

Quick review T/F questions

True or false. Justify!

- 1) If P and Q are two Sylow subgroups of a group G , then P and Q intersect trivially.
- 2) If P and Q are two Sylow p -subgroups of a group G , then P and Q intersect trivially.
- 3) If p and q are distinct primes, P is a Sylow p -subgroup of G and Q is a Sylow q -subgroup of G , then P and Q intersect trivially.
- 4) Sylow subgroups are normal.
- 5) The direct product of two groups is always abelian.
- 6) The semidirect product of two groups is always nonabelian.
- 7) There always exists a ring homomorphism between any two rings.
- 8) Given any ring R , there exists exactly one ring homomorphism $\mathbb{Z} \rightarrow R$.
- 9) Given any ring R , there exists exactly one ring homomorphism $R \rightarrow \mathbb{Z}$.
- 10) Given any ring R , there exists exactly one ring homomorphism $\mathbb{Z}/n \rightarrow R$.
- 11) Given any ring R , there exists exactly one ring homomorphism $R \rightarrow \mathbb{Z}/n$.
- 12) Every nonzero element in \mathbb{Z} is a unit.
- 13) Every ring contains at least two ideals.
- 14) Every domain is a field.
- 15) Every field is a domain.
- 16) Any ring that has only two ideals is a field.
- 17) The ring $\mathbb{Z}/n[x]$ is a domain.
- 18) If R and S are domains, then $R \times S$ is a domain.
- 19) Any subring of a domain is a domain.
- 20) Any subring of a field is a field.
- 21) The kernel of any ring homomorphism is an ideal of the domain.
- 22) The kernel of any ring homomorphism is a subring of the domain.
- 23) The image of any ring homomorphism is an ideal of the codomain.
- 24) The image of any ring homomorphism is a subring of the codomain.
- 25) Every proper ideal is the kernel of some ring homomorphism.
- 26) If R is a commutative ring and $(g) = R$, then g is a unit.
- 27) If R is a domain, then $R[x]$ is a domain.
- 28) If F is a field, then $F[x]$ is a field.
- 29) If $p \in \mathbb{Z}/2[x]$ has degree 3, then $\mathbb{Z}/2[x]/(p)$ has 4 elements.
- 30) If $p \in F[x]$ for some field F is irreducible, then $\gcd(p, f)$ is 1 or p .
- 31) In $R[x]$, the product of two monic polynomials can be zero.
- 32) If $uf + vg = 4$ in $\mathbb{Q}[x]$, then $f + (g)$ is a unit in $\mathbb{Q}[x]/(g)$.
- 33) The element $x^2 + 4 + (x^4 - x^2) \in \mathbb{Z}/5[x]/(x^4 - x^2)$ is a unit.
- 34) The element $x^3 + 2 + (x^4 - x^2) \in \mathbb{Z}/5[x]/(x^4 - x^2)$ is a unit.
- 35) The quotient ring $\mathbb{R}[x]/(x^3 - x - 6)$ is a field.
- 36) An element of a commutative ring R can be both a unit and a zerodivisor.
- 37) \mathbb{Z}/n is a domain if and only if it is a field.
- 38) Every nonzero element in $\mathbb{Z}/21$ is a unit.
- 39) In $\mathbb{Z}/77$, $(a) = (b)$ if and only if $a = b$.
- 40) Every ideal in $\mathbb{Z}/123$ is principal.
- 41) In any ring R , $ab = 0$ implies $a = 0$ or $b = 0$.
- 42) In any ring R , we can cancel addition: $a + b = a + c \implies b = c$.
- 43) In any ring R , we can cancel multiplication: $ab = ac \implies b = c$.
- 44) If I and J are ideals in a ring R , $I \cup J$ is an ideal in R .