Math 314. Quiz #1 (Chapter 1)

- Please submit your answers on Canvas no more than one hour after you open the quiz. You do not need to rewrite the questions where you write your solutions, but please write the question numbers next to your solutions.
- You may use your textbook and notes that you yourself write to prepare, but you may **not** consult with any other sources (including other students, the internet, computational software, etc.)
- Show all of your work.
- Good luck!

(1) Consider the linear system

$$\begin{cases} 2y - z & = 4 \\ 3x + 6y - 5z & = 8 \\ 6x - 2y - 3z & = -12 \end{cases}$$

(a) Write the augmented matrix of the linear system above.

$$\left[\begin{array}{ccc|c} 0 & 2 & -1 & 4 \\ 3 & 6 & -5 & 8 \\ 6 & -2 & -3 & -12 \end{array}\right]$$

(b) Row reduce this augmented matrix into reduced row echelon form.

$$\left[\begin{array}{ccc|c} 3 & 6 & -5 & 8 \\ 0 & 2 & -1 & 4 \\ 6 & -2 & -3 & -12 \end{array}\right] \to \left[\begin{array}{ccc|c} 3 & 6 & -5 & 8 \\ 0 & 2 & -1 & 4 \\ 0 & -14 & 7 & -28 \end{array}\right] \to \left[\begin{array}{ccc|c} 3 & 6 & -5 & 8 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 3 & 0 & -2 & -4 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -2/3 & -4/3 \\ 0 & 1 & -1/2 & 4/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(c) Find the general solution of the linear system.

$$X_1 = 2/3\, X_3 - 4/3$$

$$X_2 = 1/2\, X_3 + 4/3$$

X_3 free

(2) Consider the vectors

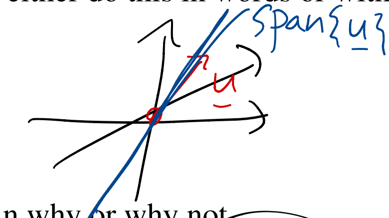
$$\mathbf{u} = \begin{bmatrix}1\\2\\3\end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix}1\\-2\\-5\end{bmatrix}$$

(a) What is $\mathbf{u}+\mathbf{v}$?

$$\begin{bmatrix}2\\0\\-2\end{bmatrix}$$

(b) Describe $\mathrm{Span}\{\mathbf{u}\}$ geometrically. You may either do this in words or with a picture.

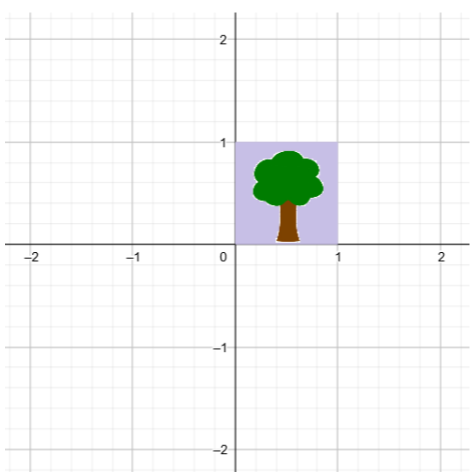
Line through origin & $\underline{u}$:

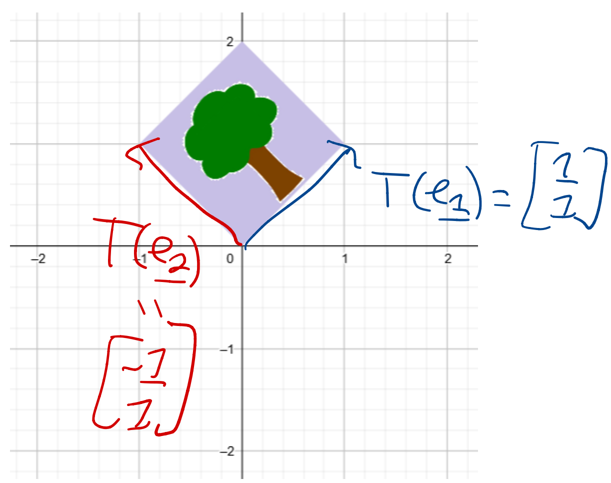


(c) Is $\{\mathbf{u}, \mathbf{v}\}$ a linearly independent set? Explain why or why not.

Yes: two vectors are linearly independent if neither is a multiple of the other.

(3) Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that transforms the image on the left to the image on the right. Find the standard matrix of T.





$$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}.$$

(4) Consider the linear system

$$\begin{bmatrix} 3 & -2 & 19 \\ -1 & 1 & -7 \\ -2 & 1 & -12 \\ -6 & -10 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}.$$

When $a = -12$, $b = 5$, $c = 7$, and $d = -18$ the general solution of this system is

$$\begin{cases} x_1 = -5x_3 - 2 \\ x_2 = 2x_3 - 3 \\ x_3 \text{ free.} \end{cases}$$

(a) Do there exist values of $a, b, c, d \in \mathbb{R}$ such that $\star$ has exactly one solution? Briefly explain why or why not.

No: there is a free variable, so if a, b, c, d make the system consistent, we will have an infinite ~~solution~~ set.

(b) Do there exist values of $a, b, c, d \in \mathbb{R}$ such that $\star$ has no solution? Briefly explain why or why not.

Yes: there cannot be a pivot in every row, hence there are constants for which the system is inconsistent.

(c) Suppose that for some particular values of a, b, c, d, we have $x_1 = 4$, $x_2 = -1$, and $x_3 = 1$ as a solution. For the same values of a, b, c, d, express the general solution of this system in parametric vector form.

For $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -12 \\ 5 \\ 7 \\ -18 \end{bmatrix}$, the general solution is $\begin{bmatrix} -2 \\ -3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix}$, $x_3 \in \mathbb{R}$.

If we change $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ the solution ~~set~~ is either empty, or is the same with a different constant vector.

The solution set is $\begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix}$, $x_3 \in \mathbb{R}$.