Math 325-001 — Problem Set #6
Due: Monday, March 22 by midnight

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like "I collaborated with Steven Smale on problems 1 and 3". If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

(1) Prove that if $\{a_n\}_{n=1}^{\infty}$ is a sequence that diverges to ∞, then every subsequence of $\{a_n\}_{n=1}^{\infty}$ diverges to ∞.

(2) Prove that for every real number x, there is a sequence of irrational numbers that converges to x.

(3) Prove that, if $\{a_n\}_{n=1}^{\infty}$ is a sequence that is not bounded below, then $\{a_n\}_{n=1}^{\infty}$ has a decreasing subsequence.

(4) Let $\{a_n\}_{n=1}^{\infty}$ be any sequence and L any real number. Prove that if $\{a_n\}_{n=1}^{\infty}$ does not converge to L, then there exists an $\epsilon > 0$ and a subsequence $\{a_{n_k}\}_{k=1}^{\infty}$ such that $|a_{n_k} - L| \geq \epsilon$ for all k.

(5) Determine whether each of the following sequences is Cauchy, and prove your answer just the definition of Cauchy (not any theorems).

(a) $\left\{\frac{(-1)^n}{n}\right\}_{n=1}^{\infty}$

(b) $\{\sqrt{n}\}_{n=1}^{\infty}$

(c) $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$