INM431: Machine Learning

On loss functions

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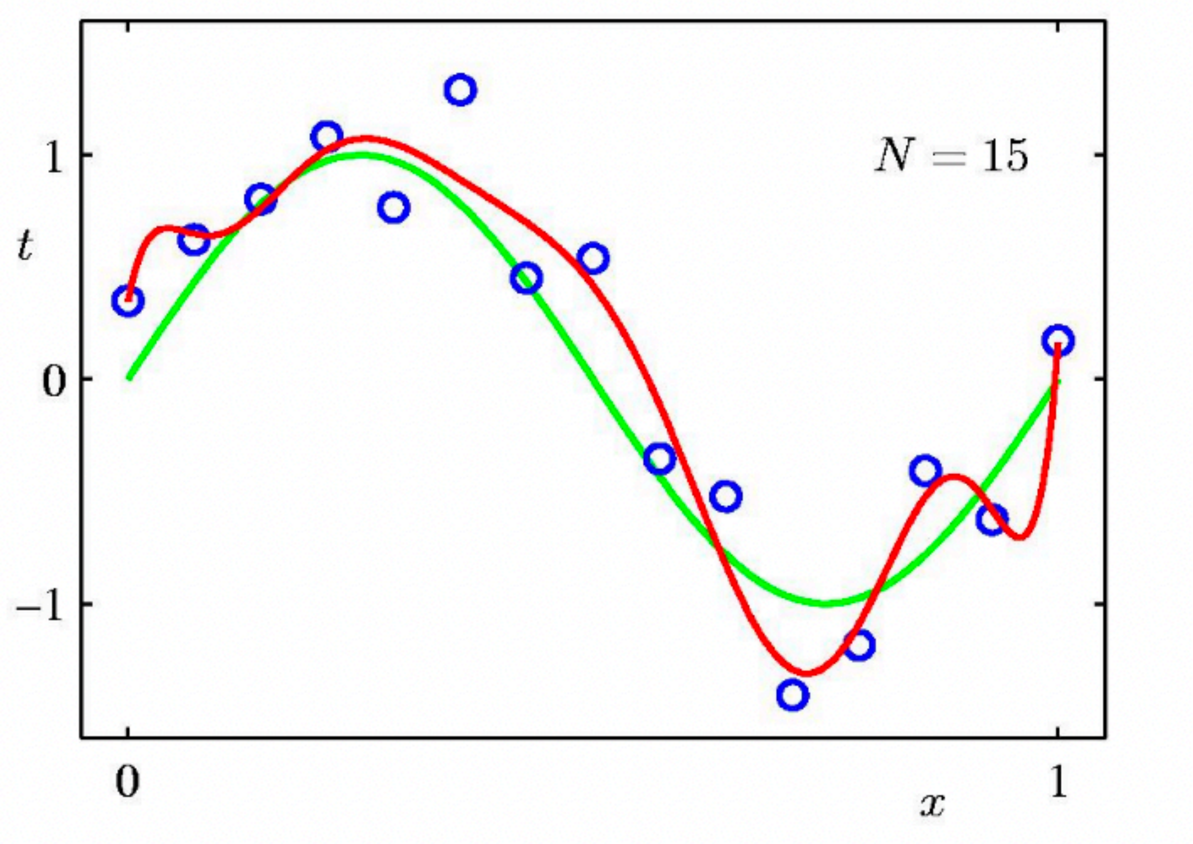
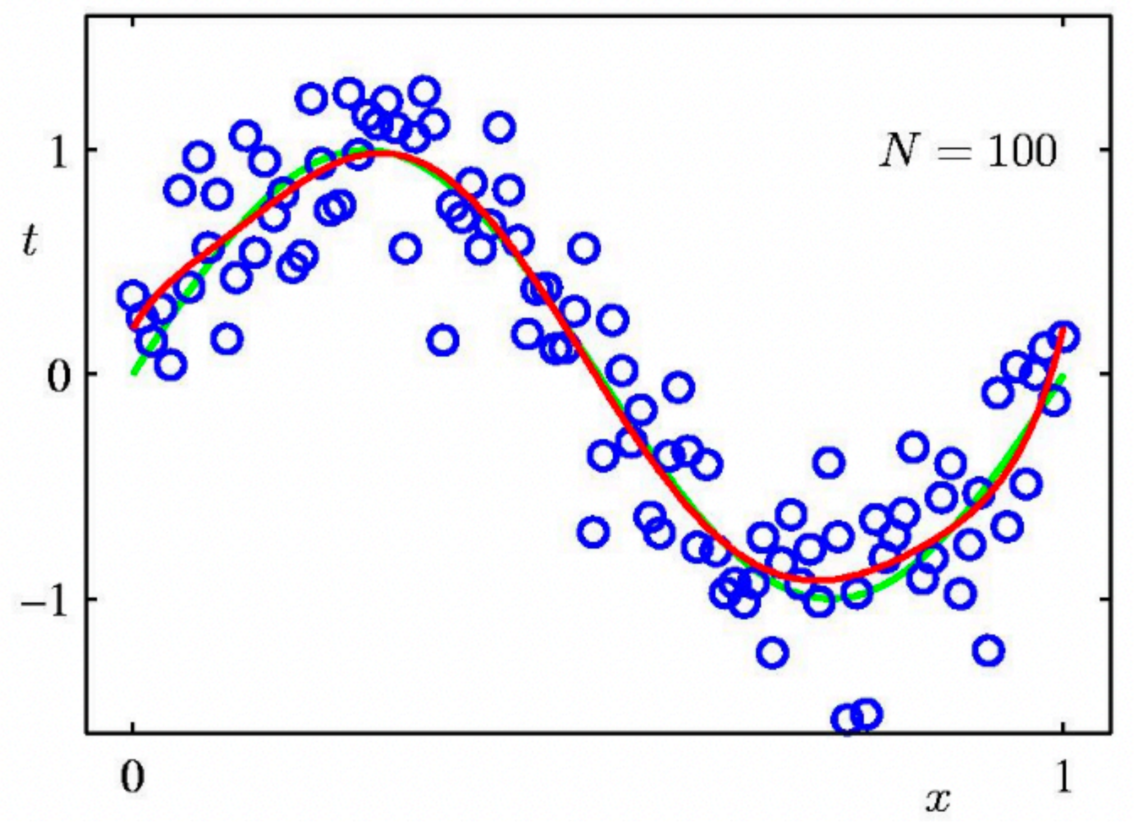
Based on materials by Artur S. d'Avila Garcez + Bishop's book + Murphy's book

Jumping back to linear regression

$$L(w) = \frac{1}{2}\sum_{n=1}^{N}(f(x_n, w) - t_n)^2$$





Least squares as a loss function?

Revisiting MLE

$$\log P(D\,|\,\theta) = \sum_c \log P(d_c\,|\,\theta)$$

Loss function:

$$-\log P(\theta\,|\,D) = -\log P(D\,|\,\theta) - \log P(\theta) + \log P(D)$$

Where, we obtain θ by optimising the log-likelihood.

Squared loss

$$L(w) = \frac{1}{2} \sum_{n=1}^{N} (f(x_n, w) - t_n)^2$$

Given a collection of input features, outputs, and a hypothesis function f (or the machine learning function of choice), we want to find the parameters w that minimises the sum of the losses.

Recipe

$$L(w) = \frac{1}{2}\sum_{n=1}^{N}(f(x_n, w) - t_n)^2$$

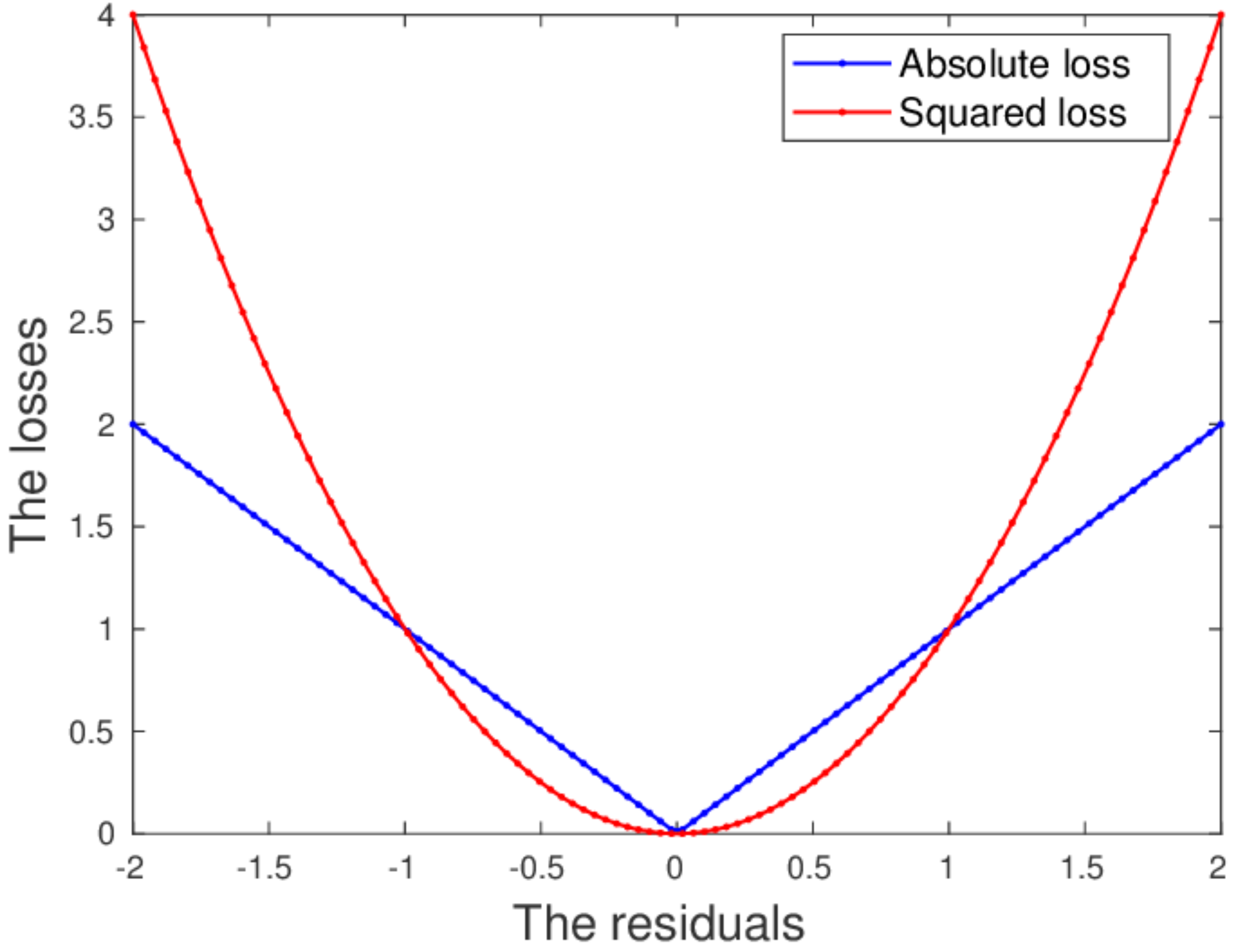
Minimising the loss:

$$\min_w \frac{1}{2}\sum_{n=1}^{N}(f(x_n, w) - t_n)^2 = 0$$

Approach 1: Gradient descent - take first order derivative

Approach 2: Analytic solution, solve for w

Visualising the losses

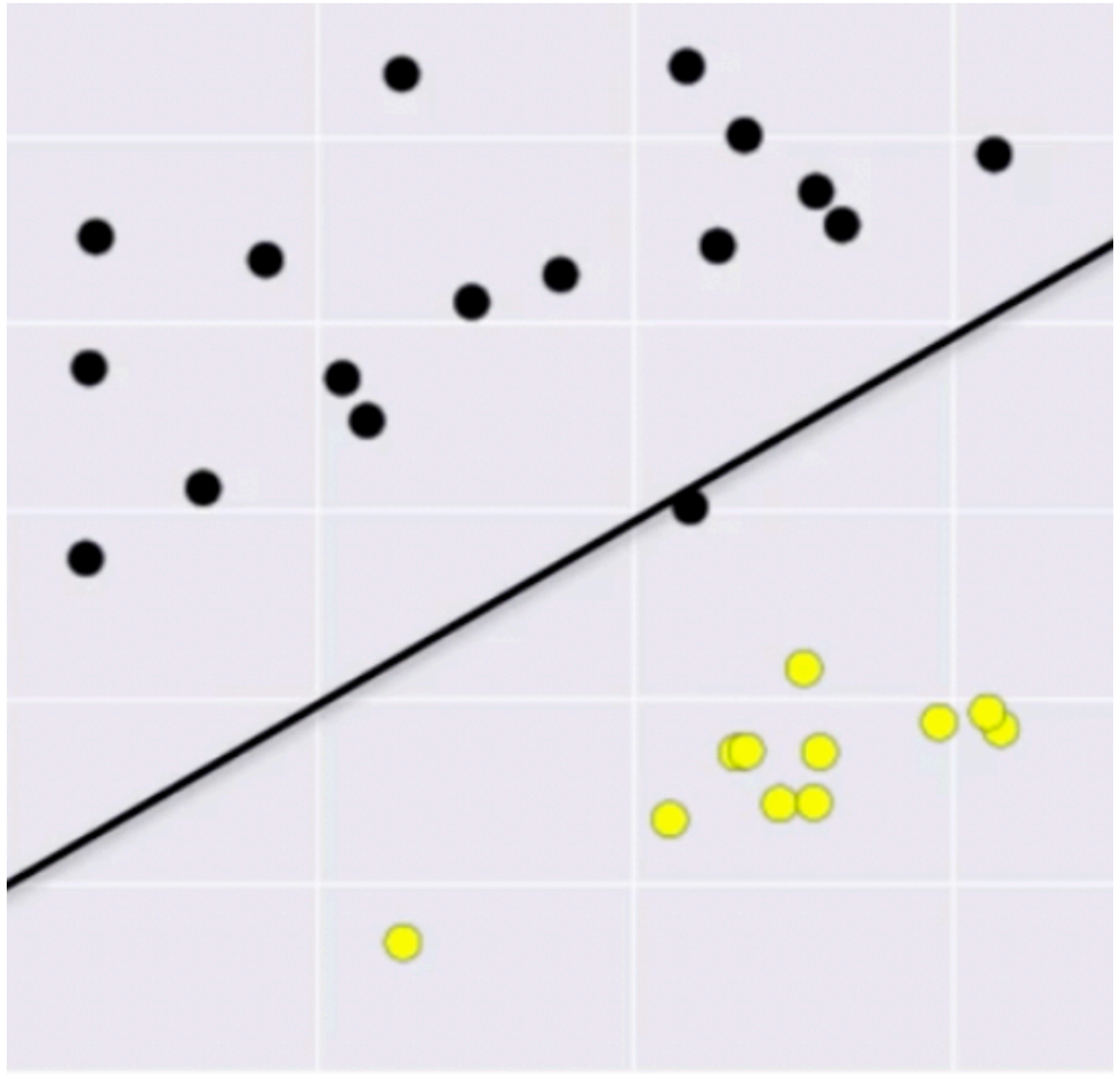


Linear classification

A linear classifier computes the linear score:

$$f(w) = w^\top x_i + b$$

Predict label of x_i based on linear score s_i

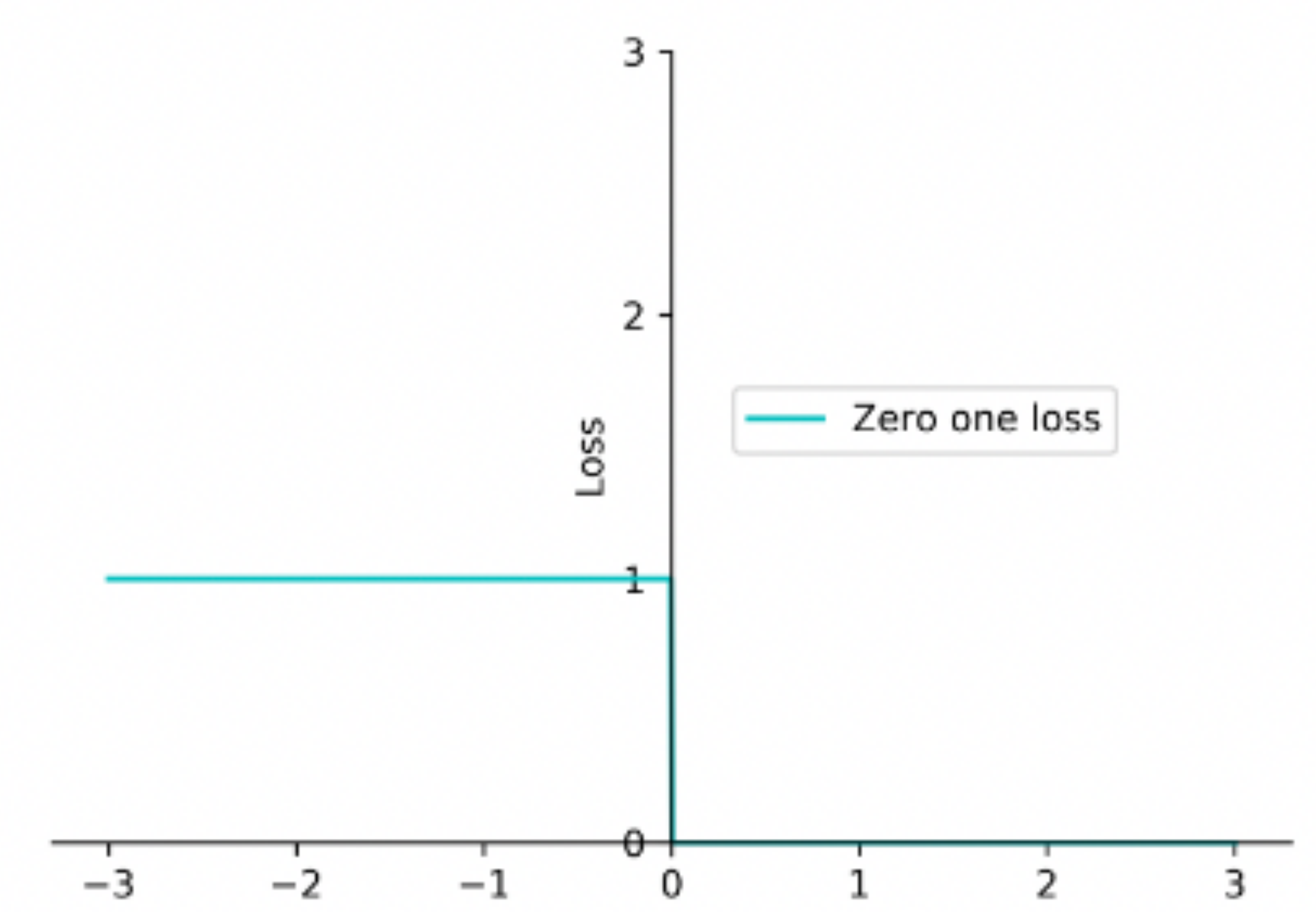


A few more loss functions for classification

Zero-one loss

Measure the total number of prediction errors for the classifier.

$$L_{0/1} = \begin{cases} 1, & \text{if } y_i f(w, x_i) < 0 \\ 0, & \text{otherwise} \end{cases}$$



when, $y_i = \pm 1$(binary case, with labels +1/-1)

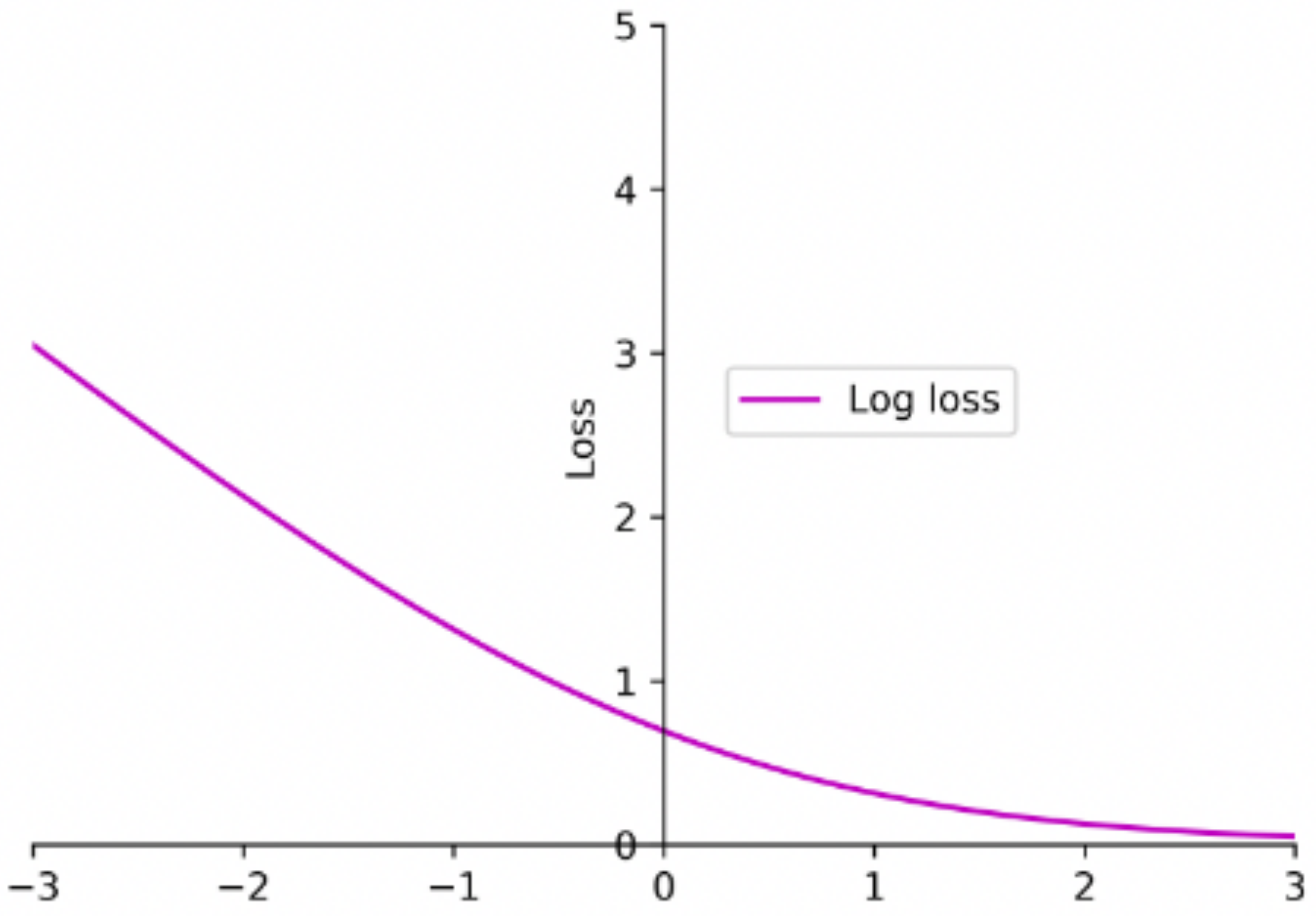
Log loss/cross-entropy loss/max-entropy loss

Log loss is one of the most popular loss functions for training:

$$\frac{1}{n}\sum_{i=1}^{n} L_{\log}(y_i, f(w, x_i))$$

where, $L_{\log}$ is defined as:

$$L_{\log}(y_i, f(w, x_i)) = \log(1 + e^{-y_i f(w, x_i)})$$



Can be used for multi-class discrete classification tasks, i.e. one-hot target vectors.

Exercise

Suppose your ML model output is the following in comparison with the respective target values as shown in the table

Calculate:

(1) the classification error,

(2) the mean square error,

(3) least squares and

(4) cross-entropy

ML output			Target		
0.1	0.3	0.6	0	0	1
0.2	0.6	0.2	0	1	0
0.3	0.4	0.3	1	0	0