

# Fundamentals of probability

Tutorial/Lab work: Week 1

Instructions: Write the solutions to the following 6 problems on a separate document. Kindly stage+commit+push the solution document to the “week01” directory on your INM431 Github repository. Grading: All attempts will get 1 point. **Please make sure that the document is pushed to Github by 13/10/2021.**

## Problem 1

Suppose we know nothing about coins except that each tossing event produces heads with some unknown probability  $p$  or tails with probability  $1 - p$ . Your model of a coin has one parameter,  $p$ . You observe 100 tosses and there are 53 heads. Please answer the following questions:

1. What is  $p$ ?
2. What if you only tossed the coin once and got heads?
3. Is it reasonable to give a single answer if we do not have sufficient amount of data?

## Problem 2

A drug manufacturer claims that its roadside drug test will detect the presence of cannabis in the blood<sup>1</sup> 90% of the time. However, the manufacturer admits that 10% of all cannabis-free drivers also will test positive. A national survey indicates that 20% of all drivers have smoked cannabis during the last 72 hours.

<sup>1</sup> i.e. show a positive test for a driver who has smoked cannabis in the last 72 hours

1. What is the probability that a driver has smoked cannabis in the last 72 hours if they have tested positive?
2. New information arrives which indicates that, while the roadside drug test will now show positive for a driver who has smoked cannabis 99.9% of the time, the number of cannabis-free drivers testing positive has gone up to 20%. What is the probability that someone smoked cannabis in the last 72 hours if they have not tested positive?

## Problem 3

X-factor viewers from 3 towns took part in a survey about how they vote: 46% of those surveyed were from Bury, 38% from Croydon and

16% from Dover. The poll showed that 61% of Bury viewers, 88% of Croydon viewers, and 51% of Dover viewers voted for the eventual winner of *X-factor*.

What is the probability that a vote for the winner was cast by a viewer from Dover?

#### *Problem 4*

You don't know if your friend, George, knows about neural networks. You assume that he has studied Artificial Intelligence, you but are not completely sure; let's say you're 50% Certain about this. Maybe he majored in Computer Science; you're only 20% sure of that.

What are the chances that George knows what a neural network is?

Assume that 80% of people who study AI know what a neural network is, 40% of people who study CS know what a neural network is, and 10% of the rest of the population know what a neural network is.

#### *Problem 5*

You have a bag containing 1000 coins. 999 are genuine, but one of them is biased, with a 'head' on both sides. You pick one of the 1000 coins at random, and flip it 10 times. All ten times it falls 'heads'.

What is the probability that this coin is the biased one?

#### *Problem 6*

(Challenge – *Prisoner's dilemma*<sup>2,3</sup>)

Suppose that, in your desperation to get rich as quickly as possible, you consider the various alternatives, infer their likely consequences and decide that the best alternative is to rob a local bank. You recruit your best friend, John, well known for his meticulous attention to detail, to help you plan and carry out the crime. Thanks to your joint efforts, you succeed in breaking into the bank in the middle of the night, opening the safe, and making your get-away with a cool million pounds in the boot of your car.

Unfortunately, years of neglect have left your car in a state of general disrepair, and you are stopped by the police for driving at night with only one headlight. In the course of a routine investigation, they discover a suitcase with the million pounds in the boot. You plead ignorance of any wrongdoing, but they arrest you both anyway on the

<sup>2</sup> For a thorough introduction, if you are interested, please follow: [https://en.wikipedia.org/wiki/Prisoner's\\_dilemma](https://en.wikipedia.org/wiki/Prisoner's_dilemma)

<sup>3</sup> For a more comic introduction have a look at: <https://www.youtube.com/watch?v=0UGuPvrsG3E>

suspicion of robbery. Without witnesses and without a confession, the police can convict you and your friend only of the lesser offense of possessing stolen property, which carries a penalty of one year in jail.

However, if one of you turns witness against the other, and the other does not, then the first will be released free of charge, and the second will take all of the blame and be sentenced to six years in jail. If you both turn witness, then you will share the blame and will be sentenced to three years in jail each. You are placed in different cells and cannot communicate with each other.

What would you do?