Linear Algebra

Math 314 Fall 2025

Lecture 5

Today's poll code:

MNE2CY

To do list:

- WebWork 2.2 due Friday
- Webwork 2.3 due Tuesday September 16
- WeBWork 2.4 due Friday September 19

Quiz 3 on Friday
on lectures 4—5

Office hours

Mondays 5–6 pm
Wednesdays 2–3 pm
in Avery 339 (Dr. Grifo)

Tuesdays 11–noon
Thursdays 1–2 pm
in Avery 337 (Kara)

Office hours

Mondays 5–6 pm and Wednesdays 2–3 pm
in Avery 339 (Dr. Grifo)

Tuesdays 11–noon and Thursdays 1–2 pm
in Avery 337 (Kara)

Homogeneous linear systems

A linear system is **homogeneous** if we can write it as

$$Ax = 0.$$

A homogeneous system always has a solution:

the **trivial solution** $x = 0$.

A solution $x \neq 0$ is called **nontrivial**.

$$
\begin{cases}
3x_1 + 5x_2 - 4x_3 = 0 \\
-3x_1 - 2x_2 + 4x_3 = 0 \\
6x_1 + x_2 - 8x_3 = 0
\end{cases}
$$

homogeneous system

$$\begin{cases} 3x_1 + 5x_2 - 4x_3 = 0 \\ -3x_1 - 2x_2 + 4x_3 = 0 \\ 6x_1 + x_2 - 8x_3 = 0 \end{cases}$$

homogeneous system

$$\underbrace{\begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & -8 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{b}$$

matrix equation

$$\begin{cases} 3x_1 + 5x_2 - 4x_3 = 0 \\ -3x_1 - 2x_2 + 4x_3 = 0 \\ 6x_1 + x_2 - 8x_3 = 0 \end{cases}$$

homogeneous system

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 3 & 0 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 - 2R_1} \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{3} R_2} \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right] \xrightarrow{R_1 \to R_1 - 5R_2} \left[\begin{array}{ccc|c} 3 & 0 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 + 9R_2} \left[\begin{array}{ccc|c} 3 & 0 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R1 \to \frac{1}{3} R1} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{4}{3} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Note that the last column doesn't really add any information!

When the system is homogeneous, can row reduce A instead.

$$\begin{cases} 3x_1 + 5x_2 - 4x_3 = 0 \\ -3x_1 - 2x_2 + 4x_3 = 0 \\ 6x_1 + x_2 - 8x_3 = 0 \end{cases}$$

homogeneous system

$$\begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & -8 \end{bmatrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{bmatrix} 3 & 5 & -4 \\ 0 & 3 & 0 \\ 6 & 1 & -8 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_1} \begin{bmatrix} 3 & 5 & -4 \\ 0 & 3 & 0 \\ 0 & -9 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \to \frac{1}{3} R_2} \begin{bmatrix} 3 & 5 & -4 \\ 0 & 1 & 0 \\ 0 & -9 & 0 \end{bmatrix} \xrightarrow{R_1 \to R_1 - 5R_2} \begin{bmatrix} 3 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & -9 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 + 9R_2} \begin{bmatrix} 3 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R1 \to \frac{1}{3} R1} \begin{bmatrix} 1 & 0 & -\frac{4}{3} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$$\begin{cases} 3x_1 + 5x_2 - 4x_3 = 0 \\ -3x_1 - 2x_2 + 4x_3 = 0 \\ 6x_1 + x_2 - 8x_3 = 0 \end{cases}$$

homogeneous system

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 3 & 0 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 - 2R_1} \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{3}R_2} \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right] \xrightarrow{R_1 \to R_1 - 5R_2} \left[\begin{array}{ccc|c} 3 & 0 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 + 9R_2} \left[\begin{array}{ccc|c} 3 & 0 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R1 \to \frac{1}{3}R1} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{4}{3} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

x_3 is free $\qquad x_1 = \frac{4}{3}x_3 \qquad x_2 = 0$

$$\underbrace{\begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & -8 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{b}$$

homogeneous system

x_3 is free $\quad x_1 = \frac{4}{3}x_3 \quad x_2 = 0$

General solution:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} \frac{4}{3} \\ 0 \\ 1 \end{bmatrix} \quad \text{or} \quad t \begin{bmatrix} \frac{4}{3} \\ 0 \\ 1 \end{bmatrix}$$

$t = 0$ gives the trivial solution

Each choice of $t \neq 0$ gives a nontrivial particular solution the general solution to a homogeneous system is

a linear combination of vectors with the free variables as coefficients

$$\underbrace{\begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & -8 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{b}$$

homogeneous system

General solution in parametric vector form

$$x = x_3 \begin{bmatrix} \frac{4}{3} \\ 0 \\ 1 \end{bmatrix} \quad \text{or} \quad x = t \begin{bmatrix} \frac{4}{3} \\ 0 \\ 1 \end{bmatrix}$$

Nonhomogeneous linear systems

A **nonhomogeneous linear system** is a linear system of the form

$$Ax = b \qquad \text{with } b \neq 0$$

Theorem. The general solution to the nonhomogeneous system $Ax = b$ is

$$x = \text{one particular solution} + \begin{array}{c}\text{general solution to the}\\ \text{homogeneous system } Ax = 0\end{array}$$

A homogeneous system always has a solution:

the **trivial solution** $x = 0$.

A nonhomogeneous system might not have a solution.

A nonhomogeneous system might not have a solution.

Example:

The nonhomogeneous system

$$0 = 1$$

has no solutions

$$\begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & 8 \end{bmatrix} x = \begin{bmatrix} 7 \\ 1 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 5 & -4 & 7 \\ -3 & -2 & 4 & 1 \\ 6 & 1 & 8 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -\frac{4}{3} & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A

$$\begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & 8 \end{bmatrix} x = \begin{bmatrix} 7 \\ 1 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 5 & -4 & 7 \\ -3 & -2 & 4 & 1 \\ 6 & 1 & 8 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -\frac{4}{3} & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

General solution in parametric vector form

$$x = \underbrace{\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}}_{\text{particular solution}} + x_3 \underbrace{\begin{bmatrix} \frac{4}{3} \\ 0 \\ 1 \end{bmatrix}}_{\text{solution to } Ax=0}$$

Important

To write the solution set of a consistent system:

1) Row-reduce the augmented matrix into reduced echelon form.
2) Write each non-free variable in terms of the free ones.
3) Write the general solution x as a vector whose entries depend on the free variables (if there are free variables).
4) Decompose this as a linear combination of vectors where each coefficient is a free variable (plus possibly one term with coefficient 1 for a particular solution).

Important

Caution! Given a linear system $Ax = b$, there is a big difference between

the coefficient matrix A and the augmented matrix $\begin{bmatrix} A & b \end{bmatrix}$.

- Is the system $Ax = b$ consistent? $\Longrightarrow$ look at the augmented matrix.
- The system $Ax = 0$ is always consistent.

  We can solve the system by focusing only on A and then finding a particular solution, but if we do so we must remember A is *not* the augmented matrix of the system.

Today’s poll code:

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A homogeneous linear system is always consistent.

A. True

B. False

Today's poll code:

MNE2CY

A nonhomogeneous linear system is always consistent.

A. True

B. False

Linear independence

$v_1, \ldots, v_p$ vectors in $\mathbb{R}^n$

$v_1, \ldots, v_p$ are **linearly independent** if the equation

$$x_1 v_1 + \cdots + x_p v_p = 0$$

has no nontrivial solutions.

$v_1, \ldots, v_p$ are **linearly dependent** if the equation

$$x_1 v_1 + \cdots + x_p v_p = 0$$

has nontrivial solutions

$v_1, \ldots, v_p$ vectors in $\mathbb{R}^n$

$v_1, \ldots, v_p$ are **linearly independent** if

$$c_1 v_1 + \cdots + c_p v_p = 0 \quad \Longrightarrow \quad c_1 = \cdots = c_p = 0$$

$v_1, \ldots, v_p$ are **linearly dependent** if

$$c_1 v_1 + \cdots + c_p v_p = 0$$

for some $c_1, \ldots, c_p$ not all zero

$v_1, \ldots, v_p$ are **linearly dependent** if

$$c_1 v_1 + \cdots + c_p v_p = 0$$

for some $c_1, \ldots, c_p$ not all zero

in this case
the equation

$$c_1 v_1 + \cdots + c_p v_p = 0$$

is an equation of **linear dependence**

among $v_1, \ldots, v_p$

$v_1, \ldots, v_p$ are **linearly independent** if

$$c_1 v_1 + \cdots + c_p v_p = 0 \quad \Longrightarrow \quad c_1 = \cdots = c_p = 0$$

$\{v\}$ is linearly independent

$v_1, \ldots, v_p$ are **linearly independent** if

$$c_1 v_1 + \cdots + c_p v_p = 0 \quad \Longrightarrow \quad c_1 = \cdots = c_p = 0$$

$\{v\}$ is linearly independent

$$\Updownarrow$$

$$v \neq 0$$

$v_1, \ldots, v_p$ are **linearly dependent** if

$$c_1 v_1 + \cdots + c_p v_p = 0$$

for some $c_1, \ldots, c_p$ not all zero

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ are

$v_1, \ldots, v_p$ are **linearly dependent** if

$$c_1 v_1 + \cdots + c_p v_p = 0$$

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$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ are linearly dependent

$v_1, \ldots, v_p$ are **linearly dependent** if

$$c_1 v_1 + \cdots + c_p v_p = 0$$

for some $c_1, \ldots, c_p$ not all zero

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ are linearly dependent

for example

$$0 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Theorem. Any set of vectors in $\mathbb{R}^n$ that contains the zero vector is linearly dependent.

$v_1, \ldots, v_p$ are **linearly dependent** if

$$c_1 v_1 + \cdots + c_p v_p = 0$$

for some $c_1, \ldots, c_p$ not all zero

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ are linearly dependent

for example

$$2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-1) \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 0$$

$v_1, \ldots, v_p$ are **linearly dependent** if

$$c_1 v_1 + \cdots + c_p v_p = 0$$

for some $c_1, \ldots, c_p$ not all zero

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ are linearly dependent

for example

$$2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-1) \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 0$$

Any two nonzero vectors that are scalar multiples of each other are linearly dependent.

Any two nonzero vectors that are scalar multiples of each other are linearly dependent.

v is any nonzero vector and $t \neq 1$

$$\Downarrow$$

v and tv are linearly dependent

$$t \cdot v + (-1) \cdot (tv) = 0$$

A set $\{v_1, \dots, v_p\}$ of two or more vectors is linearly dependent

if and only if

one of the vectors is a linear combination of the others.

Note: this does <u>not</u> say that <u>every</u> v_i is a linear combination of the rest.

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$

is linearly dependent

since

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

But $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is not a linear combination of the other two vectors

An equation of linear dependence among the vectors $v_1, \ldots, v_n$ is a nontrivial solution to the homogeneous system

$$x_1 v_1 + \cdots + x_n v_n = 0$$

To decide if the vectors $v_1, \ldots, v_n$ are linearly independent, we consider the matrix

$$A = \begin{bmatrix} v_1 & \cdots & v_n \end{bmatrix}$$

and ask whether the system

$$Ax = 0$$

has a nontrivial solution

Theorem. The columns of a matrix A are linearly independent

if and only if

the homogeneous system $Ax = 0$ has only the trivial solution.

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Are v_1, v_2, and v_3 linearly independent?

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Are v_1, v_2, and v_3 linearly independent?

$$A = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 1 \\ 3 & 6 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Are v_1, v_2, and v_3 linearly independent?

$$A = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 1 \\ 3 & 6 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 1 \\ 3 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Are v_1, v_2, and v_3 linearly independent?

$$A = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Are v_1, v_2, and v_3 linearly independent?

$$A = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

free variable

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Are v_1, v_2, and v_3 linearly independent?

$$A = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

free variable

$Ax = 0$ has nontrivial solutions $\Longrightarrow$ v_1, v_2, v_3 are linearly dependent

$$A = \begin{bmatrix} v_1 & \cdots & v_n \end{bmatrix}$$

$v_1, \ldots, v_n$ are

linearly independent

$$\Updownarrow$$

$Ax = 0$ has

only one solution

$$\Updownarrow$$

A has

pivots in every column

Theorem. Consider the matrix

$$A = \begin{bmatrix} v_1 & \cdots & v_n \end{bmatrix}.$$

The column vectors $v_1, \ldots, v_n$ are linearly independent

if and only if

A has a pivot in every column.

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

free variable

$Ax = 0$ has nontrivial solutions $\implies$ v_1, v_2, v_3 are linearly dependent

Relation of linear dependence:

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

free variable

$Ax = 0$ has nontrivial solutions $\Longrightarrow$ v_1, v_2, v_3 are linearly dependent

relation of linear dependence: any nontrivial solution to $Ax = 0$

$$\begin{cases} x_3 \text{ free} \\ x_1 = 2x_3 \\ x_2 = -x_3 \end{cases} \Longrightarrow \quad x = t\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \quad \Longrightarrow \quad x = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

general solution particular solution

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$Ax = 0$ has nontrivial solutions $\Longrightarrow$ v_1, v_2, v_3 are linearly dependent

$$x = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

particular solution

Relation of linear dependence:

$$2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 1\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Theorem. Any set of more than n vectors in $\mathbb{R}^n$ is linearly dependent.

Today's poll code:

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Today's poll code:

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$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

are

A. Linearly dependent

B. Linearly independent

Today's poll code:

MNE2CY

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

are

A. Linearly dependent

B. Linearly independent

Today's poll code:

MNE2CY

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

are

A. Linearly dependent

B. Linearly independent

Today's poll code:

MNE2CY

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$$

are

A. Linearly dependent

B. Linearly independent

Today's poll code:

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$$\begin{bmatrix} 71 \\ 135 \\ -7 \end{bmatrix} \text{ and } \begin{bmatrix} 8 \\ 42 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} -3 \\ 4 \\ 23 \end{bmatrix} \text{ and } \begin{bmatrix} 9 \\ 0 \\ -500 \end{bmatrix}$$

are

A. Linearly dependent

B. Linearly independent

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- WebWork 2.2 due Friday
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- WeBWork 2.4 due Friday Sep 19

On Friday:

Quiz 3

at the beginning

of the recitation

on Lectures 4—5

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