

# Linear Algebra

Math 314 Fall 2025

Today's poll code:

C7DJE9

Lecture 20

To do list:

- Webwork 5.2 now due today
- Webwork 6.1 due on Friday
- Webwork 6.2 due next Friday
- Study for the midterm!

## Office hours

Mondays 5–6 pm and Wednesdays 2–3 pm

Thursday 3:30 to 4:30 pm

Friday 3:30 to 4:30 pm

in Avery 339 (Dr. Grifo)

Tuesdays 11–noon

Thursdays 1–2 pm

in Avery 337 (Kara)

Quiz on Friday

on eigenvalues

# **Quick Recap on diagonalization**

$A$  square matrix

A real number  $\lambda$  is an **eigenvalue** of  $A$

if there exists a nontrivial solution  $x$  to the equation

$$Ax = \lambda x.$$

$x$  is a **eigenvector** of  $A$  associated to  $\lambda$  if

$$Ax = \lambda x.$$

Recipe to find eigenvalues:

Find the solutions to the

**characteristic equation** of  $A$

$$\det(A - \lambda I) = 0$$

$A$  and  $B$  two  $n \times n$  matrices

We say that  $A$  and  $B$  are **similar** if  
there exists an invertible matrix  $P$  such that

$$A = PBP^{-1}$$

$A$  is similar to  $B \iff B$  is similar to  $A$

take  $Q = P^{-1}$ , get  $B = QAQ^{-1}$

## Properties of similarity:

Reflexivity: every square matrix  $A$  is similar to itself.

$$A = IAI^{-1}$$

Symmetry: if  $A$  is similar to  $B$  then  $B$  is similar to  $A$ .

$$A = PBP^{-1} \quad \text{take } Q = P^{-1}, \text{ get } B = QAQ^{-1}$$

Transitivity:

$A$  is similar to  $B$  and  $B$  similar to  $C \Rightarrow A$  is similar to  $C$

$$A = PBP^{-1} \quad B = QCQ^{-1} \quad \text{Take } S = PQ, \text{ get } A = (PQ)C(PQ)^{-1}$$

**Theorem.** A  $n \times n$  matrix

$A$  is diagonalizable if and only if

the sum of the dimensions of its eigenspaces is  $n$ .

Important point:

The eigenspace associated to an eigenvalue  $\lambda$  always has dimension  $\geq 1$ .

so if all the eigenvalues of  $A$  have multiplicity 1  
then  $A$  is diagonalizable.

When  $A$  is diagonalizable:

$$A = PDP^{-1}$$

where

columns of  $P$  = linearly independent eigenvectors of  $A$

Algorithm: find a basis for each eigenspace

$D$  = diagonal with eigenvectors, in order

(the order in  $P$  and  $D$  needs to match)

# **Applications of diagonalization**

not on the midterm

**Theorem.** Let  $T: \mathbb{R}^n \rightarrow \mathbb{R}^n$  be a linear transformation with standard matrix  $A$ .

If  $A = PDP^{-1}$  with  $D$  a diagonal matrix,

and  $\mathcal{B}$  is the basis for  $\mathbb{R}^n$  whose elements are the columns of  $P$ ,  
then  $D$  is the matrix representing  $T$  in the basis  $\mathcal{B}$ .

Example:  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$        $T(x) = Ax$

$$A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}.$$

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\} \xrightarrow{\hspace{1cm}} A_{\mathcal{B} \leftarrow \mathcal{B}} = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$$

Suppose  $A$  is diagonalizable, say

$$A = PDP^{-1}$$

Then

$$A^k = \underbrace{A \cdots A}_{k \text{ times}} = PD\underbrace{(P^{-1}P)}_I D\underbrace{(P^{-1}P)}_I \cdots \underbrace{(P^{-1}P)}_I DP^{-1} = PD\underbrace{\cdots DP^{-1}}_{k \text{ times}} = PD^k P^{-1}.$$

Example:

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$



$$A^k = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 5^k & 0 \\ 0 & 3^k \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 \cdot 5^k - 3^k & 5^k - 3^k \\ 2 \cdot 3^k - 2 \cdot 5^k & 2 \cdot 3^k - 5^k \end{bmatrix}$$

Consider the sequence defined recursively by

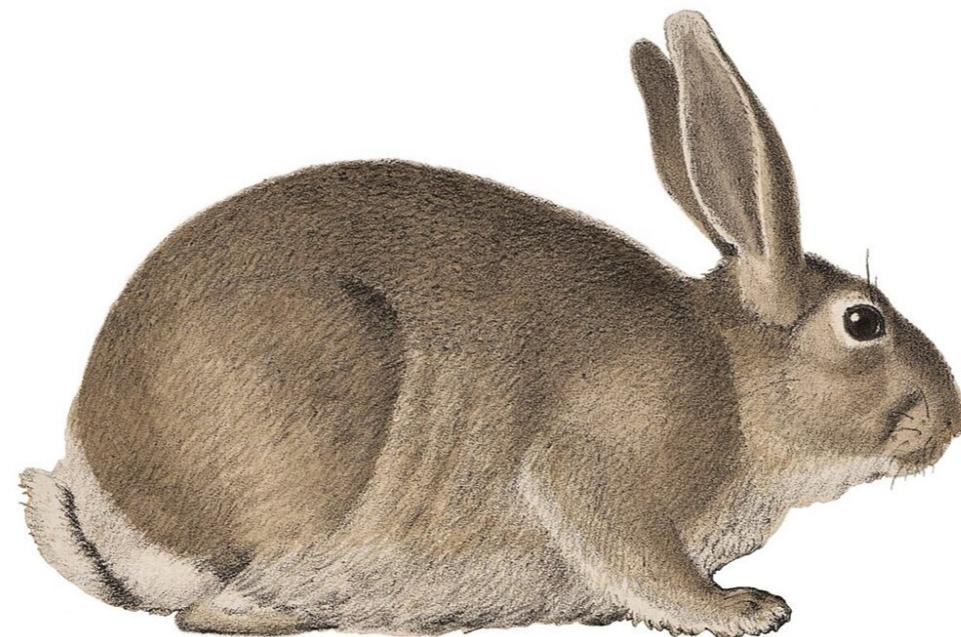
$$f_0 = 1 \quad f_1 = 1 \quad f_{n+2} = f_n + f_{n+1}$$

Consider the sequence defined recursively by

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This is called the Fibonacci sequence, and it starts with

1, 1, 2, 3, 5, 8, 13, 21, . . .



Consider the sequence defined recursively by

$$f_0 = 1 \quad f_1 = 1 \quad f_{n+2} = f_n + f_{n+1}$$

Then

$$\begin{bmatrix} f_{n+1} \\ f_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} f_n \\ f_{n-1} \end{bmatrix}$$

$$\begin{bmatrix} f_{n+1} \\ f_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^2 \begin{bmatrix} f_{n-1} \\ f_{n-2} \end{bmatrix}$$

$$\begin{bmatrix} f_{n+1} \\ f_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} f_1 \\ f_0 \end{bmatrix}$$

Consider the sequence defined recursively by

$$f_0 = 1 \quad f_1 = 1 \quad f_{n+2} = f_n + f_{n+1}$$

Then

$$\begin{bmatrix} f_{n+1} \\ f_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

How do we compute

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n ?$$

Diagonalize!

Diagonalize  $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ .

$$\begin{vmatrix} 1 - \lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\iff -\lambda(1 - \lambda) - 1 = 0$$

$$\iff \lambda^2 - \lambda - 1 = 0$$

$$\iff \lambda = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

Eigenvalues:  $\varphi_+ = \frac{1+\sqrt{5}}{2}$  and  $\varphi_- = \frac{1-\sqrt{5}}{2}$

All eigenvalues have multiplicity 1, so our matrix is diagonalizable!

Diagonalize  $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ .

Eigenvectors:

$\begin{bmatrix} \varphi_+ \\ 1 \end{bmatrix}$  is an eigenvector associated to  $\varphi_+$

$\begin{bmatrix} \varphi_- \\ 1 \end{bmatrix}$  is an eigenvector associated to  $\varphi_-$

Diagonalization: take

$$P = \begin{bmatrix} \varphi_+ & \varphi_- \\ 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} \varphi_+ & 0 \\ 0 & \varphi_- \end{bmatrix}$$

so

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} \varphi_+ & \varphi_- \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \varphi_+^n & 0 \\ 0 & \varphi_-^n \end{bmatrix} \begin{bmatrix} \varphi_+ & \varphi_- \\ 1 & 1 \end{bmatrix}^{-1}$$

Can get a formula for  $f_n$  from here:

$$\begin{bmatrix} f_{n+1} \\ f_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$f_n = \frac{1}{\sqrt{5}} \left( \frac{1+\sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left( \frac{1-\sqrt{5}}{2} \right)^n$$

# **Complex eigenvalues**

not on the midterm

Some  $n \times n$  matrices have less than  $n$  real eigenvalues  
or even none.

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or even none.

Example:

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

characteristic equation       $\lambda^2 + 1 = 0$

No real solutions  $\implies$  no real eigenvalues

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But  $A$  does have two **complex** eigenvalues:  $i$  and  $-i$

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No real solutions  $\implies$  no real eigenvalues

But  $A$  does have two **complex** eigenvalues:  $i$  and  $-i$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -i \end{bmatrix} = \begin{bmatrix} i \\ 1 \end{bmatrix} = i \begin{bmatrix} 1 \\ -i \end{bmatrix} \xrightarrow{\hspace{1cm}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \text{ is a complex eigenvector associated to } i$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} -i \\ 1 \end{bmatrix} = -i \begin{bmatrix} 1 \\ i \end{bmatrix} \xrightarrow{\hspace{1cm}} \begin{bmatrix} 1 \\ i \end{bmatrix} \text{ is a complex eigenvector associated to } -i.$$

# **Midterm 2**

## **On Monday November 10**

What is on the midterm?

- Vector spaces (chapter 4)
- Determinants (chapter 5)
- Eigenvalues and eigenvectors (6.1)

## Midterm 2

### on Monday in lecture

On the day of the midterm:

- Arrive a few minutes early if you can
- Know your NUID!
- Write your name and NUID on the cover page
- Leave one empty seat between you and the student next to you
- No calculators or notes allowed
- Write only on the front side of each page
- There are extra pages at the end of the midterm you can use
- Scratch paper will be provided for you (you cannot use your own)
- Only material you need to bring: writing utensils

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# **Review on vector spaces**

TL;DR: can add vectors and multiply by scalars, with good properties.

A **vector space** is a nonempty set  $V$ , whose elements we call **vectors**, with rules for **addition** of vectors in  $V$  and **multiplication by scalars**, satisfying the following properties:

- a) The addition  $u + v$  of any vectors  $u$  and  $v$  in  $V$  is also a vector in  $V$ .
- b) The multiplication  $cv$  of a vector  $v$  by a scalar  $c$  is a vector in  $V$ .
- c) Commutativity:  $u + v = v + u$  for all  $u, v \in V$ .
- d) Associativity:  $(u + v) + w = u + (v + w)$  for all  $u, v, w \in V$ .
- e) There is a **zero vector** in  $V$ , denoted  $0$ , such that  $0 + v = v + 0 = v$ .
- f) For every vector  $v$  there is a vector  $-v$  such that  $v + (-v) = 0$ .
- g) Distributivity:  $c(u + v) = cu + cv$  and  $(c + d)v = cv + dv$  for all  $u, v \in V$  and all scalars  $c$  and  $d$ .
- h) Associativity of multiplication by scalars:  $c(dv) = (cd)v$ .
- i)  $1v = v$ .
  - $0v = 0$ .
  - $c\mathbf{0} = \mathbf{0}$ .
  - $-v = (-1)v$ .

Consequence: we also have

$V$  vector space

$W$  subset of  $V$

$W$  is a subspace of  $V$  if

1) The zero vector  $0$  is in  $W$

2)  $W$  is closed under addition:

if  $u$  and  $v$  are in  $W$ , then  $u + v$  is in  $W$

3)  $W$  is closed under multiplication by scalars:

if  $v$  is in  $W$ , and  $c$  is any scalar, then  $cv$  is in  $W$

A subspace is also a vector space!

Example:

$$V = \mathbb{R}^4$$

$$W = \left\{ \begin{bmatrix} a - 3b \\ b - a \\ a \\ b \end{bmatrix} : a, b \text{ any scalars} \right\}$$

is a subspace of  $\mathbb{R}^4$

easier:

all vectors in  $W$  can be written as

$$\begin{bmatrix} a - 3b \\ b - a \\ a \\ b \end{bmatrix} = a \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

so  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$

is a subspace

linear combination of these vectors!

Example:

In  $M_{2 \times 2}$  (space of  $2 \times 2$  matrices)

$$D = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \text{ any scalars} \right\} \quad \text{diagonal matrices}$$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = a \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$D = \text{span} \left( \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} \right)$$

so  $D$  is a subspace of  $M_{2 \times 2}$

A **basis**<sup>\*</sup> for a vector space  $V$  is  
a spanning set of linearly independent vectors.

The **dimension** of a vector space is  
the number of vectors in a basis.

Notation:  $\dim(V)$

\* The plural of basis is bases.

$$\dim(\mathbb{R}^n) = n \quad \text{Basis: } e_1, \dots, e_n$$

$$\dim(M_{m \times n}) = mn \quad \dim(\{0\}) = 0$$

all  $m \times n$  matrices

$$\dim(\mathbb{P}_n) = n + 1$$

polynomials  $a_0 + a_1t + \dots + a_nt^n$

Think of  $\mathbb{P}_n$  as  $\mathbb{R}^{n+1}$

$$a_0 + a_1t + \dots + a_nt^n$$



$$\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}$$

$$W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 7 \end{bmatrix} \right\} \right)$$

What is the dimension of  $W$ ?

$W$  is the same as the column space of

$$\begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

$W = \text{span}(\{v_1, \dots, v_n\})$  subspace of  $\mathbb{R}^m$

How to find a basis for  $W$ ?

Find a basis for the column space of

$$A = [v_1 \quad \cdots \quad v_n]$$

Step 1: Find the RREF of  $A$ .

Step 2: Collect the pivot columns of  $A$ .

**Warning:** Make sure to use the pivot columns of  $A$ ,  
not of its reduced echelon form!

$$\dim(\text{col}(A)) = \# \text{ of pivots in } A$$

To find a basis for the null space of  $A$ :

Step 1: Find the general solution for  $Ax = 0$ .

Step 2: Write the solution in parametric vector form.

Use one vector for each free variable.

Step 2: The vectors we used form a basis for  $\text{Nul}(A)$ .

$$\dim(\text{Nul}(A)) = \# \text{ free variables}$$

$$\dim(\text{col}(A)) = \# \text{ of pivots in } A = \text{rank}(A)$$

$$\dim(\text{Nul}(A)) = \# \text{ free variables} = \text{nullity of } A$$

**Theorem** (Rank–Nullity Theorem).

For any  $m \times n$  matrix  $A$ ,

$$\text{rank}(A) + \dim(\text{Nul}(A)) = n.$$

# **Review on determinants**

## Theorem.

$A$  square matrix

$$\det(A) \neq 0$$



$A$  is invertible

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$A^{-1} = \frac{1}{\det A} \operatorname{adj}(A).$$

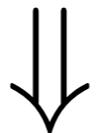
## Theorem.

If  $A$  is a triangular matrix, then

$\det(A)$  = product of the main diagonal entries

$$A = \begin{bmatrix} -1 & -1 & 7 & 5 \\ 0 & -5 & 42 & 2 \\ 0 & 0 & 2 & 13 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$A$  upper triangular



$$\det(A) = (-1) \cdot (-5) \cdot 2 \cdot 5 = 50$$

## Effect of Elementary Row Operations on the determinant:

$$A \xrightarrow{\text{Replace}} B \qquad \det(B) = \det(A)$$

add a multiple of a row of  $A$   
to another row

$$A \xrightarrow{\text{Swap}} B \qquad \det(B) = -\det(A)$$

$$A \xrightarrow{\text{Rescale}} B \qquad \det(B) = c \det(A)$$

multiply a row of  $A$  by  $c$

## Theorem.

$A$  square matrix

1.  $\det(A^\top) = \det(A).$
2.  $\det(AB) = \det(A)\det(B).$
3.  $\det(cA) = c^n \det(A).$

**Warning!** No formula for  $\det(A + B)$

# **Practice problems**

**Today's poll code:**

**C7DJE9**

Regular polls for today:

1 point total for answering at least 1 question today

## **Practice problems**

up to 6 bonus points

up to 5 points for 5 correct answers

1 bonus point if all correct

Today's poll code:

C7DJE9

$7 \times 5$  matrix  $M$  with 3 pivots

What is the dimension of the space of solutions to

$$Mx = 0$$

?

Today's poll code:

C7DJE9

Is it possible for a homogeneous system  
of 5 equations in 6 variables to have  
only the trivial solution?

A. Yes

B. No

Today's poll code:

C7DJE9

Is it possible for a homogeneous system  
of 6 equations in 5 variables to have  
only the trivial solution?

A. Yes

B. No

Today's poll code:

C7DJE9

Is it possible for the solutions to a  
homogeneous systems with 7 equations in 9 variables  
to be all contained in a plane?

A. Yes

B. No

Today's poll code:

C7DJE9

$$A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \\ 1 & -2 & 0 & 0 & -5 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 0 & 1 & -1 & 3 & 4 & -3 \\ 0 & 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The null space of  $A$  is a subspace of

A.  $\mathbb{R}^5$

B.  $\mathbb{R}^6$

C.  $\mathbb{R}^3$

D.  $M_{5 \times 6}$

E.  $M_{6 \times 5}$

Today's poll code:

C7DJE9

$$A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \\ 1 & -2 & 0 & 0 & -5 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 0 & 1 & -1 & 3 & 4 & -3 \\ 0 & 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The dimension of the null space of  $A$  is

Today's poll code:

C7DJE9

Which of these is a basis for  
the space of diagonal  $2 \times 2$  matrices?

A.  $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

B.  $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

C.  $M_{2 \times 2}$

D.  $\left\{ \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} : a \text{ any real number} \right\}$

Today's poll code:

C7DJE9

$A$  and  $B$   $n \times n$  matrices

$$\det(AB) = 17$$

- A.  $A$  is invertible
- B.  $A$  is not invertible
- C. Not enough information

Today's poll code:

C7DJE9

$A$  and  $B$   $n \times n$  matrices

$$\det(AB) = 0$$

- A.  $A$  is invertible
- B.  $A$  is not invertible
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**Lab 2**  
**(To be released)**  
**due after Thanksgiving**

**Groups of 2 or 3 students**

**Solo teams not allowed this time**

## To do list:

- Webwork 5.2 now due today
- Webwork 6.1 due on Friday
- Webwork 6.2 due Friday November 14
- Study for the midterm!



not on the midterm  
due after the midterm

**Quiz on Friday  
on eigenvalues**

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