Exercise

Suppose your ML model output is the following in comparison with the respective target values as shown in the table

ML output			Target		
0.1	0.3	0.6	0	0	1
0.2	0.6	0.2	0	1	0
0.3	0.4	0.3	1	0	0

Calculate:

(1) the classification error,

(2) the mean square error,

(3) least squares and

(4) cross-entropy

Zero/one loss for the multi class classification problems

Suppose your ML model output is the following in comparison with the respective target values as shown in the table

ML output			Target		
0.1	0.3	0.6	0	0	1
0.2	0.6	0.2	0	1	0
0.3	0.4	0.3	1	0	0

Zero/one loss can be re-written as:

$$\text{Zero/One Loss} = \frac{1}{n}\sum_{i}^{n} L_{0/1}(y_i, f(w, x_i))$$

where, $L_{0/1}(y_i, f(x_i, w)) = 1$ iff $f(x_i, w) \neq y_i$ else 0

i.e., that is, we are interested in measuring only errors when the prediction is not equal to the target label

Zero/one loss for the multi class classification problems

Lets simplify by converting ML output ($f(x_i, w)$ to one hot vectors:

We can use the argmax operator to find the argument (or the index) of the maximally predicted class:

argmax([0.1, 0.3, 0.6]) = 3; corresponding one hot vector = [0,0,1]

argmax([0.2, 0.6, 0.2]) = 2; corresponding one hot vector = [0,1,0]

argmax([0.3, 0.4, 0.3]) = 2; corresponding one hot vector = [0,1,0]

ML output			Target		
0.1	0.3	0.6	0	0	1
0.2	0.6	0.2	0	1	0
0.3	0.4	0.3	1	0	0

The above one-hot vectors are the outputs of the ML classifier, or the outputs of $f(x_i, w)$

Zero/one loss for the multi class classification problems

Rewriting it on the table to the right

$$\text{Zero/One Loss} = \frac{1}{3} \times 1$$

ML output			Target		
0	0	1	0	0	1
0	1	0	0	1	0
0	1	0	1	0	0

Error!

Least squares for classification problems

$$L(w) = \frac{1}{2}\sum_{n=1}^{N}(f(x_n, w) - t_n)^2$$

ML output			Target		
0.1	0.3	0.6	0	0	1
0.2	0.6	0.2	0	1	0
0.3	0.4	0.3	1	0	0

where, $f(x_n, w)$ is the ML output and t_n is the target labels

Least squares for classification problems

$$L(w) = \frac{1}{2}\sum_{n=1}^{N}(f(x_n, w) - t_n)^2$$

Let us apply this to each of the samples here:

Squared error for first example:

[(0.1 - 0)^2 + (0.3 - 0)^2 + (0.6 - 1)^2] = 0.26

Squared error for second example:

(0.2 - 0)^2 + (0.6 - 1)^2 + (0.2 - 0)^2 = 0.24

Squared error for second example:

(0.3 - 1)^2 + (0.4 - 0)^2 + (0.3 - 0)^2 = 0.74

Least squares = $\dfrac{[0.26 + 0.24 + 0.74]}{2}$

ML output			Target		
0.1	0.3	0.6	0	0	1
0.2	0.6	0.2	0	1	0
0.3	0.4	0.3	1	0	0

Mean squared error for classification problems

$$L(w) = \frac{1}{N} \sum_{n=1}^{N} (f(x_n, w) - t_n)^2$$

where, $f(x_n, w)$ is the ML output and t_n is the target labels

ML output			Target		
0.1	0.3	0.6	0	0	1
0.2	0.6	0.2	0	1	0
0.3	0.4	0.3	1	0	0

Mean squared error for classification problems

$$L(w) = \frac{1}{N}\sum_{n=1}^{N}(f(x_n, w) - t_n)^2$$

Let us apply this to each of the samples here:

Squared error for first example:

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Squared error for second example:

$$[(0.2 - 0)^2 + (0.6 - 1)^2 + (0.2 - 0)^2] = 0.24$$

Squared error for second example:

$$[(0.2 - 0)^2 + (0.6 - 1)^2 + (0.2 - 0)^2] = 0.24$$

Least squares $= \dfrac{[0.26 + 0.24 + 0.74]}{2}$

ML output			Target		
0.1	0.3	0.6	0	0	1
0.2	0.6	0.2	0	1	0
0.3	0.4	0.3	1	0	0