Quantitative Management Modeling - Assignment_3

```
library("lpSolveAPI")
library("lpSolve")
library("tinytex")

#Creating a table representing the data
unit.shipping.cost<- matrix(c(22,14,30,600,100,
                16,20,24,625,120,
                80,60,70,"-","-"), ncol=5,byrow=T)
colnames(unit.shipping.cost) <- c("Warehouse1", "Warehouse2",      "Warehouse3", "ProductionCost",
"ProductionCapacity")
rownames(unit.shipping.cost) <- c("PlantA", "PlantB", "Demand")
unit.shipping.cost <- as.table(unit.shipping.cost)
unit.shipping.cost
```

```
##        Warehouse1 Warehouse2 Warehouse3 ProductionCost ProductionCapacity
## PlantA 22         14         30         600            100
## PlantB 16         20         24         625            120
## Demand 80         60         70         -              -
```

#The objective function is to minimize the transportation cost

$$Z = 622X_{11} + 614X_{12} + 630X_{13} + 0X_{14} + 641X_{21} + 645X_{22} + 649X_{23} + 0X_{24}$$

#Subject to the following constraints

$$Supply \quad Constraints$$
$$X_{11} + X_{12} + X_{13} + X_{14} <= 100$$
$$X_{21} + X_{22} + X_{23} + X_{24} <= 120$$
$$Demand \quad Constraints$$
$$X_{11} + X_{21} >= 80$$
$$X_{12} + X_{22} >= 60$$
$$X_{13} + X_{23} >= 70$$
$$X_{14} + X_{24} >= 10$$
$$Non-Negativity \quad Constraints$$
$$X_{ij} >= 0 \quad \text{Where i = 1,2 and j = 1,2,3,4}$$

```
#As the demand is not equal to supply we are creating the dummy variables .
#Creating a matrix for the given objective function
transport_cost <- matrix(c(622,614,630,0,
                641,645,649,0), ncol=4, byrow=T)
transport_cost
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  622  614  630    0
## [2,]  641  645  649    0
```

```r
#Defining the column names
colnames(transport_cost) <- c("Warehouse1", "Warehouse2",
                              "Warehouse3", "Dummy")
#Defining the row names, row signs and row values
rownames(transport_cost) <- c("PlantA", "PlantB")
transport_cost
```

```
##        Warehouse1 Warehouse2 Warehouse3 Dummy
## PlantA        622        614        630     0
## PlantB        641        645        649     0
```

```r
row_signs <- rep("<=",2)
row_RHS <- c(100,120)
#It cannot be greater to the specified units as it is the supply function.

#Defining the column signs and column values
col_signs <- rep(">=",4)
col_RHS <- c(80,60,70,10)
#It can be greater than the specified units as it is the demand function.
```

```r
#Running the lp.transport function
lp_transport_cost <- lp.transport(transport_cost,"min", row_signs,row_RHS,col_signs,col_RHS)
```

```r
#Getting the objective value
lp_transport_cost$objval
```

```
## [1] 132790
```

#The resulting minimization value is $132,790***, which is the lowest total cost that can be derived from the costs of production and shipping of defibrillators.

```r
#Getting the constraints value
lp_transport_cost$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

#80 AEDs in Plant B - Warehouse1, 60 AEDs in Plant A - Warehouse2, 40 AEDsin Plant A - Warehouse3, 30 AEDs*** in Plant B - Warehouse3 should be produced in each plant and then distributed to each of the three wholesaler warehouses in order to minimize the overall cost of production as well as shipping.

#Formulate the dual of the above transportation problem.

#Since the primal was to minimize the transportation cost the dual of it would be to maximize the value added(VA).

$$Maximize \quad VA = \quad 80W_1 + 60W_2 + 70W_3 - 100P_A - 120P_B$$

#Subject to the following constraints

$$\text{Total} \quad Payments \quad Constraints$$

$$W_1 - P_A >= 622$$

$$W_2 - P_A >= 614$$

$$W_3 - P_A >= 630$$

$$W_1 - P_B >= 641$$

$$W_2 - P_B >= 645$$

$$W_3 - P_B >= 649$$

$$\text{W}here \quad W_1 = Warehouse \quad 1$$

$$W_2 = Warehouse \quad 2$$

$$W_3 = Warehouse \quad 3$$

$$P_1 = Plant \quad 1$$

$$P_2 = Plant \quad 2$$

#Economic Interpretation of the dual

$$W_1 <= 622 + P_A$$

$$W_2 <= 614 + P_A$$

$$W_3 <= 630 + P_A$$

$$W_1 <= 641 + P_B$$

$$W_2 <= 645 + P_B$$

$$W_3 <= 649 + P_B$$

$$\text{I}t \quad is \quad clear \quad from \quad the \quad above \quad that \quad W_1 - P_A >= 622, \quad can \quad be \quad exponented \quad as \quad W_1 <= 622 + P_A$$

$$\text{I}n \quad this \quad case \quad W1 \quad is \quad taken \quad into \quad account \quad as \quad payments \quad made \quad at \quad the \quad source$$

$$which \quad is \quad nothing \quad but \quad the \quad revenue$$

$$Whereas \quad P_A + 622 \quad is \quad the \quad money \quad paid \quad at \quad the \quad origin \quad at \quad Plant_A$$

$$\text{H}ence \quad the \quad equation \quad is \quad MR_1 >= MC_1.$$

$$\text{F}or \quad profit \quad maximization, \quad the \quad Marginal \quad revenue \quad should \quad be$$

$$equal \quad to \quad Marginal \quad cost.\text{T}herefore, \quad MR_1 = MC_1$$

$$\text{T}herefore \quad it \quad can \quad be \quad concluded \quad that,$$

$$Profit \quad maximization \quad takes \quad place \quad if \quad MC \quad is \quad equal \quad to \quad MR.$$

#If MR > MC, we will need to increase the production supply to meet the Marginal Revenue and if MR < MC we will have to decrease the cost at plants in order to meet the Marginal Revenue (MR).