Quick review T/F questions

True or false. Justify!

1) If P and Q are two Sylow subgroups of a group G, then P and Q intersect trivially.
2) If P and Q are two Sylow p-subgroups of a group G, then P and Q intersect trivially.
3) If p and q are distinct primes, P is a Sylow p-subgroup of G and Q is a Sylow q-subgroup of G, then P and Q intersect trivially.
4) Sylow subgroups are normal.
5) The direct product of two groups is always abelian.
6) The semidirect product of two groups is always nonabelian.
7) There always exists a ring homomorphism between any two rings.
8) Given any ring R, there exists exactly one ring homomorphism $\mathbb{Z} \longrightarrow R$.
9) Given any ring R, there exists exactly one ring homomorphism $R \longrightarrow \mathbb{Z}$.
10) Given any ring R, there exists exactly one ring homomorphism $\mathbb{Z}/n \longrightarrow R$.
11) Given any ring R, there exists exactly one ring homomorphism $R \longrightarrow \mathbb{Z}/n$.
12) Every nonzero element in $\mathbb{Z}$ is a unit.
13) Every ring contains at least two ideals.
14) Every domain is a field.
15) Every field is a domain.
16) Any ring that has only two ideals is a field.
17) The ring $\mathbb{Z}/n[x]$ is a domain.
18) If R and S are domains, then $R \times S$ is a domain.
19) Any subring of a domain is a domain.
20) Any subring of a field is a field.
21) The kernel of any ring homomorphism is an ideal of the domain.
22) The kernel of any ring homomorphism is a subring of the domain.
23) The image of any ring homomorphism is an ideal of the codomain.
24) The image of any ring homomorphism is a subring of the codomain.
25) Every proper ideal is the kernel of some ring homomorphism.
26) If R is a commutative ring and $(g) = R$, then g is a unit.
27) If R is a domain, then $R[x]$ is a domain.
28) If F is a field, then $F[x]$ is a field.
29) If $p \in \mathbb{Z}/2[x]$ has degree 3, then $\mathbb{Z}/2[x]/(p)$ has 4 elements.
30) If $p \in F[x]$ for some field F is irreducible, then $\gcd(p, f)$ is 1 or p.
31) In $R[x]$, the product of two monic polynomials can be zero.
32) If $uf + vg = 4$ in $\mathbb{Q}[x]$, then $f + (g)$ is a unit in $\mathbb{Q}[x]/(g)$.
33) The element $x^2 + 4 + (x^4 - x^2) \in \mathbb{Z}/5[x]/(x^4 - x^2)$ is a unit.
34) The element $x^3 + 2 + (x^4 - x^2) \in \mathbb{Z}/5[x]/(x^4 - x^2)$ is a unit.
35) The quotient ring $\mathbb{R}[x]/(x^3 - x - 6)$ is a field.
36) An element of a commutative ring R can be both a unit and a zerodivisor.
37) $\mathbb{Z}/n$ is a domain if and only if it is a field.
38) Every nonzero element in $\mathbb{Z}/21$ is a unit.
39) In $\mathbb{Z}/77$, $(a) = (b)$ if and only if $a = b$.
40) Every ideal in $\mathbb{Z}/123$ is principal.
41) In any ring R, $ab = 0$ implies $a = 0$ or $b = 0$.
42) In any ring R, we can cancel addition: $a + b = a + c \implies b = c$.
43) In any ring R, we can cancel multiplication: $ab = ac \implies b = c$.

44) If I and J are ideals in a ring R, $I \cup J$ is an ideal in R.