# A Circular IMM Filter Applied to a Maneuvering Target

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**Abstract-** *One of the main problem in maneuver detection in angle-only observation applications is the model change during the maneuver that creates a peak in the estimate error. We propose in this paper to estimate a vehicle's heading using a Circular IMM defined in the bayesian framework and using the von Mises distribution. The performances of this novel filter is compared with a circular filter and a classical Kalman Filter in a Y shape road.*

**Keywords-** *angular estimation; heading estimation; IMM; circular filter; circular IMM; maneuver detection*

## 1 INTRODUCTION

When using angle-only observations, it can be difficult to estimate the angle with the linear classic methods like the Kalman Filter due to the periodical nature of the angles and the noise distribution on them. In addition, detecting and estimating a mobile's maneuvers based only on angle observation can be more difficult with the conventional Kalman filter. Indeed, this filter requires an accurate model and an exact a-priori information, which is generally unavailable for a maneuvering target.

The problem of maneuver detection is an adaptive estimation problem where the mobile maneuvers imply a behavior change that the mathematical model should reflect. Many methods exist in the literature such as [1]:

- *Continuous Noise Level Adjustment* that consists in implementing a procedure to detect important variations in the innovation of the filter during the maneuver. In this case, the noise variance is increased. After the maneuver, this variance is readjusted again to its initial value.
- *Input Estimation* that estimates an additional parameter that is the unknown command through the observation of innovations in a temporal sliding window
- *Variable Dimension Filtering* where additional variables are added to the state vector during the maneuver to reflect the model change then removed afterwards according to some predefined thresholds.

These methods were improved in many recent researches. The authors in [2] use an adaptive two-stage EKF applied to an inertial navigation system loosely coupled with a system that estimates the unknown bias. In [3], the authors propose a sensor fusion along with a Kalman filter based on statistics of the mathematical expectation of the spectral norm of a normalized innovation matrix. Their method allows a simultaneous test of the mathematical expectation and the variance of the innovation sequence. In [4], the proposed algorithm reconstructs the innovation equation from the updating sensor probability in the so-called Relaying Kalman Filter. The authors in [5] propose to adjust the order of the Kalman filter according to the threshold maneuver detector.

One of the recurring problems in maneuvers detection is the high level of error on the estimate during the maneuver that is generally due to the delay in the model adjustment in the filter. It was demonstrated in [1] that the multiple model (MM) approach can bring a solution to this problem by using simultaneously a set of filters based on simple models adapted to the different dynamics of the observed system. The global estimate is the combination of the estimates provided by each simple filter. The multiple-model filter operates in a way that, based on the innovations, the most relevant estimate contributes more heavily in the final estimate thus reducing the error peak during the maneuver.

Many generations of Multiple-model filters exists on the literature. Each one of them combines differently the output of each filter and deals differently with the estimates history. An optimal MM filter cannot exist in practice due to the exponential increase of the processing load. Only sub-optimal versions can be implemented. The Interacting Multiple-Model (IMM) filter achieves a good compromise between accuracy and computational load. It also offers more robustness in case of inadequacy between the considered models and the real model. A comprehensive study of the Multiple-Model filters including the IMM can be found in [6].

The IMM was quite successfully applied to track maneuvering targets. Some applications can be found in [7], [8], [9]. The basic assumptions in these applications are linearity and the Gaussian nature of noise. In the angle-only observation like a vehicle's heading, two problems arise: the first is non-linearity and the second is the periodic nature of the observations. Many variations of the IMM were proposed such as the EFK-based IMM in [10] to deal with non-linearity, but the local linearization required by the EKF makes this approach applicable only for small variations. Besides, the periodic aspect of the angles makes the Kalman Filter (KF) behave strangely when a natural transition between $-\pi$ and $\pi$ occurs in the sensor reading.

We proposed in [11] a version of the IMM applied to the circular domain. This new version denoted CIMM uses the von Mises distribution instead of the Gaussian distribution to model the probability on the angles. This makes the CIMM insensitive to the $2\pi$ periodicity of the observed angles. The CIMM filter will be applied in this work to estimate the angle of a maneuvering target and to detect the model change as soon as possible in order to prevent the estimate error peak during the maneuver. We compare the CIMM with a circular filter and with a traditional Kalman Filter. We suppose here a vehicle equipped with a magnetometer for heading measurement.

We will follow this organization: after this brief introduction, we will present in the second part a brief summary on the mathematical basis of the used filters. In the third part, we will assess the CIMM in a synthetic framework to detect a turning maneuver of a moving vehicle.

# 2 Circular filter and Circular IMM

## 2.1 Estimation in the Circular domain

The von Mises distribution is the circular domain equivalent of the normal distribution in the linear domain. Its probability density function $f_{CN}(\theta;\mu,\kappa)$ of the circular variable $\theta$ of mean $\mu$ and concentration parameter $\kappa$ is given by:

$$f_{CN}(\theta;\mu,\kappa) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa\cos(\theta-\mu)} \tag{1}$$

where $I_0(\kappa)$ is the modified Bessel function of order 0. $\kappa$ is the equivalent of the inverse of the variance of a normal law. For high values of $\kappa$, the von Mises distribution tends to a normal law of variance $1/\kappa$. For low values of $\kappa$, it tends to a uniform distribution between 0 and $2\pi$.

The von Mises distribution is uni-modal in a $2\pi$ interval and is $2\pi$-periodic which makes it adapted to angle estimation. In the circular domain, we define the circular distance as a measure of distance between two angles with the following expression:

$$d(\phi_1,\phi_2) = 1 - \cos(\phi_1 - \phi_2) \tag{2}$$

The circular distance varies between 0 and 2 for a difference of angle that varies respectively between 0 and $\pi$.

## 2.2 The Circular filter

The circular filter is the equivalent of the classic Kalman filter applied to angle estimation. It is fully described in [12]. It relies on a scheme of Prediction/Update defined as follows:

1) *Prediction step*: The angle $\hat{\theta}_{k|k-1}$ and its concentration parameter $P_{\theta_{k|k-1}}$ are predicted at instant $k$ by the following equations:

$$\hat{\theta}_{k|k-1} = \hat{\theta}_{k-1} + \hat{\tilde{\theta}}_{k-1} \tag{3}$$

$$\hat{\tilde{\theta}}_{k|k-1} = \hat{\tilde{\theta}}_{k-1} \tag{4}$$

$$P_{\theta_{k|k-1}} = A^{-1}(A(\kappa_Q)\, A(P_{\theta_{k-1}})\, A(P_{\tilde{\theta}_{k-1}})) \tag{5}$$

$$P_{\tilde{\theta}_{k|k-1}} = A^{-1}(A(\kappa_{\tilde{Q}})\, A(P_{\tilde{\theta}_{k-1}})) \tag{6}$$

where $\hat{\theta}_{k-1}$ and $P_{\theta_{k-1}}$ are respectively the direction estimate and the concentration parameter of the distribution of $\theta_{k-1}$. $\tilde{\theta}_{k-1}$ and $P_{\tilde{\theta}_{k-1}}$ refer respectively to the change in the angle during the measurement interval and the concentration parameter of its distribution. $A(\kappa) = \mathrm{I}_1(\kappa)/\mathrm{I}_0(\kappa)$ with $\mathrm{I}_1(\kappa)$ the modified Bessel function of the first kind and order 1. Its inverse $A^{-1}(.)$ is derived by the approximation found in [13].

2) *Update step*: The estimated angle $\hat{\theta}_k$ and the estimated angle variation $\hat{\tilde{\theta}}_k$ along with their respective concentration parameters $P_{\theta_k}$ and $P_{\tilde{\theta}_k}$ are obtained by the update step. This step relies on the angle measurement $\phi_k$ through the following equations [12]:

$$\hat{\theta}_k = \mathrm{atan2}(S_{\theta_k}, C_{\theta_k}) \tag{7}$$

$$\hat{\tilde{\theta}}_k = \mathrm{atan2}(S_{\tilde{\theta}_k}, C_{\tilde{\theta}_k}) \tag{8}$$

$$P_{\theta_k} = \sqrt{C_{\theta_k}^2 + S_{\theta_k}^2} \qquad (9)$$

$$P_{\tilde{\theta}_k} = \sqrt{C_{\tilde{\theta}_k}^2 + S_{\tilde{\theta}_k}^2} \qquad (10)$$

where

$$C_{\theta_k} = \alpha_{11}\cos\hat{\theta}_{k|k-1} + \alpha_{12}\cos\phi_k \qquad (11)$$

$$S_{\theta_k} = \alpha_{11}\sin\hat{\theta}_{k|k-1} + \alpha_{12}\sin\phi_k \qquad (12)$$

$$C_{\tilde{\theta}_k} = \alpha_{21}\cos\hat{\tilde{\theta}}_{k|k-1} + \alpha_{22}\cos(\phi_k - \hat{\theta}_{k-1}) \qquad (13)$$

$$S_{\tilde{\theta}_k} = \alpha_{21}\sin\hat{\tilde{\theta}}_{k|k-1} + \alpha_{22}\sin(\phi_k - \hat{\theta}_{k-1}) \qquad (14)$$

with

$$\alpha_{11} = P_{\theta_{k|k-1}}\ ;\ \alpha_{12} = \kappa_R \qquad (15)$$

$$\alpha_{21} = P_{\tilde{\theta}_{k|k-1}}\ ;\ \alpha_{22} = A^{-1}(A(P_{\theta_{k-1}})A(\kappa_R)) \qquad (16)$$

atan2 is the quadrant specific inverse tangent function. $\kappa_R$ denotes the concentration parameter of the measurement noise that follows a zero centered von Mises distribution.

## 2.3 The circular IMM Filter

For a maneuvering mobile with angle-only observations, the Circular IMM (CIMM) offers a practical solution to model the most important maneuvers. The main difference between the CIMM and the IMM is that the Kalman filters corresponding to each model are replaced by circular filters. The full description of the CIMM can be found in our previous paper [11]. The structure of the CIMM is depicted in figure 1. The mixing step initializes each filter from all previous estimates at instant *k-1* using a weighted sum in the circular domain. The same process is applied in the combination step that generates the final estimate of the CIMM using a weighted sum of the estimates at instant $k$, the weights in this case are the model probabilities denoted $\mu_k^j$ , where $j$ is the model number.

As we are interested in turn maneuvers detection, each model $j$ is associated with a recursive circular filter that includes a prediction step and an update step. The difference between models lies in the prediction step defined as follows [11]:

$$\hat{\theta}_{k|k-1}^{j} = \hat{\theta}_{k-1}^{0j} + M_j \qquad (17)$$

$$P_{\theta_{k|k-1}}^{j} = A^{-1}(A(P_{\theta_{k-1}}^{0j})A(\kappa_{Q^j})) \qquad (18)$$

Where $\hat{\theta}_{k|k-1}^{j}$ , $P_{\theta_{k|k-1}}^{j}$ are angle and concentration parameter predicted for model $j$ and $\kappa_{Q^j}$ the state noise concentration parameter defined for model $j$.

For a straight forward movement (constant angle) : $M_j = 0$. For a left turn (decreasing angle): $M_j < 0$ and for a right turn (increasing angle): $M_j > 0$. $M_j$ is the angular rate supposed constant during the maneuver.

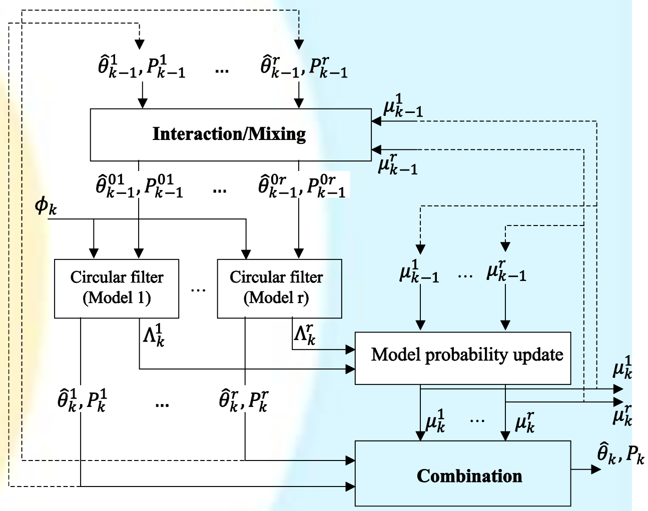


**Fig 1: CIMM filter structure**

## 3 Experiment

We consider a vehicle equipped with a magnetometer to measure its heading. This vehicle moves straight forward with a constant velocity then makes a left turn on a Y-shape roadway (see figure 2). The first aim of this experiment is to compare the estimates given by a Kalman filter, a circular filter and a CIMM. The second aim is to show the maneuver detection with the CIMM. Given this scenario, we consider two models in the CIMM architecture:

- *Model 1* reflecting a straight forward movement ($M_1 = 0$)
- *Model 2* corresponding to a left turn with an angular rate $M_2 = -0.15$ rd. This rate is an approximate turn rate for a vehicle on an urban road; it can be adjusted depending on the application.

The temporal length of the trajectory is 100s, the turn occurs at t = 50s

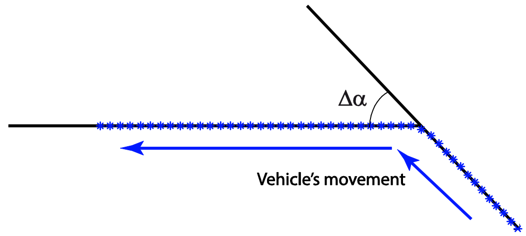


**Fig 2: Vehicle trajectory on Y-shape roadway**

Figure 3 shows that the estimate given by the Kalman filter is wrong after the turn when the vehicle's heading is equal to $\pi$. Indeed, the transitions between -$\pi$ and $\pi$ caused by the sensor noise are interpreted by the Kalman filter as 2$\pi$ jumps, whereas for the circular filter these transitions are correctly interpreted because of the use of the von Mises distribution.

In figure 4 we compare the circular filter and the CIMM by calculating the circular dispersion at each instant. The circular dispersion is the average of the circular distances between the estimate and the real heading for 1000 realization of the noise. Figure 4.(a) shows the circular dispersion over time and figure 4.(b) the circular dispersion per noise realization.

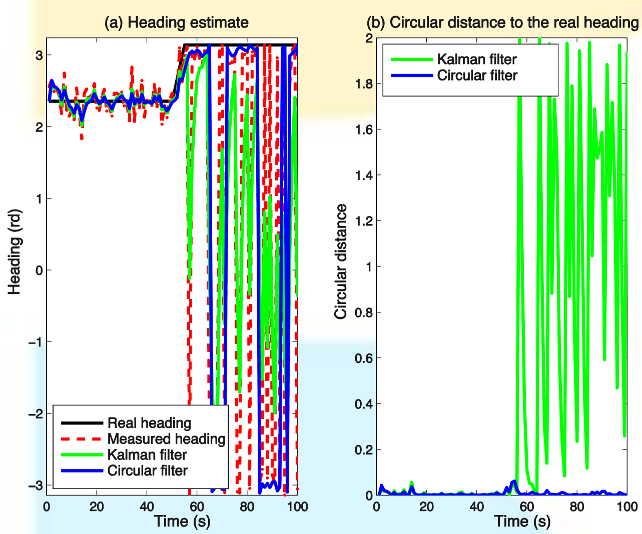


**Fig 3: Comparison between the Kalman and circular filters**

In figure 4.(b), we note that both filters reduce the noise on the measurement. We also note clearly in figure 4.(a) a peak on the circular dispersion of the circular filter estimate as a consequence of the turning maneuver, whereas the circular dispersion on the CIMM estimate shows two small peaks that stay below the measurement error dispersion. These small peaks are due to the model change within the CIMM. Figure 5 clearly shows the maneuver detection by the CIMM reflected by the increase of Model 2 (turn left model) probability at t = 50s.

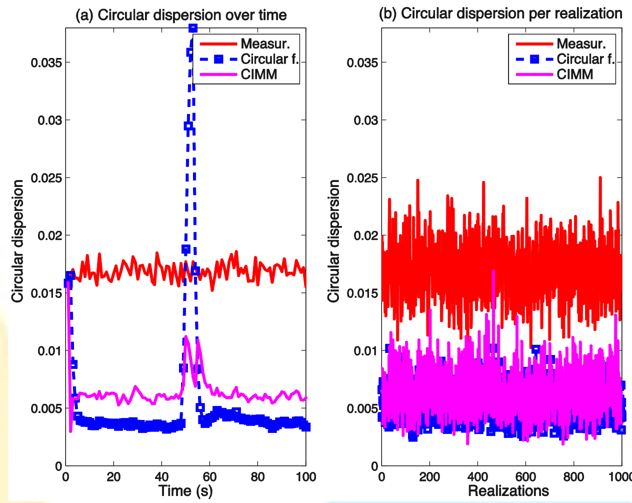


**Fig 4: Comparison between the circular filter and the CIMM**

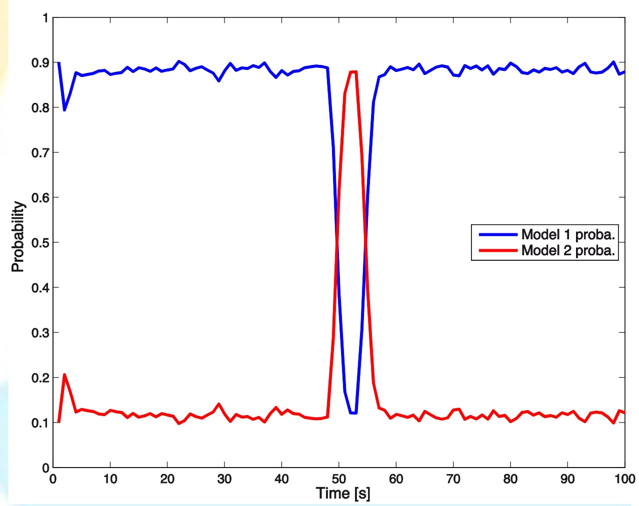


**Fig 5: Model probabilities of the CIMM**

## 4 Conclusion

In this paper, we show the contribution of the Circular IMM in detecting a turn maneuver of a vehicle in an angle only observation case. We show that the CIMM outperforms the traditional Kalman filter and also the Circular filter in a scenario of a vehicle performing an unpredicted turn. Indeed, the CIMM offers a better estimate of the observed angle by executing two parallel circular filters. We show that the CIMM is able to reduce the error peak on the estimate that is generally observed on filters that rely only on one model.

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