Exercise – 07 (total = 100’)

Due date: Apr. 21, 2022, 23:59

Part – 1: True or False (2’ x 4 = 8’)

(1) $y' + y^2 = 1$ is a linear differential equation.

(2) $x^2 y' + 2y = 1$ is a linear differential equation.

(3) $y = Ae^x + Be^{-x}$ is the general solution of the equation $y'' - y = 1$.

(4) $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial y^2} = 0$ is a second – order partial differential equation.

Part – 2: Solve PDEs (5’ x 6 = 30’)

Find the general solutions to the followings homogeneous PDEs.

(1) $\dfrac{\partial^2 u}{\partial x^2} - 2\dfrac{\partial^2 u}{\partial x \partial y} - 3\dfrac{\partial^2 u}{\partial y^2} = 0.$

(2) $\dfrac{\partial^2 u}{\partial x^2} - 2\dfrac{\partial^2 u}{\partial x \partial y} + 2\dfrac{\partial^2 u}{\partial y^2} = 0.$

(3) $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial x \partial y} = 0.$

(4) $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{c^2}{r^2}\dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial u}{\partial r}\right)$, $c \neq 0$. *Hint:* consider the substitution $u(r,t) = \frac{1}{r}\phi(r,t)$

(5) $(a^2 - b^2)\dfrac{\partial^2 u}{\partial x^2} + 2a\dfrac{\partial^2 u}{\partial x \partial t} + \dfrac{\partial^2 u}{\partial t^2} = 0,\ b \neq 0.$

(6) $\dfrac{\partial^4 u}{\partial x^4} - \dfrac{\partial^4 u}{\partial y^4} = 0.$

Part – 3: Solve PDEs (6’ x 7 =42’)

Find the general solutions to the following inhomogeneous PDEs.

Hint-1: the solution = one particular solution (no matter how you obtain it, e.g. you may even try to guess a solution that happens to satisfy the PDE) + one general solution to the corresponding homogeneous PDE.

Hint-2: For (5)-(7), refer to the example on page 27, Lecture-07.

(1) $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial x \partial y} = e^{3x+2y}.$

(2) $\dfrac{\partial^2 u}{\partial x^2} - 2\dfrac{\partial^2 u}{\partial x \partial y} - 3\dfrac{\partial^2 u}{\partial y^2} = e^{x-y}.$

(3) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial x \partial y} - 6\dfrac{\partial^2 u}{\partial y^2} = \sin(x+y).$

(4) $9\dfrac{\partial^2 u}{\partial x^2} - 6\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 u}{\partial y^2} = 2x + 6y.$

(5) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = x^2 + xy.$

(6) $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial y^2} = xy - x.$

(7) $\dfrac{\partial^2 u}{\partial x^2} - 2\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 u}{\partial y^2} = x^2 + y.$

Part – 4: Verification (5' x 2 = 10')

(1) $\dfrac{1}{\hat{L}(\hat{D}_x, \hat{D}_y)} e^{ax+by} g(x,y) = e^{ax+by} \dfrac{1}{\hat{L}(\hat{D}_x + a, \hat{D}_y + b)} g(x,y)$

(2) $\dfrac{1}{\hat{D}_y^3} y = \dfrac{1}{24} y^4$

Part – 5: Solve PDE (10' x 1 = 10')

Find the general solution to the following PDE.

(1) $x^2 \dfrac{\partial^2 u}{\partial x^2} - 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} + x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0.$