Linear Algebra

Math 314 Fall 2025

Lecture 2

Lectures:

Mondays and Wednesdays 11:30—12:20
Bessey Hall 117

Recitations: Fridays
Time and location vary

Instructor: Dr. Eloísa Grifo
Teaching assistant: Kara Fagerstrom

Download the Nebraska App

Office hours

Mondays 5–6 pm and Wednesdays 2–3 pm
in Avery 339 (Dr. Grifo)

Tuesdays 11–noon and Thursdays 1–2 pm
in Avery 337 (Kara)

To do list:

- Download the Nebraska App
- Do the first bonus assignment, so we can learn your name
- Read the syllabus (see Canvas)
- WeBWorK 1.1 due September 2
- WeBWork 1.2 due September 5

On Friday:

Quiz 1

at the beginning

of the recitation

Problems 6 and 7 discussed on Friday

Download the Nebraska App

WeBWorK

START SOON!!

Try to at least login soon, to make sure the system is working for you

- WeBWorK 1.1 due September 2
- WeBWork 1.2 due September 5

Problems 6 and 7 discussed on Friday

The textbook is free (see Canvas)

Also on Canvas:
class notes
lecture slides (after class)

A **linear equation** in the variables $x_1, x_2, \ldots, x_n$ is an equation that can be written in the form:

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, \ldots, a_n, b$ are constants (real numbers).

The constant a_i is the **coefficient** of x_i, and b is the **constant term**.

A system of linear equations

$$\begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n = b_m \end{cases}$$

has

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

coefficient matrix

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

constant vector

$$\begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n = b_m \end{cases}$$

has

coefficient matrix

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

constant vector

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$[A|\mathbf{b}] = \begin{bmatrix} a_{11} & \cdots & a_{1n} & b_1 \\ \vdots & \ddots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} & b_m \end{bmatrix}$$

augmented matrix

In general, a system of linear equations may have:

- No solutions,
- Exactly one solution, or
- Infinitely many solutions.

But it can **never** have a finite number of solutions greater than one.

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Today’s poll code:

MN9T6V

Today's poll code:

MN9T6V

Poll:

How many solutions can a linear system have?

A. 0, 1, or 2
B. 0, 1, or infinitely many
C. Any number

Theorem. Any system of linear equations can be solved using the following **elementary row operations** on the augmented matrix:

1. Replace: Replace one row by the sum of itself and a multiple of another row.

2. Swap: Swap two rows.

3. Scale: Multiply all entries of a row by a nonzero constant.

We say two $n \times m$ matrices A and B are **row equivalent** if there exists a finite sequence of row operations that converts A into B.
We will write $A \sim B$ to say that A and B are equivalent.

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 5R_1} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 10 & -10 & 10 \end{bmatrix}$$

so the matrices are equivalent!

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 10 & -10 & 10 \end{bmatrix}$$

Theorem. If the augmented matrices of two linear systems are row equivalent, then the systems have the same solution set.

$$\begin{cases} 2x_2 - 8x_3 = 8 \\ x_1 - 2x_2 = 0 \\ 5x_1 - 5x_3 = 10 \end{cases}$$

$$\begin{bmatrix} 0 & 2 & -8 & 8 \\ 1 & -2 & 0 & 0 \\ 5 & 0 & -5 & 10 \end{bmatrix}$$

$$\begin{cases} 2x_2 - 8x_3 = 8 \\ x_1 - 2x_2 = 0 \\ 5x_1 - 5x_3 = 10 \end{cases} \qquad \begin{bmatrix} 0 & 2 & -8 & 8 \\ 1 & -2 & 0 & 0 \\ 5 & 0 & -5 & 10 \end{bmatrix}$$

$$\left[\begin{array}{ccc:c} 0 & 2 & -8 & 8 \\ 1 & -2 & 0 & 0 \\ 5 & 0 & -5 & 10 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc:c} 1 & -2 & 0 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right] \xrightarrow[R_3 \to \frac{1}{5} R_3]{R_2 \to \frac{1}{2} R_2} \left[\begin{array}{ccc:c} 1 & -2 & 0 & 0 \\ 0 & 1 & -4 & 4 \\ 1 & 0 & -1 & 10 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_1} \left[\begin{array}{ccc:c} 1 & -2 & 0 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 2 & -1 & 10 \end{array}\right] \xrightarrow{R_3 \to R_3 - 2R_2} \left[\begin{array}{ccc:c} 1 & -2 & 0 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 7 & 2 \end{array}\right].$$

$$\begin{cases} 2x_2 - 8x_3 = 8 \\ x_1 - 2x_2 = 0 \\ 5x_1 - 5x_3 = 10 \end{cases} \qquad \begin{bmatrix} 0 & 2 & -8 & 8 \\ 1 & -2 & 0 & 0 \\ 5 & 0 & -5 & 10 \end{bmatrix}$$

$$\left[\begin{array}{ccc:c} 0 & 2 & -8 & 8 \\ 1 & -2 & 0 & 0 \\ 5 & 0 & -5 & 10 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc:c} 1 & -2 & 0 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right] \xrightarrow[R_3 \to \frac{1}{5} R_3]{R_2 \to \frac{1}{2} R_2} \left[\begin{array}{ccc:c} 1 & -2 & 0 & 0 \\ 0 & 1 & -4 & 4 \\ 1 & 0 & -1 & 10 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_1} \left[\begin{array}{ccc:c} 1 & -2 & 0 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 2 & -1 & 10 \end{array}\right] \xrightarrow{R_3 \to R_3 - 2R_2} \left[\begin{array}{ccc:c} 1 & -2 & 0 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 7 & 2 \end{array}\right].$$

so $$\begin{bmatrix} 0 & 2 & -8 & 8 \\ 1 & -2 & 0 & 0 \\ 5 & 0 & -5 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 7 & 2 \end{bmatrix}$$

Solve $\begin{cases} 2x_2 - 8x_3 = 8 \\ x_1 - 2x_2 = 0 \\ 5x_1 - 5x_3 = 10 \end{cases}$

$$\begin{bmatrix} 0 & 2 & -8 & 8 \\ 1 & -2 & 0 & 0 \\ 5 & 0 & -5 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 7 & 2 \end{bmatrix}$$

Solve $\begin{cases} 2x_2 - 8x_3 = 8 \\ x_1 - 2x_2 = 0 \\ 5x_1 - 5x_3 = 10 \end{cases}$

$$\begin{bmatrix} 0 & 2 & -8 & 8 \\ 1 & -2 & 0 & 0 \\ 5 & 0 & -5 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 7 & 2 \end{bmatrix}$$

$$x_3 = \tfrac{2}{7}$$

$$x_2 = 4x_3 + 4 = \tfrac{8}{7} + 4 = \tfrac{36}{7}$$

$$x_1 = 2x_2 = \tfrac{72}{7}$$

The solution set is $\{(\frac{72}{7}, \frac{36}{7}, \frac{2}{7})\}$.

Given a matrix, the **leading entry** of a particular row is the first nonzero entry in that row (from the left).

A rectangular matrix is in **row echelon form** if:

- Any rows consisting entirely of zeros are at the bottom.
- The leading entry of each nonzero row is to the right of the leading entry of the row above.
- All entries below a leading entry (in the same column) are zero.

$$\begin{bmatrix} 2 & -3 & 0 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & \\ 1 & 4 & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -3 & \\ 0 & 0 & 0 & \\ 0 & 3 & 4 & \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & \\ 1 & 4 & \end{bmatrix}$$

in row echelon form

$$\begin{bmatrix} 1 & 3 & -3 & \\ 0 & 0 & 0 & \\ 0 & 3 & 4 & \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

in row echelon form

$$\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$$

not in row echelon form

$$\begin{bmatrix} 1 & 3 & -3 \\ 0 & 0 & 0 \\ 0 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

in row echelon form

$$\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$$

not in row echelon form

$$\begin{bmatrix} 1 & 3 & -3 \\ 0 & 0 & 0 \\ 0 & 3 & 4 \end{bmatrix}$$

not in row echelon form

$$\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

in row echelon form

$$\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$$

not in row echelon form

$$\begin{bmatrix} 1 & 3 & -3 \\ 0 & 0 & 0 \\ 0 & 3 & 4 \end{bmatrix}$$

not in row echelon form

$$\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$$

not in row echelon form

A matrix is in **reduced row echelon form (RREF)** if it is in row echelon form and:

- The leading entry in each nonzero row is 1.
- Each leading 1 is the only nonzero entry in its column.

reduced row echelon form

$$\begin{bmatrix} \cdots & 0 & 1 & \star & 0 & \star & 0 & \star \\ \cdots & 0 & 0 & \star & 1 & \star & 0 & \star \\ \cdots & 0 & 0 & 0 & 0 & 1 & \star & \star \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & 0 & 1 & \star \\ \multicolumn{8}{c}{\text{all zeros in these rows}} \end{bmatrix}$$

$\star$ can be anything

$$\begin{bmatrix} 2 & -3 & 0 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 1 & 4 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 36 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

row echelon form

not reduced

$$\begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 1 & 4 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 36 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

row echelon form
not reduced

$$\begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 1 & 4 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

row echelon form
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row echelon form
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row echelon form
not reduced

$$\begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 1 & 4 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

row echelon form
not reduced

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

row echelon form
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$$\begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 36 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

reduced row echelon form

reduced row echelon form

$$\begin{bmatrix} \cdots & 0 & 1 & \star & 0 & \star & 0 & \star \\ \cdots & 0 & 0 & \star & 1 & \star & 0 & \star \\ \cdots & 0 & 0 & 0 & 0 & 1 & \star & \star \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & 0 & 1 & \star \\ \multicolumn{8}{c}{\text{all zeros in these rows}} \end{bmatrix}$$

$\star$ can be anything

Theorem. Each matrix is row equivalent to one and only one matrix in reduced echelon form.

To solve a linear system of equations, we are going to:

1. Write out the augmented matrix corresponding to the system.
2. Get the augmented matrix in row reduced echelon form.

   Remember: there is only <u>one</u> possible row reduced echelon form.
3. Read the solution to the system from the row reduced echelon form.

A **pivot position** in a matrix, often shortened to **pivot**, is a position that corresponds to a leading 1 in the reduced echelon form of the matrix.

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$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

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A **pivot column** is a column that contains a pivot position.

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

pivot columns

Gauss Elimination:

0. Start with the leftmost nonzero column.

   This will be the first pivot column, with a pivot at the very top.

1. Choose a nonzero entry in this pivot column; swap rows if needed to move it into the top position, and do nothing if the pivot is already in place. From this point on, we will <u>not</u> switch this row with another ever again.

2. Use row operations to eliminate all other entries in this pivot column.

3. Move (right) to the next pivot column and repeat.

4. Scale pivot rows so that each pivot is 1. This can be done together with the previous steps, or all together at the end.

5. Eliminate all entries *above* each pivot.

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix}$$

$$\text{first pivot column} = \text{first nonzero column}$$

Gauss Elimination:

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5. Eliminate all entries *above* each pivot.

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix}$$

pivot
column

pivot

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix}$$

pivot

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & 6 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & 6 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{R_1 \mapsto \frac{1}{2} R_1} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & 6 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{R_1 \mapsto \frac{1}{2} R_1} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

pivot column?

pivot

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & 6 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{R_1 \mapsto \frac{1}{2} R_1} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

pivot column? **no!**

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & 6 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{R_1 \mapsto \frac{1}{2} R_1} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

pivot
column

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & 6 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{R_1 \mapsto \frac{1}{2} R_1} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

pivot

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & 6 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{R_1 \mapsto \frac{1}{2} R_1} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{R_2 \mapsto \frac{1}{4} R_2} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & 6 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{R_1 \mapsto \frac{1}{2} R_1} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{R_2 \mapsto \frac{1}{4} R_2} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_1 \mapsto R_1 + R_2} \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 4 \\ 2 & 6 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & 6 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{R_1 \mapsto \frac{1}{2} R_1} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

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pivot

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$$\xrightarrow{R_2 \mapsto \frac{1}{4} R_2} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_1 \mapsto R_1 + R_2} \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reduced row echelon form

Gauss Elimination complete!

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases} \longrightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases} \longrightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

first pivot column = first nonzero column

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases} \longrightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

pivot
column

Gauss Elimination:

0. Start with the leftmost nonzero column.

   This will be the first pivot column, with a pivot at the very top.

1. Choose a nonzero entry in this pivot column; swap rows if needed to move it into the top position, and do nothing if the pivot is already in place. From this point on, we will not switch this row with another ever again.

2. Use row operations to eliminate all other entries in this pivot column.

3. Move (right) to the next pivot column and repeat.

4. Scale pivot rows so that each pivot is 1. This can be done together with the previous steps, or all together at the end.

5. Eliminate all entries *above* each pivot.

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases} \longrightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

pivot

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

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pivot

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right].$$

Gauss Elimination:

0. Start with the leftmost nonzero column.

   This will be the first pivot column, with a pivot at the very top.

1. Choose a nonzero entry in this pivot column; swap rows if needed to move it into the top position, and do nothing if the pivot is already in place. From this point on, we will <u>not</u> switch this row with another ever again.

2. Use row operations to eliminate all other entries in this pivot column.

3. Move (right) to the next pivot column and repeat.

4. Scale pivot rows so that each pivot is 1. This can be done together with the previous steps, or all together at the end.

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$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases} \longrightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

pivot

$$\left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right].$$

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases} \longrightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

pivot

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right].$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right]$$

pivot column?

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases} \longrightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

pivot

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right].$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right]$$

pivot column? **no!**

$$
\begin{cases}
x_1 - x_2 + x_3 = 2 \\
2x_1 - 2x_2 + 3x_3 = 5 \\
-x_1 + x_2 - 2x_3 = -3
\end{cases}
\longrightarrow
\begin{bmatrix}
1 & -1 & 1 & 2 \\
2 & -2 & 3 & 5 \\
-1 & 1 & -2 & -3
\end{bmatrix}
$$

$$
\left[\begin{array}{ccc:c}
1 & -1 & 1 & 2 \\
2 & -2 & 3 & 5 \\
-1 & 1 & -2 & -3
\end{array}\right]
\xrightarrow{R_2 \to R_2 - 2R_1}
\left[\begin{array}{ccc:c}
1 & -1 & 1 & 2 \\
0 & 0 & 1 & 1 \\
-1 & 1 & -2 & -3
\end{array}\right]
\xrightarrow{R_3 \to R_3 + R_1}
\begin{array}{c}
\begin{array}{cccc}\text{pivot} & & & \end{array} \\
\left[\begin{array}{ccc:c}
1 & -1 & 1 & 2 \\
0 & 0 & 1 & 1 \\
0 & 0 & -1 & -1
\end{array}\right]
\end{array}.
$$

$$
\begin{array}{c}
\left[\begin{array}{ccc:c}
1 & -1 & 1 & 2 \\
0 & 0 & 1 & 1 \\
0 & 0 & -1 & -1
\end{array}\right] \\
\begin{array}{cccc} & & \begin{array}{c}\text{pivot}\\ \text{column}\end{array} & \end{array}
\end{array}
$$

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases} \longrightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

pivot

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right].$$

pivot

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right]$$

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases} \longrightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

pivot

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right].$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

pivot

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases} \longrightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix}$$

pivot

$$\left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ -1 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right].$$

$$\left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_2} \left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1 \to R_1 - R_2} \left[\begin{array}{ccc:c} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced row echelon form

Gauss Elimination complete!

How do we read the solutions from the RREF?

Once we obtain the RREF of a system:

- Among the columns corresponding to $x_1, \ldots, x_n$, columns without pivots correspond to free variables.
- Free variables can take arbitrary values.
- Each choice of free variables gives one solution to the system.

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced row echelon form

free variable

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced row echelon form

free variable

$$\begin{cases} x_1 - x_2 = 1 \\ x_3 = 1 \\ 0 = 0 \end{cases}$$

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced row echelon form

free variable

$$\begin{cases} x_1 - x_2 = 1 \\ x_3 = 1 \\ 0 = 0 \end{cases} \iff \begin{cases} x_1 = 1 + x_2 \\ x_3 = 1 \end{cases}$$

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced row echelon form

free variable

$$\begin{cases} x_1 = 1 + t \\ x_2 = t \\ x_3 = 1 \end{cases}$$

The solutions are all the points of the form

$$(1 + t, t, 1)$$

where t can be any real number.

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ -x_1 + x_2 - 2x_3 = -3 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 3 & 5 \\ -1 & 1 & -2 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced row echelon form

free variable

$$\begin{cases} x_1 = 1 + t \\ x_2 = t \\ x_3 = 1 \end{cases} \longrightarrow$$

Solution set:

$$\{(1+t, t, 1) \mid t \in \mathbb{R}\}$$

reduced row echelon form

$$\begin{bmatrix} \cdots & 0 & 1 & \star & 0 & \star & 0 & \star \\ \cdots & 0 & 0 & \star & 1 & \star & 0 & \star \\ \cdots & 0 & 0 & 0 & 0 & 1 & \star & \star \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & 0 & 1 & \star \\ \multicolumn{8}{c}{\text{all zeros in these rows}} \end{bmatrix}$$

$\star$ can be anything

Download the Nebraska App

Today’s poll code:

MN9T6V

Today's poll code:

MN9T6V

Poll: Is this matrix in reduced row echelon form?

$$\begin{bmatrix} 2 & 4 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

A. Yes

B. No

Today's poll code:

MN9T6V

Poll: Is this matrix in reduced row echelon form?

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 7 \\ 0 & 1 & 5 & 0 & 9 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

A. Yes

B. No

A system of linear equations is:

- **Consistent** if it has at least one solution.
- **Inconsistent** if it has no solutions.

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Theorem. A linear system of equations is inconsistent if and only if the reduced echelon form of its augmented matrix has a pivot in the last column.

A system of linear equations is:

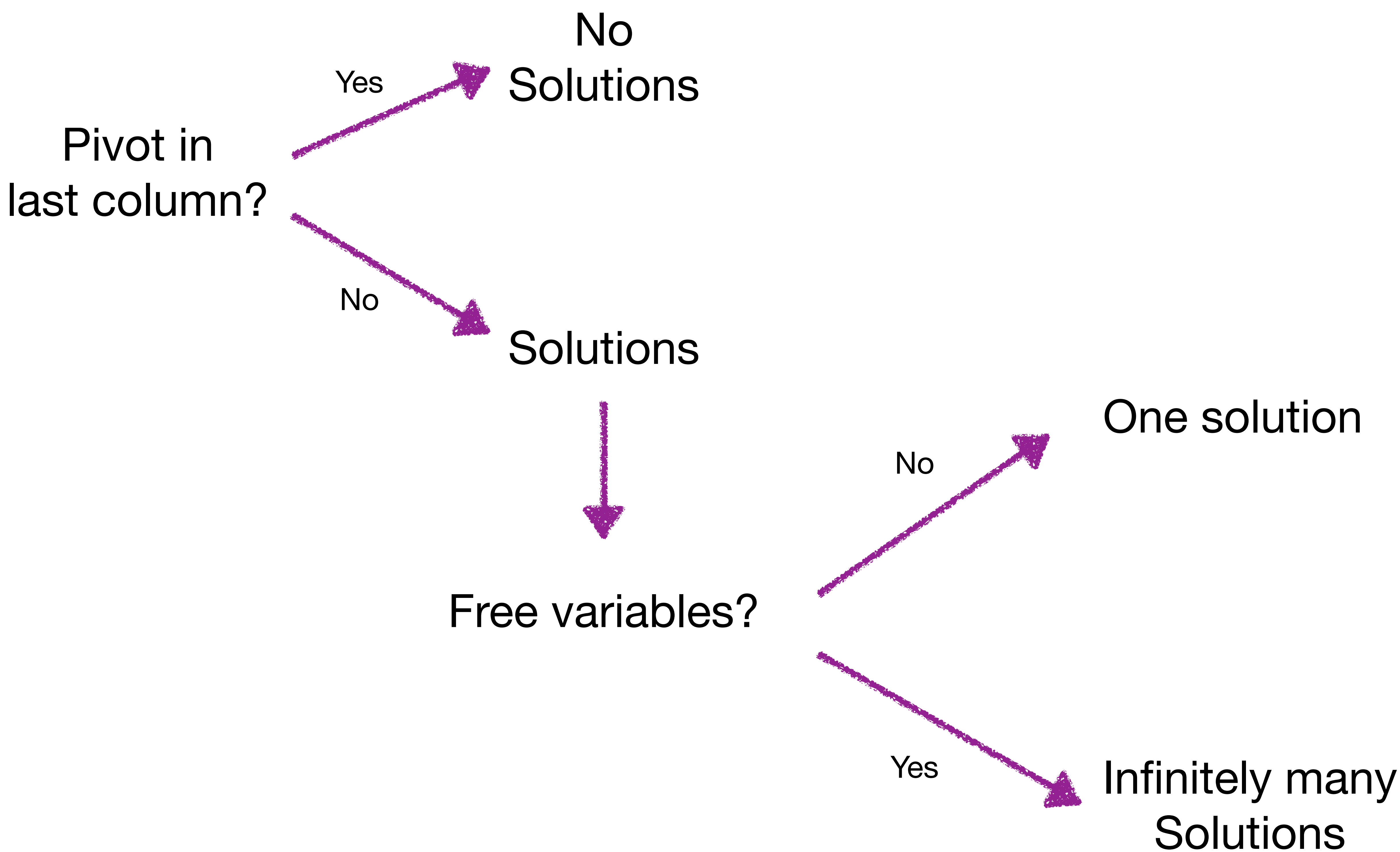
- **Consistent** if it has at least one solution.
- **Inconsistent** if it has no solutions.

Theorem. A linear system of equations is inconsistent if and only if the reduced echelon form of its augmented matrix has a pivot in the last column.

so

the system is inconsistent if the RREF has a row of the form

$$[0\ 0\ \cdots\ 0 \mid 1].$$


Pivot in
last column?
Yes
No
Solutions
No
Solutions
Free variables?
No
One solution
Yes
Infinitely many
Solutions

Pivot in last column	Yes	Yes	No	No
Free variables	Yes	No	No	Yes
Number of solutions	0	0	∞	1

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

inconsistent

no solution

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

inconsistent

no solution

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

consistent

one solution

augmented matrix

RREF

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

consistent

infinitely many solutions

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

inconsistent

no solution

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

consistent

one solution

Download the Nebraska App

Today's poll code:

MN9T6V

Today's poll code:

MN9T6V

The linear system with augmented matrix

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 7 \\ 0 & 1 & 5 & 0 & 9 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

has A. 0 B. 1 C. 2 D. infinitely many

solutions

Today’s poll code:

MN9T6V

The linear system with augmented matrix

$$\begin{bmatrix} 1 & 0 & 3 & 2 & 7 \\ 0 & 1 & 5 & 2 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

has A. 0 B. 1 C. 2 D. infinitely many

solutions

Today's poll code:

MN9T6V

The linear system with augmented matrix

$$\begin{bmatrix} 2 & 0 & 3 & 2 & 7 \\ 0 & 3 & 5 & 2 & 9 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

has A. 0 B. 1 C. 2 D. infinitely many

solutions

Directed Reading Program

The DRP pairs advanced undergraduates with a graduate student mentor to explore a mathematical topic outside the standard undergraduate curriculum. Students read weekly, meet regularly with their mentor to discuss the material, and conclude the semester by giving a short presentation to fellow DRP participants.

Applications are due September 9

https://drp-unl.github.io/projects.html

To do list:

- Download the Nebraska App
- Read the syllabus (see Canvas)
- Do the first bonus assignment, so we can learn your name
- WeBWorK 1.1 due September 2
- WeBWork 1.2 due September 5

Problems 6 and 7 discussed on Friday

On Friday:

Quiz 1

at the beginning

of the recitation

Gauss Elimination

Office hours

Mondays 5–6 pm and Wednesdays 2–3 pm
in Avery 339 (Dr Grifo)

Tuesdays 11–noon and Thursdays 1–2 pm
in Avery 337 (Kara)