Exercise – 05 (total = 100')

Due date: Mar. 27, 2022, 23:59

Note: Unless mentioned otherwise, z is treated as a complex number.

Part – 1: Calculation (5' x 2 = 10')

Find the value of integration along the given path

(1) $\int_0^{2+\mathrm{i}} \mathrm{Re}(z)dz$

(i) $0 \to 2 \to 2+\mathrm{i}$ (the path consists of two lines, the 1st is horizontal and the 2nd is vertical.)

(ii) $z = (2+\mathrm{i})t,\ 0 \le t \le 1$ (the path is an inclined line)

(2) $\int_C \frac{dz}{\sqrt{z}}$. We assume that $\sqrt{z}|_{z=1} = 1$.

(i) the upper half of a unit circle that starts from $z = 1$

(ii) the lower half of a unit circle that starts from $z = 1$

Part – 2: Calculation (5' x 4 =20')

Find the value of integration. You should keep the details of your calculation, don't just show the final answer.

(1) $\oint_{|z|=2} \frac{dz}{z}$

(2) $\oint_{|z|=2} \frac{|dz|}{z}$

(3) $\oint_{|z|=2} \frac{dz}{|z|}$

(4) $\oint_{|z|=2} \left|\frac{dz}{z}\right|$

Part – 3: Calculation (5' x 4 = 20')

Find the value of integration along the given path.

$\oint_C \frac{1}{z^2-1} \sin\frac{\pi z}{4} dz,$ where C is :

(i) $|z| = 1/2$

(ii) $|z-1| = 1$

(iii) $|z| = 3$
(iv) $|z| = 100$

Part – 4: Calculation (5' x 8 = 40')

Find the value of integration.

(1) $\oint_{|z|=2} \frac{\cos(z)}{z} dz$

(2) $\oint_{|z|=2} \frac{z^2 - 1}{z^2 + 1} dz$

(3) $\oint_{|z|=2} \frac{\sin(e^z)}{z} dz$

(4) $\oint_{|z|=2} \frac{e^z}{\cosh(z)} dz$

(5) $\oint_{|z|=2} \frac{\sin(z)}{z^2} dz$

(6) $\oint_{|z|=2} \frac{|z|e^z}{z^2} dz$

(7) $\oint_{|z|=2} \frac{\sin(z)}{z^4} dz$

(8) $\oint_{|z|=2} \frac{dz}{z^2(z^2 + 16)}$

Part – 5: Calculation (5' x 2 = 10')

Apply the formula of higher-order derivatives to answer the following questions.

(1) Find the value of integration $\oint_{|z|=1} \frac{e^z}{z^3} dz$

(2) Find the value of a such that function $F(z) = \int_{z_0}^{z} e^z \left(\frac{1}{z} + \frac{a}{z^3} \right) dz$ is single valued.