Discussion Questions

TRUE or FALSE. Justify.

(1) Let $x, y \in \mathbb{R}$. The negation of the statement "If x and y are rational, then xy is rational" is "If x and y are rational, then xy is irrational". F

(2) Let $x, y \in \mathbb{R}$. The contrapositive of the statement "If x and y are rational, then xy is rational" is "If xy is irrational, then x and y are irrational". F

(3) The associative property/axiom of addition says that $(x+y)+z = x+(y+z)$. T

(4) Every set of real numbers that is bounded above has a supremum. F

(5) There is a set S of real numbers such that $\sup(S)$ exists, but $\sup(S) \notin S$. T

(6) If $a < b$ are real numbers, there is a natural number $n \in \mathbb{N}$ such that $a < n < b$. F

(7) Every nonempty set of real numbers has a smallest element (i.e., a minimum element). F

(8) Every nonempty set of integers that is bounded below has a smallest element (i.e., a minimum element). T

(9) If $S \subseteq \mathbb{R}$ is bounded above, there there is a natural number b such that b is an upper bound for S. T

(10) It is possible to prove that there is a real number x such that $x^2 = 2$ using just the first 10 axioms (i.e., without using the Completeness Axiom). F

(11) Every set of real numbers satisfies the property that "for all $x \in S$, there exists a real number y such that $x < y^2$". T

(12) Every set of real numbers satisfies the property that "for all $x \in S$, there exists a real number y such that $y^2 < x$". F

(13) The supremum of the set $\{1/n \mid n \in \mathbb{N}\}$ is 1. T

(14) The supremum of the set $\{-1/n \mid n \in \mathbb{N}\}$ is -1. F

(15) The negation of the statement "for all $x \in S$, there exists a real number y such that $x < y^2$" is "for all $x \in S$, there exists a real number y such that $x \geq y^2$". F

(16) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to L, then there is some $N \in \mathbb{R}$ such that for all natural numbers $n > N$, $a_n = L$. F

(17) For every real number L there is a sequence that converges to L. T

(18) For every real number L there is a sequence $\{a_n\}_{n=1}^{\infty}$ such that $a_n \neq L$ for all $n \in \mathbb{N}$ and converges to L. T

(19) A sequence of positive numbers can converge to a negative number. F

(20) A sequence of positive numbers can converge to zero. T

(21) Every increasing sequence is bounded below. T

(22) Every increasing sequence is convergent. F

(23) Every convergent sequence is either increasing or decreasing. F

(24) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are sequences, $\{a_n\}_{n=1}^{\infty}$ converges to L, and there is some $N \in \mathbb{R}$ such that $a_n = b_n$ for $n > N$, then $\{b_n\}_{n=1}^{\infty}$ converges to L. T

(25) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent sequences, then $\{a_n + b_n\}_{n=1}^{\infty}$ is a convergent sequence. T

(26) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent sequences, and $b_n \neq 0$ for all $n \in \mathbb{N}$, then $\{a_n/b_n\}_{n=1}^{\infty}$ is a convergent sequence. F

(27) The sequence $\left\{ \dfrac{3n^2 - 4n + 7}{6n^2 + 1} \right\}_{n=1}^{\infty}$ converges to $1/2$. T

(28) The negation of "$\{a_n\}_{n=1}^{\infty}$ is a monotone sequence" is "there exists $n \in \mathbb{N}$ such that $a_n > a_{n+1}$ and $a_n < a_{n+1}$". F

(29) Every convergent sequence of rational numbers converges to a rational number. F

(30) Every convergent sequence of natural numbers converges to a natural number. T