Modeling and Forecasting Volatility in the Gold Market : A Markov-Switching Multifractal Approach

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Abstract

In this paper, we use Markov-Switching Multifractal (MSM) models to analyze and forecast the gold price volatility (spot and future). These models capture the complex behavior of volatility in financial time series, particularly the occurrence of extreme events and regime changes. We compare their forecasting performance against traditional GARCH family models and Markov-Switching GARCH model to determine whether MSM models provide better predictions of the gold price volatility. (Results summary to be written)

Keywords: *Financial Volatility, Modeling and Forecasting, Gold Market, GARCH-family Models, Regime-Switching Dynamics, Markov-Switching GARCH Models, Markov-Switching Multifractal, Multifractality.*

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1 Introduction

Gold is a unique asset as it is both a commodity and a monetary unit. Historically, it has been regarded as a reliable store of value, and it continues to play a significant role in modern finance. Gold is recognized as an attractive investment option, serving as a hedge against inflation, a safe haven during periods of economic uncertainty, and a tool for portfolio diversification and risk management (Baur and Lucey (2010)). Growing significantly over the past few decades, the gold market confirmed its status as a key component of the global economy. Major crises such as the 2008 financial crisis and the COVID-19 pandemic have highlighted the importance of gold as a hedge against economic uncertainties, influencing its price volatility.

Understanding the volatility in the gold market is a key factor for investors. It is essential in all financial markets, and modeling it helps assess risks and adjust investment strategies. The volatility of gold prices is particularly complex, driven by geopolitical tensions, economic shifts, monetary policies, and currency fluctuations. Changes in gold prices impact stock markets worldwide: its volatility positively impacts the stock markets of emerging BRICS economies (Raza et al. (2016)), while Stoupos and Kiohos (2020) showed that there is a negative dynamic between gold prices and advanced stock market indices. These dynamics underscore the importance of forecasting gold volatility for analysts and investors.

In the literature of finance, traditional econometric models have been widely used to analyze and capture the dynamics of volatility. These studies have identified common stylized characteristics in financial time series, such as volatility clustering, fat-tailed distribution, and leverage effects. The family of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, introduced by Engle (1982) and later generalized by Bollerslev (1986), is particularly effective in modeling these stylized facts. Over time, numerous extensions have been developed to capture additional characteristics (see for instance the Exponential GARCH (EGARCH) model proposed by Nelson (1991), the Fractional Integrated GARCH (FIGARCH) of Baillie et al. (1996), among others).

Regarding gold volatility forecasting, various GARCH models have been assessed for their effectiveness. For instance, Bentes (2015) analyzed gold spot prices from August 2, 1976, to February 6, 2015, and found that the FIGARCH(1,1) model outperformed GARCH(1,1) and IGARCH(1,1) models in capturing the linear dependence in the conditional variance of gold returns, by information criteria comparison. It was also identified as the most effective model for forecasting gold spot return volatility. Similarly, Kumari and Tan (2018) examined gold futures prices from January 1990 to June 2014. They used models with long memory and fat-tailed distributions, and among the linear and non-linear GARCH-class models, EGARCH and FIEGARCH provided the best in-sample volatility forecasts, highlighting their robustness in capturing dynamics in gold futures.

Despite their widespread use, a significant limitation of these GARCH-class models is their struggle to effectively capture abrupt changes in gold market conditions, such as those triggered by geopolitical events or financial crises. Lamoureux and Lastrapes (1990) demonstrated that these structural breaks can distort the results of standard GARCH models, as they fail to account for shifts in volatility regimes. This limitation can significantly impact the accuracy of volatility forecasts in scenarios marked by sudden market disruptions.

To address these limitations, more advanced models have been developed in the field of financial time series econometrics. Hamilton and Susmel (1994) stated that the problem of high persistence in GARCH type models, which is clearly unrealistic in any market, can be solved by combining the Markov Regime Switching model with ARCH models (SWARCH). Regime-switching models are based on the idea that changes in market conditions lead to shifts in the factors influencing volatility. These models incorporate dynamic transitions between different states, allowing the model parameters to adjust to varying market regimes. The Markov-Switching GARCH (MS-GARCH) model (e.g. Klaassen, 2002; Haas et al., 2004) is another approach including these properties. Sopipan et al. (2012) found that this model demonstrated superior performance compared to traditional GARCH models in terms of modeling and forecasting gold price volatility. This was particularly evident in certain loss functions where MS-GARCH outperformed GARCH-type models.

Numerous studies in the literature have focused on the multifractal analysis of financial markets. Multifractality refers to the existence of multiple scaling behaviors or fractal dimensions within time series data. Unlike traditional fractality, which is characterized by a single scaling exponent, multifractality involves a range of exponents that capture the data's complex and nonlinear characteristics. The gold market has been extensively studied regarding its multifractal properties. Using the multifractal detrended fluctuation analysis (MDFA) method, Wang et al. (2011) showed that the gold return series demonstrate multifractality across all time scales. They observed that for time scales smaller than a month, the main contribution of multifractality is fat-tail distribution, while at longer scales, both fat-tailed distribution and long-range correlations play significant roles in the contribution of multifractality. More recently, Wang et al. (2024) proved that during major events like the US–China trade war, the COVID-19 pandemic, and the Russia–Ukraine War, Chinese gold spot market is particularly anti-persistent and exhibits significant multifractal characteristics, suggesting that the gold spot market possesses predictability and has significant volatility.

An alternative approach to modeling and forecasting volatility dynamics, including multifractality, is the use of Markov-Switching Multifractal (MSM) models (e.g. Calvet and Fisher, 2001, 2004; Lux, 2008). These models combine the dynamic transitions between different volatility states permitted by the Markov Regime Switching framework with a multifractal process, assuming that different volatility measures exhibit different levels of long-term dependence. These properties cannot be adequately captured by the traditional GARCH-class models, and several studies have demonstrated superior forecasting performance by MSM models compared to the GARCH family and its extensions. For instance, Lux and Kaizoji (2007) showed that multifractal models largely outperform GARCH and ARMA models in forecasting volatility over long-term horizons for the Tokyo stock market. Similarly, Calvet and Fisher (2004) compared the forecasting performance of various models for four exchange rates data and found that the Binomial MSM performs comparably or slightly better than the standard GARCH, FI-GARCH, and the MS-GARCH model over short-term horizons. Another example is the work of Segnon et al. (2015), which highlighted the superiority of MSM models in forecasting the volatility of carbon dioxide emission allowance spot prices under the vast majority of criteria and forecast horizons.

While the academic literature covers a wide range of financial assets when comparing MSM models with GARCH family models, it remains somewhat limited regarding the gold market. This study aims to demonstrate the efficiency of MSM models in modeling and forecasting the volatility of gold spot and futures prices, and to compare their performance with traditional GARCH and regime-switching MS-GARCH models, thus contributing to fill this gap in the literature.

The remainder of this paper is organized as follows: In Section 2, we briefly describe the data and methodologies employed in our study, including traditional GARCH-class models, Markov-Switching GARCH model, and Markov-Switching Multifractal models. Section 3 provides an overall assessment of the descriptive statistics of gold price returns (spot and future). We then present the in-sample estimation results for the different models and evaluate their out-of-sample forecasting performance. Finally, Section 4 discusses the implications of our findings, offers concluding remarks, and suggests directions for future research.

2 Data and Methodology

2.1 Data and general definitions

2.2 Models

2.2.1 GARCH, E-GARCH, and GJR-GARCH Models

2.2.2 Markov-Switching GARCH Models

2.2.3 Markov-Switching Multifractal Model

3 Findings

3.1 Data Description and Preliminary Analysis

3.2 In-sample Analysis

3.3 Out-of-sample Forecasting Analysis

4 Concluding Remarks

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Disclosure of interest

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References

Baillie, R. T., Bollerslev, T., and Mikkelsen, H. O. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 74(1):3–30.

Baur, D. G. and Lucey, B. M. (2010). Is gold a hedge or a safe haven? an analysis of stocks, bonds and gold. *Financial Review*, 45(2):217–229.

Bentes, S. R. (2015). Forecasting volatility in gold returns under the garch, igarch and figarch frameworks: New evidence. *Physica A: Statistical Mechanics and its Applications*, 438:355–364.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3):307–327.

Calvet, L. and Fisher, A. (2001). Forecasting multifractal volatility. *Journal of Econometrics*, 105(1):27–58. Forecasting and empirical methods in finance and macroeconomics.

Calvet, L. E. and Fisher, A. J. (2004). How to forecast long-run volatility: Regime switching and the estimation of multifractal processes. *Journal of Financial Econometrics*, 2(1):49–83.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50(4):987–1007.

Haas, M., Mittnik, S., and Paolella, M. S. (2004). A new approach to Markov-switching GARCH Models. *Journal of Financial Econometrics*, 2(4):493–530.

Hamilton, J. D. and Susmel, R. (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics*, 64(1):307–333.

Klaassen, F. (2002). Improving GARCH volatility forecasts with regime-switching GARCH. *Empirical Economics*, 27:363–394.

Kumari, S. N. and Tan, A. (2018). Modeling and forecasting volatility series: with reference to gold price. *Thailand Statistician*, 16(1):77–63.

Lamoureux, C. G. and Lastrapes, W. D. (1990). Persistence in variance, structural change, and the GARCH model. *Journal of Business & Economic Statistics*, 8:225–234.

Lux, T. (2008). The Markov-switching multifractal model of asset returns. *Journal of Business & Economic Statistics*, 26(2):194–210.

Lux, T. and Kaizoji, T. (2007). Forecasting volatility and volume in the Tokyo stock market: Long memory, fractality and regime switching. *Journal of Economic Dynamics and Control*, 31(6):1808–1843.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, 59(2):347–370.

Raza, N., Jawad Hussain Shahzad, S., Tiwari, A. K., and Shahbaz, M. (2016). Asymmetric impact of gold, oil prices and their volatilities on stock prices of emerging markets. *Resources Policy*, 49:290–301.

Segnon, M., Lux, T., and Gupta, R. (2015). Modeling and forecasting carbon dioxide emission allowance spot price volatility: Multifractal vs. garch-type volatility models. FinMaP-Working Paper 46, Kiel. info:eu-repo/grantAgreement/EC/FP7/612955.

Sopipan, N., Sattayatham, P., Premanode, B., et al. (2012). Forecasting volatility of gold price using markov regime switching and trading strategy. *Journal of Mathematical Finance*, 2(01):121.

Stoupos, N. and Kiohos, A. (2020). Gold's price and advanced stock markets: A post-crisis approach. In Zopounidis, C., Kenourgios, D., and Dotsis, G., editors, *Recent Advances and Applications in Alternative Investments*, pages 143–156. IGI Global, Hershey, PA.

Wang, F., Chang, J., Zuo, W., and Zhou, W. (2024). Research on efficiency and multifractality of gold market under major events. *Fractal and Fractional*, 8(8).

Wang, Y., Wei, Y., and Wu, C. (2011). Analysis of the efficiency and multifractality of gold markets based on multifractal detrended fluctuation analysis. *Physica A: Statistical Mechanics and its Applications*, 390(5):817–827.

5 Appendix