TRUE or FALSE. Justify.

(1) Every sequence has a convergent subsequence. FALSE

(2) Every bounded sequence is a convergent sequence. FALSE

(3) Every convergent sequence is a bounded sequence. TRUE

(4) To prove that the formula

$$1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^{n-1}}=2-\frac{1}{2^{n-1}}$$

is true for every natural number $n \in \mathbb{N}$ by the Principle of Mathematical Induction, it is logically sufficient to show that

- $1 = 2 - \frac{1}{2^{1-1}}$, and
- $1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^{k-1}} \neq 2-\frac{1}{2^{k-1}}$ for some natural number $k$. FALSE

(5) To prove that a sequence $\{a_n\}_{n=1}^{\infty}$ is bounded above by 10 by the Principle of Mathematical Induction, it is logically sufficient to show that

- $a_1 < 10$, and
- if $a_k < 10$ for some natural number $k$, then $a_{k+1} < 10$. TRUE

(6) Every sequence has a bounded subsequence. FALSE

(7) If a sequence has a divergent subsequence, then it diverges. TRUE

(8) Every Cauchy sequence converges. TRUE

(9) Every convergent sequence is Cauchy. TRUE

(10) If $\{a_n\}_{n=1}^{\infty}$ is a bounded sequence, then there is a number $L \in \mathbb{R}$ and a strictly increasing sequence of natural numbers $\{n_k\}_{k=1}^{\infty}$ such that $\{a_{n_k}\}_{k=1}^{\infty}$ converges to $L$. TRUE

(11) There is a sequence without any monotone subsequence. FALSE

(12) If $\{a_n\}_{n=1}^{\infty}$ is Cauchy, then the sequence $\{a_n - a_{2n}\}_{n=1}^{\infty}$ converges to 0. TRUE

(13) If $\{a_n\}_{n=1}^{\infty}$ converges to 2, then $\{a_n^2+1\}_{n=1}^{\infty}$ converges to 5. TRUE

(14) If $\{a_n^2+1\}_{n=1}^{\infty}$ converges, then $\{a_n\}_{n=1}^{\infty}$ converges. FALSE

(15) The limit of $f(x) = \frac{x^2-2x+3}{x-7}$ as $x$ approaches 3 is $-3/2$. TRUE

(16) The function $f(x) = \cos(1/x)$ has a limit as $x$ approaches 0. FALSE

(17) If $\lim_{x\to-1} f(x)/g(x) = 2$, then $\lim_{x\to-1} f(x)$ and $\lim_{x\to-1} g(x)$ both exist. FALSE

(18) If $\lim_{x\to-1} f(x)$ and $\lim_{x\to-1} g(x)$ both exist, then $\lim_{x\to-1} f(x)g(x)$ exists. TRUE

(19) If $\lim_{x\to -1} f(x)$ and $\lim_{x\to -1} g(x)$ both exist, then $\lim_{x\to -1} f(x)/g(x)$ exists. FALSE

(20) If $\lim_{x\to 0} f(x) = 3$, then the sequence $\{f(1/n)\}_{n=1}^{\infty}$ converges to 3. TRUE

(21) If $f$ is a function defined on $\mathbb{R}$ and $\{f(1/n)\}_{n=1}^{\infty}$ converges to 3, then $\lim_{x\to 0} f(x) = 3$. FALSE

(22) If $f$ is a function defined on $\mathbb{R}$, $\{f(1/n)\}_{n=1}^{\infty}$ converges to 3, and $\lim_{x\to 0} f(x) = L$, then $L = 3$. TRUE

(23) If $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 1} g(x) = 2$, then $\lim_{x\to 1}(f \circ g)(x) = 3$. FALSE

(24) If $f$ is continuous at 2, $f(2) = 3$, and $\lim_{x\to 1} g(x) = 2$, then $\lim_{x\to 1}(f \circ g)(x) = 3$. TRUE

(25) The sequence $\{\sin(n^2)\}_{n=1}^{\infty}$ has a convergent subsequence. TRUE

(26) For a given sequence, there are at most two real numbers that occur as limits of subsequences of the sequence. FALSE

(27) The function $f(x) = \frac{x^2-2x+3}{x-7}$ is continuous on $(7, \infty)$. TRUE

(28) The function $f(x) = \frac{x^2-2x+3}{x-7}$ is continuous on $\mathbb{R}$. FALSE

(29) The function $f(x) = \sqrt{|x^3 - 7x + 1|}$ is continuous on $\mathbb{R}$. TRUE

(30) If $\lim_{x\to a} f(x)$ exists, then $f(x)$ is continuous at $x = a$. FALSE

(31) If $f$ is continuous at $a$ and $\lim_{x\to a} f(x) \neq 0$, then there is some $\delta > 0$ such that $f(x) \neq 0$ for all $x \in (a - \delta, a + \delta)$. TRUE

(32) There is some $c \in [-1, 0]$ such that $c^5 + c^3 + 1 = 0$. TRUE

(33) There is some $c \in (-1, 0)$ such that $c^5 + c^3 + 1 = 0$. TRUE

(34) If $f$ is continuous on $\mathbb{R}$ and $a < b$, and $y$ is between $f(a)$ and $f(b)$, then there is some $c \in [a, b]$ such that $f(c) = y$. TRUE

(35) If $f$ is continuous on $\mathbb{R}$ and $a < b$, and $y$ is between $f(a)$ and $f(b)$, then there is exactly one $c \in [a, b]$ such that $f(c) = y$. FALSE

(36) If $f$ is defined on $\mathbb{R}$ and $f$ has the property that for every $a < b$ if $y$ is between $f(a)$ and $f(b)$ then there is some $c \in [a, b]$ such that $f(c) = y$, then $f$ is continuous on $\mathbb{R}$. FALSE