

Linear Algebra

Math 314 Fall 2025

Today's poll code:

KJNMQ5

Lecture 14

Office hours

Mondays 5–6 pm

Wednesdays 2–3 pm

in Avery 339 (Dr. Grifo)

Tuesdays 11–noon

Thursdays 1–2 pm

in Avery 337 (Kara)

To do list:

- Webwork 4.1 due tomorrow
- Lab 1 due on Friday
- Webwork 4.2 due next Wednesday

Quiz on Friday

on vector spaces

Bonus poll points

Miss at most 2 lectures between Lecture 14 and Lecture 30



3 bonus poll points

Quick Recap

TL;DR: can add vectors and multiply by scalars, with good properties.

A **vector space** is a nonempty set V , whose elements we call **vectors**, with rules for **addition** of vectors in V and **multiplication by scalars**, satisfying the following properties:

- a) The addition $u + v$ of any vectors u and v in V is also a vector in V .
- b) The multiplication cv of a vector v by a scalar c is a vector in V .
- c) Commutativity: $u + v = v + u$ for all $u, v \in V$.
- d) Associativity: $(u + v) + w = u + (v + w)$ for all $u, v, w \in V$.
- e) There is a **zero vector** in V , denoted 0 , such that $0 + v = v + 0 = v$.
- f) For every vector v there is a vector $-v$ such that $v + (-v) = 0$.
- g) Distributivity: $c(u + v) = cu + cv$ and $(c + d)v = cv + dv$ for all $u, v \in V$ and all scalars c and d .
- h) Associativity of multiplication by scalars: $c(dv) = (cd)v$.
- i) $1v = v$.
 - $0v = 0$.
 - $c\mathbf{0} = \mathbf{0}$.
 - $-v = (-1)v$.

Consequence: we also have

Vector space: can add vectors and multiply by scalars, with good properties.

Example: \mathbb{R}^n with

(component-wise)
addition:

$$\begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} u_1 + v_1 \\ \vdots \\ u_n + v_n \end{bmatrix}$$

(component-wise)
multiplication
by scalars

$$c \cdot \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} c u_1 \\ \vdots \\ c u_n \end{bmatrix}$$

Vector space: can add vectors and multiply by scalars, with good properties.

$$\mathbf{M}_{n \times m} = \{\text{all } m \times n \text{ matrices}\}$$

(component-wise)

addition:

$$A + B := [a_{ij} + b_{ij}]$$

(component-wise)

multiplication
by scalars

$$cA = [ca_{ij}]$$

V vector space

W subset of V

W is a subspace of V if

1) The zero vector 0 is in W

2) W is closed under addition:

if u and v are in W , then $u + v$ is in W

3) W is closed under multiplication by scalars:

if v is in W , and c is any scalar, then cv is in W

A subspace is also a vector space!

V vector space

v_1, \dots, v_p vectors in V

The **span** of v_1, \dots, v_p is

the set of all linear combinations of v_1, \dots, v_p

$$\text{span}(\{v_1, \dots, v_p\}) = \{c_1v_1 + \dots + c_pv_p \mid c_i \in \mathbb{R}\}$$

Let v_1, \dots, v_p be vectors in \mathbb{R}^n .

span of v_1, \dots, v_p = set of all linear combinations of v_1, \dots, v_p

$$\text{span}(\{v_1, \dots, v_p\}) = \{c_1v_1 + \dots + c_pv_p \mid c_i \in \mathbb{R}\}$$

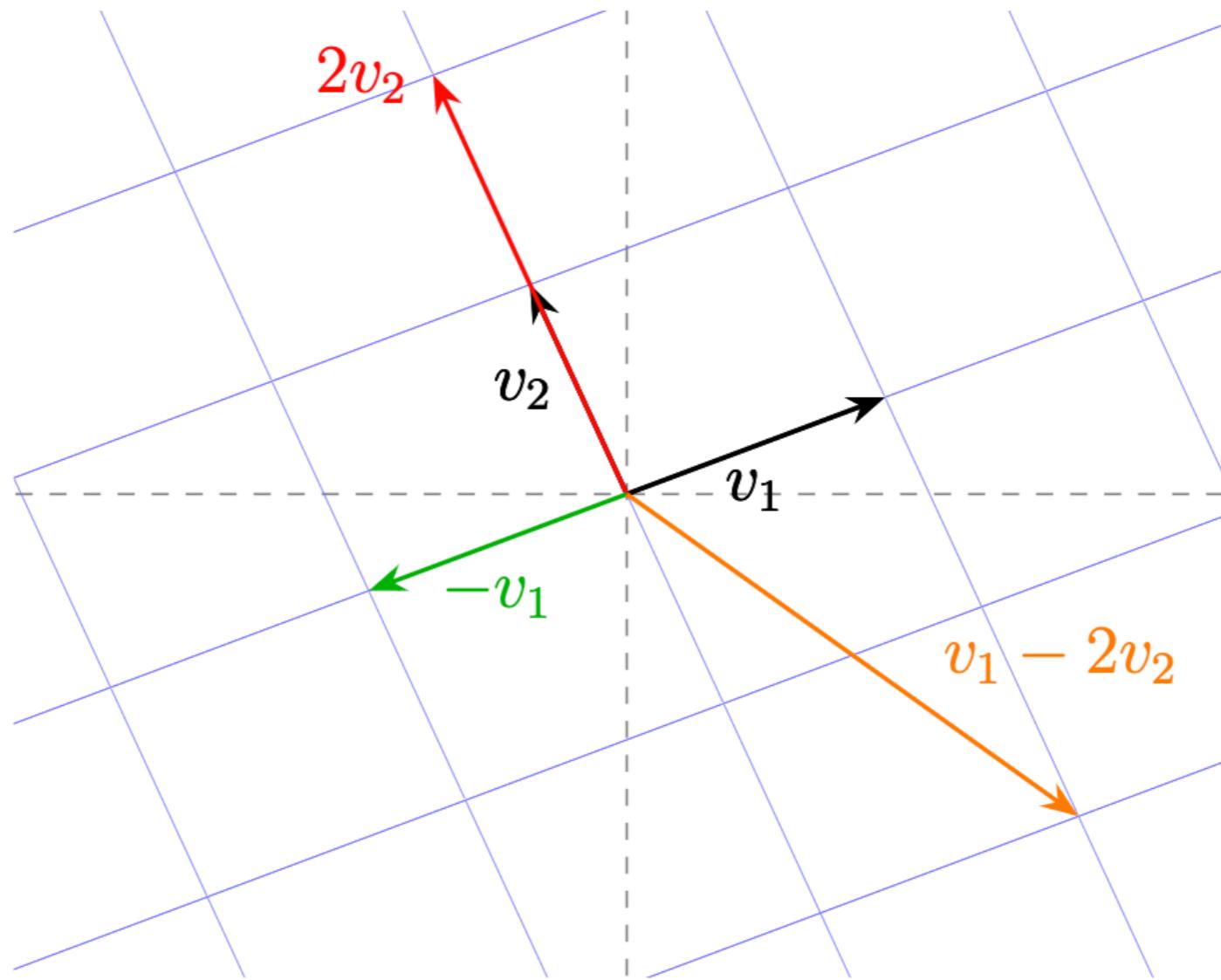
$\text{span}(\{v_1\})$ = line determined by v_1

if $v_1 \neq 0$

Let v_1, \dots, v_p be vectors in \mathbb{R}^n .

span of v_1, \dots, v_p = set of all linear combinations of v_1, \dots, v_p

$$\text{span}(\{v_1, \dots, v_p\}) = \{c_1v_1 + \dots + c_pv_p \mid c_i \in \mathbb{R}\}$$



$\text{span}(\{v_1, v_2\})$ = plane determined by v_1 and v_2
if $v_1 \neq 0$ and $v_2 \neq 0$ and v_1 and v_2 not multiples of each other

Theorem.

Given any vector space V , and vectors v_1, \dots, v_n in V ,

the set $\text{span}(\{v_1, \dots, v_n\})$ is a subspace of V .

Conversely, all subspaces of V are spanned by some collection of vectors in V .

Example:

$$V = \mathbb{R}^4$$

$$W = \left\{ \begin{bmatrix} a - 3b \\ b - a \\ a \\ b \end{bmatrix} : a, b \text{ any scalars} \right\}$$

is a subspace of \mathbb{R}^4

easier:

all vectors in W can be written as

$$\begin{bmatrix} a - 3b \\ b - a \\ a \\ b \end{bmatrix} = a \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

so $W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$

is a subspace

linear combination of these vectors!

Example:

In $M_{2 \times 2}$ (space of 2×2 matrices)

$$D = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \text{ any scalars} \right\} \quad \text{diagonal matrices}$$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = a \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$D = \text{span} \left(\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} \right)$$

so D is a subspace of $M_{2 \times 2}$

V vector space

v_1, \dots, v_n vectors in V

v_1, \dots, v_n form a **spanning set** for V if

$$V = \text{span} (\{v_1, \dots, v_n\})$$

In \mathbb{R}^n

$$\mathbf{e}_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \xrightarrow{\text{position } i}$$

*i*th standard vector

any vector v can be written as

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = v_1 e_1 + \cdots + v_n e_n$$

so

$$\mathbb{R}^n = \text{span} (\{e_1, \dots, e_n\})$$

$\{e_1, \dots, e_n\}$ forms a spanning set for \mathbb{R}^n

Linearly independent vectors

V vector space

v_1, \dots, v_p vectors in V

v_1, \dots, v_p are **linearly independent** if the equation

$$x_1v_1 + \cdots + x_pv_p = 0$$

has no nontrivial solutions.

v_1, \dots, v_p are **linearly dependent** if the equation

$$x_1v_1 + \cdots + x_pv_p = 0$$

has nontrivial solutions

V vector space

v_1, \dots, v_p vectors in V

v_1, \dots, v_p are **linearly independent** if

$$c_1v_1 + \cdots + c_pv_p = 0 \implies c_1 = \cdots = c_p = 0$$

v_1, \dots, v_p are **linearly dependent** if

$$c_1v_1 + \cdots + c_pv_p = 0$$

for some c_1, \dots, c_p not all zero

Theorem. Let V be a vector space.

Any set of vectors that contains the zero vector
is linearly dependent.

v_1, \dots, v_p are **linearly dependent** if

$$c_1v_1 + \cdots + c_pv_p = 0$$

for some c_1, \dots, c_p not all zero

Example:

in \mathbb{R}^2 $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ are linearly dependent

for example

$$2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-1) \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 0$$

v_1, \dots, v_p are **linearly dependent** if

$$c_1v_1 + \cdots + c_pv_p = 0$$

for some c_1, \dots, c_p not all zero

Example:

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for example

$$2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-1) \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 0$$

Any two nonzero vectors that are scalar multiples of each other
are linearly dependent.

In any vector space

Any two nonzero vectors that are scalar multiples of each other
are linearly dependent.

v is any nonzero vector and $t \neq 1$



v and tv are linearly dependent

$$t \cdot v + (-1) \cdot (tv) = 0$$

In any vector space

A set $\{v_1, \dots, v_p\}$ of two or more vectors is linearly dependent
if and only if
one of the vectors is a linear combination of the others.

Note: this does not say that every v_i is a linear combination of the rest.

Example:

In $M_{2 \times 2}$ (space of 2×2 matrices)

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ are linearly independent

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ are linearly dependent

$$-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - 2 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

spanning set for \mathbb{R}^2

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

spanning set for \mathbb{R}^2 linearly dependent

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

not spanning set for \mathbb{R}^2

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

not spanning set for \mathbb{R}^2

linearly independent

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

spanning set for \mathbb{R}^2

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

spanning set for \mathbb{R}^2

linearly independent

Bases and dimension

A **basis**^{*} for a vector space V is
a spanning set of linearly independent vectors.

The **dimension** of a vector space is
the number of vectors in a basis.

Notation: $\dim(V)$

* The plural of basis is bases.

The **dimension** of a vector space is
the number of vectors in a basis.

Notation: $\dim(V)$

Theorem. Let V be a vector space.
Every basis for V has the same number of elements.

$$\dim(\mathbb{R}^2) =$$

$$\dim(\mathbb{R}^3) =$$

$$\dim(\mathbb{R}^n) =$$

$\dim(V)$ = number of vectors in a basis for V

$$\dim(\mathbb{R}^2) = 2$$

Basis: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\dim(\mathbb{R}^3) =$$

$$\dim(\mathbb{R}^n) =$$

$\dim(V)$ = number of vectors in a basis for V

$$\dim(\mathbb{R}^2) = 2$$

Basis: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\dim(\mathbb{R}^3) = 3$$

Basis: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\dim(\mathbb{R}^n) =$$

$\dim(V)$ = number of vectors in a basis for V

$$\dim(\mathbb{R}^2) = 2$$

Basis: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\dim(\mathbb{R}^3) = 3$$

Basis: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\dim(\mathbb{R}^n) = n$$

Basis: e_1, \dots, e_n

$\dim(V)$ = number of vectors in a basis for V

Warning:

The same vector space can have many different basis!

Example: \mathbb{R}^2

Basis 1: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Basis 2: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Basis 3: $\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Basis 4: $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\dim(M_{2 \times 2}) =$$

all 2×2 matrices

$$\dim(M_{m \times n}) =$$

all $m \times n$ matrices

$$\dim(M_{2 \times 2}) = 4$$

all 2×2 matrices

Basis: $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

$$\dim(M_{m \times n}) =$$

all $m \times n$ matrices

$$\dim(M_{2 \times 2}) = 4$$

all 2×2 matrices

Basis: $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

$$\dim(M_{m \times n}) = mn$$

all $m \times n$ matrices

Basis: all the matrices with one 1
and zeroes everywhere else

$$\dim(\{0\})=\;0$$

$$\dim(\mathbb{P}^1) =$$

polynomials $a + bt$

$$\dim(\mathbb{P}^2) =$$

polynomials $a + bt + ct^2$

$$\dim(\mathbb{P}^n) =$$

polynomials $a_0 + a_1t + \cdots + a_nt^n$

$$\dim(\mathbb{P}) =$$

polynomials of any degree

$$\dim(\mathbb{P}^1) = 2$$

Basis: $1, t$

polynomials $a + bt$

$$\dim(\mathbb{P}^2) =$$

polynomials $a + bt + ct^2$

$$\dim(\mathbb{P}^n) =$$

polynomials $a_0 + a_1t + \cdots + a_nt^n$

$$\dim(\mathbb{P}) =$$

polynomials of any degree

$$\dim(\mathbb{P}^1) = 2$$

polynomials $a + bt$

Basis: $1, t$

$$\dim(\mathbb{P}^2) = 3$$

polynomials $a + bt + ct^2$

Basis: $1, t, t^2$

$$\dim(\mathbb{P}^n) =$$

polynomials $a_0 + a_1t + \cdots + a_nt^n$

$$\dim(\mathbb{P}) =$$

polynomials of any degree

$$\dim(\mathbb{P}^1) = 2$$

polynomials $a + bt$

Basis: $1, t$

$$\dim(\mathbb{P}^2) = 3$$

polynomials $a + bt + ct^2$

Basis: $1, t, t^2$

$$\dim(\mathbb{P}^n) = n$$

polynomials $a_0 + a_1t + \cdots + a_nt^n$

Basis: $1, t, \dots, t^n$

$$\dim(\mathbb{P}) =$$

polynomials of any degree

$$\dim(\mathbb{P}^1) = 2$$

Basis: $1, t$

polynomials $a + bt$

$$\dim(\mathbb{P}^2) = 3$$

Basis: $1, t, t^2$

polynomials $a + bt + ct^2$

$$\dim(\mathbb{P}^n) = n$$

Basis: $1, t, \dots, t^n$

polynomials $a_0 + a_1t + \dots + a_nt^n$

$$\dim(\mathbb{P}) = \infty$$

Basis: $1, t, t^2, \dots$

polynomials of any degree

A vector space is **finite-dimensional** if its dimension is finite.

\mathbb{R}^n is finite-dimensional

\mathbb{P} is infinite-dimensional

Today's poll code:

KJNMQ5

Today's poll code:

KJNMQ5

What is the dimension of \mathbb{R}^{2025} ?

Today's poll code:

KJNMQ5

What is the dimension of the space $M_{2 \times 3}$ of 2×3 matrices?

Theorem.

Linearly independent vectors can always be extended to a basis.

A spanning set always contains a basis.

Theorem (Basis theorem).

Let V be a vector space of dimension n .

Any set of n linearly independent vectors is a basis for V .

Any set of n vectors that spans V is a basis for V .

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

spanning set for \mathbb{R}^2

linearly dependent

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

~~$$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$~~

$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

basis for \mathbb{R}^2

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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spanning set for \mathbb{R}^2

linearly dependent

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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~~$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$~~

basis for \mathbb{R}^2

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

spanning set for \mathbb{R}^2

linearly dependent

~~$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$~~

$$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

basis for \mathbb{R}^2

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

spanning set for \mathbb{R}^2

linearly dependent

~~$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$~~

$$v_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

basis for \mathbb{R}^2

Example:

in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

spanning set for \mathbb{R}^2

linearly dependent

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

~~$$v_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$~~

$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

basis for \mathbb{R}^2

Example: in \mathbb{R}^2

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

spanning set for \mathbb{R}^2

linearly dependent

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

~~$$v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$~~

NOT a basis for \mathbb{R}^2

Example: Consider the subspace of \mathbb{R}^4

$$W = \left\{ \begin{bmatrix} a - 3b + 6c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} : a, b, c, d \text{ any real numbers} \right\}$$

Example: Consider the subspace of \mathbb{R}^4

$$W = \left\{ \begin{bmatrix} a - 3b + 6c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} : a, b, c, d \text{ any real numbers} \right\}$$

$$W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\} \right)$$

Example: Consider the subspace of \mathbb{R}^4

$$W = \left\{ \begin{bmatrix} a - 3b + 6c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} : a, b, c, d \text{ any real numbers} \right\}$$

$$W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\} \right)$$

$$\dim(W) \leq 4 \quad (\text{since } W \text{ is spanned by 4 vectors})$$

But what is the dimension of W ?

Example: Consider the subspace of \mathbb{R}^4

$$W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\} \right)$$

$$-2 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix} \Rightarrow W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\} \right)$$

Example: Consider the subspace of \mathbb{R}^4

$$W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\} \right)$$

$$-2 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix} \Rightarrow W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\} \right)$$

Is $\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\}$ a basis for W ?

Example: Consider the subspace of \mathbb{R}^4

$$W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\} \right)$$

Is $\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\}$ a basis for W ?

Example: Consider the subspace of \mathbb{R}^4

$$W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\} \right)$$

Is $\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\}$ linearly independent?

Example: Consider the subspace of \mathbb{R}^4

$$W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\} \right)$$

Is $\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\}$ linearly independent?

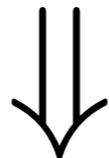
$$\begin{bmatrix} 1 & -3 & 0 & 4 \\ 5 & 0 & 0 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

pivot in every
column
 \Downarrow
yes

Example: Consider the subspace of \mathbb{R}^4

$$W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\} \right)$$

$$\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix} \right\} \text{ is a basis for } W$$



$$\dim(W) = 3$$

To do list:

- Webwork 4.1 due Tuesday Quiz on Friday
- Lab 1 due Friday on subspaces
- Webwork 4.2 due next Wednesday

Office hours this week

Mondays 5–6 pm and Wednesdays 2–3 pm
in Avery 339 (Dr Grifo)

Tuesdays 11–noon and Thursdays 1–2 pm
in Avery 337 (Kara)