

Exercise

Suppose your ML model output is the following in comparison with the respective target values as shown in the table

Calculate:

- (1) the classification error,
- (2) the mean square error,
- (3) least squares and
- (4) cross-entropy

ML output				Target		
0.1	0.3	0.6		0	0	1
0.2	0.6	0.2		0	1	0
0.3	0.4	0.3		1	0	0

Zero/one loss for the multi class classification problems

Suppose your ML model output is the following in comparison with the respective target values as shown in the table

Zero/one loss can be re-written as:

$$\text{Zero/One Loss} = \frac{1}{n} \sum_i^n L_{0/1}(y_i, f(w, x_i))$$

where, $L_{0/1}(y_i, f(x_i, w)) = 1$ iff $f(x_i, w) \neq y_i$ else 0

i.e., that is, we are interested in measuring only errors when the prediction is not equal to the target label

ML output				Target		
0.1	0.3	0.6		0	0	1
0.2	0.6	0.2		0	1	0
0.3	0.4	0.3		1	0	0

Zero/one loss for the multi class classification problems

Lets simplify by converting ML output $f(x_i, w)$ to one hot vectors:

We can use the argmax operator to find the argument (or the index) of the maximally predicted class:

$\text{argmax}([0.1, 0.3, 0.6]) = 3$; corresponding one hot vector = $[0, 0, 1]$

$\text{argmax}([0.2, 0.6, 0.2]) = 2$; corresponding one hot vector = $[0, 1, 0]$

$\text{argmax}([0.3, 0.4, 0.3]) = 2$; corresponding one hot vector = $[0, 1, 0]$

ML output				Target		
0.1	0.3	0.6		0	0	1
0.2	0.6	0.2		0	1	0
0.3	0.4	0.3		1	0	0

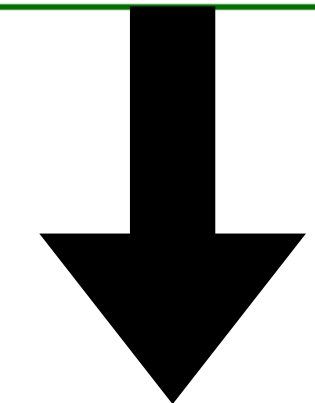
The above one-hot vectors are the outputs of the ML classifier, or the outputs of $f(x_i, w)$

Zero/one loss for the multi class classification problems

Rewriting it on the table to the right

$$\text{Zero/One Loss} = \frac{1}{3} \times 1$$

ML output	Target
0 0 1	0 0 1
0 1 0	0 1 0
0 1 0	1 0 0



Error!

Least squares for classification problems

$$L(w) = \frac{1}{2} \sum_{n=1}^N (f(x_n, w) - t_n)^2$$

where, $f(x_n, w)$ is the ML output and t_n is the target labels

ML output				Target		
0.1	0.3	0.6		0	0	1
0.2	0.6	0.2		0	1	0
0.3	0.4	0.3		1	0	0

Least squares for classification problems

$$L(w) = \frac{1}{2} \sum_{n=1}^N (f(x_n, w) - t_n)^2$$

Let us apply this to each of the samples here:

Squared error for first example:

$$[(0.1 - 0)^2 + (0.3 - 0)^2 + (0.6 - 1)^2] = 0.26$$

Squared error for second example:

$$(0.2 - 0)^2 + (0.6 - 1)^2 + (0.2 - 0)^2 = 0.24$$

Squared error for second example:

$$(0.3 - 1)^2 + (0.4 - 0)^2 + (0.3 - 0)^2 = 0.74$$

$$\text{Least squares} = \frac{[0.26 + 0.24 + 0.74]}{2}$$

ML output				Target		
0.1	0.3	0.6		0	0	1
0.2	0.6	0.2		0	1	0
0.3	0.4	0.3		1	0	0

Mean squared error for classification problems

$$L(w) = \frac{1}{N} \sum_{n=1}^N (f(x_n, w) - t_n)^2$$

where, $f(x_n, w)$ is the ML output and t_n is the target labels

ML output				Target		
0.1	0.3	0.6		0	0	1
0.2	0.6	0.2		0	1	0
0.3	0.4	0.3		1	0	0

Mean squared error for classification problems

$$L(w) = \frac{1}{N} \sum_{n=1}^N (f(x_n, w) - t_n)^2$$

Let us apply this to each of the samples here:

Squared error for first example:

$$[(0.1 - 0)^2 + (0.3 - 0)^2 + (0.6 - 1)^2] = 0.26$$

Squared error for second example:

$$[(0.2 - 0)^2 + (0.6 - 1)^2 + (0.2 - 0)^2] = 0.24$$

Squared error for second example:

$$[(0.2 - 0)^2 + (0.6 - 1)^2 + (0.2 - 0)^2] = 0.24$$

$$\text{Least squares} = \frac{[0.26 + 0.24 + 0.74]}{2}$$

ML output				Target		
0.1	0.3	0.6		0	0	1
0.2	0.6	0.2		0	1	0
0.3	0.4	0.3		1	0	0