

$$\text{depth}(R) \leq \dim(R) \quad \text{More generally, } \text{depth}(M) \leq \dim(R)$$

(R, m) Noetherian local ring

R is Cohen-Macaulay if $\text{depth}(R) = \dim(R)$

M is Cohen-Macaulay if $\text{depth}(M) = \dim(M)$

Examples

- Every regular ring is Cohen-Macaulay
- Every 1-dimensional domain is Cohen-Macaulay
- $k[x]/(x^2)$ is Cohen-Macaulay but not regular
- $k[x, y, z]/(xy, yz)$ is not Cohen-Macaulay
- Rings with nicे singularities are Cohen-Macaulay
eg rings of invariants of a finite group G acting on $k[x_1, \dots, x_d]$, char $k[G]$

$$\begin{array}{c} \text{depth } R \leq \dim(R) \leq \text{embdim}(R) \\ \downarrow \quad \quad \quad \downarrow \\ \text{Cohen-Macaulay} \quad \text{regular} \end{array}$$

- M Cohen-Macaulay
 $\Rightarrow \text{depth}(M) = \dim(R/\mathfrak{P})$
for all $\mathfrak{P} \in \text{Ass}(R)$
- R has no embedded primes
- Cohen-Macaulayness localizes
- R Cohen-Macaulay
 \mathfrak{P} prime
 $\dim(R) - \text{height}(\mathfrak{P}) = \dim(R/\mathfrak{P})$