across Age Groups in 2005 vs. 2018



Notes: These figures display the average occupational job skills over the life cycle of workers, for both model predictions and empirical data. Model simulated or empirical data points for the year 2005 are represented by circles, and for the year 2018 are represented as squares. The empirical data observations are drawn from the ACS for the years 2005 and 2018, with an age range restricted to 20 to 60 years.

technological advancements and worker sorting shape the job ladder.

VI Drivers of Skill Mixing and Labor Market Impacts

What drives the observed increase in skill mixing, and what are their implications for aggregate labor market outcomes? In this section, I employ the model to perform a series of counterfactual experiments to assess the relative significance of each model channel in explaining the shifts in the degree of skill mixing. I then evaluate the influence of these channels on the changes in earnings and employment distributions.

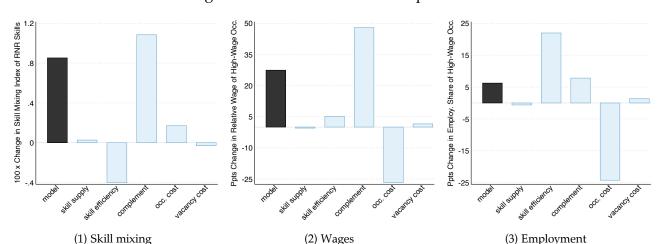


Figure 9: Counterfactual Decomposition

Notes: These figures plot the model-generated changes in skill mixing in low-skill occupations (panel A), changes in the relative wage of high-wage occupations (panel B), and changes in the employment share of high-wage occupations (panel C). Different model channels are shut down in various sequences, and the effect of each channel is calculated by averaging across those sequences. The full model includes all features. The values of skill complementarity in production, cost of skills in occupation operation, efficiency differentials, and vacancy posting costs across the two periods are shown in Table 6. Worker skill supply distribution variation across the periods is calibrated as in Figure 7.

For the counterfactual analysis, I take the 2018 economy and then sequentially remove shifts in calibrated parameters representing different channels, setting their values to that in the 2005 economy. Specifically, I examine the roles of changes in skill supply $(G(\mathbf{x}))$, skill efficiencies (α_k) , skill complementarity in production (σ) , occupation operation cost (τ, ϕ) , and job posting cost (c) in generating moment variations. Given the non-linear interplay of these forces, I remove these elements in different sequences and calculate the effect of each channel by averaging across those sequences.

Counterfactual Skill Mixing: I begin by assessing how different channels contribute to the growth in skill mixing within low-wage occupations, which has been noticeably observed in the data.⁶⁴ The first panel of Figure 9 illustrates that the full model predicts a rise in skill mixing within low-wage occupations over the two periods consistent with the observed data. Changes in the supply of worker skills and vacancy posting costs during these periods did not play a significant role in this rise. On the other hand, shifts in skill efficiency have had a negative contribution to the change in the degree of skill mixing. The latter result arises because, as the efficiency of analytical/computer and interpersonal skills rises and of routine skill declines, firms are incentivized to redesign occupations to shift towards either

⁶⁴Online appendix B.6 shows the results for high-wage occupations.

analytical/computer or interpersonal skill away from routine skill, to a degree that it leads to a slight increase in specialization towards the skills that become more efficient.

The subsequent counterfactual results indicate that the rise in the complementarity of skills in production and the costs of skills account for the increase in skill mixing. The increase in skill complementarity contributes to 86 percent of the increase, while changes in occupational operation cost account for 12 percent. These results are consistent with the predictions in Section IV and highlight the importance of skill complementarity and their costs in driving firms' endogenous skill demand specialization.⁶⁵

Wage and Employment Effects: I proceed to investigate how the same model channels that influence skill mixing also impact wage and employment distributions. Panel B of Figure 9 illustrates the changes in the relative wage between high-wage and low-wage occupations from 2005 to 2018, with the model predicting a 27 percentage points (ppts) increase. As observed with skill mixing, changes in worker skill supply and vacancy posting costs have negligible impacts. In contrast, changes in operation costs significantly reduce the relative wage gap, decreasing it by nearly the same as the net increase. However, the growing complementarity and changing efficiency of skills notably increase wage disparities, contributing to 88 and 9 percent, respectively, of the overall rise in wage premiums for high-wage occupations.

I further decompose the influence of various model factors on the increase in employment in high-wage occupations, as depicted in Figure 9 panel C. The full model indicates a 7 ppts rise in the employment share of high-wage occupations. Skill supply has only a marginal impact, similar to its effect on relative wages. The most significant factor is changes in skill efficiencies, accounting for 73 percent of the increase, while changes in skill complementarity contribute 27 percent. However, increased operation costs notably reduce the employment gap. Online Appendix B.6 further breaks down the contributions of individual skills to changes in wages and employment. The results show that the increased efficiency of analytical/computer skill drives the wage and employment gaps, while the decline in routine skill efficiency narrows the wage gap.

In summary, the counterfactual analysis highlights that the increasing complementarity of skills in production and changes in the cost of skills for occupation operation are key drivers of skill mixing. Further, growing skill complementarity and shifts in skill efficiency contribute significantly to the wage and employment gains in high-wage occupations. Conversely, changes in skill operation costs help reduce the wage and employment gaps.

⁶⁵Further analysis of the implications of τ and ϕ individually for skill mixing changes shows that ϕ plays a bigger role.

Discussions:

Task-biased vs. Skill Complementarity & Cost. Biased technological change, especially task-biased technological change (TBTC), is shown to be a key driver of the recent trends in wage inequality in developed countries. Studies by Costinot and Vogel (2010) and Acemoglu and Zilibotti (2001) employ one-dimensional assignment models, while Lindenlaub (2017) uses a multi-dimensional assignment model to examine this phenomenon. This change is characterized by increasing complementarities and efficiency in cognitive tasks and a decline in routine tasks. Such a shift leads to the replacement of workers in medium-wage occupations and an increase in wages and employment in high-wage occupations.

In my model, I incorporate both changes in skill efficiency representing TBTC, as well as variations in skill complementarity and cost. My counterfactual analysis first confirms the significance of TBTC, which accounts for 73 percent of the employment gains in highwage occupations, whereas skill complementarity and vacancy posting cost account for the rest. However, for the wage premium in high-wage occupations, skill complementarity is more crucial than TBTC. For the increase in skill mixing, it is entirely attributed to skill complementarity and cost. Overall, the results indicate that while TBTC is crucial for employment distribution, skill complementarity, and cost are more influential for wage distribution and also in shaping firms' endogenous skill specialization.

Implications for Education. In the counterfactual analysis, evolving skill supply $(G(\mathbf{x}))$ across two periods highlights the potential impact of education on labor market outcomes under skill mixing. Figure 9 shows that skill supply has not significantly affected wage and employment distributions. This is due to two factors: first, the directed search model with perfect information contains submarkets for different workers, where firms tailor the occupations, reducing the worker distribution's impact on labor outcomes. Second, workers' skills adjust to meet occupational requirements, minimizing the influence of initial skill levels. Although skill supply does not notably change aggregate distributions, it decreases returns to specialization by 1.2 to 2.8 ppts, as shown in online Appendix Table B2. This suggests that while education may have marginal effects on overall distributions under directed search, it reduces wage disparities between skill experts and non-experts.

VII Concluding Remarks

Skills are inevitably embedded in workers and understanding the demand for skill *mixtures* is important in studying the dynamics of the labor market. I present a rich set of empirical findings on the phenomenon of "skill mixing" and demonstrate that between 2005 and

2018, there was significant growth in skill mixing in the U.S. economy, particularly for non-routine skills such as analytical, computer, and interpersonal skills, especially in medium-to lower-paid occupations. To understand the heterogeneous within-occupation variation in skill mixing across occupations, I provide a multi-dimensional skill directed search model integrating endogenous occupation design and worker skill investment. Bringing the model to the data, I show that technological change as reflected in the increased skill complementarity and costs are the main drivers of skill mixing, and they also significantly affect wage and employment distribution changes.

The phenomenon of skill mixing brings forth very different policy implications for worker training and college education. Using NLSY 79 & 97 combined with O*NET data, I show that workers in occupations that become more mixed in non-routines skills earn a positive wage premium. Further, workers who possess a more mixed set of these skills, or those who study a college more mixed in these skills earn 3 to 6 percent more. A quantitative counterfactual exercise shows that increasing the mixing of non-routine skills among workers reduces wage disparities between skill specialists and non-specialists.

The empirical evidence and theoretical framework presented in this paper open up several avenues for future research.

First, there is an exciting opportunity to explore educational interventions that emphasize skill mixing. For example, a college computer science course that includes a section on communicating technical results to non-experts, or an English writing class that integrates writing data analysis, will help students develop a more mixed skill set. Future research could compare these interdisciplinary courses with traditional, single-skill-focused courses to evaluate their impact on employment and wages. These types of programs are promising to help workers reap the benefits of skill mixing through a tailored educational experience.

Moreover, while this paper models skill investment through learning by doing, studies that expand this paper's multi-dimensional skill search framework to include skill portfolio choices under uncertainty may provide new insights into education policies. Early models, such as those by Becker (1964) and Ben-Porath (1967), focused on homogeneous human capital investments over the life cycle. More recent studies (Wasmer 2006; Gervais, Livshits, and Meh 2008; Silos and Smith 2015) examine the trade-offs between specific and general human capital under economic turbulence and labor market uncertainty. Extending the analysis of skill mixing to incorporate these considerations would be a valuable addition.

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Appendix for Online Publication

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A ADDITIONAL EMPIRICAL RESULTS

A.1 Data Construction

In this section, I give more details on data construction for the two primary datasets on job skill demand employed in Section II and III, namely O*NET (Occupation Information Network) and Lightcast (previously known as "Burning Glass"). Specifically, I discuss strategies for leveraging the longitudinal information in these datasets with higher precision. I also present an overview of their inherent characteristics, advantages and disadvantages, and how they are cross-walked with other datasets used in the analysis.

O*NET: Administered by the U.S. Department of Labor, O*NET is a replacement for the Dictionary of Occupational Titles (DOT). It is more comprehensive and more frequently updated and has been used widely to analyze occupation skill requirements and work settings (i.e., Acemoglu and Autor 2011; Yamaguchi 2012; Autor and Price 2013).

Nonetheless, to use the longitudinal variation from O*NET, the key challenge concerns partial updating – each new version of O*NET only updates an average of 110 targeted occupations among the 970 7-digit occupations. Online Appendix Table A1 lists different versions of O*NET, the release year, and the year composition for 3 of the modules. Specifically, for each release of O*NET, I assign a "Considered Year" such that at least 55% to 60% of occupations are updated after that year.

Moreover, I use 4-year intervals. The last column of online Appendix Table A1 shows the percent of occupations that are updated from the last considered year of data included in the analysis. On average, more than 50 percent of the occupations are updated across the succeeding years included in the analysis.

O*NET contains around 270 descriptors about occupations that are grouped into 9 modules: abilities, knowledge, skills, work context, work activities, experience/education requirement, job interest, work values, and work styles. For my main analysis, I only use descriptors from 3 modules: work context, work activities, and knowledge that are more interpretable as the skill requirements and are consistently evaluated by incumbent workers for each new release. These descriptors come as importance, level, extent, and relevance. To interpret the skill measures as gauging the intensity, I use the importance information, similar to i.e., Acemoglu and Autor (2011) and Guvenen et al. (2020), but the level and importance pieces of information are highly correlated and do not affect the qualitative patterns of skill mixing shown in the paper.

In Section II, I show the longitudinal changes in skill mixing by combining O*NET and ACS datasets. O*NET uses SOC 2000 occupation classification for releases between 2000 and 2010 and SOC 2010 for years after 2010. To link O*NET and ACS, I first bridge SOC codes to the census' OCC 2000 and OCC 2010 codes respectively using crosswalks provided by the Analyst Resource Center and the Bureau of Labor Statistics. Then different years of OCC codes are homogenized using a balanced and consistent panel of occupation codes developed by Autor and Dorn (2013) and updated by Deming (2017). The same code is also used for combining all years of ACS and O*NET data.

Lightcast: Lightcast (formerly "Burning Glass Technologies") is an analytics software company that has developed a comprehensive and detailed dataset derived from online job postings, capturing real-time labor market information, and reflecting the current demand for skills and occupations. One of the key advantages of Lightcast data is its extensive coverage and high-frequency updates. By examining over 40000 online job boards and company websites, it provides a near universe of online posted vacancies; moreover, it provides a level of detail that is rarely matched by other sources of labor market data, such as job titles, employer information, and specific skill requirements. This allows for a very granular analysis of job skill requirements and labor market dynamics across different industries and regions.

The information that Lightcast collected is then parsed and deduplicated into a systematic list of thousands of codified skills. Similar to Hershbein and Kahn (2018) and Braxton and Taska (2023), the dataset that this study uses defines different skills if the codified skills from Lightcast contain relevant keywords. Specifically, the keywords used to capture analytical skill are: "research", "analy", "decision", "solving", "math", "statistic", and "thinking". The keywords used to capture interpersonal skills are "communication", "teamwork", "collaboration", "negotiation", and "presentation". For each occupation, the share of posted vacancies that require a particular skill is then the measure of skill for that occupation, capturing the extensive margin of firm skill demand.

However, like any data source, Lightcast data also has its limitations. For instance, it only covers online job postings, which may not represent the entire labor market, especially for low-skilled jobs or jobs in small firms that do not typically advertise online. It may also have a bias towards certain types of jobs or industries that use online job advertisements more frequently, and online vacancies by nature overrepresent growing firms (Davis, Faberman, and Haltiwanger 2013). One note of Lightcast data is that the measure of skill as introduced above focuses on the extensive margin – whether a job uses a skill or not – this is very different than the level and importance information that O*NET contains.

Table A1: O*NET Versions and Corresponding Years

Version	Released Year	Division	Work Context	Work Activities	Knowledge	Skills	Abilities	Considered Year
O*NET 13.0	2008	Post 2005 Before 2005	73.79% 26.21%	73.79% 26.21%	73.79% 26.21%	73.79% 26.21%	73.79% 26.21%	2005
O*NET 18.0	2013	Post 2009 Before 2009	57.15% 42.85%	57.21% 42.79%	57.21% 42.79%	99.89% 0.11%	57.21% 42.79%	2009
O*NET 22.0	2017	Post 2013 Before 2013	57.84% 42.16%	57.67% 42.33%	57.67% 42.33%	57.67% 42.33%	57.67% 42.33%	2013
O*NET 25.0	2022	Post 2018 Before 2018	54.52% 45.48%	54.52% 45.48%	54.52% 45.48%	54.52% 45.48%	54.52% 45.48%	2018

Notes: The table summarizes different versions of the O*NET (Occupational Information Network) database, along with their released year, year division for the 5 modules (work context, work activities, knowledge, skills, abilities), and the considered year for each version. The "Post" and "Before" rows indicate whether the data in each version was collected post or before a particular year. The "Considered Year" column represents the year considered to be corresponding to each release of O*NET based on the year division of data.

A.2 Details of Skill Measures

In this section, I discuss the choice of skill measures used in the main analysis. Specifically, I show the composition of descriptors of each skill used in the main analysis. I also discuss the composite skill measures' validity and correlation with other measures used in the literature.

Table A2 lists the O*NET descriptors for each of the constructed composite skill measures. The analytical measure corresponds to "non-routine cognitive analytic" and the interpersonal measure corresponds to "non-routine interpersonal" from Acemoglu and Autor (2011). I collapse Acemoglu and Autor (2011)'s "routine cognitive" (the first three items under Routine) and "routine manual" (the last three items under Routine) into a big routine skill, as occupations using these skills have been shown to have had similar labor market dynamics (Autor, Levy, and Murnane 2003; Acemoglu and Autor 2011). I didn't include the "non-routine manual" from Acemoglu and Autor (2011), since it includes descriptors from the "Abilities" module of O*NET that is evaluated solely by job analysts, and for consistency purposes I focus on occupation descriptors that are evaluated incumbents workers.

Further, I include two additional composite skills that are considered to be non-routine. First, I include a "leadership" composite skill that is comprised of descriptors of problem-solving, strategic thinking, teamwork, and communication. They all demand an ability to guide and manage teams, strategize and plan, solve problems, coordinate activities, and communicate effectively within a team or organizational context. Second, I include a "design" composite skill measure centering around technical proficiency and creativity. The composing descriptors entail a strong understanding of design principles, and the ability to draft and layout specifications for technical devices.

Table A2: O*NET Skill Measures and Composing Descriptors

Skill Category	Task Descriptors				
Non-routine analytical	Analyzing data/information Thinking creatively Interpreting information for others				
Non-routine interpersonal	Establishing and maintaining personal relationships Guiding, directing and motivating subordinates Coaching/developing others				
Computer	Interacting with computers Programming Computers and electronics				
Routine	Importance of repeating the same tasks Importance of being exact or accurate Structured vs. unstructured work (reverse) Pace determined by speed of equipment Controlling machines and processes Spend time making repetitive motions				
Design	Design Drafting, laying out, and specifying technical devices, parts, and equipment				
Leadership	Making decisions and solving problems Developing objectives and strategies Organizing, planning, and prioritizing work Coordinating the work and activities of others Developing and building teams Guiding, directing, and motivating subordinates Provide consultation and advice to others				

Notes: This table shows the detailed O*NET descriptors for skill measures. The Non-routine Analytical and Non-routine Interpersonal skills align with Acemoglu and Autor (2011)'s "non-routine cognitive analytic" and "non-routine interpersonal" skills. A unified Routine skill measure combines Acemoglu and Autor (2011)'s "routine cognitive" and "routine manual" skills, reflecting their similar market trends. The study omits "non-routine manual" to maintain consistency with incumbent worker-evaluated descriptors. Two additional skills, 'leadership' and 'design', are included to capture managerial and creative competencies.

Table A3: Correlations Among Skill Measures

Skill Measures	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1) Analytical	1.00								
(2) Routine	-0.45	1.00							
(3) Interpersonal	0.44	-0.49	1.00						
(4) Computer	0.92	-0.27	0.25	1.00					
(5) Math skill	0.50	-0.11	0.12	0.46	1.00				
(6) Social skill	0.34	-0.54	0.61	0.24	0.09	1.00			
(7) Analytical (broader)	0.84	-0.59	0.55	0.68	0.63	0.57	1.00		
(8) Mechanical (broader)	-0.43	0.58	-0.24	-0.38	-0.11	-0.38	-0.49	1.00	
(9) Interpersonal (broader)	0.10	-0.35	0.73	0.02	-0.09	0.70	0.28	-0.22	1.00

Notes: This table reports the correlation among different skill measures constructed using O*NET data from 2000-2020. The first four skills measures in rows (1) to (4) are the ones used in the main text and are constructed using the O*NET descriptors shown in Table A1. The next two measures in rows (5) to (6), math skill and social skill are constructed based on Deming (2017). Math skill is the average of 1) mathematical reasoning ability, 2) mathematics knowledge, and 3) mathematics skill. Social skill consists of the average of four variables, 1) social perceptiveness, 2) coordination, 3) persuasion, and 4) negotiation. Rows (7) to (9) contain the broader analytical, mechanical, and interpersonal skills constructed using factor analysis as discussed in online Appendix A.6 with their specific component variables.

Table A3 shows the correlation among the chosen skills used in the main analysis, as well as math skill and social skill, which are constructed based on Deming (2017), and broader skill measures skills constructed using factor analysis as discussed in online Appendix A.6. It reveals the analytical skill (row 1), exhibits a strong positive correlation with computer skills (0.92) and a moderate correlation with math skills (0.50). This pattern suggests that positions requiring analytical skills frequently necessitate computer and mathematical proficiency. Interpersonal skills (row 3) indicate a moderate-to-strong positive correlation with social skills (0.61) and broader interpersonal skills (0.73). This correlation suggests that occupations demanding interpersonal skill also emphasize social abilities. These results validate the interpretation of the analytical and interpersonal skills with a strong positive correlation with math and social skills used in other studies.

On the other, a strong negative correlation exists between routine and interpersonal skills (-0.49) and between routine and interpersonal skills (-0.45), indicating that these skill sets rarely overlap in job requirements. The broader skill categories (rows 7 to 9) align well with their narrower counterparts, reinforcing the validity of these categorizations. In sum, there exist specific, identifiable skills in the labor market, some of which are more aligned with each other, but they tend not to overlap, reflecting distinct competencies.

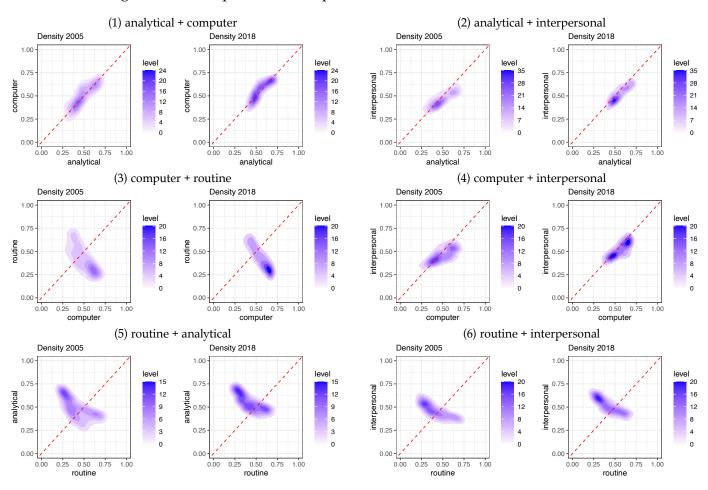
A.3 Alternative Non-parametric Examination of Trend

An intuitive alternative check of the changes in the degree of skill mixing across periods is to non-parametrically plot the density of skill intensities in different skill dimensions. Figure A1 depicts the density of skill requirements of six skill pairs out of the four constructed skills in 2005 and 2018 respectively using O*NET data combined with ACS. As in previous studies of job attributes, I aggregate the ACS to sex-education-industry cells that implicitly control for changes in task inputs due to variations in skill and industry mixes in the U.S. economy. Employment weights are obtained as the total hours of work aggregated to each cell. The ACS then supplies the O*NET data with employment across worker types to present an overarching picture of skill intensities in the economy.

From the figure, there is a clear shift towards mixed skill requirements in panel (1) pertaining to analytical and computer skills where these skills are positively correlated. Two salient changes happened in this period: first, the entire distribution of skill intensities moves near the 45-degree line; second, there is a significant increase in density around the 45-degree line. Both of these changes will lead to an increased degree of skill mixing, according to how it is defined based on the position of skill vectors relative to the 45-degree line. Such a change is also salient for other non-routine skill combinations: in the analytical and interpersonal skills space (panel 2), as well as in the computer and interpersonal skills space (panel 4).

On the other hand, one can scarcely observe changes in mixing in the routine skill spaces, as shown in panels (3),(5), and (6). From these three plots, there is an increase in density towards the non-routine direction, losing density in routine skill, and the resulting change in relationship with the diagonal does not indicate a strengthening of mixing.

Figure A1: Non-parametric Depiction of Skill Intensities, 2005 vs. 2018



Notes: These density plots show the intensity of occupation skill requirements across the U.S. economy in 2005 (column 1) and 2018 (column 2) in six two-dimensional skill spaces, as illustrated in the six panels. Darker colors indicate higher density and the 45-degree line is also plotted. O*NET and ACS data are combined for the construction of these plots. The two datasets are merged using consistent occupation codes constructed by Autor and Price (2013) and further developed by Deming (2017). Skill measures are constructed using the O*NET descriptors shown in Table A1. All measures are normalized to [0,1].

A.4 Robustness of Trend Results to Different Weights and Groupings

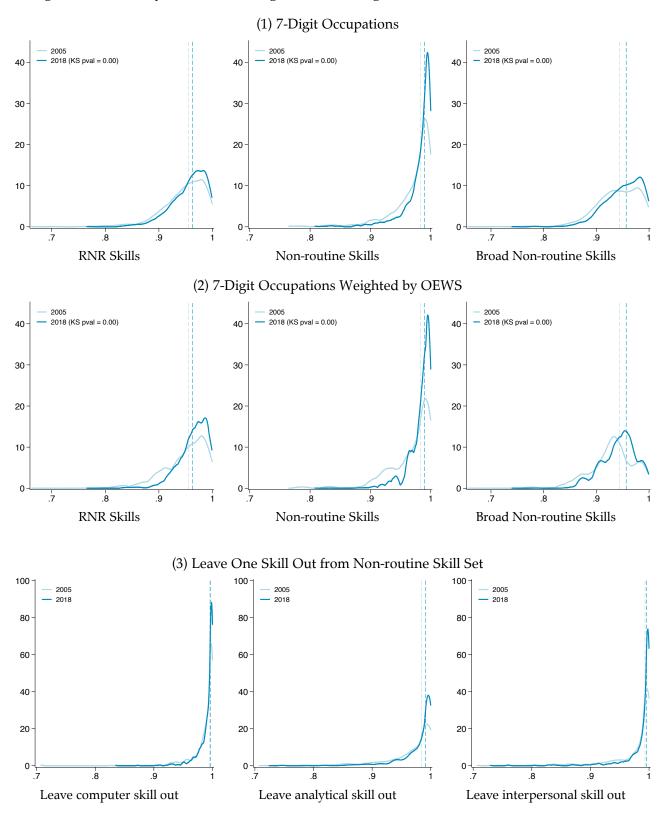
In this section, I discuss the robustness of the trend results in different weighting, granularities, and groupings. In particular, I show the density results using weighted skill mixing indexes instead of unweighted ones in the main analysis, as well as at a higher occupations level; the trend of skill mixing using indexes for different skill pairs, instead of high-dimensional indexes; the heterogeneity of skill mixing increases across occupations using indexes for different skill pairs; and the differential changes in skill mixing across industries.

One concern of the analysis of skill mixing shown in Figure 3 is that as it shows the changes in the density of skill mixing indexes without weighting, it might not accurately represent the overall picture of mixing in the whole economy. In Figure A2 panel B, I weigh the skill mixing indexes using employment weight from the OEWS. The results show a similar message that there is a sizable increase in skill mixing particularly for non-routine skills. The only difference is that with employment weighting, the increase in the skill mixing of RNR skills is more discernable. This implies a relatively higher weight of occupations intensive in RNR skills that also increase skill mixing in these skills. In Figure A2 panel B, I show the density results at a higher 4-digit occupation level, and a similar trend holds.

Next, I discuss the changes in skill mixing using indexes of different skill pairs instead of high-dimensional indexes. Figure A7 panel (1) shows the results. The figure shows similar results as the main analysis: there is a stronger increase in skill mixing among non-routine skills. For the skill combinations involving routine skills, the change in skill mixing is negligible.

In Table A6, I show the decomposition results of the changes in the skill mixing indexes for different skill pairs across different datasets. A similar pattern as the main analysis in 2, that is within-occupation variation surpassed across-occupation variation in accounting for the increase in skill mixing. This is particularly true using constantly updated occupations at 6-digit occupation level for non-routine skill pairs, and also quite apparent in the Lightcast data. The only slight difference is that for full O*NET data at the 7-digit level, across-occupation variation does contribute to a comparable amount to the change in skill mixing for skill pairs with routine skill.

Figure A2: Density for Skill Mixing Indexes (Weighted Cosine Distances), 2005 vs. 2018



Notes: These figures plot the PDF of different mixing indexes in 2005 (light blue line) and 2018 (dark blue line). The x-axis displays the value of mixing indexes with a maximum of 1 by construction. These plots are created using O*NET and ACS data merged with occupation codes constructed by Autor and Price (2013) and developed by Deming (2017).

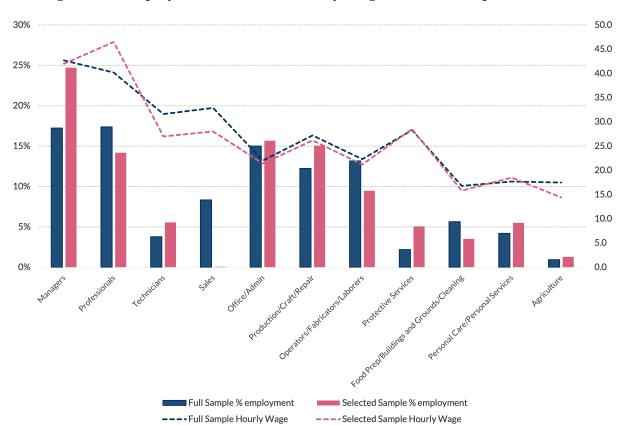


Figure A3: Employment Share and Hourly Wage of Full and Updated O*NET

Additionally, in Figure A3, I show employment percentages and hourly wages across various job categories in the full and the sample for constantly updated occupations. This information gives the occupational structure and returns for these two samples. It can be seen that while professionals make up a smaller percentage in the selected sample, they exhibit a higher average wage, suggesting a focus on higher-earning professionals in the selected sample. Conversely, the sales category shows a drastic reduction in the selected sample, indicating its limited representation. The hourly wage rates across the categories seem fairly consistent between the full and selected samples, with minor discrepancies.

Table A4: Top Occupations in Skill Mixing Growth

Top Occupations	Year	Analytical	Computer	Inter- personal	Routine	Mixing Index	Percentile
Panel A. Mix of Non-routine Skills							
Packers, fillers, and wrappers	2005	0.58	0.44	0.16		0.915	1
(Operators/Fabricators/Laborers)	2018	0.52	0.40	0.42		0.994	99
Housekeepers, maids, cleaners	2005	0.00	0.10	0.24		0.753	0
(Personal Care and Services)	2018	0.28	0.20	0.25		0.990	96
Sales counter clerks	2005	0.13	0.32	0.30		0.946	7
(Sales)	2018	0.50	0.52	0.39		0.993	99
Recreation facility attendants	2005	0.24	0.18	0.39		0.947	7
(Personal Care and Services)	2018	0.38	0.40	0.35		0.998	99
Janitors	2005	0.10	0.07	0.21		0.913	1
(Food Prep/Buildings and Grounds)	2018	0.15	0.16	0.21		0.987	93
Carpenters	2005	0.50	0.14	0.44		0.915	1
(Production/Craft/Repair)	2018	0.59	0.38	0.53		0.985	90
Cashiers	2005	0.08	0.41	0.33		0.892	0
(Sales)	2018	0.31	0.41	0.49		0.984	87
Packers and packagers by hand	2005	0.16	0.16	0.30		0.951	12
(Operators/Fabricators/Laborers)	2018	0.49	0.40	0.54		0.992	99
Data entry keyers	2005	0.56	0.77	0.27		0.935	3
(Office/Admin)	2018	0.55	0.66	0.43		0.985	90
Sales supervisors and proprietors	2005	0.40	0.39	0.79		0.943	6
(Sales)	2018	0.49	0.57	0.74		0.985	92
		nel B. Mix o	of RNR Skill	S			
Packers and packagers by hand	2005	0.16	0.16	0.30	0.71	0.824	0
(Operators/Fabricators/Laborers)	2018	0.49	0.40	0.54	0.70	0.979	99
Cashiers	2005	0.08	0.41	0.33	0.71	0.863	2
(Sales)	2018	0.31	0.41	0.49	0.61	0.973	99
Assemblers of electrical equipment		0.35	0.25	0.34	0.82	0.894	5
(Operators/Fabricators/Laborers)	2018	0.44	0.43	0.40	0.65	0.979	99
Equipment cleaners	2005	0.23	0.24	0.26	0.63	0.896	5
(Operators/Fabricators/Laborers)	2018	0.41	0.32	0.52	0.54	0.981	99
Cooks	2005	0.24	0.16	0.34	0.59	0.899	6
(Food Prep/Buildings and Grounds)	2018	0.46	0.33	0.46	0.64	0.974	99
Painters, construction, maintenance		0.29	0.12	0.28	0.56	0.892	5
(Production/Craft/Repair)	2018	0.53	0.30	0.56	0.72	0.962	94
Hairdressers and cosmetologists	2005	0.52	0.10	0.38	0.35	0.912	11
(Personal Care and Services)	2018	0.51	0.30	0.44	0.46	0.985	99
Accounting and auditing clerks	2005	0.26	0.72	0.33	0.28	0.905	7
(Office/Admin)	2018	0.40	0.69	0.33	0.44	0.960	93
Packers, fillers, and wrappers	2005	0.58	0.44	0.16	0.44	0.900	6
(Operators/Fabricators/Laborers)	2018	0.52	0.40	0.42	0.82	0.954	90
Punching and stamping operatives	2005	0.25	0.40	0.15	0.86	0.809	0
(Operators/Fabricators/Laborers)	2018	0.42	0.27	0.15	0.74	0.948	81
(Operators) rabificators/ Laborers)	2010	0.12	0.07	0.55	0.7 1	0.710	

Notes: This table presents specific O*NET occupations at census SOC levels that have the greatest growth in skill mixing from 2005 to 2018. It provides details on compositions of skills within these occupations and the corresponding changes in skill mixing indexes. The last column translates skill mixing levels into percentiles relative to their 2005 distributions.

Table A5: R-Squared Values for Non-Routine Skills' Mixing Index by Polynomial Order

	Analytical	Computer	Interpersonal
	1st Order Po	olynomial	
All occupations	0.11	0.12	0.15
High-wage	0.00	0.02	0.25
White-collar	0.17	0.00	0.38
Blue-collar	0.02	0.22	0.03
Service	0.22	0.37	0.18
	3rd Order P	olynomial	
All occupations	0.15	0.48	0.21
High-wage	0.03	0.45	0.55
White-collar	0.21	0.20	0.52
Blue-collar	0.05	0.56	0.15
Service	0.30	0.62	0.20
	5th Order P	olynomial	
All occupations	0.18	0.50	0.22
High-wage	0.04	0.46	0.55
White-collar	0.22	0.21	0.53
Blue-collar	0.07	0.57	0.16
Service	0.38	0.73	0.26

The table presents the R-Squared values from a polynomial regression analysis, assessing the relationship between non-routine skills' mixing index and each composing skill's polynomials up to order N over the period from 2000 to 2020. The regression formula used is $Mix(\mathbf{y})^{percentile}_{ijt} = \beta_1 y^1_{ijt} + \beta_2 y^2_{ijt} + ... + \beta_N y^N_{ijt}$, where $Mix(\mathbf{y})^{percentile}_{ijt}$ indicates the percentile rank of an individual's i mixing index of non-routine skills in occupation j at time t, and y_{ijt} is the measure of a specific composing skill for the same individual and occupation at time t. The R-Squared values for polynomial orders N=1,3,5 are provided, illustrating the degree to which each composing skill explains the variance in skill mixing.

Table A6: Decomposition of Mixing Indexes' Changes by Skill Pairs

	6-digit Occupations			3-digit Occupations		
Skill Groups	total	within	across	total	within	across
Full O*NET						
Analytical + Computer	10.52	6.40	4.12	8.13	6.71	1.42
Analytical + Interpersonal	5.36	2.90	2.46	6.42	4.21	2.21
Computer + Routine	4.38	2.41	1.97	2.65	3.03	-0.37
Computer + Interpersonal	7.23	3.60	3.63	10.28	7.67	2.60
Routine + Analytical	4.00	2.29	1.71	1.52	3.26	-1.75
Routine + Interpersonal	1.93	0.12	1.81	-1.25	1.13	-2.38
Constant Updates						
Analytical + Computer	5.59	6.03	-0.44	6.75	5.96	0.79
Analytical + Interpersonal	3.53	4.58	-1.05	4.24	3.15	1.09
Computer + Routine	2.88	3.69	-0.81	0.77	1.97	-1.20
Computer + Interpersonal	0.78	1.86	-1.09	7.24	6.06	1.17
Routine + Analytical	2.04	2.13	-0.09	1.72	3.69	-1.96
Routine + Interpersonal	0.81	0.82	-0.01	-0.08	1.53	-1.61
Lightcast						
Analytical + Computer				13.20	11.74	0.90
Analytical + Interpersonal		_		2.73	2.20	0.31
Computer + Interpersonal		_		-3.90	-3.79	-0.39

Notes: This table shows the shift-share decomposition of changes in the average level of different mixing indexes between 2005-2018 in percentile units. Specifically, for a change in the percentile of a mixing index h over two periods t and τ , its change $\Delta T_{h\tau} = T_{\tau} - T_{t}$ which can be decomposed to $\Delta T_{h} = \sum_{j} \left(\Delta E_{j\tau} \alpha_{jh} \right) + \sum_{j} \left(E_{j} \Delta \alpha_{jh\tau} \right) = \Delta T_{h}^{a} + \Delta T_{h}^{w}$ where $E_{j\tau}$ is employment weight in occupation j in year τ , and $\alpha_{jh\tau}$ is the level of mixing index h in occupation j in year τ , $E_{j} = \frac{1}{2}(E_{jt} + E_{j\tau})$ and $\alpha_{jh} = \frac{1}{2}(\alpha_{jht} + \alpha_{jh\tau})$. ΔT_{h}^{a} and ΔT_{h}^{w} then represent across-occupation and within-occupation change.

Table A7: Relationship between Robotics, IT Capital, and Skill Mixing Shifts

	Non-rou	tine Skills	RNR	Skills
	(1)	(2)	(3)	(4)
A. Skill Mixing Inc	dex, 2005-2	018 (O*NET))	
IT capital stock	-0.00 [0.02]	0.09*** [0.03]	0.12** [0.04]	-0.09** [0.03]
Observations R-squared	821,030 0.07	821,030 0.19	821,030 0.11	821,030 0.24
B. Skill Mixing Indo	ex, 2007-20)17 (Lightcast	•)	
IT capital stock	0.02 [0.07]	0.06 [0.07]		
Observations R-squared	518,520 0.09	518,520 0.26		
C. Change in Skill Mixing Index	., 2005-201	0 and 2010-2	015 (O*NE	T)
Δ industrial robots Observations R-squared	-1.38*** [0.36] 97,650 0.02	-0.41 [0.38] 97,650 0.12	-1.77** [0.54] 97,650 0.04	-1.24*** [0.31] 97,650 0.14
Year FE Experience and education controls	X X	X X	X X	X X
Gender \times education FE Gender \times education \times industry FE	X	Χ	X	Χ

Notes: This table provides regression results on the changes in skill mixing indexes for non-routine and RNR skills between 2005-2018, measured in percentile units based on their distributions in 2005. The analysis integrates data from O*NET and Lightcast to derive skill intensities for calculating skill mixing, which are then merged with ACS data. The data on IT capital stock is sourced from the Bureau of Labor Statistics Total Multifactor Productivity tables. It reflects the productive capital stock for "Total information processing equipment" in billions of 2017 dollars, which is then converted into logarithmic values. The information on the number of industrial robots per thousand workers is sourced from the International Federation of Robotics (IFR), which covers seven industries including Manufacturing, Agriculture, Mining, Utilities, Construction, Education, and Services, and covers the periods 2004-2010 and 2010-2014. Following the methodology in Acemoglu and Restrepo (2020), I use IFR data from Denmark, Finland, France, Italy, and Sweden to assess the influence of global technological progress. Robust standard errors are reported in brackets.

**** p<0.01, *** p<0.05, and * p<0.1.

A.5 Additional Results on Trend Heterogeneity

Now, I turn to discuss the robustness of the occupation heterogeneity in skill mixing changes. Figure A5 provides a detailed view of the changes in the skill mixing of different skill pairs across various occupations. Overall, the increase in the degree of mixing of non-routine skill pairs is higher than the increase in the mixing of skill pairs that include routine skills. Service and blue-collar occupations experience the highest increases in skill mixing of different skills, surpassing white-collar and high-wage occupations. When it comes to routine skills, blue-collar jobs lead other occupations in terms of increase in mixing.

Figure A5 also provides a detailed view of the changes in the skill mixing of RNR and non-routine skills across various industries. The main patterns indicate that the private service sector, followed by retail trade and construction, leads others in the growth of skill mixing, while public, education, social, and professional services experience the least increases in skill mixing. There is also noticeable heterogeneity across industries in terms of the skills that are mixed. For instance, in finance, real estate, and professional services, there is much higher mixing in non-routine skills relative to RNR skills; conversely, in industries like mining, transportation, public utilities, and construction, RNR skills are mixed in a higher degree.

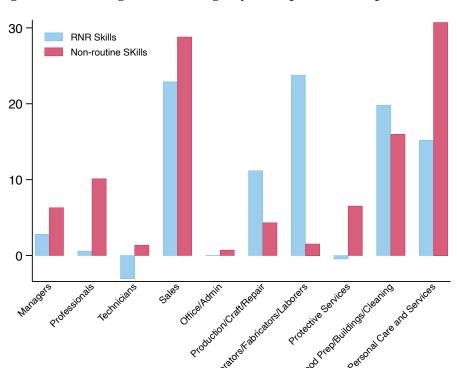
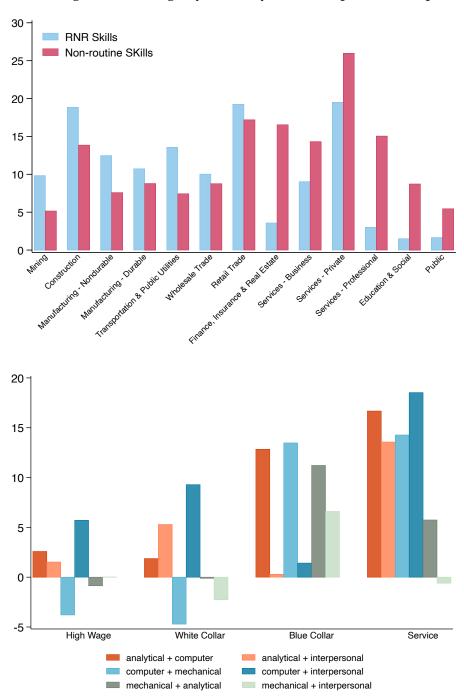


Figure A4: Mixing Index Change by Occupation Groups, 2005-2018

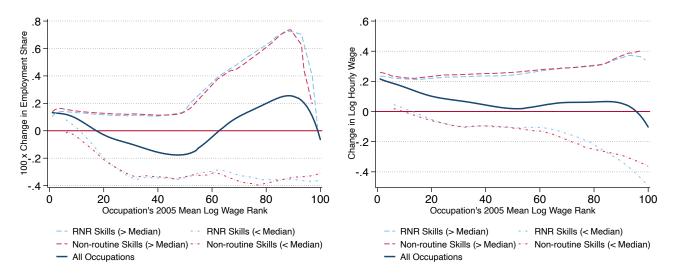
Notes: This figure plots the changes in skill mixing indexes across different occupation groups. The units of the index changes are percentiles of their distributions in 2005. Workers are categorized into 10 1-digit occupational groups that cover the entirety of US non-agricultural employment following Acemoglu and Autor (2011). O*NET and ACS data are combined for these figures with consistent occupation codes from Autor and Price (2013) and developed by Deming (2017).

Figure A5: Mixing Index Change by Industry and Occupation Groups, 2005-2018



Notes: These two figures plot the changes in mixing indexes across different industry and occupation groups. The units of the index changes are percentiles of their distributions in 2005. The industry grouping is based on the industrial classification from the 1990 census. The occupation groups (High-wage, White-collar, Blue-collar, Service) follow Acemoglu and Autor (2011). O*NET and ACS data are combined for these figures with consistent occupation codes from Autor and Price (2013) and developed by Deming (2017).

Figure A6: Smoothed Employment and Wage Changes by Skill Percentile, 2005-2018



Notes: These figures plot the smoothed changes of share of total hours workded (Panel A) and hourly wage (Panel B) for occupations between 2005-2018. On the x-axis, occupations are ranked into 100 percentiles by the average log wages of workers in those occupations in 2005. The changes in the share of hours worked and percent wage growth are then calculated for each percentile, which fit into smoothed lines using cubit polynomial fit. Solid lines depict the smoothed employment/wage changes for all the occupations, while dashed (or dotted) lines depict the changes for occupations with above-median (or below-median) increases in the skill mixing indexes.

Additional Results on Distributional Implications: One of the key structural changes in the U.S. labor market post-1980 is the pronounced job polarization or hollowing out of middle-skill employment and wage growth, due potentially to the routine-biased technological change and offshoring (Acemoglu and Autor 2011; Goos, Manning, and Salomons 2014). To see how much skill mixing can relate to these distributional dynamics, Figure A6 depicts the smoothed observed changes in both the share of total hours worked and log wage in 2005-2018 for occupations ranked by their hourly wage percentiles in 2005. I reconstruct these smoothed employment/wage changes for two groups of occupations: those with above-median increases in skill mixing indexes and those below the median.

Figure A6 first confirms the inverted bell shape (polarization) of observed employment and to a lesser extent, the change in wages. Furthermore, it illustrates key differences in occupations that have become more skill mixed. For occupations within similar wage ranks in 2005, those that become more mixed in skills have a higher increase in employment share and wage growth. In fact, almost the entirety of employment and wage growth is accounted for by occupations that have become more skill mixed during this period. Therefore, relating to polarization, the differential growth in employment and wage among occupations at the top and bottom end of the 2005 wage distribution is entirely accounted for by skill-mixing

occupations during this period. Besides being an important phenomenon for labor market dynamics, skill mixing also provides a unified and multi-dimensional perspective of the polarization changes.

A.6 Robustness of Trend Results to Measures of Skills

In this section, I discuss the robustness of the trend results to using alternative measures of skills. Specifically, I present alternative trend results using different ways of processing skill descriptors from O*NET, such as not using PCA, and standardizing rather than rescaling. I also show the robustness using broader skill measures than those applied in the main analysis.

Alternative Construction of Skills: Since O*NET contains a large number of descriptors, many of which capture the same dimensions of skill requirements, it becomes standard practice to first abstract useful information from the descriptors to construct lower-dimensional measures of skills. The first approach, as in Autor, Levy, and Murnane (2003), Acemoglu and Autor (2011) and Deming (2017), takes the average of a subset of variables and assumes that such average represents a particular broader skill intensity and not others. The other approach, as in Lise and Postel-Vinay (2020), applies PCA to the entire set of variables, which assumes that each of the variables contains information about underlying components that are orthogonally distributed. Both approaches impose different assumptions, with the first one giving more easily interpretable skill groups while the second being more data-driven. A third approach, as in Yamaguchi (2012), first picks descriptors that are ex-ante most easily interpretable with respect to each skill dimension, and then conducts PCA on those descriptors to abstract the most relevant variation. The main body of the paper adopts the third approach; here I show robustness checks using alternative skill measures.

Online Appendix Figure A7 presents the trend results using skill measures constructed by taking an average of the descriptors without imposing PCA (panel 2) and using skill measures normalized by standard deviation rather than linearly scaled to [0,1] (panel 3). Normalizing by standard deviation necessarily creates negative values for the skills; since the mixing index is defined based on positive real values, having these negative values invalidates the mixing index in measuring skill mixing. One solution is to add a positive number to the skill measures. As any number chosen is essentially arbitrary, here I added the negative of the smallest value such that the re-scaled measure lies exactly above 0. For both of these robustness exercises, the main message is similar to the main text: there is a significant increase in mixing for non-routine skills, and less so for RNR skills.

Skill Measures: Another concern is that by using skill measures from Acemoglu and Autor (2011), each of which is constructed from a few descriptors, the resulting skill measures could be relatively "narrow" and do not provide a comprehensive depiction of the skill spaces. To alleviate this concern, I construct skill measures using a broader set of descriptors, similar to that of Lise and Postel-Vinay (2020). I first select descriptors from abilities, knowledge, skills, and work activities files that are more relevant for job skill demand, leaving me with around 163 descriptors. I then combine each year's O*NET data with ACS and conduct PCA on the merged data from the years 2005-2018.

The result from this approach supports the choice of analytical, routine, and interpersonal skills in the main text The first three factors out of PCA explain around 60% of the variation across all the descriptors for years. The first factor has a strong positive association with reason and math skills, such as "Deductive Reasoning", "Inductive Reasoning" and "Mathematics", while the second factor relates more to motor coordination and mechanical work, such as "Multi-limb Coordination", "Mechanical" and "Equipment Maintenance". The third factor is clearly more associated with interacting with other people, such as "Selling or Influencing Others" and "Resolving Conflicts and Negotiating with Others". I interpret the second factor as "mechanical" rather than routine for the broader skill measures.

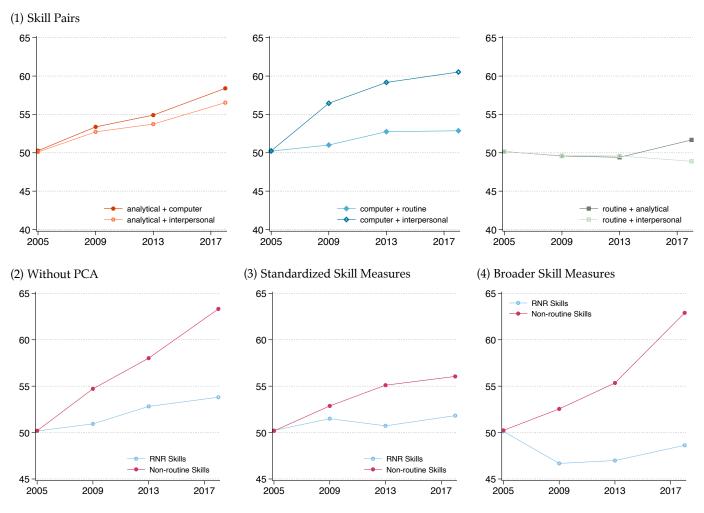
After conducting PCA, one could directly extract the factors imposing the assumptions that these factors are orthogonal to each other. While this is obviously quite convenient, it nevertheless creates the challenge of interpretability, since each of the factors has been constructed such that it is positively or negatively correlated with all of the 163 descriptors, and the assumption of orthogonality appears strong if the underlying skills are complementary in production across occupations. To take a fine balance between comprehensibility and interpretability, I adopt an approach similar to the measurement validation literature (Costello and Osborne 2005; Thompson and Daniel 1996), where I first conduct PCA/factor analysis to reveal the underlying dimensionality and structure of the measure (as has been done in the previous step). Guided by the factor loadings, I then hand-pick the skill descriptors into three broad groups "analytical", "mechanical" and "interpersonal" without imposing the orthogonality assumption.

Online Appendix Table A8 illustrates the selected descriptors for each of the composite skill measures. These descriptors are broadly in line with Acemoglu and Autor (2011) but have several distinctions. First, the descriptors coming from factor analysis lean more toward reasoning, comprehension, and expression. Second, the mechanical skill used in the main text is the average of two ASVAB test scores that are constructed by the weighted

average of 26 O*NET descriptors. The ASVAB "Mechanical Comprehension" tests contestants' "understanding of the principles of mechanical devices, structural support, and properties of materials" and the ASVAB Electronics Information tests contestants' "understanding of electrical current, circuits, devices, and systems", both stressing one's knowledge basis. On the other hand, the descriptors chosen by conducting PCA relate more to physical control, coordination, and machine operation aspects rather than mental perception. Third, the descriptor choices for interpersonal skill from factor analysis also emphasize interactions with others as in Acemoglu and Autor (2011) but are more comprehensive.

Online Appendix Figure A7 panel (4) illustrates the trend results using these broader skill measures. The message on the growth of skill mixing remains the same as the main text, that is there is strong growth of skill mixing for non-routine skills. Nonetheless, for RNR skills, the degree of skill mixing has decreased using the broader measures.

Figure A7: Trend of Skill Mixing with Alternative Skill Measures



Notes: These three panels plot the employment-weighted mixing indexes of different skills in the U.S. economy from 2005-2018 using O*NET and ACS data. Panel (1) shows the changes in skill mixing indexes of 6 distinct skill pairs of the 4 skills. In panel (2), skill mixing indexes are calculated using skill measures without using PCA, and in panel (3), skill measures are normalized to have mean 0 and standard deviation 1. Panel (4) shows the changes in mixing indexes using broader skill measures as described in online Appendix A.6.

Table A8: Components of Broader Skill Measures

Skill Category	Task Descriptors
Analytical	Deductive Reasoning Inductive Reasoning Mathematical Reasoning Number Facility Mathematics Economics and Accounting Reading Comprehension Writing Speaking Oral Comprehension Written Comprehension Oral Expression Written Expression
Routine	Multilimb Coordination Speed of Limb Movement Mechanical Performing General Physical Activities Handling and Moving Objects Controlling Machines and Processes Operate Vehicles, Mechanized Devices or Equipment Repairing and Maintaining Mechanical Equipment Repairing and Maintaining Electronic Equipment Installation Equipment Maintenance Repairing Production and Processing
Interpersonal	Assisting and Caring for Others Selling or Influencing Others Resolving Conflicts and Negotiating Coaching and Developing Others Staffing Organizational Units Service Orientation Administration and Management Customer and Personal Service

Notes: This table lists the O*NET descriptor components for each of the constructed (broader) composite skill groups as discussed in online Appendix A.6.

A.7 Robustness of Trend Results to Measures of Skill Mixing

I introduce two additional measures and show the robustness of the trend results using these alternative mixing measures.

A first commonly used measure for concentration or specialization based on the share of a total quantity is the Herfindahl–Hirschman Index (HHI).⁶⁶ Equation (7) shows how to use inverse HHI to measure skill mixing for an occupation represented by $(\alpha_{ja}, \alpha_{js})$. Observe that this index is maximized when $\alpha_{ja} = \alpha_{js}$, exactly corresponding to the case when the skill vector lies on the unit vector and becomes most mixed. If one skill's intensity is greater than the other, the occupation becomes less mixed and this index becomes smaller. Similar to an angle-based mixing index, this measure is insensitive to the length of a skill vector, since each skill is normalized by the total quantity of skills in that occupation.

$$\left[\left(\frac{\alpha_{ja}}{\alpha_{ja} + \alpha_{js}} \right)^2 + \left(\frac{\alpha_{js}}{\alpha_{ja} + \alpha_{js}} \right)^2 \right]^{-1} \tag{7}$$

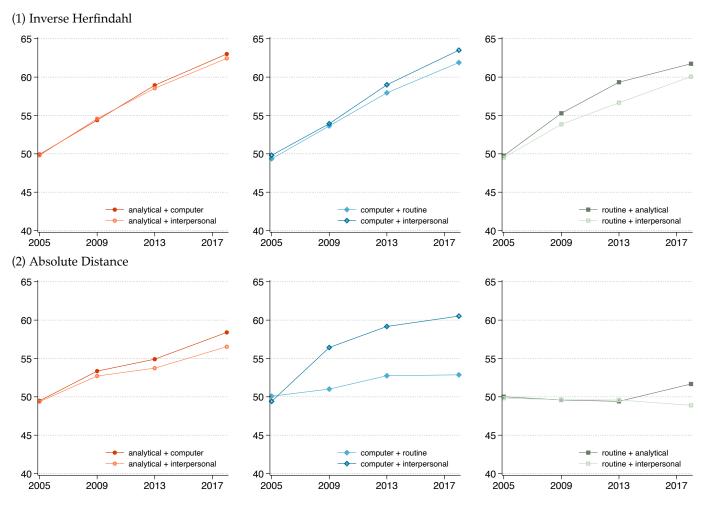
$$-\frac{|\alpha_{ja} - \alpha_{js}|}{\alpha_{ja} + \alpha_{js}} \tag{8}$$

In a similar vein, the degree of skill mixing could also be measured by normalizing the absolute distance between skill intensities for a skill vector: as this distance decreases, the overall skill portfolio becomes more balanced; normalization then eliminates the effect of the length of the skill vector. Equation (8) gives a particular specification of such a measure. As can be seen from this construction, as the absolute distance between skill intensities decreases and the degree of mixing increases, this measure also increases, though from the direction of $(-\infty,0)$.

In Online Appendix Figure A8, I show the robustness of the trend results using these alternative measures in panels (1) and (2). Both measures deliver the same message as the cosine mixing index in the main text, that is, there is a sizable increase in skill mixing, particularly for non-routine skills. The only difference is that for the HHI skill index, there is also a comparable increase in skill mixing for RNR skills.

⁶⁶For applications in the labor literature, Ransom and Phipps (2017) and Jin (2017) use the inverse of HHI as the "variety index" to examine how diverse the jobs held for students who graduated from a certain major. Similar logic can be applied to the measurement of skill mixing: in the context of 2-dimensional skill space, the more "varied" skills an occupation uses essentially means that skills are more mixed.

Figure A8: Trend of Skill Mixing with Alternative Indexes



Notes: These three panels plot the employment-weighted mixing indexes of different skills in the U.S. economy from 2005-2018 using O*NET and ACS data. In panels (1) and (2), mixing indexes are calculated using the Inverse Herfindahl index and Absolute Distance as discussed in online Appendix A.7.

A.8 Additional Results on Wage and Employment Returns

In this section, I provide more detailed results on wage returns, and results relating employment to occupation skill mixing. I also provide robustness results to the analysis of the returns to skill mixing in the main paper.

Detailed Results on Wage and Employment Returns: I first check the returns to individual skills and how they interact with the returns to skill mixing. Table A9 column (1) shows that in a cross-sectional regression analytical and computer skills both have significant positive returns. Workers employed in occupations requiring a higher degree of these two skills earn more. Nonetheless, workers in occupations that require a higher level of interpersonal skills have a wage reduction.

Column (2) of Table A9 shows that by restricting to within-occupation variation and including skill mixing measures an important pattern appears: the coefficients for most individual skills become slightly more negative (except for routine skill),⁶⁷ while the skill mixing indexes of analytical paired with interpersonal skills, as well as routine and interpersonal skills show significant positive returns. Such a pattern persists in columns (3) and (4) including worker skills and fixed effects, only that the skill mixing of analytical and computer skills is more precisely estimated to have a positive return. This indicates that the mixing of skills earns separate and additional rewards beyond those predicted by individual skills.

Turning to employment, there is also a positive employment premium for workers with a more mixed skill set. Column (6) of Table A9 shows that workers with a more mixed level of computer and interpersonal routine skills, or computer and interpersonal skills, or routine and interpersonal skills, are more likely to move from unemployment to employment. Workers with a more mixed level of analytical and computer, or analytical and interpersonal skills, are also more likely to exit unemployment, but the results are not precisely estimated. On the other hand, workers with a more mixed level of routine and interpersonal skills are less likely to find employment.

Robustness Checks: Table A10 shows the robustness checks to the results in Table 4. Specifically, Columns (1) and (2) utilize the Absolute Distance and Inverse Herfindahl measures to formulate mixing indexes (refer to online Appendix A.7 for details), while Columns (3)

⁶⁷The insignificant or even negative return to analytical skill over time also finds support from the literature. Lise and Postel-Vinay (2020) shows a strong negative 14.4 percent return on cognitive skill using NLSY data with 3-digit occupation fixed effects. Deming (2017) found that the return to cognitive skills has declined across the NLSY79 and NLSY97 cohorts, similar to Castex and Kogan Dechter (2014).

and (4) employ standardized and broader measures of skills (refer to online Appendix A.6 for details).

The findings presented in Table A10 clearly indicate a consistent trend: workers experience a positive return when they are employed in occupations that are more mixed with analytical with computer skills, analytical with interpersonal skills, and routine with interpersonal skills. Specifically, a notable increase in wage is observed with workers in occupations more mixed of analytical and computer skills, especially when applying standardized and broad skill measures; similarly, occupations becoming more mixed of analytical and interpersonal skills, when assessed using the Absolute Distance and Inverse Herfindahl measures, also show a significant positive return. The mixing of routine and interpersonal skills exhibits a positive return as well across the different measures.

On the other hand, the mixing of computer and routine skills, computer and interpersonal skills, and routine and analytical skills all exhibit significant negative wage returns at the occupational level. These negative coefficients may indicate that the combination of these particular skills is less beneficial or leads to inefficiency

Table A9: Return to Skill Mixing Full Table

Dependent Variable:	ln(hourly wage)				Employed	
	(1)	(2)	(3)	(4)	(5)	(6)
Occupation Skills						
Analytical	-0.023**	-0.023**	-0.022**	-0.026*	-0.015*	
,	[0.009]	[0.010]	[0.010]	[0.014]	[0.008]	
Computer	-0.008	-0.014	-0.015	-0.019	-0.009	
	[0.010]	[0.011]	[0.011]	[0.016]	[0.009]	
Interpersonal	-0.009	-0.014	-0.015*	-0.002	-0.013*	
	[0.009]	[0.009]	[0.009]	[0.012]	[0.008]	
Mechanical	0.021**	0.029***	0.028***	0.034*	0.019**	
	[0.010]	[0.011]	[0.011]	[0.018]	[0.009]	
Mix (analytical + computer + interpersonal)	0.017***	0.015***	0.001	0.005	0.014***	
	[0.005]	[0.005]	[0.006]	[0.009]	[0.005]	
Mix (routine + computer)	-0.035***	-0.045***	-0.044***	-0.045***	-0.037***	
	[0.008]	[0.008]	[0.008]	[0.013]	[0.007]	
Mix (routine + analytical)	-0.041***	-0.045***	-0.042***	-0.007	-0.039***	
	[0.007]	[0.008]	[0.008]	[0.013]	[0.007]	
Mix (routine + interpersonal)	0.029***	0.035***	0.033***	0.014	0.025***	
	[0.009]	[0.009]	[0.009]	[0.015]	[0.008]	
Worker Skills						
AFQT (analytical)		0.074***	0.073***		-0.048*	-0.009**
		[0.011]	[0.011]		[0.028]	[0.004]
Computer		0.045***	0.044***		0.031	0.056***
		[0.006]	[0.006]		[0.025]	[0.002]
Social (interpersonal)		0.016***	0.015***		0.032	-0.001
		[0.005]	[0.005]		[0.030]	[0.002]
ASVAB mechanical (routine)		-0.015	-0.014		0.015	-0.002
		[0.015]	[0.015]		[0.024]	[0.005]
Mix (AFQT + computer + social)		0.065***	0.070***		0.030**	0.135***
		[0.017]	[0.017]		[0.013]	[0.009]
Mix (ASVAB mechanical + computer)		0.029*	0.024		-0.004	0.038***
N. (ACMAR 1 1 A FOTT)		[0.017]	[0.017]		[0.018]	[0.010]
Mix (ASVAB mechanical + AFQT)		0.006	0.007		-0.013	0.000
M' (ACYAD		[0.008]	[0.008]		[0.026]	[0.004]
Mix (ASVAB mechanical + social)		-0.039***	-0.038***		0.011	0.030***
Interaction		[0.008]	[0.008] 0.032***		[0.017]	[0.004]
Interaction						
		_	[0.008]	_	_	
Ethnicity*Gender, Age/Year, Region, Edu FE	X	X	X	X	X	
Occupation FE	X	X	X	X	X	
Worker FE	00.501	5 0.5.15	= 0.5.15	Χ	0400	
Observations	88,391	79,343	79,343	88,391	94,062	
R-squared	0.416	0.430	0.431	0.756	0.136	

Notes: See Table 4 notes.

Table A10: Robustness Checks of Return to Skill Mixing

Dependent Variable:	ln(hourly wage)		
	(1)	(2)	(3)
Occupation Skills			
Analytical	-0.014*	-0.008	-0.013
	[0.008]	[0.033]	[0.008]
Computer	-0.002	0.069**	-0.038***
	[0.009]	[0.027]	[0.010]
Interpersonal	-0.019**	-0.118***	-0.014*
	[0.008]	[0.030]	[0.008]
Routine	0.026***	0.091***	0.010
	[0.009]	[0.017]	[0.008]
Mix (analytical + computer)	0.007	-0.040	0.020***
	[0.005]	[0.036]	[0.007]
Mix (analytical + interpersonal)	0.010**	0.156***	0.025***
	[0.004]	[0.042]	[0.005]
Mix (computer + routine)	-0.028***	-0.045***	-0.087***
	[0.007]	[0.015]	[0.013]
Mix (computer + interpersonal)	-0.011**	-0.019	-0.021***
	[0.005]	[0.033]	[0.008]
Mix (routine + analytical)	-0.033***	-0.080***	-0.041**
	[0.007]	[0.015]	[0.018]
Mix (routine + interpersonal)	0.010	0.033**	0.026**
	[0.007]	[0.016]	[0.012]
Ethnicity*Gender, Age/Year, Region, Edu FE	X	X	X
Occupation FE	X	X	X
Worker FE	X	Χ	Χ
Observations	87,655	87,655	87,655
R-squared	0.757	0.757	0.758

Notes: This table reports the robustness checks to the results in Table 4. Columns (1) and (2) use Absolute Distance and Inverse Herfindahl measures to construct mixing indexes (see online Appendix A.7 for details) and Columns (3) and (4) use standardized and broad measures of skills (see online Appendix A.6 for details). Log hourly wages are the outcome variables and person-year is the unit of observation. The occupational skill and skill mixing measures come directly from O*NET and are merged to NLSY79&97 based on census occupation codes. All measures of skill and skill mixing are normalized to have mean 0 and standard deviation 1. Ethnicity-by-gender, age, year, census region, urbanicity, and a 5-category (no high-school, high-school graduate, some college, college graduate, post-college) education fixed effects are included for all regressions, with additional fixed effects as indicated in the table. Standard errors are clustered at the individual level. *** p<0.01, ** p<0.05, * p<0.10.

A.9 Additional Results on College Major's Skill Mixing

In the online Appendix Table A11, I list the top majors both in terms of the levels and changes in the degree of skill mixing for different skill pairs. Architecture and Environmental Design stands out as the highest major in mixing the three non-routine skills, followed by Computer and Information Sciences, and Communications. Two other majors: Social Sciences and Agriculture and Natural Resources are among the top majors in mixing routine and non-routine skills.

In Table A9 column (4), I represent a worker's human capital by the skill content of a worker's accumulated education experience.⁶⁸ Such a designation necessarily restricts the analysis to those who have entered college and brings up selection concerns; however, controlling for worker fixed effects and fixed and time-varying occupation attributes, the estimates show whether it is rewarding to study a more skill-mixed major conditional on one's job choices. The result in column (4) shows a positive return of around 3 percent studying a college major that is associated with a standard deviation higher mixing of non-routine skills. Interestingly, when taking into account the skill mixing of a worker's college major, the wage premium to occupational skill mixing becomes insignificant. This is due to the correlation between the skill mixing of college majors and subsequent occupational choices and shows that the former plays a more significant role in driving the wage returns.

⁶⁸This is determined using rolling averages of skill and mixing measures from the worker's entire educational history, since workers may have studied multiple majors.

Table A11: Top College Majors in Skill Mixing Growth

Skill Combination	Fields
Analytical + Computer + Interpersonal	Architecture and Environmental Design Computer and Information Sciences Communications
Analytical + Computer	Interdisciplinary Studies Area Studies Computer and Information Sciences
Analytical + Interpersonal	Architecture and Environmental Design Computer and Information Sciences Communications
Computer + Interpersonal	Architecture and Environmental Design Computer and Information Sciences Engineering
Routine + Computer	Social Sciences Agriculture and Natural Resources Foreign Languages
Routine + Analytical	Agriculture and Natural Resources Social Sciences Foreign Languages
Routine + Interpersonal	Agriculture and Natural Resources Architecture and Environmental Design Social Sciences

Notes: This table lists the top 3 college majors for each mixing index both in terms of levels and in terms of changes from 2005 to 2019. To calculate the degree of skill mixing for college majors, I first map the occupation level degree of skill mixing contained in the O*NET data to NLSY, and then calculate for each college major's students, the employment weighted average of skill intensities and mixing indexes of their occupations. I use both NLSY79&97 to get the employment weight on occupations.

Table A12: Return to Skill Mixing Full Table with Individual Skills

NLSY97 Code (before 2010)	Major Field of Study	NLSY79 Code	NLSY97 Code (CM10)	Major Field of Study	NLSY79 Code	NLSY79 Code	Major Field of Study
0	None, no major yet (didn't/don't) have to declare yet;		1	Agriculture, agriculture operations, & related sciences	1	0	None, General Studies
1	Agriculture/Natural resources	1	3	Natural resources and conservation	1	1	Agriculture and Natural Resources
2	Anthropology	22	4	Architecture and related services	2	2	Architecture and Environmental Design
3	Archaeology	22	5	Area, ethnic, cultural, gender, and group studies	3	3	Area Studies
4	Architecture/Environmental design	2	9	Communications, journalism, and related programs	6	4	Biological Sciences
5	Area studies	3	10	Communications technologies/technicians & support services	6	5	Business and Management
6	Biological sciences	4	11	Computer & information sciences & support services	7	6	Communications
7	Business management	5	12	Personal and culinary services	49	7	Computer and Information Sciences
8	Communications	6	13	Education	8	8	Education
9	Computer/Information science	7	14	Engineering	9	9	Engineering
10	Criminology	22	15	Engineering technologies & engineering-related fields	9	10	Fine and Applied Arts
11	Economics	22	16	Foreign languages, literatures, and linguistics	11	11	Foreign Languages
12	Education	8	19	Family and consumer sciences/human sciences	13	12	Health Professions
13	Engineering	9	22	Legal professions and studies	14	13	Home Economics
14	English	15	23	English language and literature/letters	15	14	Law
15	Ethnic studies	3	24	Liberal arts and sciences, general studies & humanities	49	15	Letters
16	Fine and applied arts	10	25	Library science	16	16	Library Science
17	Foreign languages	11	26	Biological and biomedical sciences	4	17	Mathematics
18	History	22	27	Mathematics and statistics	17	18	Military Sciences
19	Home economics	13	28		18	19	Physical Sciences
20		49	29	Military science, leadership, and operational art	18	20	-
	Interdisciplinary studies			Military technologies and applied sciences			Psychology
21	Mathematics	17	30	Multi/interdisciplinary studies	49	21	Public Affairs and Services
22	Nursing	12	31	Parks, recreation, leisure, and fitness studies	21	22	Social Sciences
23	Other health professions	12	32	Basic skills development/remedial education	8	23	Theology
24	Philosophy	15	33	Citizenship activities	21	24	Mechanics
25	Physical sciences	19	34	Health-related knowledge and skills	12	25	Transportation
26	Political science and government	21	35	Interpersonal and social skills	6	49	Interdisciplinary Studies
27	Pre-dental	4	36	Leisure and recreational activities	49	99	Other
28	Pre-law	14	37	Personal awareness and self-improvement	8		
29	Pre-med	4	38	Philosophy and religious studies	15		
30	Pre-vet	4	39	Theology and religious vocations	23		
31	Psychology	20	40	Physical sciences	19		
32	Sociology	22	41	Science technologies/technicians	24		
33	Theology/religious studies	23	42	Psychology	20		
36	Nutrition/Dietetics	4	43	Homeland security, law enforcement, firefighting, and related protective services	18		
37	Hotel/Hospitality management	5	44	Public administration and social service professions	21		
38	Other - Recoded to Liberal Arts and Sciences	49	45	Social sciences	22		
39	Other - Recoded to Automobile/Automotive Mechanics Technology/Technician	24	46	Construction trades	24		
40	Other - Recoded to Human Services, General	21	47	Mechanic and repair technologies/technicians	24		
41	Other - Recoded to Social Work	21	48	Precision production	24		
42	Other - Recoded to Electrical/Electronics Maintenance and Repair Technology	24	49	Transportation and materials moving	25		
43	Other - Recoded to Geography	22	50	Visual and performing arts	10		
44	Other - Recoded to International Relations & Affairs	21	51	Health professions and related programs	12		
45	Other - Recoded to transportation & materials moving	25	52	Business, management, marketing, & related support services	5		
46	Other - Recoded to security and protective services	21	53	High school/secondary programs and certificates	8		
47	Other - Recoded to legal support services	14	54	History	22		
48	Other - Recoded to other sciences/applied sciences	49	60	Residency programs Uncodable	12		
99	UNCODABLE	99	999	Uncodable	99		

B THOERY AND QUANTITATIVE

B.1 Propositions and Proofs

Lemma 1. An occupation $\mathbf{y}^j = \{y_1^j, ..., y_k^j, ..., y_k^j\} \in S \subset \mathbb{R}^{K+}$ within a closed skill space S of dimension K is more mixed in skills based on Definition 1 if for any pair of skills (h, k), the ratio of $\frac{y_h}{y_k}$ becomes closer to 1.

Proof of Lemma 1: For the occupation \mathbf{y} we want to establish how the degree of skill mixing changes if the skill dimensions for j and k are to vary. The lemma can be simply proved by considering the skill mixing index for this occupation. Let $y_k = ry_h$ and denote y_h by y, the mixing index for \mathbf{y} is:

$$\frac{y+ry+A}{\sqrt{K}\sqrt{y^2+r^2y^2+B}},$$

where A and B are two constants that do not depend on y_k and y_h . The above equation is maximized at r = 1. Therefore, for any y_h , the occupation is more skill-mixed if the ratio r is close to 1. This completes the proof. Q.E.D.

Proposition 1 (Changes in Skill Mixing). Consider an occupation $\mathbf{y}^j = \{y_1^j, ..., y_k^j, ..., y_K^j\} \in S \subset \mathbb{R}^{K+}$ within a closed skill space S of dimension K. Assume that firms operate the occupation with a production technology as described by equation (2) and under an occupation operation cost defined in Section V. Under these conditions, occupation \mathbf{y}^j will show an increased degree of skill mixing given the following conditions:

(i) The skills within the vector \mathbf{y}^j demonstrate a rise in complementarity in production (a decrease in σ), provided that σ does not undergo a change in sign.

- (ii) The skills within the vector \mathbf{y}^j exhibit an higher increasing marginal cost (an increase in ρ), under the condition that $\rho > \sigma^j$.
- (iii) Additionally, occupation \mathbf{y}^j will exhibit a increased degree of skill mixing in the (y_k^j, y_h^j) dimension if the ratio between (x_k, x_h) approaches unity.

Proof of Proposition 1: Lemma 1 posits that an occupation \mathbf{y}^j exhibits greater skill mixing if the ratio across all skill dimensions approximates 1. Therefore, establishing the influence of the ratio on the degree of skill mixing suffices. The initial step concentrates on any two skills within the vector (y_k^j, y_h^j) . I subsume occupation superscript j in the proof below.

The firm value function indicates that the firm re-optimizes the choice of \mathbf{y} in each period. Consequently, within a given submarket at a particular time instance $(\mathbf{x}, \mathbf{y}, \omega)$, the firms' choices of \mathbf{y} remain uninfluenced by the continuation value, rendering it a static problem. Time subscript is subsumed in the subsequent proof.

By deriving the first-order condition of firms' optimization problems in the submarket (\mathbf{x}, \mathbf{y}) and taking ratios, one obtains the following condition: $\frac{y_h}{y_k} = (\frac{x_h}{x_k})^{\frac{\sigma}{\rho-\sigma}} (\frac{\alpha_h}{\alpha_k})^{\frac{\sigma}{\rho-\sigma}}$. Therefore, the ratio of firms' optimal skill requirement choices for any two skills (y_h, y_k) is influenced by four variables: the elasticity parameter of substitution in production σ , the degree of increasing marginal occupation operation cost ρ , the ratio of worker skills in the submarket (x_h, x_k) , and the ratio of skill efficiencies (α_h, α_k) .

From the equation, it is evident that as σ decreases, indicating an increase in skill complementarity in production, $\frac{y_h}{y_k}$ will converge to 1 for any two skills (y_h, y_k) , under the assumption that σ does not change sign. Similarly, as ρ increases, $\frac{y_h}{y_k}$ will approximate 1 for any two skills (y_h, y_k) , given that $\rho - \sigma$ does not change sign.

The influence of worker skill bias on the degree of skill mixing of \mathbf{y} presents a more complex scenario, as a change in the ratio $\frac{x_h}{x_k}$ does not directly imply a change in the ratio of other skill pairs. Consequently, to gauge its impact on the degree of skill mixing, the focus must remain on the (y_h, y_k) dimension. For this specific dimension, if $\frac{x_h}{x_k}$ converges to 1, then $\frac{y_h}{y_k}$ also approaches 1. *Q.E.D.*

Proposition 2 (Changes in Wage and Job Finding). Consider an occupation $\mathbf{y} \in S \subset \mathbb{R}^K$ within a closed skill space S of dimension K. Assume that firms operate the occupation with a production technology as described by equation (2) and under an occupation operation cost defined by equation (??). Also, these firms offer an output share ω to workers and have value functions described by equation (5). Further, let worker value be described by equation (4). Under these conditions, workers in occupation \mathbf{y} will earn a higher wage, and unemployed workers will have a higher job-finding probability under conditions (i) and (ii) of Proposition 1

Proof of Proposition 2:

Wages: To establish the change in wages, one needs to show that the output of the worker-firm match increases as the elasticity parameter σ decreases and approaches 0 from 1, or if σ decreases in the negative range, consistent with skills becoming more complementary in production. At a particular output share rate ω , such value changes of σ will lead to higher wages.

Now, let us obtain the first derivative of σ for the production function 2. WLOG, let's consider the case of two skills, and express y_1x_1 and y_2x_2 as m and n. The output of a worker-firm match can be expressed as $q = (m^{\sigma} + n^{\sigma})^{1/\sigma}$. We can take log of the production function $\ln(q) = \frac{1}{\sigma} \ln(m^{\sigma} + n^{\sigma})$ and then take logarithmic differentiation that gives the following:

$$\frac{1}{q}\frac{\partial q}{\partial \sigma} = -\frac{1}{\sigma^2}\ln(m^{\sigma} + n^{\sigma}) + \frac{1}{\sigma}\frac{1}{m^{\sigma} + n^{\sigma}}(m^{\sigma}\ln(m) + n^{\sigma}\ln(n))$$

Solving for $\frac{\partial q}{\partial \sigma}$ gives:

$$\frac{\partial q}{\partial \sigma} = q\left[-\frac{1}{\sigma^2}\ln(m^{\sigma} + n^{\sigma}) + \frac{1}{\sigma}\frac{1}{m^{\sigma} + n^{\sigma}}(m^{\sigma}\ln(m) + n^{\sigma}\ln(n))\right]$$
$$\frac{\partial q}{\partial \sigma} = q\left[-\frac{1}{\sigma}\ln(q) + \frac{1}{\sigma}q^{-\sigma}(m^{\sigma}\ln(m) + n^{\sigma}\ln(n))\right]$$

In the case of the calibration of the model, since m, n, and y are all in the range of [0,1], one can show that the above derivative is negative when $0 < \sigma < 1$ or when $\sigma < 0$.

With respect to (ii) of Proposition 1, it is easy to see that since for the analysis of this paper, both (x, y) are in the range [0, 1], therefore the occupation operation cost is decreasing in ρ , so wage should increase as marginal cost increases.

Employment: For job finding probability, it suffices to show that $p(\theta_t(\mathbf{x}, \mathbf{y}, \omega))$ is increasing in σ and ρ . This becomes simpler since the above proof establishes that worker-firm output is increasing in both σ and ρ , and so does the firm's value $J(\mathbf{x}, \mathbf{y}, \omega)$. By the free entry condition in equation (6), at a fixed vacancy posting cost, an increase in $J(\mathbf{x}, \mathbf{y}, \omega)$ implies a decrease in $J(\mathbf{x}, \mathbf{y}, \omega)$ and therefore implies an increase in $J(\mathbf{x}, \mathbf{y}, \omega)$ under constant return to scale matching technology. *Q.E.D.*

B.2 Equilibrium Definition and Block Recursivity

In this section, I define a block-recursive equilibrium (BRE) for the economy following Menzio and Shi (2011). I further show that the equilibrium of the economy is unique and is block-recursive.

Definition 2 (Block-recursive Equilibrium). Let $\psi \in \Psi$ be the aggregate state of the economy, which is a distribution of agents across employment status e = U, W, skill profiles \mathbf{x} , occupational skill requirements \mathbf{y} , and output shares ω .

A block-recursive equilibrium for this economy consists of value functions for both unemployed and employed workers $U(\mathbf{x}): S \to \mathbb{R}$, $W(\mathbf{x}, \mathbf{y}, \omega): S \times S \times [0,1] \to \mathbb{R}$, and their respective policy functions $y'_U(\mathbf{x}): S \to S \times [0,1]$, $y'_W(\mathbf{x}, \mathbf{y}, \omega): S \times S \times [0,1] \to S \times S \times [0,1]$; firms' policy function $J(\mathbf{x}, \mathbf{y}, \omega): S \times S \times [0,1] \to \mathbb{R}$ and corresponding policy function $y'_J(\mathbf{x}, \mathbf{y}, \omega): S \times S \times [0,1] \to S \times S \times [0,1]$; labor market tightness $\theta(\mathbf{x}, \mathbf{y}, \omega): S \times S \times [0,1] \to \mathbb{R}_+$; and aggregate state $\psi \in \Psi$ such that:

- 1. The worker's value functions $U(\mathbf{x})$ and $W(\mathbf{x}, \mathbf{y} \text{ satisfy } (4) \text{ for all states } \psi \in \Psi \text{ and } y'_U(\mathbf{x}),$ $y'_W(\mathbf{x}, \mathbf{y}, \omega)$ are the associated policy functions respectively
- 2. Firms' value function $J(\mathbf{x}, \mathbf{y}, \omega)$ satisfy (5) for all states $\psi \in \Psi$ and $y'_J(\mathbf{x}, \mathbf{y}, \omega)$ is the associated policy function
- 3. The labor market tightness $\theta(\mathbf{x}, \mathbf{y}, \omega)$ in each submarket $(\mathbf{x}, \mathbf{y}, \omega)$ for all states $\psi \in \Psi$ is consistent with free-entry condition in equation (6)

From the above definition of block-recursive equilibrium agents' value functions and policy functions, as well as the market tightness are independent of the aggregate state, only requiring that they are consistent with the aggregate state distribution of agents. Such an equilibrium is easier to characterize analytically and solve numerically. Note a key difference between the above definite of BRE and the one defined in Menzio and Shi (2011). In the economy studied in this paper, because I use the model to study the steady-state equilibrium, the value functions, policy functions, and market tightness are entirely independent of the aggregate state. Whereas Menzio and Shi (2011) studies out-of-steady-state dynamics, the value functions, policy functions, and market tightness still depend on the aggregate productivity shocks but are independent of the distribution of agents across employment status and match-specific shocks.

Now, I show that a block-recursive equilibrium exists and is unique.

Proposition 3 (Existence and Uniqueness of BRE). *Under the model specification of linear utility and invertible and constant returns to scale matching function, also assume that the support for worker and occupation skill profiles S has bounded, then: i) all equilibria are block recurve as defined in definition 2; ii) there exists a unique block-recursive equilibrium.*

Proof of Proposition 3:

The proof first establishes the uniqueness of value functions (U, W, J), as well as policy functions and market tightness $(y'_U, \omega'_U, y'_W, \omega'_W, y'_J, \theta)$; then, the proof establishes their independence from the aggregate state.

Uniqueness: I first show that the value functions for workers and firms as defined in equation (4) and (5) are contractions. Let $\Theta = S \times S \times [0,1]$, which is bounded based on the assumption that S is bounded. Let $B(\Theta)$ the space of bounded functions $V: \Theta \to \mathbb{R}$ and the operator associated with the worker or firm value functions denoted by $T: B(\Theta) \to B(\Theta)$. It is straightforward to verify that T satisfies monotonicity and discounting properties:

- 1. (monotonicity) For $V, V' \in B(\Theta), V \leq V'$ implies $T(V) \leq T(V')$
- 2. (discounting) For $V \in B(\Theta)$ and $\epsilon > 0$, $T(V + \epsilon) =$

The above conditions establish that the operator *T* associated with either firm or worker values functions is a contraction bunder Blackwell's sufficient conditions. Therefore, the optimal values workers and firms obtain through dynamic optimization problems are unique.

Next, I show that the policy functions and market tightness are also unique. Since the optimal values firms and workers obtain for their dynamic optimization problems (4) and (5) are unique, the associated policy functions $(y'_U, \omega'_U, y'_W, \omega'_W, y'_J)$ are also unique due to concavity of the production function defined in equation (2) and workers have linear utility over consumption. To show the uniqueness of market tightness, first note that since it is assumed that the matching function is invertible, one may directly obtain market tightness through the market clearing condition (6) with $\theta > 0$. The uniqueness of θ then follows from the uniqueness of firms' value function.

Independence of Aggregate State: In the model economy, workers with different skill profiles \mathbf{x} search in their own market, and firms with different skill requirements \mathbf{y} post jobs in these separated markets, therefore, one can establish that the value functions of firms, workers and the market tightness are all independent of the aggregate state ψ . I establish this argument more rigorously through a backward induction argument as in Braxton and Taska (2021). For this purpose, I introduce back time subscript in the notation.

At the terminal period t = T, for an employed worker, the continuation value is zero for T + 1 onward, so the worker's dynamic programming problem does not depend on the aggregate distribution across states, and is equal to the worker's share of output $W_T(\mathbf{x}, \mathbf{y}, \omega) = \omega f(\mathbf{x}, \mathbf{y})$.

Similarly, the firm's value function also remains independent of the aggregate distribution $J_T(\mathbf{x}, \mathbf{y}, \omega) = (1 - \omega) f(\mathbf{x}, \mathbf{y})$. As a result, through the free entry condition in equation (6), the market tightness $\theta_T(\mathbf{x}, \mathbf{y}, \omega)$ is also independent of the aggregate distribution.

Firms at T-1 make occupation design choices \mathbf{y} to solve the firm dynamic programming problem in equation (5); workers at T-1 make labor market search choices over occupations \mathbf{y} to solve the worker dynamic programming problem in equation (4); As long as \mathbf{y} is within a bounded interval, the extreme value theorem assures at least one solution to this problem. This process is repeated stepping back from t=T-1,...,1, which completes the proof. *Q.E.D.*

B.3 Identification of Parameters

I begin by estimating the elasticity parameters in production and occupation operation cost, denoted by σ and ρ . As highlighted by Caselli and Coleman (2006), the challenge arises when allowing for the endogenous choice of the efficiency of inputs under constraints, as the elasticity parameters cannot be separately identified. To overcome this challenge, I estimate σ using the *relative wage within occupation* instead of relying on absolute wage levels.

Specifically, based on the model, the wage that workers receive per period is given by the share ω of the output of the worker-firm match, reduced by the occupation design cost, formulated as $w(\mathbf{x}, \mathbf{y}) = \omega f(\mathbf{x}, \mathbf{y}) - C(\mathbf{y})$. Consequently, within each occupation, the difference in wage relative to a base worker type $\Delta w(\mathbf{x}, \mathbf{y})$ can be articulated as follows:

$$\Delta w(\mathbf{x}, \mathbf{y}) = \omega \left[\sum_{k=1}^{K} (x^k y^k)^{\sigma} \right]^{\frac{1}{\sigma}} - A, \tag{9}$$

where A is occupation-specific and does not depend on the cost parameter τ or ρ . This formulation enables the identification of σ independent of the cost parameters. To carry out the estimation equation (9), I first adjust the wage for occupation fixed effects in order to account for A and ω . Next, I compute the within-occupation difference of the adjusted wage relative to the lowest skill type worker.⁶⁹ Last, I target the correlation between this adjusted within-occupation relative wage and worker abilities \mathbf{x} .⁷⁰

I now turn to the identification of the cost parameters ρ and τ . To begin with, note that the first-order condition of firms' optimization problems in the submarket (\mathbf{x},\mathbf{y}) can be simplified in ratios to $\frac{y_h}{y_k} = \left(\frac{x_h}{x_k}\right)^{\frac{\sigma}{\rho-\sigma}}$, a relationship that exclusively depends on the parameters σ and ρ . With σ already estimated, I then target the skill ratio y_j/y_k , which aligns with the moment of the degree of hybridization of occupations. Further, for employed workers, the distribution of employment across various occupations is governed by wages $w(\mathbf{x},\mathbf{y})$. Given the parameters described above, this functional relationship allows the estimation of τ .

Lastly, given the calibrated unemployment benefits b, the parameters of the matching, production, and cost functions, equation (4) reveals that the probability of exiting unemployment only depends on the vacancy posting cost. By targeting the unemployment level, c is identified.

⁶⁹Refer to Section V for an in-depth discussion on how worker skill types are calibrated.

 $^{^{70}}$ According to equation (9), σ can be identified from the correlation of any skill with the adjusted wage, which is what I use as the target.

B.4 Calibration of Skill Supply

I carry out the calibration of two key aspects of skill supply variation: the Markov probability of worker skill adjustment in a steady state equilibrium and the variation in worker skill supply spanning two data periods that the model aims to align with two steady states. I will first delve into the details of the skill variation between data periods and then explore the skill evolution within a model period as guided by the Markov process, following the approach of Lise and Postel-Vinay (2020).

Across-period Skill Supply Variation: Considering the potential influence of skill supply variation on skill mixing, I calibrate the model to reflect workers' choices in occupation, college major (if attended), and employment status, in line with the approach of Lise and Postel-Vinay (2020). This calibration introduces variation in worker skill supply across two periods. Worker skills are adjusted based on the requirements of an occupation or a college major; they increase if the requirements exceed the original skills and decrease if the requirements are lower or if the worker is unemployed. The speed of this adjustment is asymmetric and skill-specific.

Specifically, following the estimates from Lise and Postel-Vinay (2020), as presented in online Appendix Table B1, a worker's skills accumulate at a rate of γ_j times the gap between the worker's skill j and the occupation's requirement for that skill each year. The value of γ_j depends on whether it relates to learning or depreciation (upward or downward accumulation). Additionally, workers can lose skills when not employed, with unemployment treated as requiring a zero level for all skills. However, I specify such that a worker's skill level cannot fall below their initial endowments. For changes in skills while in school, I specify that workers spend an average of three years learning the skills of their majors.

I incorporate two modifications into this framework. First, since Lise and Postel-Vinay (2020)'s estimates are based on weekly data, I adjust them by multiplying by the number of working weeks, set at 47. Second, I align Lise and Postel-Vinay (2020)'s estimates of cognitive, interpersonal, and manual skills with my analysis's categories of analytical, interpersonal, and routine skills. Since Lise and Postel-Vinay (2020)'s estimates do not include computer skills, I use their cognitive skill estimates as a proxy.

In calculating the skill adjustment, I first standardize both worker skills and occupation skill requirements. Then, for example, if a worker is employed in an occupation that requires

⁷¹Their exclusion restriction imposes that (i) the ASVAB mathematics knowledge score only reflects cognitive skills; (2) the ASVAB automotive and shop information score only reflects manual skills; (3) the Rosenberg self-esteem score only reflects interpersonal skills.

a standard deviation higher in analytical skill compared to the worker's analytical skill, the worker will accumulate 0.36 standard deviations of analytical skill in a year due to learning on the job. Conversely, if a worker's interpersonal skill is higher than required, it will decrease by only 3×10^4 standard deviations, almost remaining unchanged, as interpersonal skills are estimated to be very hard to lose.

Markov Skill Supply Adjustment: I now discuss the Markov process of skill adjustment. Specifically, considering each skill j in the worker's skill profile \mathbf{x} as an element of the finite set S, the evolution of this skill follows a Markov process $\pi(x_j'|x_j,y_j)$, conditional on the worker's current skill level and employed occupation. If a worker is matched with an occupation that requires a skill level exceeding his or her own $(x_j < y_j)$, the worker's skill j will adjust upward in the next period: $x_j' > x_j$, and the inverse applies for a worker whose skill is lower than the requirements of their current occupation.

The calibration of the Markov adjustment probability is conducted in a similar manner to that of the across-period skill supply variation. The annual adjustment rates for different skills γ represent the rate at which worker skills approach occupation skill requirements, and it is regarded as the probability that a worker's skill j will adjust to the corresponding value.⁷²

The key challenge in this calibration process arises when quantifying the model: both worker skill and occupation skill requirements are discretized as grid values. To accommodate this discretization, the probability that a worker moves up or down a grid for skill j based on the occupation is scaled as below.

The Markov probability of upward adjustment is determined by:

$$\frac{y_j - x_j}{x_j^{up} - x_j} \mathbf{1}(x_j^{up} < y_j) \times \gamma_j^{up}$$

Similarly, the Markov probability of downward adjustment is given by:

$$\frac{y_j - x_j}{x_j^{down} - x_j} \mathbf{1}(y_j < x_j^{down}) \times \gamma_j^{down}$$

Here, x_j represents the current grid value of worker skill j, while x_j^{up} or x_j^{down} denotes the value of worker skill j up or down a grid, respectively. The indicator variables $\mathbf{1}(y_j < x_j^{down})$ or $\mathbf{1}(x_j^{up} < y_j)$ evaluates whether the skill j grid value of the worker's current employed occupation is greater or smaller than the value of the worker's skill j grid. This means that a

⁷²Here γ_j mirrors the annual skill adjustment rates as in Lise and Postel-Vinay (2020), scaled by the gap between a worker's skills and occupational demands to give the Markov probability.

worker will only adjust up or down a grid if the occupation's skill is larger or smaller than the corresponding up or down grid value for the worker's skill. This process specifies the interplay between skill adjustment and occupation requirements and allows for a precise calibration within the model's framework.

Table B1: Annual Skill Learning and Depreciation Rate

O*NET Measures	NLSY Measure	$\gamma_{ m school}^{ m learn}$	γ_j^{up}	$\gamma_j^{ m down}$
Analytical	AFQT score	0.33	0.36	0.10
Interpersonal	Deming (2017) social skill	0.33	0.05	0.00003
Routine	ASVAB mechanical	0.33	1.00	0.36
Computer	OCC/Major's 2005 value	0.33	0.36	0.10

Notes: This table illustrates for each O*NET skill measure, its corresponding skill measure using NLSY79&97 data, and the learning and depreciation rate for these different skills. The AFQT is the same as the one used by Altonji, Bharadwaj, and Lange (2012) followed by Deming (2017), which controls for age-at-test, test format, and other idiosyncrasies. Deming (2017)'s social skill measure consists of sociability in childhood and sociability in adulthood in NLSY79, and two questions from the Big 5 inventory gauging the extraversion in NLSY97. The average of workers' ASVAB mechanical orientation and electronics test scores are used for mechanical skill. Since ASVAB scores are not available for the NLSY97 survey, they are imputed based on predictive regression using the NLSY79 survey. Workers' occupations' or college majors' O*NET computer skill scores in the year 2000 are used as their endowed computer skill. The skill accumulation/depreciation rate is directly from Lise and Postel-Vinay (2020)'s estimates based on monthly data converted to annual values. Skill learning/depreciating while attending college is specified to be 33% per year.

Returns to Specialization: Table B2 analyzes the drivers of wage returns for specializing in different skills. The full model shows a 12.7 ppts increase in wage returns for specializing in interpersonal skills, while returns for analytical/computer skills and routine skills decrease by 21.5 and 1.7 ppts, respectively. Variations in skill supply reduce these returns by 1.1 to 2.8 ppts. For analytical/computer skill, the increase in skill complementarity leads to a 60.3 ppts rise in returns, whereas changes in operation costs and skill efficiency reduce the returns. For interpersonal skill, the increase in complementarity is the principal driver of the positive returns, leading to an 11.5 ppts rise. In contrast, for routine skills, all factors except changes in operation costs lead to reduced returns in specialization, with a decline of 27.9 ppts due to decreased skill efficiency.

Table B2: Returns to Skill Specialization Decomposition

Decomposition	Analytical/ Computer	Interpersonal	Routine
Full model	-21.46	12.65	-1.67
Skill supply	-2.26	-1.18	-2.79
Skill efficiency	8.18	-0.62	-27.87
Complementarity	-60.27	11.53	-6.22
Occ. cost	31.77	0.77	35.50

Notes: This table shows the model-generated changes in relative wages of high-type workers for the three skills. The first row shows the changes with all model channels, corresponding to the first three rows of Table 5. The following rows then show the variation attributable to different model channels. See the footnote of Figure 9 for details.

B.5 Algorithm and Solution Method

The quantitative method used for estimation is SMM. Given the parameters in the model that are internally estimated $\Theta = \{\sigma, \rho, \tau, c, \alpha_k\}$, each iteration of SMM first solves the steady state firm and worker policy function, after which a panel of worker is simulated to obtain the equilibrium distribution of labor market outcomes.

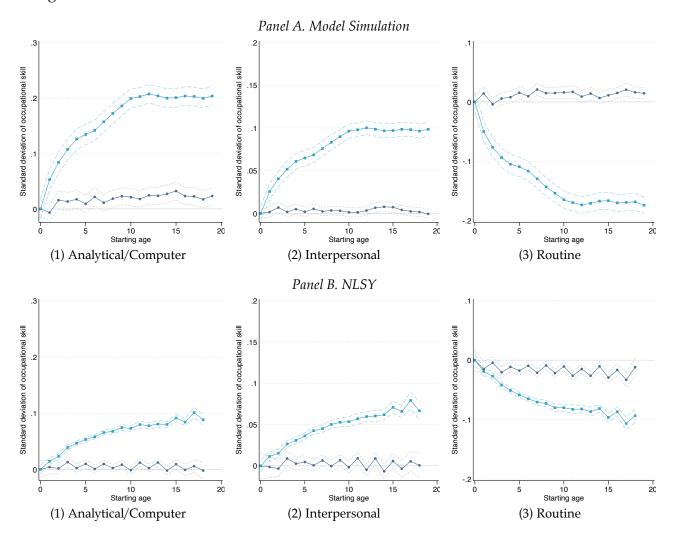
Specifically, to find the steady state policy of agents, I use value function iteration:

- 1. Fix the number of periods *T*
- 2. Starting from the terminal period T, solve the firm problem as in equation (5)
- 3. Use the free entry condition in equation (6) to obtain the market tightness $\theta_T(\mathbf{x}, \mathbf{y}, \omega)$
- 4. With the market tightness, solve the worker dynamic programming problem in equation (4)
- 5. Repeated stepping back from t = T 1, ..., 1
- 6. Check if the difference in worker value $U_{t+1} U_t$, $W_{t+1} W_t$ and the firm value $J_{t+1} J_t$ is less than a predetermined tolerance level. If yes stop, if not increase T and go back to the first step

Next, I simulate 10,000 workers to obtain a distribution of labor market outcomes across different occupations and worker types. Finally, the SMM procedure minimizes the Euclidean distance between the model-implied moments and the same data moments.

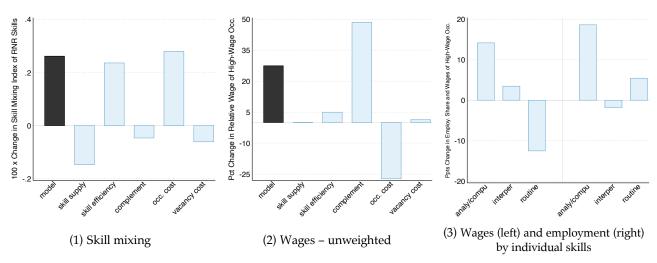
B.6 Additional Quantitative Results

Figure B1: Predicted and Observed Occupational Skills Across Age Groups in 2005 vs. 2018 Using NLSY Data



Notes: These figures display the average occupational job skills over the life cycle of workers, for both model predictions and empirical data. Model simulated or empirical data points for the year 2005 are represented by circles, and for the year 2018 are depicted as squares. The empirical observations are drawn from the NLSY 79 cohort for year 2005 and NLSY 97 cohort for year 2018. I use the 2005 O*NET values to align with observations from NLSY 79 and the 2018 values to align with NLSY 97. The age restrictions are 41 to 60 for NLSY 79 and 21 to 40 for NLSY 97, and similar constraints apply for the model-simulated paths.





Notes: These figures plot the model generated changes in skill mixing in high-skill occupations (column 1), changes in wages unweighted by employment (column 2), and changes in wages and employment from individual skills (column 3). Different model channels are shut down individually by eliminating the relative calibrated values to highlight the contribution of each channel. The full model has all the model features. The values of skill complementarity in production, cost of skills in occupation operation, efficiency differential, and vacancy posting cost across the two periods are shown in Table 6. Worker skill supply distribution variation across the periods are calibrated according to Table B1. Panel (3) and (4) depict the model generated changes in skill mixing in low-skill occupation and the relative wage of high-skill occupations by shutting down the skill efficiency differential for analytical/computer, interpersonal, and routine skills individually; also by shutting down τ and ϕ individually.