CS251 Sample Midterm Answers

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Part I. True-False and Multiple-Choice Questions

1. $\sum_{i=1}^{n} i^2$ is $\Theta(n^3)$
   True
   Answer:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

TABLE 2 Some Useful Summation Formulae.

Sum	*Closed Form*
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - a}{r-1}, r \neq 1$
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^{\infty} x^k, \lvert x \rvert < 1$	$\frac{1}{1-x}$
$\sum_{k=1}^{\infty} kx^{k-1}, \lvert x \rvert < 1$	$\frac{1}{(1-x)^2}$

notation	provides	example	shorthand for	used to
Tilde	leading term	~ 10 N^2	10 N^2 10 N^2 + 2 N + 37	provide approximate model
Theta	asymptotic growth rate	$\Theta(N^2)$	½ N^2 10 N^2 5 N^2 + 22 N log N	classify algorithms
Big-O	$\Theta(N^2)$ and smaller (bounded by N^2)	$O(N^2)$	10 N^2 100 N 22 N log N + 3 N	develop upper bounds
Omega	$\Theta(N^2)$ and larger	$\Omega(N^2)$	N^5 N^3 + 22 N log N + 3 N	develop lower bounds

2. The worst-case number of comparisons used by Merge Sort to sort n elements is $O(n \log n)$.
True
Answer:

Array Sorting Algorithms

Algorithm	Time Complexity			Space Complexity
	Best	Average	Worst	Worst
Quicksort	Ω(n log(n))	θ(n log(n))	O(n^2)	O(log(n))
Mergesort	Ω(n log(n))	θ(n log(n))	O(n log(n))	O(n)
Timsort	Ω(n)	θ(n log(n))	O(n log(n))	O(n)
Heapsort	Ω(n log(n))	θ(n log(n))	O(n log(n))	O(1)
Bubble Sort	Ω(n)	θ(n^2)	O(n^2)	O(1)
Insertion Sort	Ω(n)	θ(n^2)	O(n^2)	O(1)
Selection Sort	Ω(n^2)	θ(n^2)	O(n^2)	O(1)
Tree Sort	Ω(n log(n))	θ(n log(n))	O(n^2)	O(n)
Shell Sort	Ω(n log(n))	θ(n(log(n))^2)	O(n(log(n))^2)	O(1)
Bucket Sort	Ω(n+k)	θ(n+k)	O(n^2)	O(n)
Radix Sort	Ω(nk)	θ(nk)	O(nk)	O(n+k)
Counting Sort	Ω(n+k)	θ(n+k)	O(n+k)	O(k)
Cubesort	Ω(n)	θ(n log(n))	O(n log(n))	O(n)

3. There is a binary tree with height 4 and exactly 16 nodes.
False
Answer: Assuming that a tree with a single node has height of 1, max amount of nodes in a tree is $2^h - 1$.
$2^4 - 1 = 16 - 1 = 15$

4. The number of exchanges performed by Selection Sort in the worst case when sorting an n-element sequence is $O(n)$.
True
Answer: One of the simplest sorting algorithms works as follows: First, find the smallest item in the array and exchange it with the first entry (itself if the first entry is already the smallest). Then, find the next smallest item and exchange it with the sec- ond entry. Continue in this way until the entire array is sorted. This method is called selection sort because it works by repeatedly selecting the smallest remaining item.

5. A Radix Sort can sort any n-element sequence of 9-digit (decimal) integers in $O(n)$ steps.
True
Answer: $O(kn) = O(9n) = O(n)$

6. $43n^3 \log n + 27n^2 \log \log n$ is $O(n^3)$
False
Answer: We cannot discard $\log n$ from $43n^3 \log n$. This equation is $O(n^3 \log n)$.

7. Insertion Sort is a stable sort.
True

Multiple-Choice

Question 8. You are given a list of 800 student names in alphabetical order together with which class section they are in. The section is a number between 1 and 6. You wish to form a list of the names in order by section number and alphabetized within each section. Which of these algorithms would be most efficient to perform this task?

a **Bucket Sort.**

b Heap Sort.

c Quicksort.

d Priority Queue.

e Radix Sort.

Answer: Bucket sort, or bin sort, is a sorting algorithm that works by distributing the elements of an array into a number of buckets. Each bucket is then sorted individually, either using a different sorting algorithm, or by recursively applying the bucket sorting algorithm.

Question 9. Which one of these data structures uses the "swim up" and "sink down" operations?

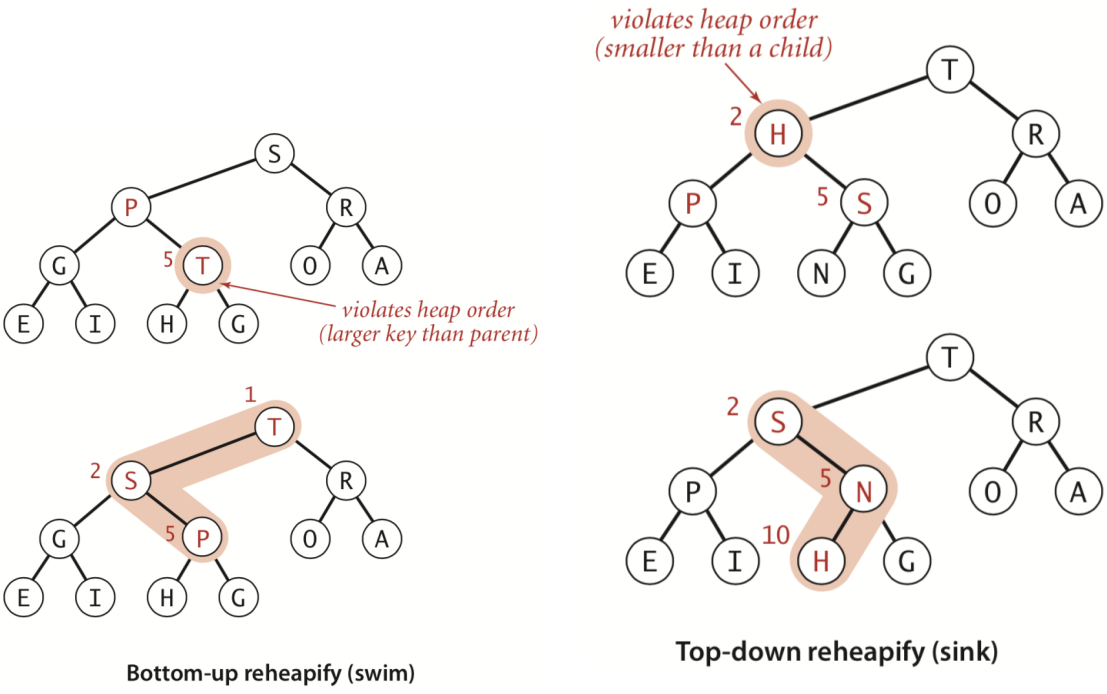
a Queue.

b Stack.

c **Heap.**

d Hash table.

e Binary Search Tree.



Question 10. Which of the following functions would be the best hash function for Social Security Numbers (which are 9-digit integers) to use for a hash table of size about 1000 with linear probing?

a The sum of the digits.

b The first three digits.

c The middle three digits.

d The three digit number formed from the first, fourth and seventh digits.

e **The SSN$\%p$, where p is a prime number near 1000.**

Question 11. If $3N^3 + 2N^2 + 6N \log N - 3 \sim N^e$, then $e =$

a 1.

b 2.

c 3.

d 4.

e **There is no integer e that makes this statement true.**

Answer: Correct $\sim$ notation will be $\sim 3N^3$.

Part II.

Question 12. A small Hash table uses linked lists and Hashcode(x) = $x\%7$ to store integers x. Show the contents of the Hash table after these operations.

- Insert(13) Hash: 6
- Insert(16) Hash: 2
- Insert(2) Hash: 2
- Insert(9) Hash: 2
- Delete(16) Hash: 2
- Insert(6) Hash: 6
- Delete(13) Hash: 6

Answer:

Hashcode	Content
0	
1	
2	2, 9
3	
4	
5	
6	6

Question 13. Give the output of the following series of Priority Queue operations. The Priority Queue has space for 100 elements and is initially empty. (The output may be an exception or void. If void, you must write "void" for the output.)

Answer:

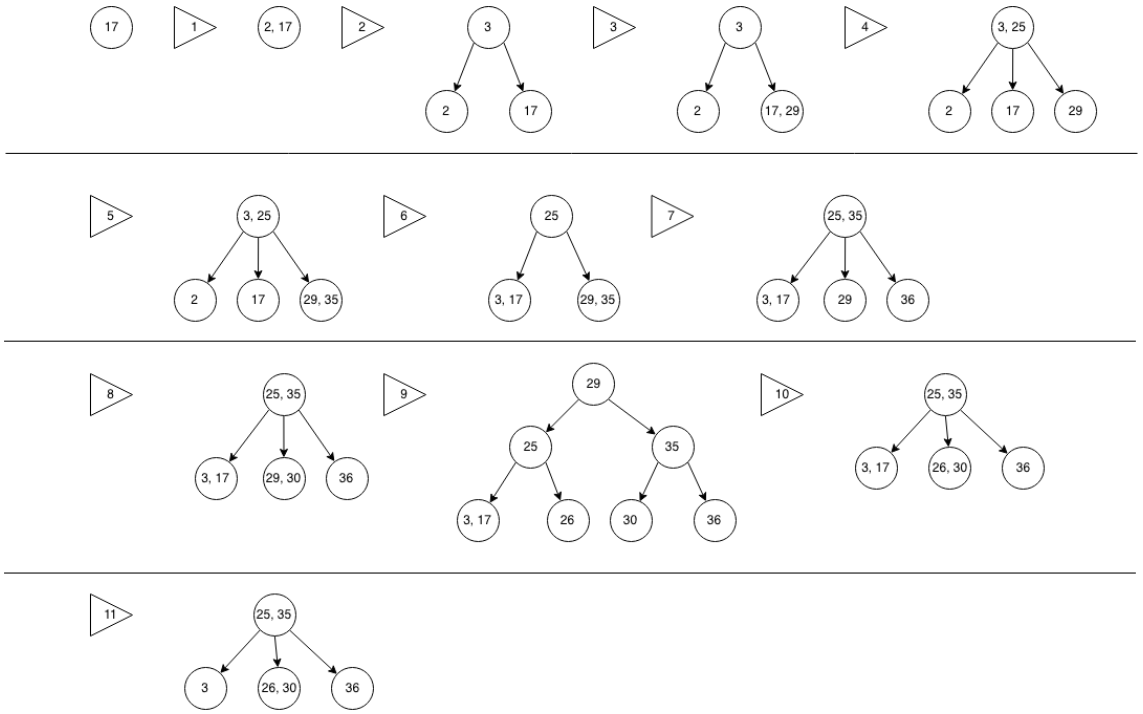
Operation	Output	Contents (ordered)
insert(9)	void	9
insert(13)	void	9, 13
insert(19)	void	9, 13, 19
insert(2)	void	2, 9, 13, 19
max()	19	2, 9, 13, 19
insert(7)	void	2, 7, 9, 13, 19
insert(1)	void	1, 2, 7, 9, 13, 19
size()	6	1, 2, 7, 9, 13, 19
delMax()	19	1, 2, 7, 9, 13
delMax()	13	1, 2, 7, 9
isEmpty()	False	1, 2, 7, 9
insert(4)	void	1, 2, 4, 7, 9

Question 14. A 2-3 Tree has exactly one node, with key value 17. Draw the tree after the following operations are performed on it.

1. insert(2)
2. insert(3)
3. insert(29)
4. insert(25)
5. insert(35)
6. deleteMin()
7. insert(36)
8. insert(30)

9. insert(26)
10. delete(29)
11. delete(17)

Answer:



Question 15. For each of the following sorting algorithms, give the requested running times using big-Oh notation. There are N items to sort. Assume that a single comparison takes constant time. Omit the cases labeled "Omit".

Answer:

Algorithm	**Best Case**	**Average Case**	**Worst Case**
Heap Sort on a Priority Queue based on a Heap	$n \log n$	$n \log n$	Omit
Selection Sort as in class	Omit	n^2	n^2
Insertion Sort as in class	n	Omit	n^2
Merge Sort as in class	$n \log n$	$n \log n$	Omit
In Place QuickSort (not Randomized)	$n \log n$	Omit	n^2

Question 16. Order the following 16 functions by big-Oh notation, with the slowest-growing one (as $n \to \infty$) at the top and the fastest-growing one (as $n \to \infty$) at the bottom. List on the same line all functions that are big-Theta of one another. Here "$\log x$" means "$\log_2 x$".

$8n \log n$	4^n	$\log \log n$	$2^{\log n} = n$
2^{1000}	2^n	$n^{0.01}$	$6n^{3/2}$
2^{2^n}	$3^{\log n}$	$n/100$	$\sqrt{\log n}$
$3\sqrt{n}$	$53n$	$1/n$	$(\log n)^2$

Answer:

Slowest-growing one (as $n \to \infty$).

1. $1/n$
2. 2^{1000}
3. $\log \log n$
4. $n^{0.01}$

5. $\sqrt{\log n}$
6. $(\log n)^2$
7. $3\sqrt{n}$
8. $2^{\log n} = n$, $n/100$, $53n$
9. $6n^{3/2}$
10. $3^{\log n}$
11. $8n \log n$
12. 2^n
13. 4^n
14. 2^{2^n}

Fastest-growing one (as $n \to \infty$).