MA 351, HW 3

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January 31, 2019

Section 2.1: True/False: 2.2, 2.3, 2.4; Exercises: 2.1, 2.7, 2.8
Section 2.2: True/False: 2.10, 2.11, 2.12, 2.15, 2.16, 2.17, 2.19; Exercises: 2.32, 2.35, 2.36

1 Section 2.1

1.1 True/False

Question 2.2. I start with a certain 4×6 matrix A and reduce, obtaining

$$M = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 3 & -2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Which of the following statements about A are guaranteed to be true, where A_i denotes the ith column of A.

(a) The columns of A are linearly independent
Answer: False, since we got a couple of free variables in M

(b) A_4 is a linear combination of A_1, A_2, and A_5
Answer: True, $A_4 = A_1 + 3A_2 + 0A_5$

(c) A_3 is a linear combination of A_1, A_2, and A_5
Answer: True, $A_3 = A_1 + A_2 + 0A_5$

(d) A_3 is a linear combination of A_1 and A_2
Answer: True, $A_3 = A_1 + A_2$

(e) $A_4 = A_1 - 2A_2$
Answer: False, $A_4 \neq A_1 - 2A_2$

Question 2.3. Suppose that A and B are n × n matrices that both have linearly independent columns. Then A and B have the same reduced echelon form.
Answer: False,

$$rref \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \neq rref \begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2.4. Suppose that A is an n×n matrix and B is an n×1 column vector such that the equation $AX = B$ has an infinite number of solutions. Then the columns of A are linearly dependent.
Answer: True, in order of the equation to have an infinite number of solutions it should have at least one free variable in its row echelon form, which means that colums of A are linearly dependent

1.2 Exercises

Question 2.1. Test the given matrices for linear dependence using the test for linear independence. Then find a basis for their span and express the other vectors (if there are any) as linear combinations of the basis elements.

(a)

$$\begin{bmatrix}1\\2\\1\\2\end{bmatrix}, \quad \begin{bmatrix}2\\3\\-1\\0\end{bmatrix}, \quad \begin{bmatrix}1\\0\\1\\0\end{bmatrix}$$

Answer:

$$\begin{bmatrix}1 & 2 & 1\\2 & 3 & 0\\1 & -1 & 1\\2 & 0 & 0\end{bmatrix} \to \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}$$

Basis: A_1, A_2, A_3
Independent

(b)

$$\begin{bmatrix}1\\0\\0\end{bmatrix}, \quad \begin{bmatrix}0\\1\\0\end{bmatrix}, \quad \begin{bmatrix}0\\0\\1\end{bmatrix}$$

Answer:

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \to \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Basis: A_1, A_2, A_3
Independent

(c)

$$\begin{bmatrix}1 & 2 & 1\end{bmatrix}, \quad \begin{bmatrix}3 & -1 & 2\end{bmatrix}, \quad \begin{bmatrix}7 & -7 & 4\end{bmatrix}$$

Answer:

$$\begin{bmatrix}1 & 3 & 7\\2 & -1 & -7\\1 & 2 & 4\end{bmatrix} \to \begin{bmatrix}1 & 0 & -2\\0 & 1 & 3\\0 & 0 & 0\end{bmatrix}$$

Basis: A_1, A_2
Linear combination: $A_3 = -2A_1 + 3A_2$
Dependent

(d)

$$\begin{bmatrix}2 & 3\\0 & 1\end{bmatrix}, \quad \begin{bmatrix}1 & 3\\0 & 0\end{bmatrix}, \quad \begin{bmatrix}17 & 0\\9 & 1\end{bmatrix}, \quad \begin{bmatrix}0 & 5\\0 & 6\end{bmatrix}$$

Answer:

$$\begin{bmatrix}2 & 1 & 17 & 0\\3 & 3 & 0 & 5\\0 & 0 & 9 & 0\\1 & 0 & 1 & 6\end{bmatrix} \to \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Basis: A_1, A_2, A_3, A_4
Independent

(e)

$$\begin{bmatrix} 2 & -2 \\ 3 & -1 \\ 0 & 2 \end{bmatrix}, \quad \begin{bmatrix} 4 & -1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 8 & -2 \\ 4 & 6 \\ 2 & 0 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 2 & 4 & 8 \\ -2 & -1 & -2 \\ 3 & 2 & 4 \\ -1 & 3 & 6 \\ 0 & 1 & 2 \\ 2 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis: A_1, A_2
Linear combination: $A_3 = 0A_1 + 2A_2$
Dependent

(f)

$$\begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 6 & 5 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 4 & 3 & 2 & 6 \\ 3 & 0 & -1 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis: A_1, A_2, A_3
Linear combination: $A_4 = A_1 + 2A_2 - 2A_3$
Dependent

(g)

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, \quad \begin{bmatrix} 5 & -2 \\ 3 & 10 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 1 & -2 \\ 3 & 3 & 3 \\ 2 & 4 & 10 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis: A_1, A_2
Linear combination: $A_3 = -3A_1 + 4A_2$
Dependent

(h)

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Basis: A_1, A_2, A_3, A_4
Independent

(i)

$$\begin{bmatrix}1\\2\\3\\1\end{bmatrix}, \quad \begin{bmatrix}2\\0\\1\\0\end{bmatrix}, \quad \begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

Answer:

$$\begin{bmatrix}1 & 2 & 1\\2 & 0 & 1\\3 & 1 & 1\\1 & 0 & 1\end{bmatrix} \to \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}$$

Basis: A_1, A_2, A_3
Independent

(j)

$$\begin{bmatrix}4 & 2 & -1\end{bmatrix}, \quad \begin{bmatrix}3 & 3 & 2\end{bmatrix}, \quad \begin{bmatrix}1 & 0 & 1\end{bmatrix}$$

Answer:

$$\begin{bmatrix}4 & 3 & 1\\2 & 3 & 0\\-1 & 2 & 1\end{bmatrix} \to \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Basis: A_1, A_2, A_3
Independent

(k)

$$\begin{bmatrix}3\\2\\4\\5\end{bmatrix}, \quad \begin{bmatrix}-4\\-3\\-5\\-6\end{bmatrix}, \quad \begin{bmatrix}2\\-3\\7\\12\end{bmatrix}$$

Answer:

$$\begin{bmatrix}3 & -4 & 2\\2 & -3 & -3\\4 & -5 & 7\\5 & -6 & 12\end{bmatrix} \to \begin{bmatrix}1 & 0 & 18\\0 & 1 & 13\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

Basis: A_1, A_2
Linear combination: $A_3 = 18A_1 + 13A_2$
Dependent

Question 2.7. Use the test for linear independence to prove that the rows of the following 3×6 matrix are linearly independent:

$$A = \begin{bmatrix}1 & a & b & c & d & e\\0 & 0 & 1 & f & g & h\\0 & 0 & 0 & 0 & 1 & k\end{bmatrix}$$

Answer:

$$\begin{bmatrix}1 & 0 & 0\\a & 0 & 0\\b & 1 & 0\\c & f & 0\\d & g & 1\\e & h & k\end{bmatrix} \xrightarrow{rref} \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

Question 2.8. Let $A = [A_1, A_2, A_3]$ be a 3×3 matrix with linearly independent columns A_i .

(a) Explain why the row reduced form of A is the following matrix R

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Answer: Since columns are linearly independent, by presenting the matrix in rref, we solve dependency equation. Matrix R shows that solution to that equation is all zeroes which means that columns are linearly independent

(b) Let $A = [A_1, A_2, A_3, A_4, A_5]$ be a 3×5 matrix such that the first three columns are linearly independent. Explain why the pivot columns must be the first three.

Answer: Since the first three columns are linearly independent, rref of $[A_1, A_2, A_3]$ will result in no free variables, which means that the pivot columns will be the first three

2 Section 2.2

2.1 True/False

Question 2.10. $\{[17, 6, -4]^t, [2, 3, 3]^t, [19, 9, -1]^t\}$ does not span $\mathbb{R}^3$

Answer: True, because matrices are not linearly independent

$$\begin{bmatrix} 17 & 2 & 19 \\ 6 & 3 & 9 \\ -4 & 3 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis: A_1, A_2

Question 2.11. $\{[1, 1]^t, [1, 2]^t, [4, 7]^t\}$ spans $\mathbb{R}^2$

Answer: True, because first two matrices are linearly independent

$$\begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 7 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

Basis: A_1, A_2

Question 2.12. Let $\mathcal{W}$ be a two-dimensional subspace of $\mathbb{R}^3$. Then two of the following three vectors span $\mathcal{W}$: $X = [1, 0, 0]^t, Y = [0, 1, 0]^t, Z = [0, 0, 1]^t$

Answer: False, for example, let $\mathcal{W}$ be a two-dimensional subspace spanned by vectors $\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$

Question 2.15. Suppose that $\mathcal{W}$ is a four-dimensional subspace of $\mathbb{R}^7$ and X_1, X_2, X_3, and X_4 are vectors that belong to $\mathcal{W}$. Then $\{X_1, X_2, X_3, X_4\}$ spans $\mathcal{W}$.

Answer: False, since vectors X_1, X_2, X_3, and X_4 can be linearly dependent and still belong to $\mathcal{W}$

Question 2.16. Suppose that $\{X_1, X_2, X_3, X_4, X_5\}$ spans a four-dimensional vector space $\mathcal{W}$ of $\mathbb{R}^7$. Then $\{X_1, X_2, X_3, X_4\}$ also spans $\mathcal{W}$

Answer: False, since vector space $\mathcal{W}$ is four-dimensional, one of the vectors $\{X_1, X_2, X_3, X_4, X_5\}$ is a linear combination of the others, but it is not guaranteed that it is X_5.

Question 2.17. Suppose that $S = \{X_1, X_2, X_3, X_4, X_5\}$ spans a four-dimensional subspace $\mathcal{W}$ of $\mathbb{R}^7$. Then S contains a basis for $\mathcal{W}$.

Answer: True

Question 2.19. Suppose that $\mathcal{W}$ is a four-dimensional subspace of $\mathbb{R}^7$ that is spanned by $\{X_1, X_2, X_3, X_4\}$. Then one of the X_i must be a linear combination of the others.

Answer: False, since $\mathcal{W}$ is a four-dimensional subspace, so its basis has exactly 4 elements.

2.2 Exercises

Question 2.32. Prove that the given sets $\mathcal{W}$ are subspaces of $\mathbb{R}^n$ for the appropriate n. Find spanning sets for these spaces and find at least two different bases for each space. Give the dimension of each space.

(a) $\mathcal{W} = \{[a+b+2c, 2a+b+3c, a+b+2c, a+2b+3c]^t | a,b,c \in \mathbb{R}\}$

Answer:

$$\begin{bmatrix}1\\2\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\2\end{bmatrix}, \begin{bmatrix}2\\3\\2\\3\end{bmatrix}$$

$$\begin{bmatrix}1&1&2\\2&1&3\\1&1&2\\1&2&3\end{bmatrix} \to \begin{bmatrix}1&0&1\\0&1&1\\0&0&0\\0&0&0\end{bmatrix}$$

Dimension: 2

Basis 1: $\begin{bmatrix}1\\2\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\2\end{bmatrix}$

Basis 2: $\begin{bmatrix}2\\4\\2\\2\end{bmatrix}, \begin{bmatrix}2\\2\\2\\4\end{bmatrix}$

(b) $\mathcal{W} = \{[a+2c, 2a+b+3c, a+b+c]^t | a,b,c \in \mathbb{R}\}$

Answer:

$$\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}2\\3\\1\end{bmatrix}$$

$$\begin{bmatrix}1&0&2\\2&1&3\\1&1&1\end{bmatrix} \to \begin{bmatrix}1&0&2\\0&1&-1\\0&0&0\end{bmatrix}$$

Dimension: 2

Basis 1: $\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}$

Basis 2: $\begin{bmatrix}2\\4\\2\end{bmatrix}, \begin{bmatrix}0\\2\\2\end{bmatrix}$

(c) $\{[a+b+2c, 2a+b+3c, a+b+c]^t | a,b,c \in \mathbb{R}\}$

Answer:

$$\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\3\\1\end{bmatrix}$$

$$\begin{bmatrix}1&1&2\\2&1&3\\1&1&1\end{bmatrix} \to \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Dimension: 3

Basis 1: $\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\3\\1\end{bmatrix}$

Basis 2: $\begin{bmatrix}2\\4\\2\end{bmatrix}, \begin{bmatrix}2\\2\\2\end{bmatrix}, \begin{bmatrix}4\\6\\2\end{bmatrix}$

(d) $\mathcal{W} = \{[a+2b-4c+5d, -2a-2b+2c-6d, 6a+4b+14d, 3a+b+3c+5d] | a,b,c,d \in \mathbb{R}\}$

Answer:

$$\begin{bmatrix}1\\-2\\6\\3\end{bmatrix}, \begin{bmatrix}2\\-2\\4\\1\end{bmatrix}, \begin{bmatrix}-4\\2\\0\\3\end{bmatrix}, \begin{bmatrix}5\\-6\\14\\5\end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -4 & 5 \\ -2 & -2 & 2 & -6 \\ 6 & 4 & 0 & 14 \\ 3 & 1 & 3 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Dimension: 2
Basis 1: $\begin{bmatrix} 1 \\ -2 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 4 \\ 1 \end{bmatrix}$
Basis 2: $\begin{bmatrix} 2 \\ -4 \\ 12 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ 8 \\ 2 \end{bmatrix}$

Question 2.35. Find a basis and give the dimension for the following spaces of matrices A

(a) $2 \times 2, A = A^t$

Answer:

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

Dimension: 2
Basis: $\left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]$

(b) $3 \times 3, A = A^t$

Answer:

$$\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$$

Dimension: 6
Basis: $\left[\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{smallmatrix}\right]$

(c) $2 \times 2, A = -A^t$

Answer:

$$\begin{bmatrix} 0 & -b \\ b & 0 \end{bmatrix}$$

Dimension: 1
Basis: $\left[\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right]$

(d) 3×3 upper triangular.

Answer:

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

Dimension: 6
Basis: $\left[\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{smallmatrix}\right]$

(e) 3×3 lower triangular.

Answer:

$$\begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix}$$

Dimension: 6
Basis: $\left[\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{smallmatrix}\right]$

Question 2.36. Find a basis for the subspace of $M(2,2)$ spanned by the following matrices. What is the dimension of this subspace?

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, \quad \begin{bmatrix} -1 & -2 \\ -3 & -5 \end{bmatrix}, \quad \begin{bmatrix} -1 & -2 \\ 0 & -2 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 2 & 2 & -2 & -2 \\ 1 & 2 & -3 & 0 \\ 3 & 4 & -5 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis: A_1, A_2
Dimension: 2