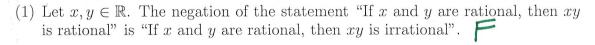
DISCUSSION QUESTIONS

TRUE or FALSE. Justify.



- (2) Let $x, y \in \mathbb{R}$. The contrapositive of the statement "If x and y are rational, then xy is rational" is "If xy is irrational, then x and y are irrational".
- (3) The associative property/axiom of addition says that (x+y)+z=x+(y+z).
- (4) Every set of real numbers that is bounded above has a supremum.
- (5) There is a set S of real numbers such that $\sup(S)$ exists, but $\sup(S) \notin S$.
- (6) If a < b are real numbers, there is a natural number $n \in \mathbb{N}$ such that a < n < b.
- (7) Every nonempty set of real numbers has a smallest element (i.e., a minimum element).
- (8) Every nonempty set of integers that is bounded below has a smallest element 7 (i.e., a minimum element).
- (9) If $S \subseteq \mathbb{R}$ is bounded above, there there is a natural number b such that b is an \mathcal{T} upper bound for S.
- (10) It is possible to prove that there is a real number x such that $x^2 = 2$ using just the first 10 axioms (i.e., without using the Completeness Axiom).
- (11) Every set of real numbers satisfies the property that "for all $x \in S$, there exists a real number y such that $x < y^2$ ".
- (12) Every set of real numbers satisfies the property that "for all $x \in S$, there exists a real number y such that $y^2 < x$ ".
- (13) The supremum of the set $\{1/n \mid n \in \mathbb{N}\}$ is 1.
- (14) The supremum of the set $\{-1/n \mid n \in \mathbb{N}\}$ is -1.
- (15) The negation of the statement "for all $x \in S$, there exists a real number y such that $x < y^2$ " is "for all $x \in S$, there exists a real number y such that $x \ge y^2$ ".

(16) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to L, then there is some $N \in \mathbb{R}$ such that for all natural numbers n > N, $a_n = L$. (17) For every real number L there is a sequence that converges to L. (18) For every real number L there is a sequence $\{a_n\}_{n=1}^{\infty}$ such that $a_n \neq L$ for all $n \in \mathbb{N}$ and converges to L. (19) A sequence of positive numbers can converge to a negative number. (20) A sequence of positive numbers can converge to zero. (21) Every increasing sequence is bounded below. (22) Every increasing sequence is convergent. (23) Every convergent sequence is either increasing or decreasing. (24) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are sequences, $\{a_n\}_{n=1}^{\infty}$ converges to L, and there is some $N \in \mathbb{R}$ such that $a_n = b_n$ for n > N, then $\{b_n\}_{n=1}^{\infty}$ converges to L. (25) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent sequences, then $\{a_n+b_n\}_{n=1}^{\infty}$ is a convergent sequence. (26) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent sequences, and $b_n \neq 0$ for all $n \in \mathbb{N}$, then $\{a_n/b_n\}_{n=1}^{\infty}$ is a convergent sequence. (27) The sequence $\left\{\frac{3n^2-4n+7}{6n^2+1}\right\}_{n=1}^{\infty}$ converges to 1/2. (28) The negation of " $\{a_n\}_{n=1}^{\infty}$ is a monotone sequence" is "there exists $n \in \mathbb{N}$ such that $a_n > a_{n+1}$ and $a_n < a_{n+1}$ ". (29) Every convergent sequence of rational numbers converges to a rational number.

(30) Every convergent sequence of natural numbers converges to a natural number.