Discussion Questions

- Write the contrapositive, and the converse of each statement. Is the statement true or false? Is the converse true or false? Explain why (but don't write a full proof). For each statement below, a, b are real numbers.
  - ♢ If a is irrational, then $1/a$ is irrational.
  - ♢ If a and b are both irrational, then ab is irrational.
  - ♢ If $x > 3$ then $x^2 > 9$.
- Write the negation of each statement. Is the statement true or false? Explain why (but don't write a full proof).
  - ♢ $\exists x \in \mathbb{Q}$: $x^2 = 2$.
  - ♢ $\forall x \in \mathbb{Q}$, $x^2 > 0$.
  - ♢ $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$: $xy = 1$.
  - ♢ $\exists x \in \mathbb{R}$: $\forall y \in \mathbb{R}$, $e^y < x$.
  - ♢ $\exists x \in \mathbb{R}$: $\forall y \in \mathbb{R}$, $\sin(y) < x$.
- Prove the following statements using the axioms of $\mathbb{R}$ and facts we have proven in class.
  - ♢ Let x be a real number. If there is a real number y such that $xy = 1$, then x is nonzero.[1]
  - ♢ If x is a nonzero real number, then x^2 is also nonzero.[2]
  - ♢ For any real number x, $x \geq 0$ if and only if $-x \leq 0$.[3]
  - ♢ $0 \leq 1$. [4]
  - ♢ For any real number x, $(-1) \cdot x = -x$.
  - ♢ The product of two negative real numbers is negative.

[1] Hint: Consider the contrapositive of this statement.

[2] Hint: Use x^{-1} and the previous statement.

[3] Hint: Add something to both sides.

[4] Hint: Try a proof by contradiction and use the previous fact.