

# Indoor Localization by Particle Map Matching

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**Abstract**—This article presents the implementation of an indoor localization approach that combines map matching and a circular particle filter defined in a Bayesian framework. The technique relies only on velocity and heading observations coupled with a map of the road network. No prior knowledge of the initial position is given. A circular distribution is used to match the vehicle's heading with the roads direction. This allows to detect turns and provide a more accurate position estimate. The algorithm is assessed with a synthetic dataset in a real context.

**Keywords**—Indoor localization, Particle filter, Map matching, Circular Filter

## I. INTRODUCTION

Indoor localization has gained an increasing importance in the research area on positioning and tracking. Over the last decade, a lot of efforts have been devoted to indoor localization problems. One of the widely known approaches is fingerprinting where a pre-recorded radio map of the zone of interest is used to compute the best matching of the position. This matching uses the model constructed in a previous training phase. Many wave sources have been experimented like WiFi [1], [2], [3], GSM [4], [5] and sound signals [6]. Other techniques known as “Propagation based” compute distances through the calculation of the Radio Signal Strength Indications (RSSI) but are generally less reliable as they are compromised by interferences, multipath and shadowing [7].

Other approaches use a pre-deployed active beacon infrastructure at known locations with technologies like Bluetooth [8], infrared [9], ultrasound [10], RFID [11] or FM [12]. These short-range signals, however, need an important effort of deployment and a constant labor of monitoring and maintenance.

Indoor navigation has also been driven by the advancement in the technology of imaging and camera sensors (e.g. CCD). The authors in [13] use panoramic landscape images to localize pedestrians using 3D models. In [14], images of points markers taken from a phone camera are used to compute the location indoors.

In this research paper, we are particularly interested in the problem of localizing a vehicle operating in a logistic area such as a warehouse. In this kind of applications, the vehicle navigates in a predefined road network. However, the existence of metallic structures and obstacles makes difficult the use of technologies based on radio or light waves because of frequent non line-of-sight, multiple reflections and shadowing. In

addition, the constant change in the area content configuration (movement of boxes and goods) may decrease significantly the accuracy of fingerprinting.

One of the common techniques applied in such conditions is inertial navigation along with a dead reckoning algorithm, however the proprioceptive sensors are not reliable as they their accuracy drifts quickly through time. The fusion of positioning data with the map of the area can bring a solution to this problem. In fact, the localization quality could be improved by the additional layer of information on the road network. This process, known as map matching, identifies the estimated position with the map information according to known criteria in order to improve the estimation accuracy. These criteria can be geometric, such as proximity of the estimated position to the road [15], [16] or topological taking into account the structure of the road network [17]. This last kind of approaches proved to be more accurate since it remedies to many limitations of the geometric algorithms [18]. Other approaches have been applied with different degrees of success such as probabilistic modeling [19], [20], fuzzy logic [21], [22] or belief theory [23]. Refer to [18] for a comparative study of the different map matching algorithms.

The method that we propose in this paper can be classified in the topological map matching category. The vehicle uses only two sensors to calculate its position: speed and heading. The initial position of the vehicle is supposed to be completely unknown. At first, there's a seeking phase that aims to reduce the positioning error to an acceptable level by identifying sequentially the most probable areas of the vehicle's existence. Once the position is found, the method tracks the vehicle mainly by detecting the turn maneuvers. The algorithm can switch back to the seeking phase if it loses track of the correct vehicle's position.

The algorithm uses a particle filter defined in the circular domain. It was initially described in our previous paper [24] as a backup positioning method during GPS masking outdoors and showed interesting performances. This method relies mainly on the vehicle's heading which has the advantage of being an absolute measurement. The aim is to match the measured heading to the roads direction on the map by using a circular statistical distribution.

The structure of this article is as follows. The principle of the circular estimation and the presentation of the localization

method are described in Section 2. The experimental setup and results are discussed in Section 3. A conclusion is found in the last section.

## II. PARTICLE CIRCULAR MAP MATCHING

### A. Circular Bayesian estimation

When estimating angle observations, the von Mises distribution is used because it is considered the equivalent of the normal distribution in the circular domain. The pdf of a von Mises distribution of the variable  $\theta$  is defined by [25]:

$$f_{CN}(\theta; \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(\theta - \mu)} \quad (1)$$

$\mu$  is the mean of the distribution and  $\kappa$  its concentration parameter.  $\kappa$  is the equivalent of the inverse of a Gaussian distribution variance.  $I_0$  is the modified Bessel function of the first kind and order zero. Figure 1 clearly shows that when the mean  $\mu$  is not centered on the measurement interval, in this case  $[0, 2\pi]$ , the bell shape of the distribution is adjusted to the  $2\pi$  interval to take into account the periodic nature of angle.

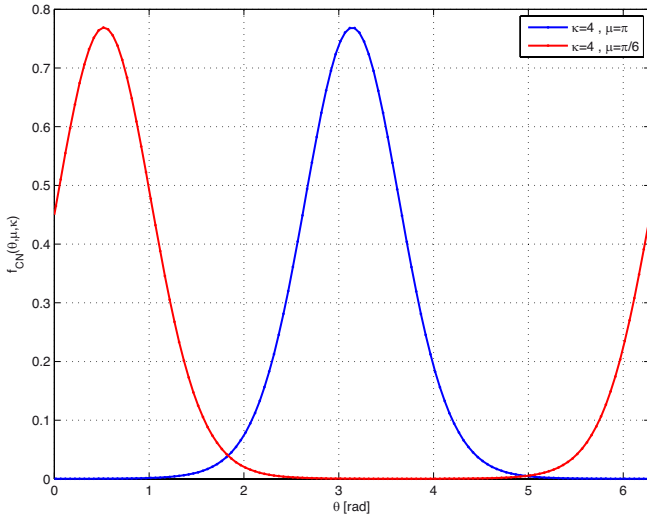


Fig. 1. The pdf of a von Mises distribution

### B. Principle of localization

To localize a vehicle using its velocity and direction, we use a dead reckoning system described by the following state model:

$$x(t) = x(t-1) + v(t) \Delta t \cos(\theta(t)) + \nu_x \quad (2)$$

$$y(t) = y(t-1) + v(t) \Delta t \sin(\theta(t)) + \nu_y \quad (3)$$

where  $v(t)$  and  $\theta(t)$  are the velocity and the direction respectively considered as commands.  $\Delta t$  is the time step.  $\nu_x$  and  $\nu_y$  are centered Gaussian noises.

To include the map data, the state model is augmented by an additional variable  $s(t)$  that stands for the road segment identity. We suppose here that the map database is a set of

linear segments identified by their identity index, the coordinates of their ends and their direction  $\beta$  with respect to the horizontal axis. The augmented state model is then:

$$x(t) = x(t-1) + v(t) \Delta t \cos(\beta(s(t-1))) + \nu_x \cos(\beta(s(t-1))) \quad (4)$$

$$y(t) = y(t-1) + v(t) \Delta t \sin(\beta(s(t-1))) + \nu_y \sin(\beta(s(t-1))) \quad (5)$$

$$s(t) = s(t-1) + \nu_r \quad (6)$$

where  $\nu_r$  is a discrete uniform noise on the road segments index connected to the current segment  $s(t)$ . It models the error on the segment identification.

The measurement equation is defined by:

$$\phi(t) = \beta(s(t)) + \nu_\phi \quad (7)$$

where  $\nu_\phi$  is the measurement noise assumed to follow a von Mises circular distribution.  $\phi(t)$  is an observation of direction provided by the on-board magnetometer.

Therefore, we can consider a recursive filter that uses the state equations 4, 5 and 6 for the state prediction. The measurement equation 7 is used to correct the road segment identity. In this case, the estimate of  $s(t)$  would be the value that minimizes the filter's innovation  $\phi(t) - \beta(s(t))$ .

This system has a strong non-linearity in its state model and measurement equations. Therefore we propose to implement it with a circular particle filter. The circular framework is motivated by the angular observations that vary in the interval  $]-\pi, \pi]$  [26].

### C. Map matching Implementation

The particle map matching algorithm is used with velocity and direction observations [24]. The circular particle filter estimates the vehicle's position on a digital map by matching the vehicle's direction with the roads direction on the map. The particles represent the posterior distribution of the state vector. Each particle has a weight proportional to its probability.

In our scenario, the initial position of the vehicle is unknown, thus, the initial set of particles is generated along all the segments. In the prediction step, we propagate each particle on the road segment at a distance  $d = v(t)\Delta t$  from its last position.  $v(t)$  is the velocity and  $\Delta t$  the time between two measurements. When a particle reaches the end of a segment, we choose randomly among the segments connected to the current one and we place the particle on it.

In the update step, each particle is affected a weight that is the likelihood of the observed direction  $\phi(t)$  compared to the predicted direction  $\beta(s(t))$ . Consequently a particle that exists on a segment whose direction is similar to the vehicle's direction receives a more important weight and has a higher probability to live.

The implementation of the filter is described as follows:

#### 1) Initialization:

Since we have no prior knowledge on the initial position of the vehicle, the particles are placed on all segments separated by the same distance. In case a prior knowledge is available, more particles should

be placed on the considered area since particles stand for probable spots of the vehicle's position.

2) *Prediction:*

Each particle moves following the state model equations described by (4), (5) and (6). It is identified by its coordinates  $(x_k^i, y_k^i)$  and the road segment on which it exists  $S_k^i$ :

$$\begin{aligned} x_k^i &= x_{k-1}^i + v_{k-1} \Delta t \cos(\beta(S_{k-1}^i)) \\ &\quad + \eta_k^i \cos(\beta(S_{k-1}^i)) \end{aligned} \quad (8)$$

$$\begin{aligned} y_k^i &= y_{k-1}^i + v_{k-1} \Delta t \sin(\beta(S_{k-1}^i)) \\ &\quad + \eta_k^i \sin(\beta(S_{k-1}^i)) \end{aligned} \quad (9)$$

$$S_k^i = \psi(S_{k-1}^i, x_k^i, y_k^i) \quad (10)$$

where:

- $\eta_k^i$  is a random noise following a centered uniform distribution of variance  $\eta_Q$ . It models the uncertainty on velocity  $v_{k-1}$ .
- $S_{k-1}^i$  is the segment index of the particle  $i$  at instant  $k-1$ . The particles are not corrected in position and are propagated according to the noise on the command  $v_{k-1} \Delta t$ .
- $\psi(\cdot)$  is a function that decides whether particle  $i$  will stay on the same road segment  $S_{k-1}^i$  or not. The segment change occurs when the particle reaches the segment end, *i.e.* the new calculated position  $(x_k^i, y_k^i)$  is located outside the current segment  $S_{k-1}^i$ . In this case, the function  $\psi(\cdot)$  selects randomly one of the segments connected to the current segment  $S_{k-1}^i$  in the movement direction.

3) *Update:*

In order to estimate the current road identity, we calculate the weight of each particle with the following von Mises likelihood distribution:

$$\tilde{w}_k^i = w_{k-1}^i f_{CN}(\phi_k; \beta(S_k^i), \kappa_P) \quad (11)$$

This weight  $\tilde{w}_k^i$  is higher for particles moving on roads whose direction  $\beta(S_k^i)$  is close to the vehicle's direction  $\phi_k$ . The concentration parameter  $\kappa_P$  controls the particles dispersion around the direction  $\beta(S_k^i)$ . The adjustment of  $\kappa_P$  is crucial for the algorithm, indeed, a more selective  $f_{CN}$  (high values of  $\kappa_P$ ) restricts the particles only on the highly probable segments which incurs the risk to uncover some potentially interesting areas, mainly turns. In the other hand, low values of  $\kappa_P$  allow the particles to explore less probable areas which delays the convergence of the algorithm and increases unnecessarily the computational cost.

The weights are normalized as follows:

$$w_k^i = \frac{\tilde{w}_k^i}{\sum_{j=1}^N \tilde{w}_k^j} \quad (12)$$

4) *Position estimation:*

The following weighted sum of the particles gives the position coordinates  $(\hat{x}_k^p, \hat{y}_k^p)$ :

$$\hat{x}_k^p = \sum_{i=1}^N w_k^i x_k^i ; \quad \hat{y}_k^p = \sum_{i=1}^N w_k^i y_k^i \quad (13)$$

This position is then projected on the nearest road segment  $\hat{s}_k$  to generate the estimate of the vehicle's position  $(\hat{x}_k, \hat{y}_k)$ . Where  $\hat{s}_k$  is the estimated road segment index.

5) *Particle resampling:*

We resample the particles according to the Sequential Importance Resampling (SIR) method described in [27] with an effective number of particles defined as follows:

$$N_{eff} = \frac{1}{\sum_{i=1}^N w_k^i{}^2} \quad (14)$$

In the resampling process, we keep unchanged the position  $(x_k^i, y_k^i)$  and the segment identity  $S_k^i$  of each particle.

Finally, the particle map matching filter at instant  $k$  is defined with the following algorithm:

**for**  $i=1:N$  **do**

    Draw particle:

$(x_k^i, y_k^i) \sim g((x_k, y_k) | (x_{0:k-1}^i, y_{0:k-1}^i))$

    Draw segment identity:  $S_k^i \sim \psi(s_k | (x_k^i, y_k^i))$

    Compute the particle's weight:

$\tilde{w}_k^i = w_{k-1}^i f_{CN}(\phi_k | \beta(S_k^i))$

**end**

Compute  $\hat{x}_k^p = \sum_{i=1}^N w_k^i x_k^i$  ;  $\hat{y}_k^p = \sum_{i=1}^N w_k^i y_k^i$ , for  $i = 1, \dots, N$

Find the nearest segment to  $(\hat{x}_k^p, \hat{y}_k^p) \rightarrow$  segment identity estimate  $\hat{s}_k$

Project  $(\hat{x}_k^p, \hat{y}_k^p)$  on  $\hat{s}_k \rightarrow$  position estimate  $(\hat{x}_k, \hat{y}_k)$

Normalize weights:  $w_k^i = \tilde{w}_k^i / \sum_{i=1}^N \tilde{w}_k^i$ , for  $i = 1, \dots, N$

**if**  $1 / \sum_{i=1}^N (w_k^i)^2 < N_T$  **then**

    Resample the particles with the SIR algorithm

    Homogenize weights:  $w_k^i = 1/N$ , for  $i = 1, \dots, N$

**end**

**Algorithm 1:** Particle map matching algorithm

Where  $g(\cdot)$  is a uniform law,  $f_{CN}(\cdot)$  a von Mises law and  $\psi(\cdot)$  the function that predicts the segment identity defined by (10).

### III. EXPERIMENTATION

We consider a vehicle such as forklift that moves inside a covered warehouse. The movement is restricted to the road network depicted in figure 2. The segments represent the different corridors and can be vertical or horizontal. Each segment holds a number between 1 to 26.

The vehicle is equipped with two sensors : an odometer and a digital compass for velocity and heading measurement respectively. In this experimentation, we suppose that the vehicle starts from an unknown position then moves through the road network, when it reaches the end of a segment, it chooses randomly to turn left or right. The maximum velocity authorized for a forklift in a warehouse is 8mph or 3.58m/s, we choose a constant velocity of 2m/s. We assume that the error on the velocity measurement is Gaussian and biased. The

distribution standard deviation is set to 0.6 m/s and the bias is assumed to be following a uniform distribution between 0.5 m/s and -0.5 m/s.

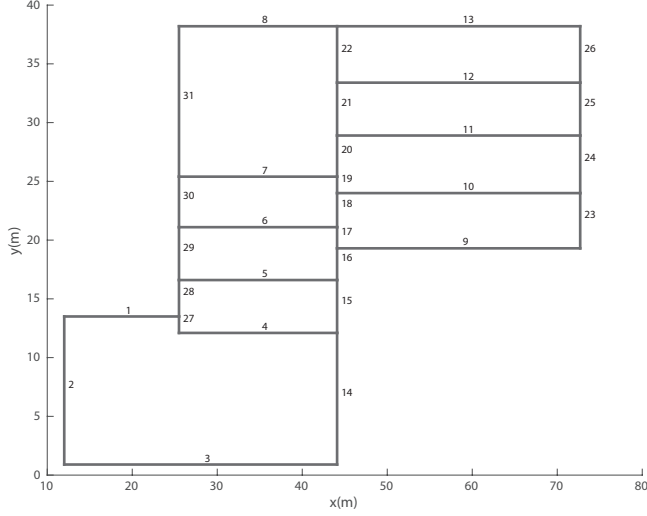


Fig. 2. Warehouse Road network

We run 100 realizations of the algorithm with a random starting point and trajectory. The duration of a simulation is 200s (3min20s). We suppose that the algorithm is in the seeking phase when the positioning error (distance between the real and the estimated position) is above 2m. When it falls below this threshold, the algorithm is in the tracking phase.

First, we are interested in the error of positioning over time for different values of  $\kappa_P$ . This error is depicted in Figure 3. We observe that the error is high in the seeking phase at the start then falls to a quasi-constant value for each  $\kappa_P$ . We also show in Table I the global positioning error which is the mean of the errors for all the simulation interval. We can easily observe that  $\kappa_P = 0.5$  and  $\kappa_P = 1$  are the best settings. Indeed they have the smallest global positioning error (around 3.3m) and their positioning error falls below 2m after 80s and 120s respectively. For the other values of  $\kappa_P$  the positioning error never reaches 2m because the particles are too dispersed (low  $\kappa_P$ ) or too compressed (high  $\kappa_P$ ).

$\kappa_P$	10.0	5.0	1.0	0.5	0.1
Global positioning error (m)	13.4	9.1	3.3	3.4	7.2

TABLE I. GLOBAL POSITIONING ERROR AS A FUNCTION OF  $\kappa_P$

A second important measure of the algorithm's performance is the probability of correct segment identification. We compare the segment number of the real and estimated positions. We affect a score of one when they are the same and zero otherwise. For 100 Monte Carlo runs, we compute the probability of correct segment identification that equals the mean of these scores for each instant. We show in figure 4 this probability for each value of  $\kappa_P$ . We observe that for  $\kappa_P = 0.5$  and  $\kappa_P = 1$  the probability increases and stays above 0.8 after 120s. For  $\kappa_P = 1$ , the performances are better during the seeking phase. Table II shows the global probability of correct segment identification which is the mean of the probability during the whole simulation interval of 200s. We also notice that the best value is around 0.7 and is obtained for  $\kappa_P = 0.5$  and  $\kappa_P = 1$ .

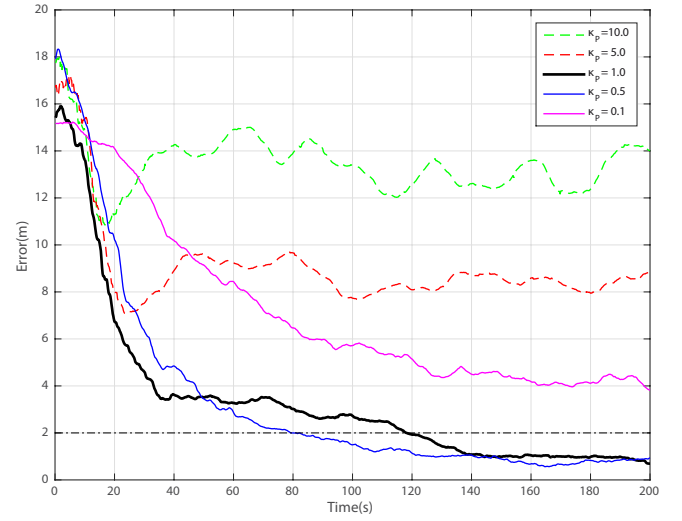


Fig. 3. Position Error over time for different values of  $\kappa_P$

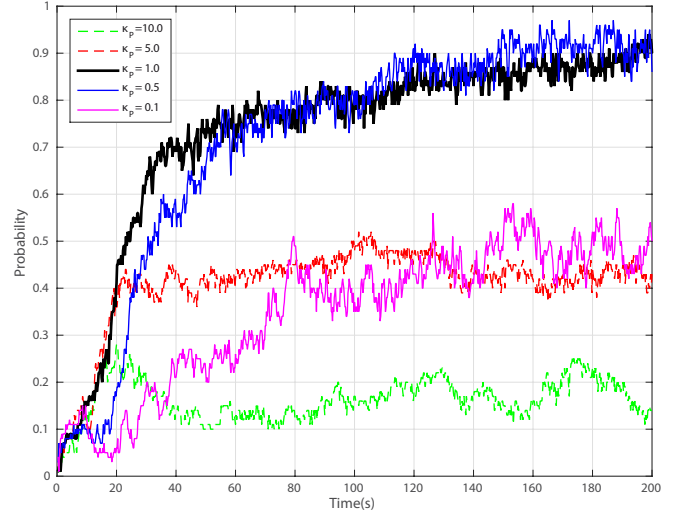


Fig. 4. Probability of Correct segment identification over time for different values of  $\kappa_P$

$\kappa_P$	10.0	5.0	1.0	0.5	0.1
Probability of identification	0.16	0.41	0.73	0.72	0.35

TABLE II. GLOBAL PROBABILITY OF SEGMENT IDENTIFICATION AS A FUNCTION OF  $\kappa_P$

In Figures 5 to 8, we illustrate the evolution of the algorithm during the seeking phase until the correct position is reached for  $\kappa_P = 1$ . In the initial state (Figure 5), the particles are uniformly distributed on all the segments. In Figure 6, the cloud of particles moves to the right after a movement of the vehicle in this direction. After several turns, we notice in Figure 7 that the particles begin to concentrate on the most probable segments mainly around the real position. Finally, after 54s of movement (Figure 8), most of the particles with less probability are eliminated and recreated around the correct position. The algorithm is now in the tracking phase.

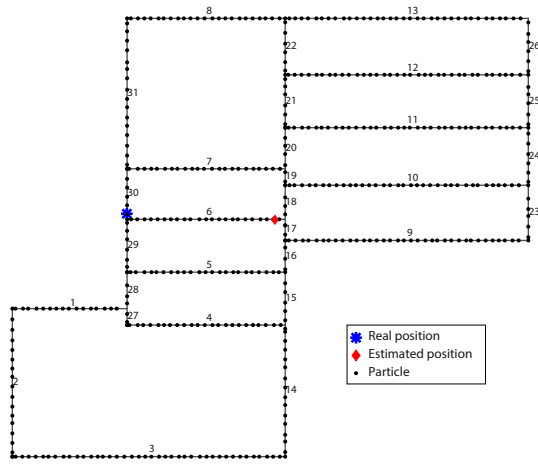


Fig. 5. Initial state of the algorithm

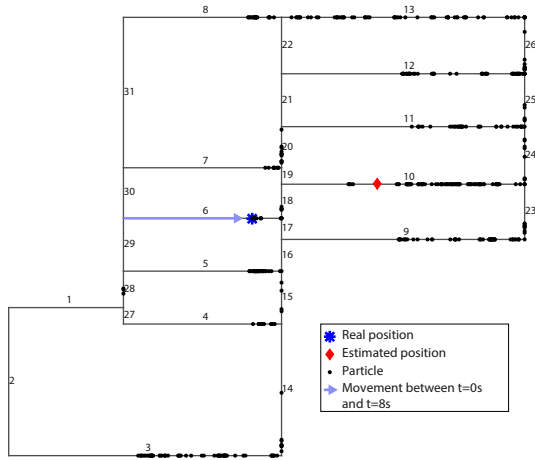


Fig. 6. Evolution of the algorithm between  $t=0s$  and  $t=8s$

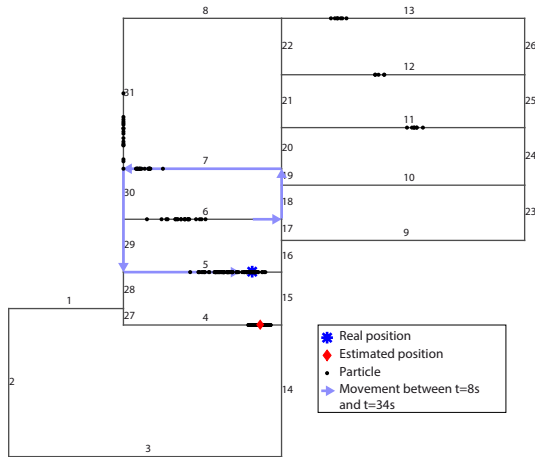


Fig. 7. Evolution of the algorithm between  $t=8s$  and  $t=34s$

#### IV. CONCLUSION

In this paper we present an approach of indoor localization based on a topological map matching algorithm and a circular particle filter defined in a Bayesian framework. The studied scenario is the localization of a vehicle moving in a

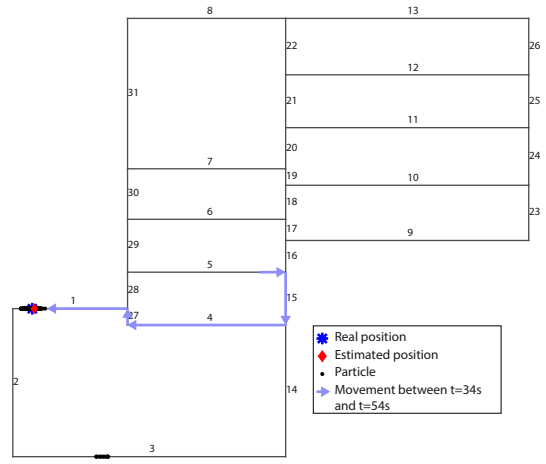


Fig. 8. Evolution of the algorithm between  $t=34s$  and  $t=54s$

covered warehouse on a predefined road network. The vehicle's position is observed with measurements of velocity and heading. The particles represent the posterior distribution of the vehicle's position in a segment. They are propagated by the velocity and their weight calculated by the circular distribution of von Mises that matches the direction of movement with the direction of the road segments.

The experimental results show that after a seeking phase where the positioning error is high, the proposed algorithm is able to localize the vehicle after 80s of movement in average with a positioning error less than 2m. The algorithm switches to the tracking phase where the error is reduced progressively to 1m with a probability of correct segment identification of around 0.9.

Future work will involve the fusion of this method with a second approach of indoor localization like fingerprinting in order to improve the performances mainly reducing the seeking phase duration.

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