Problem Set 7

Exercises for the Lecture Fundamentals of Simulation Methods

Prof. Dr. Ralf Klessen (Lecture Tuesday 9h - 11h and Thursday 9h - 11h)
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Submit the solution to your tutor in electronic form by **Wednesday December 4, 2019**.

1. Sound waves

1.1. Simple advection problem (*2 points*)

Consider the most simple 1D advection problem. It is define by a partial differential equation (PDE) of the form

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} = 0, \tag{1}$$

where $u = u(x,t)$ is a function of x and t, and v is a constant parameter. Think of u being the density and v being the sound speed.

Show that if we are given any function $q(x)$, then

$$u(x,t) = q(x - vt) \tag{2}$$

is a solution of the advection equation.

1.2. Derivation of the wave equation (*4 points*)

Now we turn to the Euler equations as defined in the lecture. Take a simple isothermal equation of state, $P = c_\mathrm{s}^2\rho$, with c_s being the isothermal sound speed. Show that they lead to a simple wave equation for density perturbations.

To do so, look at the behavior of small perturbations around the equilibrium solution in all relevant quantities. For example, the density can be written as $\rho(\vec{x},t) = \rho_0 + \delta\rho(\vec{x},t)$, and so forth. Recall that the equilibrium state is homogeneous and stationary. Only consider terms up to linear order in the perturbation.

1.3. Sound waves (*4 points*)

Assume that seismic waves can be approximated as simple sound waves in the Earth's mantle.

1. How long will it take for the earthquake from a massive volcano eruption on the other side of the Earth to be measured by a seismic station in Europe?
2. What is the travel time to this station for the corresponding sound wave through the atmosphere? Will it be noticed in Europe? Justify your answer.

Make reasonable assumptions for the velocity with which these waves travel.