1. A, B, D

2. B

3. $P = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ -7 & -1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

other answers are possible, e.g.

$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & -1 & -1 \end{bmatrix}$ $D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

4. A invertible $\Rightarrow Ax = b$ has exactly one solution

$x = A^{-1}b = \begin{bmatrix} 11 \\ -12 \\ -4 \\ 16 \end{bmatrix}$

5. a) • $F(p(t) + q(t)) \overset{?}{=} F(p(t)) + F(q(t))$

           $\|$ yes $\|$

      $p(3) + q(3)$ $p(3) + q(3)$

    • $F(cp(t)) \overset{?}{=} cF(p(t))$

       $\|$ yes $\|$

      $c\,p(3)$ $cp(3)$

F satisfies the two properties of
a linear transformation (so it is one).

b) $[D]_\beta = \left[\ [D(\sin(2x))]_\beta \quad [D(\cos(2x))]_\beta\ \right]$

     $= \left[\ [2\cos(2x)]_\beta \quad [-2\sin(2x)]_\beta\ \right]$

     $= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$