A Circular Interacting Multi-Model Filter Applied to Map Matching

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***Abstract*—The multi-model interacting algorithm, based on the Kalman filter, is defined in the linear domain. In this paper, we propose a multi-model interacting filter for the circular domain. The proposed algorithm is defined in a Bayesian framework with a von Mises circular distribution. It is used to estimate the direction of a vehicle and to define its dynamic behavior. The different models are a right turn and a left turn. The proposed circular interacting multi-model filter is applied to Map-matching. The filter processes the sensor heading measurements. For this application we assess the proposed filter for the change detection and identification of the road on which the vehicle is traveling.**

I. Introduction

The multi-model algorithms (MM) execute in parallel a set of statistical filters associated with all possible system dynamics at a given moment. The system evolution is represented by a set of state equations. The multi-model algorithms estimate the state and its covariance matrix by merging, according to a makovian model, the estimate provided by each filter. Several tracking algorithms (MM) were proposed and each of them merges differently the estimates. Among these algorithms, we find the Interacting Multi-Model Filter (IMM) which is based on a Kalman filter [1]. It was designed to track a target in movement. For example, in [2] the IMM algorithm is applied to the tracking of a vehicle with a video-camera. In [3], the IMM filter detects when the vehicle deviates from its path crossing the roadside. In this case, the measures are provided by a LIDAR, a radar and a camera. In [4] the Interacting Multi-Model Filter (IMM) is used to define the dynamic of a maneuvering target. We find in [5] a comparative study of different IMM filters.

For these applications the different movements are supposed to be linear and the noise to be Gaussian. In this case, the Interacting Multi-Model Filter (IMM) has a good accuracy for a minimal computation load. However, when the models are no longer linear, the IMM algorithm is less efficient and can rapidly diverge [14]. Many IMM filters have been proposed in the published works for non linear models. These filters use in most of the cases an extended Kalman filter EKF [6]. However, in practice, EKF has two drawbacks [7], [8]. First, the filters are designed for a local linearisation of the problem. These filters are unstable for large variations of the state. Second, the Jacobian matrix derivation is not trivial in all applications and its use leads usually to computational complexity difficulties.

In our approach, we consider a mobile which can make a linear displacement following a fixed direction, a left or a right turn. The rotations can have different angular speeds. The considered models are, therefore, circular and we want to estimate an angle: the vehicle's heading. When the state is an angle, defined on a periodic domain, the extended Kalman filter is used. In this case the Gaussian approximation is valid for small angles because it is a good approximation of the periodic circular normal distribution. Nevertheless, this approximation is no more valid for large angles and in the case of angular transitions between 0 and 2π. These natural transitions in the circular domain are seen in the linear domain as abrupt changes in the heading's values. The von Mises distribution in the circular domain is equivalent to the Gaussian distribution in the linear domain. The use of a circular distribution to model the problem allows to get solutions not depending on the periodic nature of the angles. In this article, we present a Circular Interacting Multi-Model (CIMM) algorithm defined in a Bayesian framework with the circular von Mises distribution. This algorithm is based on a circular weighted sum operator and a circular recursive filter [9],[10],[11].

The proposed CIMM algorithm is applied to Map-matching. Map-matching algorithms integrate positioning data with spatial road network described by a set of segments in a database. The goal of these algorithms is to identify the correct road (a segment defined in a database) on which a vehicle is traveling and to determine the vehicle's location on that road (a point of the segment). A number of map-matching algorithms have been developed by researchers using different informations (topological and geometric) with direct and recursive approaches. These informations are distances processes with the observations provided by the sensors and the spatial road network database. The point-to-point distance, the point-to-curve distance, the curve-to-curve distance and the curve-to-curve angle are integrated as input information by the Map-matching algorithms. The direct approach uses all the information processed at time t to perform the Map-matching while the recursive approach uses the map-matching's result at time $t-1$ to process the observations obtained at time t. The performances of these algorithms have improved over the years due to the use of accurate multi-sensors positioning system and to improvements in the spatial road network data. However, these algorithms are not always capable to reach the high navigation performance required by intelligent transportation system, especially in difficult and complex environments such as dense urban areas and in ports freight storage areas. We can find in [16] an in-depth literature review of the different Map matching approaches.

One of the task of the Map matching process is to select the correct segment on which the vehicle is traveling. In the recursive approach, this task can be separated in two sequential sub-tasks: detecting a segment change and identifying this new segment in the database. Most of the modern multi-sensors navigation systems are composed of a heading sensor (magnetometer/gyroscope) and a positioning sensor (GNSS). These two sub-tasks are performed separately with the observations provided by the two sensors and fused to get a final decision. In this paper we propose to use a CIMM filter in order to detect a change and to select a segment in the database with the heading observations.

This paper is organized as follows: after this brief introduction, we will present in the second part the CIMM algorithm. The third part is experimental and the proposed algorithm is assessed on synthetic data.

II. Circular Interacting Multi-Model (CIMM) Filter

A. IMM Filter principle

In many practical cases, it is assumed that the model can change with time. The different dynamics of a mobile are indeed difficult to define with a unique model. We have to take into account at each time all the possible models and compute their probability in order to estimate the system state.

Let's consider a system and its model defined in the following r models $\{M^1, \dots, M^r\}$. We suppose that we know the initial probability $\mu_0^j = P\{M_0^j\}$ of each model M^j. The transition probabilities $p_{ij} = P\{M_k^j | M_{k-1}^i\}$ from a model i at time $k-1$ to a model j at time k are also supposed to be known. These transition probabilities completely characterize a Markov chain and define a Markovian switching system. Optimal state filtering in the case of multi-model requires the execution of each filter for every possible model transition. Thus, in the case of r models, r^k optimal filters must be executed at time k. In order to decrease the computation load, some approximations have been proposed such as the IMM approach [1].

An IMM filter is processed in three main steps: interaction/mixing, filtering and combination. At each time-step k the previous states are mixed then used by the Kalman filters associated to the different models to provide r estimates. These estimates are combined to produce a unique final estimate state and its covariance matrix. The weights used in the combination are defined with the probability of each model.

As we process in our problem angular data defined by the vehicle's heading, the Kalman filters are replaced by circular recursive filters defined in a Bayesian framework with a von Mises distribution. The global architecture of the CIMM filter is depicted in figure 1

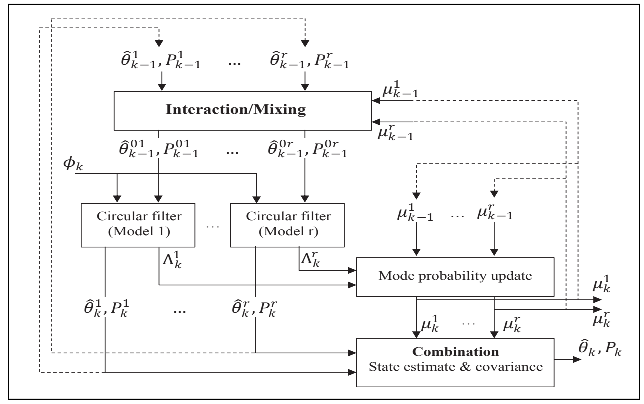

Fig. 1. CIMM filter structure

B. Circular estimation

The von Mises distribution $f(\theta, \mu, \kappa)$ describes realisations of a circular random variable of mean μ and a concentration parameter κ (this parameter is homogeneous with the inverse of the variance). The parameter κ takes its value between zero and infinity. For a low value of κ, the von Mises distribution looks like a uniform random variable between 0 and 2π. For a high value of κ, the von Mises distribution tends to a Gaussian distribution with a variance $1/\kappa$. The expression of a von Mises distribution for an angle θ is given by:

$$f(\theta, \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} \ \exp\left(\kappa \cos(\theta - \mu)\right) \tag{1}$$

I_0 is the modified Bessel function of order 0. Its expression is given by:

$$I_0(\kappa) = \frac{1}{2\pi} \int_0^{2\pi} \exp\left(\kappa \cos(\phi)\right) d\phi \tag{2}$$

We notice :

$$\theta \sim CN(\mu, \kappa) \tag{3}$$

a random variable θ, distributed according to a von Mises distribution of mean μ and a concentration parameter κ [13]. Let's define n angular measurements $\phi_1, \dots, \phi_n$ distributed according to a von Mises distribution of mean μ and a concentration parameter κ. It is shown in [13] that the maximum likelihood estimate of μ is defined as follow :

$$\hat{\mu} = \arctan^* \left(\frac{S}{C}\right) \tag{4}$$

with $C = \sum_{i=1}^n \cos\left(\phi_i\right)$, $S = \sum_{i=1}^n \sin\left(\phi_i\right)$. The maximum likelihood estimate of κ is the solution of the following equation :

$$A(\kappa) = \frac{R}{n} \ \text{ with } \ R = \sum_{i=1}^n \cos\left(\phi_i - \hat{\mu}\right) \tag{5}$$

$\arctan^*$ is the "quadrant-specific" inverse of the tangent [13], commonly known as $atan2$. It takes into account the signs of C and S and returns the angle in one of the four quadrants of the unit circle.

$A(\ldots)$ is the function defined by :

$$A(\kappa) = \frac{I_1(\kappa)}{I_0(\kappa)} \tag{6}$$

where I_0 and I_1 are respectively the modified Bessel functions of order 0 and 1. $A^{-1}(\ldots)$, the inverse function of $A(\ldots)$, is computed by approximation as defined in [10].

We define in the circular domain the distance between two angles with the following expression:

$$d(\phi_1, \phi_2) = 1 - \cos(\phi_1 - \phi_2) \tag{7}$$

The circular distance takes its values between 0 and 2 for a difference of angle that varies respectively between 0 and π.

C. State and measurement equation

Each model of the CIMM filter is described by a state equation and a measurement equation.

State equation :

$$\theta_k = \theta_{k-1}(mod\ 2\pi) + \nu_{k-1} \tag{8}$$

where the heading angle θ_k is the state at time k. ν_{k-1} is the state model noise on θ_k. It represents the error of the state model. This additive noise is distributed according to a 0-centered von Mises distribution with a concentration parameters κ_Q.

Measurement equation : We have a measurement ϕ_k on the angle θ_k, thus the measurement equation is given by:

$$\phi_k = \theta_k(mod\ 2\pi) + w_k \tag{9}$$

w_k is the measurement noise, it is distributed according to a 0-centered von Mises distribution with concentration parameter κ_R. We denote P_{θ_k} the concentration parameter of the estimate direction.

D. Step 1 : Interacting/Mixing

1) Step 1a : Processing of the conditional model probabilities: We compute the conditional model probability $\mu^{i|j}$ that represents the probability that model M_i was in effect at time $k-1$ given that M_j is in effect at time k [1].

$$\mu^{i|j}_{k-1} = \frac{1}{\bar{c}_j} p_{ij} \mu^i_{k-1} \tag{10}$$

$$\bar{c}_j = \sum_{i=1}^{r} p_{ij} \mu^i_{k-1} \tag{11}$$

With :

- μ^i_{k-1} : Probability of model i at time $k-1$
- $\bar{c}_j$: Normalisation factor
- p_{ij} : Transition probability from model i to model j

2) Step 1b : Mixing: In this stage the estimates provided by each filter at time $k-1$ are combined. The weights are defined with the conditional model probabilities $\mu^{i|j}_{k-1}$. We compute the initial data for all the filters with a weighted sum defined in the circular domain [9],[14] :

$$\hat{\theta}^{0j}_{k-1} = \arctan^*\Big(\frac{\sum_{i=1}^{r} \mu^{i|j}_{k-1} \sin(\hat{\theta}^i_{k-1})}{\sum_{i=1}^{r} \mu^{i|j}_{k-1} \cos(\hat{\theta}^i_{k-1})}\Big) \tag{12}$$

$$P^{0j}_{\theta_{k-1}} = \sqrt{\left(\sum_{i=1}^{r} \mu^{i|j}_{k-1} \sin(\hat{\theta}^i_{k-1})\right)^2 + \left(\sum_{i=1}^{r} \mu^{i|j}_{k-1} \cos(\hat{\theta}^i_{k-1})\right)^2} \tag{13}$$

With $\hat{\theta}^{0j}_{k-1}$, $P^{0j}_{\theta_{k-1}}$ the initial angle value and concentration parameter for filter j.

E. Step 2 : Filtering

1) Step 2a : Circular filter for each model: Each model j is associated with a recursive circular filter defined by a step of prediction and a step of correction [10],[14].

Prediction

$$\hat{\theta}^j_{k|k-1} = \hat{\theta}^{0j}_{k-1} + M_j \tag{14}$$

$$P^j_{\theta_{k|k-1}} = A^{-1}\Big(A(P^{0j}_{\theta_{k-1}})A(\kappa_{Q^j})\Big) \tag{15}$$

With :

- $\hat{\theta}^j_{k|k-1}$, $P^j_{\theta_{k|k-1}}$: Angle and concentration parameter predicted for model j
- κ_{Q^j} : State noise concentration parameter defined for model j

We want to estimate the heading of a mobile vehicle that can go straight ahead and make right and left turn. During a turn the angular rate is supposed to be constant and is defined by $|M_j|$ in [rd/s] for a right turn and $-|M_j|$ for a left turn.

Correction

$$P^j_{\theta_k} = \sqrt{C^{j2}_{\theta_k} + S^{j2}_{\theta_k}} \tag{16}$$

$$\hat{\theta}^j_k = \arctan^*(S^j_{\theta_k}/C^j_{\theta_k}) \tag{17}$$

With :

$$C^j_{\theta_k} = P^j_{\theta_{k|k-1}} cos(\hat{\theta}^j_{k|k-1}) + \kappa_R cos(\phi_k) \tag{18}$$

$$S^j_{\theta_k} = P^j_{\theta_{k|k-1}} sin(\hat{\theta}^j_{k|k-1}) + \kappa_R sin(\phi_k) \tag{19}$$

- ϕ_k : Measurement obtained at time k
- κ_R : Concentration parameter of the angle measurement noise.

2) Step 2b : Processing of the likelihood for each filter : In our application we only measure the angle. The likelihood $v^j_{\theta_k}$ is defined for each model j by :

$$v^j_{\theta_k} = \phi_k - \hat{\theta}^j_{k|k-1} \tag{20}$$

Its variance is given by :

$$S^j_{v_{\theta_k}} = A^{-1}(A(P^j_{\theta_{k|k-1}})A(\kappa_R)) \tag{21}$$

The likelihood function is a von Mises distribution. We compute the likelihood value for each model as follow:

$$\Lambda^j_k = CN(v^j_{\theta_k}|0, S^j_{v_{\theta_k}}) \tag{22}$$

The value Λ^j_k is as high as the measure ϕ_k is close to the estimate $\hat{\theta}^j_{k|k-1}$ provided by the filter associated with the model j.

3) Step 2c : Processing of the models' probability: The probability of each model is computed with the likelihood function obtained in the previous stage.

$$\mu^j_k = \frac{1}{c}.\Lambda^j_k\ \bar{c}_j \tag{23}$$

With :

- $\bar{c}_j$: is given by (10)
- c : a normalization factor defined by :

$$c = \sum_{j=1}^{r} \Lambda^j_k \bar{c}_j \tag{24}$$

F. Step 3 : Combination

The estimates provided by each filter are combined using a weighted sum defined in the circular domain. The weights are the models probability. This allows to promote the most probable model and increase its contribution in the estimation of the global state. The global estimate is process as follow :

$$\hat{\theta}_k = \arctan^*\Big(\frac{\sum_{j=1}^{r}\mu^j_k \sin(\hat{\theta}^j_k)}{\sum_{j=1}^{r}\mu^j_k \cos(\hat{\theta}^j_k)}\Big) \tag{25}$$

Its concentration parameter is :

$$P_{\theta_k} = \sqrt{\left(\sum_{j=1}^{r}\mu^j_k \sin(\hat{\theta}^j_k)\right)^2 + \left(\sum_{j=1}^{r}\mu^j_k \cos(\hat{\theta}^j_k)\right)^2} \tag{26}$$

III. EXPERIMENT

In this experiment, we consider a vehicle with a digital map and a magnetometer measuring the heading. The goal of this experimentation is to assess the proposed filter for the vehicle direction's estimation and for the change detection in the road direction.

The digital Map is described in a spatial road network database composed of vertices, nodes and segments. The nodes define the road intersections whereas vertices are points which separate two segments in the same road. A classical map-matching algorithm uses the vehicle's position and its direction to identify the segment to match. It is shown, in this experimentation that a CIMM filter that processes heading data improves the segment selection. The obtained results with the proposed filter are compared with those obtained with a classical IMM kalman filter.

The vehicle's trajectory is assumed to be made up of linear parts on which the vehicle can move in a straight line (constant heading angle), turn right (increasing heading angle) or turn left (decreasing heading angle). In this context we have chosen two models for the CIMM filter to characterize the vehicle's behavior. The Measurement equation is the same for both of them because we only measure the heading angle with a rate of $10Hz$. For the first model we assume a right turn $M_1 = 0.02$ and for the second model we assume a left turn $M_2 = -0.02$. For a map-matching in urban area we observe in our experimentation that two models defined for a rate of turn of $2[rd/s]$ are sufficient.

A recursive map-matching algorithm select a segment in a spatial road network database from its current position. A complete map-matching process combines the segment selection obtained with the GPS position and obtained with the heading information. With the heading data this selection process is carried out in two steps. In the first step we detect a turn (right or left turn). In the second step we select among all the connected segment the one who has a direction close to the estimated direction after the turn. The CIMM filter plays a double role. It provides an estimate of the heading angle and it gives a knowledge on the vehicle behavior thanks to the probability computation of each model. In the following we will assess the proposed CIMM filter for the estimation of the direction used in the third step. We will also assess the proposed filter for the detection of turn as described in the second step.

A. Direction estimation

We show in Figure 2 a simulated trajectory represented by dots on the map. The measured headings are depicted as red vectors. The road direction is indicated by a blue vector on each segment of the road. We show in Figure 3 the estimated

Fig. 2. Vehicle's trajectory

heading obtained with the proposed CIMM filter and with the classical IMM Kalman filter. We can notice in Figure 3 that the IMM Kalman filter is less accurate when the angle is close to the transition $(-\pi, \pi)$. In the linear domain these transitions are indeed seen as abrupt change in the heading direction.

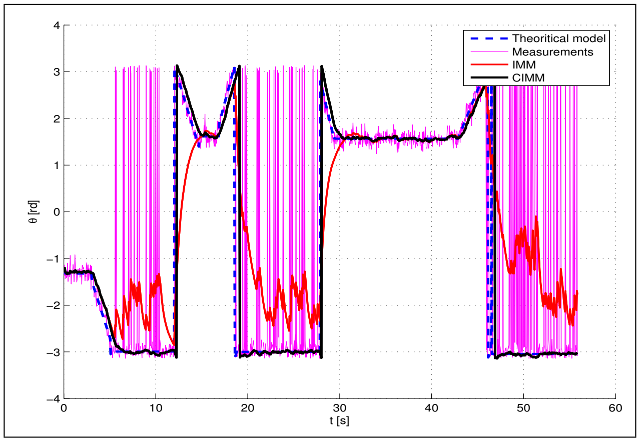

Fig. 3. Estimate Heading

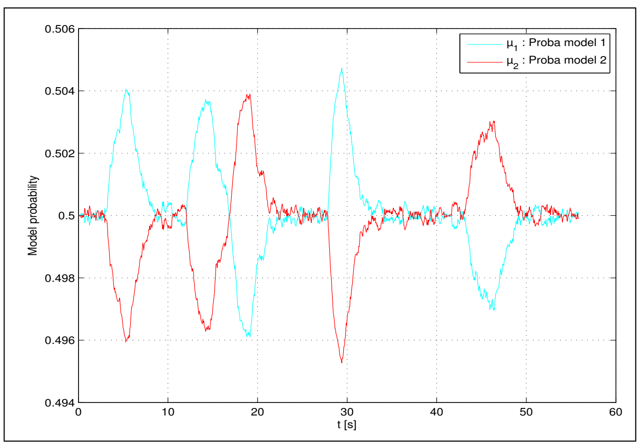

Fig. 4. Models' probability

In order to compare the results obtained with these methods we use the circular dispersion D_v defined for measurements in the circular domain. This measurement's dispersion is computed with the following expression [13]:

$$D_v = \frac{1}{n}\left(n - \sum_{k=1}^{n}\cos(\theta - \phi_k)\right) \tag{27}$$

where n is the number of observations ϕ_k of θ. We report in Table I the dispersion error between the true and the estimated direction for the different parts of the trajectory described in Figure 3. For this experimentation the parameters of the filter are : $\kappa_R = 25, \kappa_{Q^1} = \kappa_{Q^2} = 1000$.
We can conclude that the proposed CIMM filter is more accurate than the classical IMM Kalman filter because the circular error dispersion is lower.

B. Change detection in heading

As it is noticed in [16], the techniques used in existing map-matching algorithms may fail to identify the correct

TABLE I. CIRCULAR ERROR DISPERSION AS A FUNCTION OF THE OBSERVATION NOISE ERROR DISPERSION

Observation error	0.053	0.016	0.008
IMM Kalman Filter	0.778	0.523	0.354
CIMM Filter	0.01	0.007	0.006

road segment at or near a Y-junction as shown in Figure 5. This type of hypothetical road network may be observed in motorway diverging scenarios. One possible suggestion is to consider for this case a change in the heading direction as an additional input to the map-matching algorithm. The CIMM

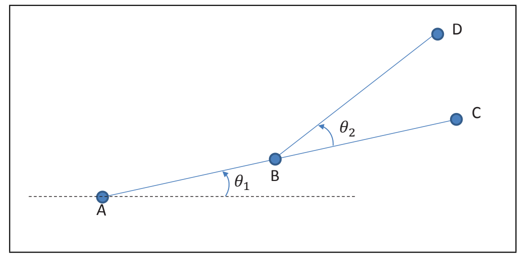

Fig. 5. Y-junction in a road network

filter provides the probability of each model. The model that owns the maximum probability at a given time informs us on the vehicle's behavior. When the vehicle travels with a constant heading direction, the probability of the two models is $1/2$ and the Kullback-Leibler divergence is zero. When a left or right turn occurs, the absolute value of the Kullback-Leibler divergence increases. We decide to detect a turn when the distance exceeds a threshold.

In this experimentation we consider two scenarios. In the first, the vehicle goes straightforward from A to C with no turn in B. We process the heading sensor observations with the filters and detect a turn. From one hundred realizations of this trajectory we define the probability of false alarm. In fact the segment change in this case can not be detected with the heading observations and will be detected in a complete map-matching algorithm with the GPS positions. In the second scenario the vehicle goes from A to D with a turn in B. From one hundred realizations of this trajectory we define the probability to detect a turn.

We show in Figure 6 the probability of detection as a function of the probability of false alarm. In this experimentation the circular noise error dispersion is 0.008 and the probabilities are processed for different probability thresholds. For the proposed filter the probability of detection is, for a given probability of false alarm, always superior to the probability of detection obtained with an IMM Kalman filter. Furthermore for a value of $\theta_1 = -\pi$ the IMM filter shows very bad performances because the transitions between $-\pi$ and π generate false detection even for high threshold (red star at the upper right corner of Figure 6), whereas, in this case, the CIMM filter is not disturbed and the results are the same as in the case of $\theta_1 = 0$.

We can conclude that the proposed CIMM filter, compared to a classical IMM Kalman filter, provides a higher probability of turn detection. Furthermore the proposed filter provides a

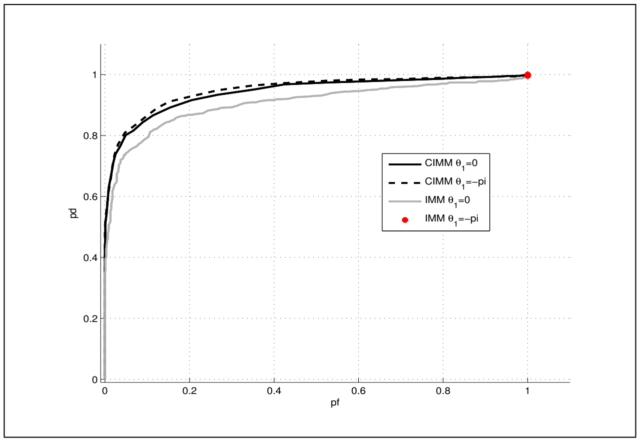

Fig. 6. Probability of detection as a function of the probability of false alarm for a turn of angle $\theta_2 = 0.14[rd]$.

more accurate heading estimate that will allow to identify more accurately, with the circular distance (7), a segment in the spatial road network database.

IV. Conclusion

In this paper, a Circular Interacting Multiple Model (CIMM) filter is developed in a Bayesian framework with a von Mises circular distribution. The proposed filter uses a circular recursive algorithm and a circular fusion operator for the mixing and combination steps. The proposed algorithm estimates the vehicle's heading and detects the right and left turns. These informations are used to detect road change and to identify a road in a map-matching process. We show in the experimentation that the circular approach outperforms the classical linear Kalman approach on angular heading data processing. The prospects of this work are mainly: the assessment of the proposed method in terms of robustness and precision to track freight engines in ports with a complete map-matching algorithm that processes the whole sensors' measurements.

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