

## Quick review T/F questions

True or false. Justify!

- 1) If  $P$  and  $Q$  are two Sylow subgroups of a group  $G$ , then  $P$  and  $Q$  intersect trivially. **FALSE**
- 2) If  $P$  and  $Q$  are two Sylow  $p$ -subgroups of a group  $G$ , then  $P$  and  $Q$  intersect trivially. **FALSE**
- 3) If  $p$  and  $q$  are distinct primes,  $P$  is a Sylow  $p$ -subgroup of  $G$  and  $Q$  is a Sylow  $q$ -subgroup of  $G$ , then  $P$  and  $Q$  intersect trivially. **TRUE**
- 4) Sylow subgroups are normal. **FALSE**
- 5) The direct product of two groups is always abelian. **FALSE**
- 6) The semidirect product of two groups is always nonabelian. **FALSE**
- 7) There always exists a ring homomorphism between any two rings. **FALSE**
- 8) Given any ring  $R$ , there exists exactly one ring homomorphism  $\mathbb{Z} \rightarrow R$ . **TRUE**
- 9) Given any ring  $R$ , there exists exactly one ring homomorphism  $R \rightarrow \mathbb{Z}$ . **FALSE**
- 10) Given any ring  $R$ , there exists exactly one ring homomorphism  $\mathbb{Z}/n \rightarrow R$ . **FALSE**
- 11) Given any ring  $R$ , there exists exactly one ring homomorphism  $R \rightarrow \mathbb{Z}/n$ . **FALSE**
- 12) Every nonzero element in  $\mathbb{Z}$  is a unit. **FALSE**
- 13) Every ring contains at least two ideals. **TRUE**
- 14) Every domain is a field. **FALSE**
- 15) Every field is a domain. **TRUE**
- 16) Any ring that has only two ideals is a field. **FALSE**
- 17) The ring  $\mathbb{Z}/n[x]$  is a domain. **FALSE**
- 18) If  $R$  and  $S$  are domains, then  $R \times S$  is a domain. **FALSE**
- 19) Any subring of a domain is a domain. **TRUE**
- 20) Any subring of a field is a field. **FALSE**
- 21) The kernel of any ring homomorphism is an ideal of the domain. **TRUE**
- 22) The kernel of any ring homomorphism is a subring of the domain. **FALSE**
- 23) The image of any ring homomorphism is an ideal of the codomain. **FALSE**
- 24) The image of any ring homomorphism is a subring of the codomain. **TRUE**
- 25) Every proper ideal is the kernel of some ring homomorphism.
- 26) If  $R$  is a commutative ring and  $(g) = R$ , then  $g$  is a unit. **TRUE**
- 27) If  $R$  is a domain, then  $R[x]$  is a domain. **TRUE**
- 28) If  $F$  is a field, then  $F[x]$  is a field. **FALSE**
- 29) If  $p \in \mathbb{Z}/2[x]$  has degree 3, then  $\mathbb{Z}/2[x]/(p)$  has 4 elements. **TRUE**
- 30) If  $p \in F[x]$  for some field  $F$  is irreducible, then  $\gcd(p, f)$  is 1 or  $p$ . **TRUE**
- 31) In  $R[x]$ , the product of two monic polynomials can be zero. **TRUE**
- 32) If  $uf + vg = 4$  in  $\mathbb{Q}[x]$ , then  $f + (g)$  is a unit in  $\mathbb{Q}[x]/(g)$ . **TRUE**
- 33) The element  $x^2 + 4 + (x^4 - x^2) \in \mathbb{Z}/5[x]/(x^4 - x^2)$  is a unit. **FALSE**
- 34) The element  $x^3 + 2 + (x^4 - x^2) \in \mathbb{Z}/5[x]/(x^4 - x^2)$  is a unit. **TRUE**
- 35) The quotient ring  $\mathbb{R}[x]/(x^3 - x - 6)$  is a field. **FALSE**
- 36) An element of a commutative ring  $R$  can be both a unit and a zerodivisor. **FALSE**
- 37)  $\mathbb{Z}/n$  is a domain if and only if it is a field. **TRUE**
- 38) Every nonzero element in  $\mathbb{Z}/21$  is a unit. **FALSE**
- 39) In  $\mathbb{Z}/77$ ,  $(a) = (b)$  if and only if  $a = b$ . **FALSE**
- 40) Every ideal in  $\mathbb{Z}/123$  is principal. **TRUE**
- 41) In any ring  $R$ ,  $ab = 0$  implies  $a = 0$  or  $b = 0$ . **FALSE**
- 42) In any ring  $R$ , we can cancel addition:  $a + b = a + c \implies b = c$ . **TRUE**
- 43) In any ring  $R$ , we can cancel multiplication:  $ab = ac \implies b = c$ . **FALSE**
- 44) If  $I$  and  $J$  are ideals in a ring  $R$ ,  $I \cup J$  is an ideal in  $R$ . **FALSE**