TRUE or FALSE. Justify.

(1) There is a continuous function $f : [2, 3] \to \mathbb{R}$ with range $[3, 5]$.

(2) There is a continuous function $f : [2, 3] \to \mathbb{R}$ with range $(3, 5]$.

(3) There is a continuous function $f : [2, 3] \to \mathbb{R}$ with range $[3, 4] \cup [5, 6]$.

(4) Every continuous function is differentiable.

(5) Every differentiable function is continuous.

(6) If $f'(r) < 0$, then there is some $\delta > 0$ such that $f(x) > f(r)$ whenever $r - \delta < x < r$.

(7) If $f'(r) < 0$, then there is some $\delta > 0$ such that $f(x) > f(r)$ whenever $r < x < r + \delta$.

(8) The function $f(x) = x^3 - 2x^2 + 5$ is increasing on the interval $(1, 2)$.

(9) The function $f(x) = x^3 - 2x^2 + 5$ is decreasing on the interval $(1, 2)$.

(10) If f is differentiable on $\mathbb{R}$, and $f'(x) > 0$ for all $x \in \mathbb{R}$, then $f(a) < f(b)$ for all $a < b$.

(11) The function $f(x) = \begin{cases} x^3 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$ is differentiable on $\mathbb{R}$.

(12) If $f'(x) = 0$ for all x in a subset $S \subseteq \mathbb{R}$, then there is some $c \in \mathbb{R}$ such that $f(x) = c$ for all $x \in S$.

(13) The function $f(x) = \begin{cases} \frac{x}{2} + x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is differentiable on all of $\mathbb{R}$.

(14) There is some $\delta > 0$ such that $\frac{x}{2} + x^2 \sin(\frac{1}{x}) > 0$ whenever $0 < x < \delta$. (Use derivatives to explain.)

(15) There is some $\delta > 0$ such that the function $f(x) = \begin{cases} \frac{x}{2} + x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is increasing on $(-\delta, \delta)$.

(16) Let $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$, and $\{c_n\}_{n=1}^{\infty}$ be sequences. The negation of the statement "If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ converge, then $\{c_n\}_{n=1}^{\infty}$ converges" is "If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ converge, then $\{c_n\}_{n=1}^{\infty}$ diverges".

(17) The commutative property/axiom of addition says that $(x + y) + z = x + (y + z)$.

(18) Every nonempty set of real numbers that is bounded above has a maximum element.

(19) If S is a set of real numbers and $\sup(S) \in S$ then $\sup(S)$ is the maximum element of S.

(20) Every nonempty set of natural numbers that is bounded below has a minimum element.

(21) Every nonempty set of integers that is bounded above has a maximum element.

(22) The supremum of the set $\{x \in \mathbb{Q} \mid x < \pi\}$ is π.

(23) The supremum of the set $\{x \in \mathbb{Q} \mid x > \pi\}$ is π.

(24) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to L, then there is some $N \in \mathbb{R}$ such that for all natural numbers $n > N$, $a_n = L$.

(25) For every real number L there is a strictly decreasing sequence of rational numbers that converges to L.

(26) A sequence of negative numbers can converge to a positive number.

(27) Every decreasing sequence is convergent.

(28) Every convergent sequence is bounded.

(29) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent sequences, then $\{3a_n^2 + a_n b_n - 6b_n\}_{n=1}^{\infty}$ is a convergent sequence.

(30) If for every $\varepsilon > 0$, there is some $N \in \mathbb{R}$ such that $|a_n - a_{n+1}| < \varepsilon$ for all $n > N$, then $\{a_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

(31) If $\lim_{x \to -1} f(x) > 5$, then there is some $\delta > 0$ such that $f(x) > 5$ whenever $x \in (-1 - \delta, -1 + \delta) \setminus \{-1\}$.

(32) If $\lim_{x \to -1} f(x) \geq 5$, then there is some $\delta > 0$ such that $f(x) \geq 5$ whenever $x \in (-1 - \delta, -1 + \delta) \setminus \{-1\}$.

(33) If $\{a_n\}_{n=1}^{\infty}$ converges to 1 and $\{b_n\}_{n=1}^{\infty}$ converges to -2, then $\{a_{3n-1}b_n - b_{n^2}/4\}_{n=1}^{\infty}$ converges to $-5 = (3 \cdot 1 - 1)(-2) - (-2)^2/4$.

(34) There is a sequence without any convergent subsequence.

(35) Every convergent sequence is a bounded sequence.

(36) If a sequence diverges to ∞, then it is not bounded above.

(37) If a sequence is not bounded above, then it diverges to ∞.

(38) If f and g are functions defined on $\mathbb{R}$, and $f(x) = g(x)$ for all $x \neq a$, and f has a limit as x approaches a, then $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$.

(39) If $\lim_{x\to 0} f(x) = 3$, then the sequence $\{a_n\}_{n=1}^{\infty}$ converges to 0, where $a_n = f(1/n)/n$.

(40) If f and g are continuous on $(-7, 7)$, and $g(4) = -1$, then $\lim_{x\to 4}(f \circ g)(x) = f(-1)$.

(41) The sequence $\{a_n\}_{n=1}^{\infty}$ where

$$a_n = \quad n \cdot \sqrt{2} \; - \; \text{the largest integer that is less than } n \cdot \sqrt{2}$$

has a convergent subsequence.

(42) If two different subsequences of $\{a_n\}_{n=1}^{\infty}$ converge, then $\{a_n\}_{n=1}^{\infty}$ converges.

(43) The function $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$ is continuous on $\mathbb{R}$.

(44) If the domain of f is $\mathbb{R}$, then f is continuous at some point.

(45) The function $f(x) = |x^2 - |x^3 - 3||$ is continuous on $\mathbb{R}$.

(46) If $f(x)$ is continuous at $x = a$ then $\lim_{x\to a} f(x)$ exists.

(47) If f is continuous on $\mathbb{R}$ and $a < b$, and $y > f(a) > f(b)$, then there is no $c \in [a, b]$ such that $f(c) = y$.

(48) If f is continuous on $\mathbb{R}$ and $a < b$, and $f(a) > y > f(b)$, then there is no $c \in [a, b]$ such that $f(c) = y$.

(49) There is a sequence $\{a_n\}_{n=1}^{\infty}$ such that $\{r \in \mathbb{R} \mid r \text{ is a limit of a subseq. of } a_n\} = [0, 7]$.

(50) There is a sequence $\{a_n\}_{n=1}^{\infty}$ such that $\{r \in \mathbb{R} \mid r \text{ is a limit of a subseq. of } a_n\} = (0, 7)$.