Exercise – 02 (total = 100’)

Due date: Mar. 8, 2022, 23:59

Note: Unless mentioned otherwise, z is treated as a complex number, while x and y are treated as real numbers.

Part – 1: True or False (3’ x 5 = 15’)

(1) Like real trigonometric functions, complex trigonometric functions are also bounded.

(2) For a complex exponential function, $e^{z} = e^{z+i(2n\pi)}$; where n is an integer.

(3) The relation $\sin^2(z) + \cos^2(z) = 1$ is valid only when z is a real number.

(4) $\cos\,(\sqrt[4]{z})$ is a single-valued function.

(5) $\sin\,(\sqrt{z})$ is a multi-valued function.

Part – 2: Proof (5’ x 10 = 50’)
Please prove the following relations. Hint: Start with the basic definition of each function.

(1) $[\sin(z)]' = \cos\,(z)$

(2) $\sin(z_1 + z_2) = \sin(z_1)\cos(z_2) + \cos\,(z_1)\sin\,(z_2)$

(3) $\cos(z_1 - z_2) = \cos(z_1)\cos(z_2) + \sin\,(z_1)\sin\,(z_2)$

(4) $\tan(z_1 + z_2) = \frac{\tan(z_1)+\tan\,(z_2)}{1-\tan\,(z_1)\tan\,(z_2)}$

(5) $\sinh(z) = -i\,\sin(iz)$

(6) $\cosh(z) = \cos(iz)$

(7) $\tanh(z) = -i\,\tan(iz)$

(8) $1 - \tanh^2(z) = \text{sech}^2(z)$

(9) $\sinh(z_1 + z_2) = \sinh(z_1)\cosh(z_2) + \cosh\,(z_1)\sinh\,(z_2)$

(10) $|\sinh\,(y)| \le |\cos(x + iy)| \le \cosh\,(y)$

Part – 3: Find the solution (5' x 3 = 15')
Please find the solution(s) to the following equations.

(1) $\cos(z) = 4$

(2) $\sin(z) = \frac{3}{4} + \frac{i}{4}$

(3) $2\cosh^2(z) - 3\cosh(z) + 1 = 0$

Part – 4: Derivation (10' x 2 = 20')
Please derive the definitions for the following inverse trigonometric functions.

(1) $\arccos(z) = \frac{1}{i}\ln\left(z + \sqrt{z^2 - 1}\right)$

(2) $\arctan(z) = \frac{1}{2i}\ln\left(\frac{1+iz}{1-iz}\right)$