Exercise – 01 (total = 100')

Due date: Mar. 1, 2022, 23:59

Note: Unless mentioned otherwise, z is treated as a complex number, x and y are treated as real numbers, θ is treated as an angle (with a unit of rad, not degree).

Part – 1: True or False (3' x 10 = 30')

(1) Complex numbers include real numbers.

(2) The imaginary part of a complex number is a pure imaginary number.

(3) $-i \cdot (0, 1) = -1$

(4) $|2 + 3i| = |3 - 2i|$

(5) Suppose $z = x + iy$, then $((-z)^*)^* = x - iy$

(6) The geometric representation of $z \cdot (-i)$ is to rotate z clockwisely by $\pi/2$.

(7) Suppose $z = x + iy$. If $z = 0$, then $x = 0$ and $y = 0$.

(8) The limit of series $\left\{z_n = \frac{i^n}{n}\right\}$ does not exist.

(9) $(1 - 3i) - (1 + 3i)^* = 0$

(10) Suppose $z = x + iy$. $\left|e^{-iz}\right| = e^{-x}$

Part – 2: Calculation (5' x 5 = 25')

Please find the real part, the imaginary part, the modulus, and the argument (its principal value in the interval $(-\pi, \pi]$) of the following expressions.

(1) $\sqrt{3}i - 1$

(2) $i \sin(\theta) + [1 + \cos(\theta)]$, where $0 \leq \theta \leq 2\pi$

(3) $\sin(\theta) + i[1 - \cos(\theta)]$, where $0 \leq \theta \leq 2\pi$

(4) e^{-z}

(5) $e^{i \sin(x)}$, where x is a real number

Part – 3: Graphing (5' x 5 = 25')
Please draw the corresponding graphs that satisfy the following relations.
(1) $|z-1+i|=\sqrt{2}$

(2) $0<\arg(z+i)<\frac{\pi}{4}$

(3) $0<\arg(i-z)<\frac{\pi}{4}$

(4) $-2<\mathrm{Im}\{z-1\}<2$

(5) $|z+2|=|z-4|$

Part – 4: Re-expression (10' x 2 = 20')
Please apply Euler's formula and De Moivre's theorem to simplify the following expressions.

(1) $\cos(\theta)+\cos(2\theta)+\cos(3\theta)+\cdots\cos(n\theta)$

(2) $\sin(\theta)+\sin(2\theta)+\sin(3\theta)+\cdots\sin(n\theta)$