Linear Algebra

Math 314 Fall 2025

Lecture 3

Today's poll code:

K3NARW

Office hours

Mondays 5–6 pm
Wednesdays 2–3 pm
in Avery 339 (Dr. Grifo)

Tuesdays 11–noon
Thursdays 1–2 pm
in Avery 337 (Kara)

To do list:

- Webwork 1.2 due Friday Sep 5
- WebworK 1.3 due Tuesday Sep 9
- Webwork 2.1 due Tuesday Sep 9
- Webwork 2.2 due Friday Sep 12

Section 1.3
discussed on Friday

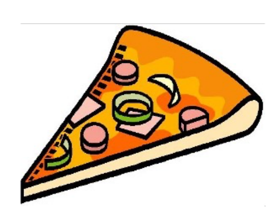
Math Club

Math Department Majors Welcome Event

For Math and CAS Data Science Majors

Calling all current and prospective Math and Data Science Majors! Join us to connect with fellow students, explore opportunities in mathematics, and kickstart your semester. Members of the math department advising team, the Math Club, and other organizations will share information about their programs.

Thursday, September 4, at 5:30 pm
in 115 Avery Hall
Refreshments served at 6:30 pm.

+ {  ,  }

www.math.unl.edu/math-club

Directed Reading Program

The DRP pairs advanced undergraduates with a graduate student mentor to explore a mathematical topic outside the standard undergraduate curriculum. Students read weekly, meet regularly with their mentor to discuss the material, and conclude the semester by giving a short presentation to fellow DRP participants.

Applications are due September 9

https://drp-unl.github.io/projects.html

Office hours

Mondays 5–6 pm and Wednesdays 2–3 pm
in Avery 339 (Dr. Grifo)

Tuesdays 11–noon and Thursdays 1–2 pm
in Avery 337 (Kara)

Quick Recap

$$\begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n = b_m \end{cases} \quad \text{has}$$

coefficient matrix

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

constant vector

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$[A|\mathbf{b}] = \begin{bmatrix} a_{11} & \cdots & a_{1n} & b_1 \\ \vdots & \ddots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} & b_m \end{bmatrix}$$

augmented matrix

reduced row echelon form

$$\begin{bmatrix}
\cdots & 0 & 1 & \star & 0 & \star & 0 & \cdots & 0 & \star & \star \\
\cdots & 0 & 0 & 0 & 1 & \star & 0 & \cdots & 0 & \star & \star \\
\cdots & 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 & \star & \star \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & \star & \star \\
\multicolumn{11}{c}{\text{if there are zero rows}} \\
\multicolumn{11}{c}{\text{they are at the bottom}}
\end{bmatrix}$$

$\star$ can be anything

In general, a system of linear equations may have:

- No solutions,
- Exactly one solution, or
- Infinitely many solutions.

But it can **never** have a finite number of solutions greater than one.

How do we read the solutions from the RREF?

Once we obtain the RREF of a system:

- Among the columns corresponding to $x_1, \ldots, x_n$, columns without pivots correspond to free variables.
- Free variables can take arbitrary values.
- Each choice of free variables gives one solution to the system.

A system of linear equations is:

- **Consistent** if it has at least one solution.
- **Inconsistent** if it has no solutions.

A system of linear equations is:

- **Consistent** if it has at least one solution.
- **Inconsistent** if it has no solutions.

Theorem. A linear system of equations is inconsistent if and only if the reduced echelon form of its augmented matrix has a pivot in the last column.

A system of linear equations is:

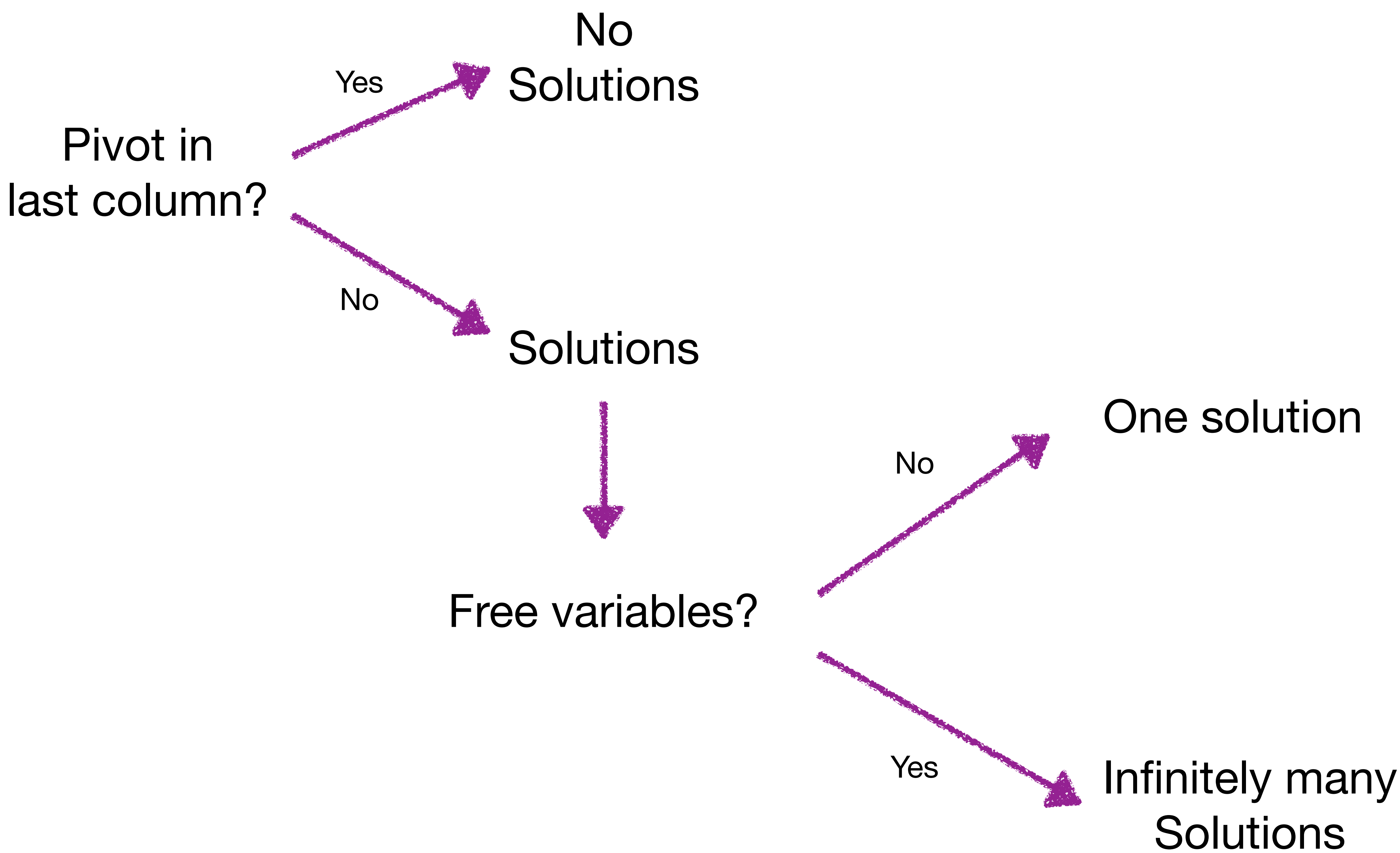
- **Consistent** if it has at least one solution.
- **Inconsistent** if it has no solutions.

Theorem. A linear system of equations is inconsistent if and only if the reduced echelon form of its augmented matrix has a pivot in the last column.

so

the system is inconsistent if the RREF has a row of the form

$$[0\ 0\ \cdots\ 0 \mid 1].$$


Pivot in last column?
Yes
No Solutions
No
Solutions
Free variables?
No
One solution
Yes
Infinitely many Solutions

Pivot in last column	Yes	Yes	No	No
Free variables	Yes	No	No	Yes
Number of solutions	0	0	1	∞

pivot

free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

pivot

free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

pivot free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

pivot

free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

pivot   free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

pivot

free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

pivot

free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

inconsistent

no solution

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

pivot

free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

inconsistent

no solution

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

pivot free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

inconsistent

no solution

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

pivot

free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

inconsistent

no solution

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

consistent

pivot

free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

inconsistent

no solution

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

consistent

no free vars!

pivot free variable

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

inconsistent

no solution

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

consistent

one solution

no free vars!

Finding all solutions:

pivot

free variable

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

solution set:

Finding all solutions:

pivot

free variable

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

solution set:

$$\{(0, 3)\}$$

Finding all solutions:

pivot

free variable

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 3x_2 = 5$$

Finding all solutions:

pivot free variable

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix} \qquad x_1 + 3x_2 = 5 \quad \Leftrightarrow x_1 = -3x_2 + 5$$

Finding all solutions:

pivot

free variable

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 3x_2 = 5 \quad \Leftrightarrow x_1 = -3x_2 + 5$$

$$x_2 = t$$

Finding all solutions:

pivot

free variable

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 3x_2 = 5 \quad \Leftrightarrow x_1 = -3x_2 + 5$$

$$x_2 = t$$

solution set:

$$\{(5 - 3t, t) \mid t \in \mathbb{R}\}$$

Today’s poll code:

K3NARW

Today's poll code:

K3NARW

The linear system with augmented matrix

$$\begin{bmatrix} 1 & 0 & 3 & 2 & 7 \\ 0 & 1 & 5 & 2 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

has A. 0 B. 1 C. 2 D. infinitely many

solutions

Today's poll code:

K3NARW

The linear system with augmented matrix

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 7 \\ 0 & 1 & 5 & 0 & 9 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

has A. 0 B. 1 C. 2 D. infinitely many

solutions

Today's poll code:

K3NARW

The linear system with augmented matrix

$$\begin{bmatrix} 2 & 0 & 3 & 2 & 7 \\ 0 & 3 & 5 & 2 & 9 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

has A. 0 B. 1 C. 2 D. infinitely many

solutions

reduced row echelon form

$$
\left[\begin{array}{ccccccccccc}
\cdots & 0 & 1 & \star & 0 & \star & 0 & \cdots & 0 & \star & \star \\
\cdots & 0 & 0 & 0 & 1 & \star & 0 & \cdots & 0 & \star & \star \\
\cdots & 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 & \star & \star \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & \star & \star \\
\multicolumn{11}{c}{\text{if there are zero rows}} \\
\multicolumn{11}{c}{\text{they are at the bottom}}
\end{array}\right]
$$

$\star$ can be anything

Chapter 2

vector = matrix with only one column

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

vector = matrix with only one column

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

first component

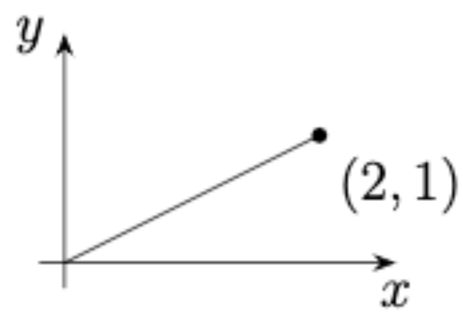
nth component

vector = matrix with only one column

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \begin{array}{l} \longleftarrow \text{first component} \\ \\ \longleftarrow n\text{th component} \end{array}$$

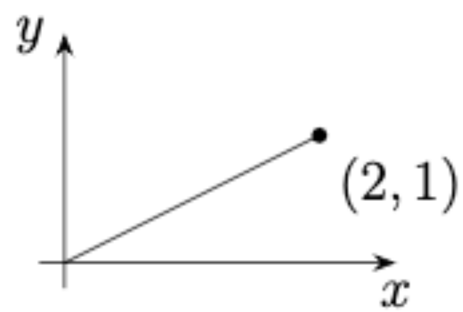
$\mathbb{R}^n$ = set of all vectors with n components

$\mathbb{R}^2$ = 2-dimensional plane



$$\longrightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\mathbb{R}^2$ = 2-dimensional plane



$$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

In $\mathbb{R}^n$:

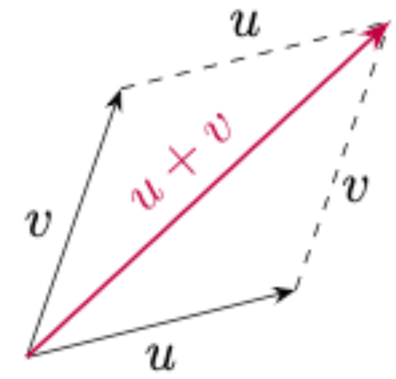
$$\overrightarrow{AB} = \begin{bmatrix} b_1 - a_1 \\ \vdots \\ b_n - a_n \end{bmatrix}$$

$$A = (a_1, \ldots, a_n)$$

$$B = (b_1, \ldots, b_n)$$

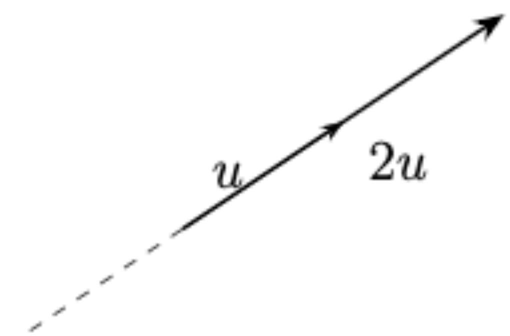
We can add vectors:

$$\begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} u_1 + v_1 \\ \vdots \\ u_n + v_n \end{bmatrix}$$



We can multiply a scalar by a vector:

$$c \cdot \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} c\, u_1 \\ \vdots \\ c\, u_n \end{bmatrix}$$



Example:

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

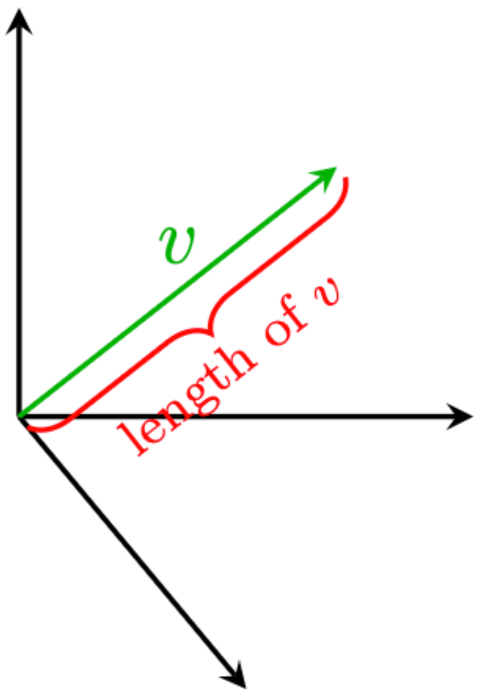
$$2 \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

For all vectors $u, v, w \in \mathbb{R}^n$ and scalars c, d:

1. $u + v = v + u$
2. $(u + v) + w = u + (v + w)$
3. $u + 0 = 0 + u = u$
4. $u + (-u) = -u + u = 0$
5. $c(u + v) = cu + cv$
6. $(c + d)u = cu + du$
7. $c(du) = (cd)u$
8. $1u = u$

v vector in $\mathbb{R}^n$

The **length** or **norm** of v is the nonnegative real number



$$\|v\| := \sqrt{v_1^2 + \cdots + v_n^2}.$$

If v is a vector in $\mathbb{R}^n$ and c is any scalar

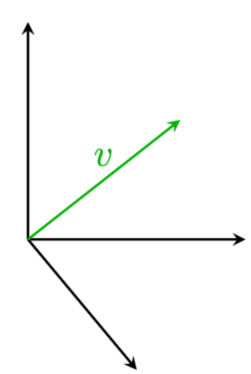
$$\|cv\| = |c| \cdot \|v\|$$

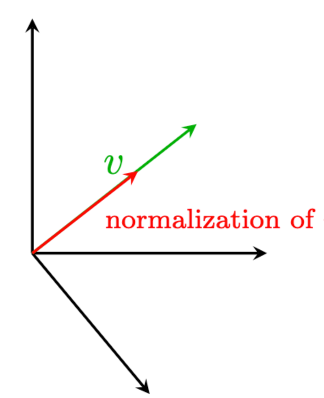
unit vector = vector with length 1

$v \neq 0$

normalization of $v = \dfrac{v}{\|v\|}$

= the unit vector with same direction as v





ith standard vector in $\mathbb{R}^n$

$$\mathbf{e}_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \longrightarrow \text{position } i$$

standard basis for $\mathbb{R}^n$: $\mathbf{e}_1, \ldots, \mathbf{e}_n$

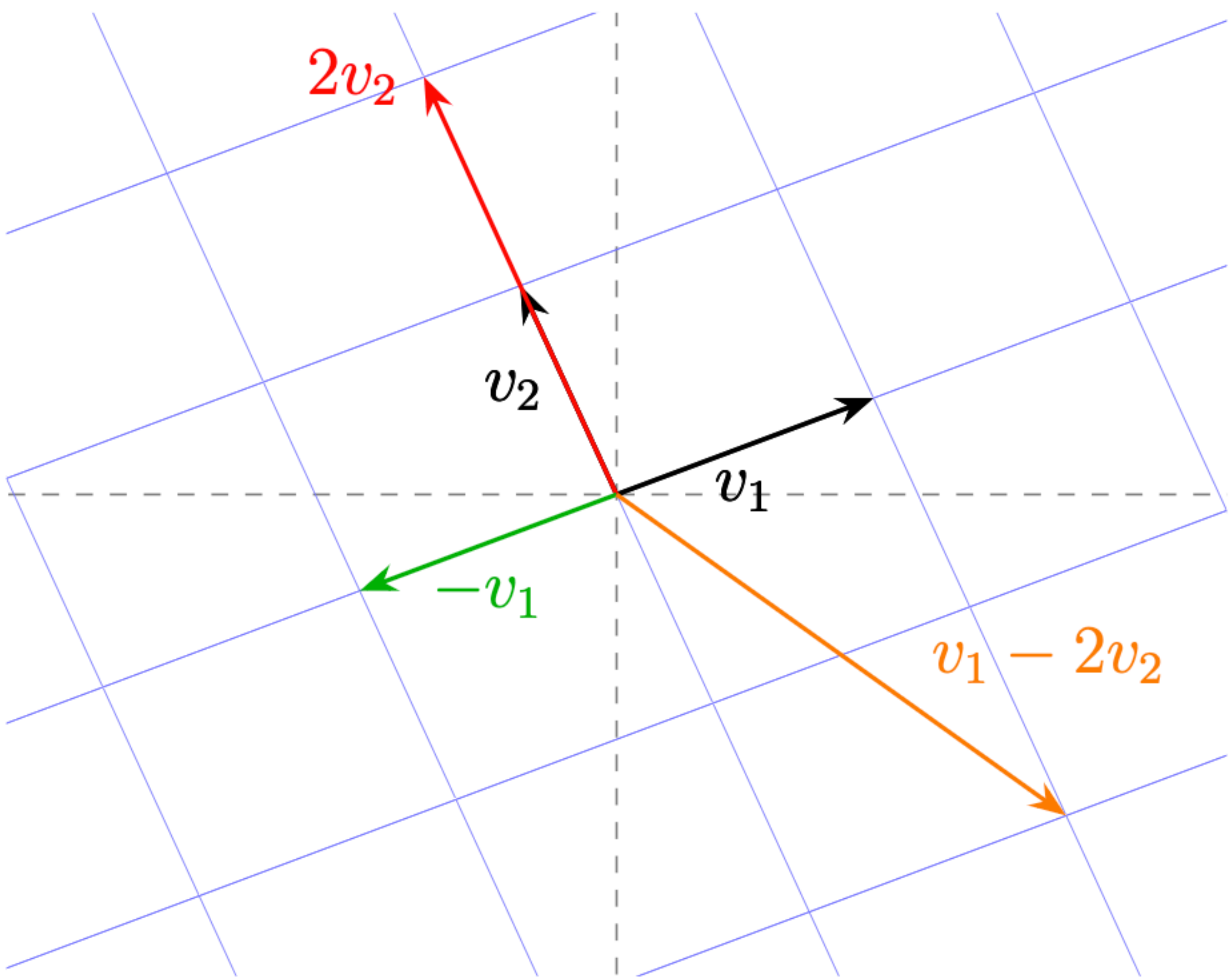
in $\mathbb{R}^3$ we sometimes write

$$\mathbf{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \mathbf{e}_1 \qquad \mathbf{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \mathbf{e}_2 \qquad \mathbf{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \mathbf{e}_3$$

Given vectors $v_1, \ldots, v_p$ and scalars $c_1, \ldots, c_p$, the vector

$$c_1 v_1 + \cdots + c_p v_p$$

is a **linear combination** of $v_1, \ldots, v_p$ with coefficients $c_1, \ldots, c_p$.



Any point on the plane determined by v_1 and v_2 is a linear combination of v_1 and v_2

Question:

$$\text{Is } \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ a linear combination of } \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}?$$

Question:

$$\text{Is } \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ a linear combination of } \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}?$$

Equivalently:

$$\text{Does the system } x_1 \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ have a solution?}$$

Question:

$$\text{Is } \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ a linear combination of } \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}?$$

Equivalently:

$$\text{Does the system } x_1 \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ have a solution?}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Question:

$$\text{Is } \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ a linear combination of } \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}?$$

Equivalently:

$$\text{Does the system } x_1 \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ have a solution?}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

No pivots in the last column

system is consistent

Answer: yes.

Let $v_1, \ldots, v_p$ be vectors in $\mathbb{R}^n$.

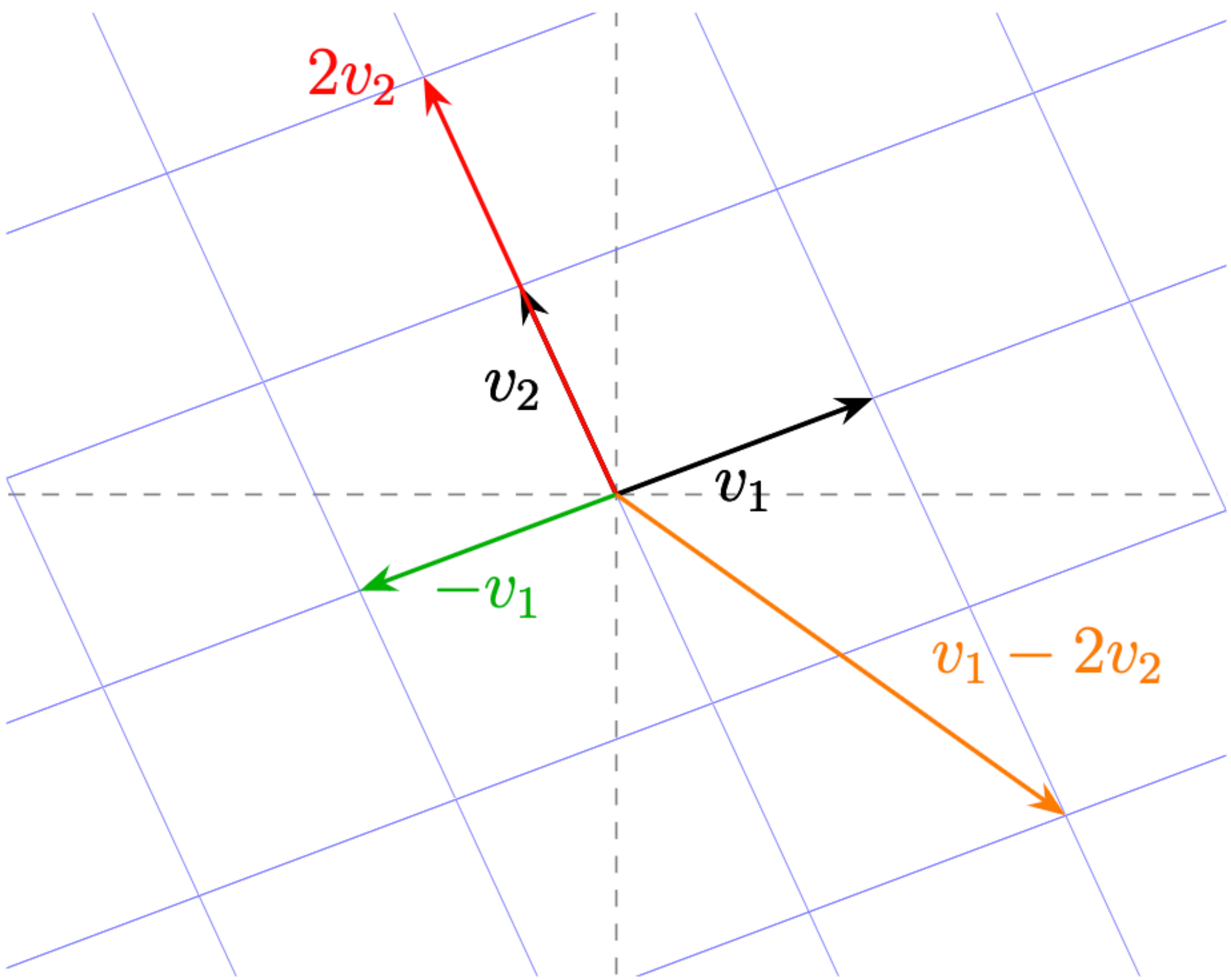
span of $v_1, \ldots, v_p$ = set of all linear combinations of $v_1, \ldots, v_p$

$$\operatorname{span}\{v_1, \ldots, v_p\} = \{c_1 v_1 + \cdots + c_n v_n \mid c_i \in \mathbb{R}\}.$$

Let $v_1, \ldots, v_p$ be vectors in $\mathbb{R}^n$.

span of $v_1, \ldots, v_p$ = set of all linear combinations of $v_1, \ldots, v_p$

$$\text{span}\{v_1, \ldots, v_p\} = \{c_1 v_1 + \cdots + c_n v_n \mid c_i \in \mathbb{R}\}.$$



$$\text{span}(\{v_1, v_2\}) = \text{plane determined by } v_1 \text{ and } v_2$$

if $v_1 \neq 0$
and $v_2 \neq 0$

Let $v_1, \dots, v_p$ be vectors in $\mathbb{R}^n$.

span of $v_1, \dots, v_p$ = set of all linear combinations of $v_1, \dots, v_p$

$$\text{span}\{v_1, \dots, v_p\} = \{c_1 v_1 + \cdots + c_n v_n \mid c_i \in \mathbb{R}\}.$$

$$\text{span}(\{v_1\}) = \text{line determined by } v_1$$

if $v_1 \neq 0$

Let $v_1, \ldots, v_p$ be vectors in $\mathbb{R}^n$.

span of $v_1, \ldots, v_p$ = set of all linear combinations of $v_1, \ldots, v_p$

$$\text{span}\{v_1, \ldots, v_p\} = \{c_1 v_1 + \cdots + c_n v_n \mid c_i \in \mathbb{R}\}.$$

$$\text{span}\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} =$$

Let $v_1, \ldots, v_p$ be vectors in $\mathbb{R}^n$.

span of $v_1, \ldots, v_p$ = set of all linear combinations of $v_1, \ldots, v_p$

$$\text{span}\{v_1, \ldots, v_p\} = \{c_1 v_1 + \cdots + c_n v_n \mid c_i \in \mathbb{R}\}.$$

$$\text{span}\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$$

Let $v_1, \ldots, v_p$ be vectors in $\mathbb{R}^n$.

span of $v_1, \ldots, v_p$ = set of all linear combinations of $v_1, \ldots, v_p$

$$\operatorname{span}\{v_1, \ldots, v_p\} = \{c_1 v_1 + \cdots + c_n v_n \mid c_i \in \mathbb{R}\}.$$

$$\operatorname{span}\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$$

$$\operatorname{span}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\} =$$

Let $v_1, \ldots, v_p$ be vectors in $\mathbb{R}^n$.

span of $v_1, \ldots, v_p$ = set of all linear combinations of $v_1, \ldots, v_p$

$$\text{span}\{v_1, \ldots, v_p\} = \{c_1 v_1 + \cdots + c_n v_n \mid c_i \in \mathbb{R}\}.$$

$$\text{span}\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$$

$$\text{span}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} a \\ 0 \end{bmatrix} : a \text{ any value} \right\}$$

Question:

$$\text{Is } \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ in span} \left\{ \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \right\}?$$

Equivalently:

$$\text{Does the system } x_1 \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ have a solution?}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

No pivots in the last column

system is consistent

Answer: yes.

Today’s poll code:

K3NARW

Today's poll code:

K3NARW

$$\text{Fact:} \quad \begin{bmatrix} 3 & -1 & 1 & 3 \\ 0 & 1 & 2 & 6 \\ 1 & 2 & 2 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\text{Is } \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \text{ in span} \left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}?$$

A. Yes

B. No

Today's poll code:

K3NARW

What is $\text{span}\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$?

A. $\left\{\begin{bmatrix}a\\a\end{bmatrix} : a \text{ is any value}\right\}$

B. $\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$

C. $\mathbb{R}^2$

Question:

$$\text{Is } \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ in span} \left\{ \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \right\}?$$

Equivalently:

$$\text{Does the system } x_1 \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ have a solution?}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

No pivots in the last column

system is consistent

Answer: yes.

Question:

$$\text{Is } \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix} \text{ in span} \left\{ \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \right\}?$$

$$\begin{bmatrix} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

No pivots in the last column

system is consistent

Answer: yes.

Solution set:

$$\{(3, 2)\}$$

$$3 \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}.$$

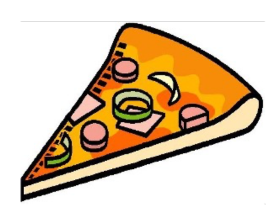
√ Math Club

Math Department Majors Welcome Event

For Math and CAS Data Science Majors

Calling all current and prospective Math and Data Science Majors! Join us to connect with fellow students, explore opportunities in mathematics, and kickstart your semester. Members of the math department advising team, the Math Club, and other organizations will share information about their programs.

Thursday, September 4, at 5:30 pm
in 115 Avery Hall
Refreshments served at 6:30 pm.

+ {  ,  }

www.math.unl.edu/math-club

Directed Reading Program

The DRP pairs advanced undergraduates with a graduate student mentor to explore a mathematical topic outside the standard undergraduate curriculum. Students read weekly, meet regularly with their mentor to discuss the material, and conclude the semester by giving a short presentation to fellow DRP participants.

Applications are due September 9

https://drp-unl.github.io/projects.html

To do list:

- WebWork 1.2 due Friday Sep 5
- WeBWorK 1.3 due Tuesday Sep 9
- WeBWork 2.1 due Tuesday Sep 9

Section 1.3
discussed on Friday

On Friday:

Quiz 2

at the beginning

of the recitation

on Lectures 1—3

Office hours

Mondays 5–6 pm and Wednesdays 2–3 pm
in Avery 339 (Dr Grifo)

Tuesdays 11–noon and Thursdays 1–2 pm
in Avery 337 (Kara)