Lecture 1

Model Solutions

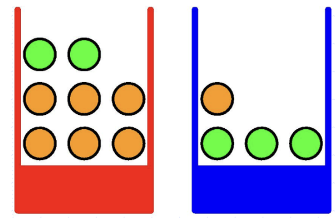

Figure 1: Oranges, apples,the two boxes

Problem 1

Question: What is the probability that the fruit was from BLUE box if an orange was picked?

$$
\begin{aligned}
P(\text{BOX=blue}|\text{FRUIT=orange}) &= \frac{P(\text{FRUIT=orange}\cap\text{BOX=blue})}{P(\text{FRUIT=orange})} \\
P(\text{FRUIT=orange} \cap \text{BOX=blue}) &= P(\text{FRUIT=orange} \mid \text{BOX=blue})P(\text{BOX=blue}) \\
P(\text{FRUIT=orange}|\text{BOX=blue}) &= \frac{1}{4} \quad \text{(from Figure 1)} \\
P(\text{FRUIT=orange} \cap \text{BOX=blue}) &= \frac{1}{4} \times \frac{1}{2} \quad \text{(under the assumption that } P(\text{BOX=orange}) = P(\text{BOX=red}) = \frac{1}{2}) \\
P(\text{FRUIT=orange}) &= P(\text{FRUIT=orange} \cap \text{BOX=blue}) + P(\text{FRUIT=orange} \cap \text{BOX=red}) \\
P(\text{FRUIT=orange} \cap \text{BOX=red}) &= P(\text{FRUIT=orange} \mid \text{BOX=red})P(\text{BOX=red}) \\
P(\text{FRUIT=orange} \mid \text{BOX=red}) &= \frac{6}{8} \\
P(\text{FRUIT=orange}) &= \frac{6}{8} \times \frac{1}{2} \\
P(\text{BOX=blue}|\text{FRUIT=orange}) &= \quad ? \quad \text{(substitute values we have computed above)}
\end{aligned}
$$

Problem 2

Question: A test for salmonella is made available to chicken farmers. The test will correctly show a positive result for salmonella 95% of the time. However the test also shows positive results 15% of the time in salmonella free chickens. 10% of chickens have salmonella.

If a chicken tests positive, what is the probability that it has salmonella?

Let us begin by defining the notation for the given information:

Let A be the presence of salmonella
Let B be a positive test
We are asked for:

$$P(A|B) = \ ?$$

(the probability that a chicken has salmonella given a positive test)

We have:

$$
\begin{aligned}
P(A) &= 0.1 \\
P(B|A) &= 0.95 \\
P(B|\neg A) &= 0.15
\end{aligned}
$$

Using the sum and the product rules:

$$P(B) = (P(B|A)P(A)) + (P(B|\neg A)P(\neg A))$$

Using Bayes' theorem, we have:

$$\begin{aligned} P(A|B) &= \frac{P(B|A)P(A)}{P(B)} \\ &= \frac{(0.95x0.1)}{((0.95x0.1) + (0.15x0.9))} \\ &= 0.413 \end{aligned}$$

There is a 41% chance that the chicken has salmonella.