Chap 11

$\boxed{dm = \rho d\tau}$

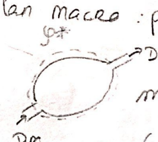
$\boxed{D_m = \iint \rho \vec{v} \, d\vec{S} \quad D_v = \iint \vec{v} \cdot d\vec{S}}$

$\boxed{dm = D_m dt}$

I- Bilan de masse

bilan local : $\boxed{\dfrac{\partial \rho}{\partial t} + div(\rho \vec{v}) = 0 \qquad \dfrac{D\rho}{Dt} + \rho \, div(\vec{v}) = 0}$

bilan macro : pr sys ouvert :

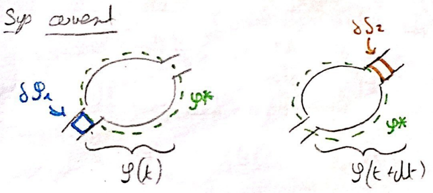


$m_{\mathcal{S}^*}(t+dt) = m_{\mathcal{S}^*}(t) + \delta m_{\text{entrant pdt } dt}$

$m_{\mathcal{S}^*}(t+dt) - m_{\mathcal{S}^*}(t) = D_{m_1} dt - D_{m_2} dt$

$\dfrac{d}{dt} m_{\mathcal{S}^*} \cancel{dt} = (D_{m_1} - D_{m_2}) \cancel{dt}$

Sys ouvert



$\mathcal{S}(t) = \mathcal{S}^*(t) + \delta\mathcal{S}_1$

$\mathcal{S}(t+dt) = \mathcal{S}^*(t+dt) + \delta\mathcal{S}_2$

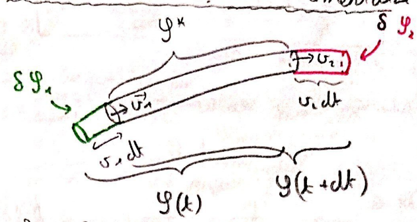
Sys fermé $\Rightarrow$ $\boxed{m_{\mathcal{S}}(t+dt) = m_{\mathcal{S}}(t)}$

$m_{\mathcal{S}^*}(t+dt) + \underbrace{\delta m(\delta\mathcal{S}_2)}_{D_{m_2} dt} = m_{\mathcal{S}^*}(t) + \underbrace{\delta m(\mathcal{S}_1)}_{D_{m_1} dt}$

$m_{\mathcal{S}^*}(t+dt) - m_{\mathcal{S}^*}(t) = (D_{m_1} - D_{m_2}) dt$

$\dfrac{d}{dt} m_{\mathcal{S}^*} \cancel{dt} = (D_{m_1} - D_{m_2}) \cancel{dt}$

Cas d'un tube de courant élémentaire



Sys fermé :

$m_{\mathcal{S}^*}(t+dt) + \underbrace{\delta m(\delta\mathcal{S}_2)}_{\rho_2 D_{v_2} dt = \rho_2 v_2 S_2 dt} = m_{\mathcal{S}^*}(t) + \underbrace{\delta m(\delta\mathcal{S}_1)}_{\rho_1 D_{v_1} dt = \rho_1 v_1 S_1 dt}$

$\dfrac{d m_{\mathcal{S}^*}}{dt} \cancel{dt} = (\rho_1 v_1 S_1 - \rho_2 v_2 S_2) \cancel{dt}$

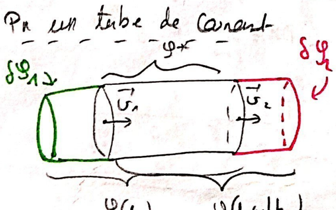
En Régime stationnaire : $\dfrac{d m_{\mathcal{S}^*}}{dt} = 0$ donc $D_{m_1} = D_{m_2}$.

II- Bilan de qté de mvt

Pr un sys fermé : $\dfrac{d\vec{p}}{dt} = \sum \vec{F}_{ext \to \mathcal{S}}$

$\vec{p}_{\mathcal{S}}(t+dt) - \vec{p}_{\mathcal{S}}(t) = d\vec{p} = \sum \vec{F}_{ext} \, dt$

- Pr un tube de courant



$\mathcal{S}(t) = \mathcal{S}^*(t) + \delta\mathcal{S}_1(t)$

$\mathcal{S}(t+dt) = \mathcal{S}^*(t+dt) + \delta S_2$

$d\vec{p}_{\mathcal{S}} = \vec{p}_{\mathcal{S}}(t+dt) - p_{\mathcal{S}}(t)$

$= \vec{p}_{\mathcal{S}^*}(t+dt) + \vec{p}(\delta S_2) - \vec{p}_{\mathcal{S}^*}(t) - \vec{p}(\delta S_1)$

$= \dfrac{d}{dt} \vec{p}_{\mathcal{S}^*} dt + \delta m_2 \vec{v}_2 - \delta m_1 \vec{v}_1$

$\sum \vec{F} dt = \dfrac{d}{dt} \vec{p}_{\mathcal{S}^*} \cancel{dt} + D_{m_2} dt \vec{v}_2 - D_{m_1} dt \vec{v}_1$

$\sum \vec{F}_{ext} = \dfrac{d}{dt} \vec{p}_{\mathcal{S}^*} + D_{m_2} \vec{v}_2 - D_{m_1} \vec{v}_1$

Si régime stationnaire : $\dfrac{d}{dt} \vec{p}_{\mathcal{S}^*} = \vec{0}$

$D_{m_1} = D_{m_2} = D_m$ donc $\sum \vec{F}_{ext} = D_m(\vec{v}_2 - \vec{v}_1)$

Exemples

* fusée × liquide sur plaque

III- Bilan en Ⓔ

1) Théorème

Pr un sys fermé $\mathcal{S}$:

$dE_{m\mathcal{S}} = E_{m\mathcal{S}}(t+dt) - E_{m\mathcal{S}}(t)$

$\boxed{dE_{m\mathcal{S}} = P_{ext}^{NC} dt + P_{int}^{NC} dt}$

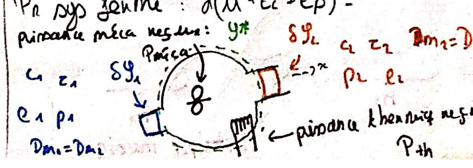
$= \delta W_{ext}^{NC} + \delta W_{int}^{NC}$

Rappel : pr translation : $P = \vec{F} \cdot \vec{v}$

pr Rotation : $P = \vec{m} \cdot \vec{\Omega}$

→ 1er principe de la ODIQ pr un écoulement permanent = 1er principe industriel :

Pr sys fermé : $d(U + E_c + E_p) = \delta W + \delta Q$



$\mathcal{S}$ fermé : $\mathcal{S}(t+dt) = \mathcal{S}^*(t+dt) + \delta S_2$

$\mathcal{S}(t) = \mathcal{S}^*(t) + \delta S_1$

varia°

• $dU = U(t+dt) - U(t)$

$= \cancel{U_{\mathcal{S}^*}(t+dt)} + U(\delta \mathcal{S}_2) - \cancel{U_{\mathcal{S}^*}(t)} - U(\delta \mathcal{S}_1)$

$= \delta m_2 u_2 - \delta m_1 u_1$ — © int massiq

$= D_m dt\,(u_2 - u_1)$

• $dE_c = D_m dt\,(e_{c2} - e_{c1})$ • $dE_p = D_m dt\,(e_{p2} - e_{p1})$

$\delta W = \delta W_{pression\,1} + \delta W_{p2} - \delta W_{autr}$

$\delta W_{p1} = \vec{F}_1 \cdot \vec{c}_1 dt = p_1 S_1 \vec{u}_x\, c_1 \vec{u}_x\, dt$

$= \underbrace{p_1 S_1 c_1 dt}_{\text{volume}}$

$\delta W_{p1} = p_1 D_m dt\, v_1$ — volume massiq

$\delta W_{p2} = -p_2 D_m dt\, v_2$

$\delta W_{autr} = P_{méca}\, dt$

$\delta W = D_m dt\,(p_1 v_1 - p_2 v_2) + P_{méca}\, dt$

$\delta Q = P_{th}\, dt$

On a donc :

$(U + E_c + E_p) = \delta W + \delta Q$

$D_m dt\,(u_2 - u_1 + e_{c2} - e_{c1} + e_{p2} - e_{p1})$

$= P_{méca} dt + P_{th} dt + D_m dt\,(p_1 v_1 - p_2 v_2)$

$D_m \left[(\underbrace{u_2 + p_2 v_2}_{h_2} + e_{c2} + e_{p2}) - (\underbrace{u_1 + p_1 v_1}_{h_1} + e_{c1} + e_{p1}) \right]$

$= P_{méca} + P_{th}$

enthalpie massiq

$$\boxed{D_m \Delta (h + e_c + e_p) = P_{méca} + P_{th}}$$

1er principe industriel

Autres écriture : si w_u le travail utile massiq

$\left.\begin{aligned}\delta W_{autr} &= P_{méca}\, dt \\ &= w_u D_m dt\end{aligned}\right\}$ $P_{méca} = w_u D_m$

de m : $q D_m = P_{th}$

D'où $\boxed{\Delta (h + e_c + e_p) = w_u + q}$

2) Pts thermo important pour GP :

Modèle du GP :

$C_p = \dfrac{dH}{dT}$ et $C_v = \dfrac{dU}{dT}$

$\boxed{\gamma = \dfrac{C_p}{C_v}}$ $\boxed{\text{Rela° de Mayer : } C_p - C_v = nR}$

$H = U + pV$
$= U + nRT$

donc $\boxed{C_v = \dfrac{nR}{\gamma - 1} = \dfrac{mR}{M(\gamma - 1)}}$

$\left(C_{p\,mass} = \dfrac{\gamma}{\gamma - 1} \dfrac{R}{M}\right)$ $\boxed{C_p = \dfrac{\gamma}{\gamma - 1} nR = \dfrac{\gamma}{\gamma - 1} \dfrac{mR}{M}}$

D'où $dh = \dfrac{\gamma}{\gamma - 1} \dfrac{R}{M} dT$

si γ indpdt de T : $h = \dfrac{\gamma}{\gamma - 1} \dfrac{RT}{M}$

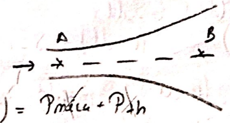
$\Delta h = \dfrac{\gamma}{\gamma - 1} \dfrac{R}{M} \Delta T$

de plus $p = \rho \dfrac{RT}{M}$

d'où $\boxed{h = \dfrac{\gamma}{\gamma - 1} \dfrac{p}{\rho} + cst}$

3) Applica°

• Tuyère :



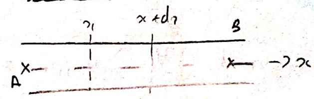
$D_m \Delta (h + e_c + \cancel{e_p}) = \cancel{P_{méca}} + \cancel{P_{th}}$

$\boxed{\Delta (h + e_c) = 0}$

$(h + e_c)_B = (h + e_c)_A$ → transfert

(lié à T°) en © cinétique.

• Echangeur thermiq.



$D_m \Delta (h + \cancel{e_c} + \cancel{e_p}) = \cancel{P_{méca}} + P_{th}$

$D_m \Delta h = P_{th}$ ⇒ transforme enthalpie en transphère thermiq

$D_m C_{p\,mass} \Delta T = P_{th}$

$D_m C_{p\,mass} (T_B - T_A) = P_{th}$ — général

⇝ entre x et x+dx :

$D_m [h(x + dx) - h(x)] = dP_{th}$

$D_m \dfrac{dh}{dx} dx = dP_{th}$

$D_m \dfrac{dh}{dT} \dfrac{dT}{dx} dx = dP_{th}$

$D_m \dfrac{C_{p\,mass}\, dT}{dT} \dfrac{dT}{dx} dx = dP_{th}$

Modèle de Newton pr le transfert th :

$dP = K (T_{ext} - T(x))\, dS$ — $dx\,(L)$ — périmètre

Soit

$D_m C_{p\,mass} \dfrac{dT}{dx} \cancel{dx} = K (T_{ext} - T_x)\, dx\, L$

$\dfrac{dT}{dx} + (T(x) - T_{ext}) \dfrac{KL}{D_m C_{p\,mass}} = 0$ — $\dfrac{1}{\text{longueur}} = \dfrac{1}{a}$

T_{ext} cst

$\dfrac{d(T - T_{ext})}{dx} + \dfrac{T_{(x)} - T_{ext}}{a} = 0$

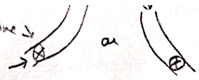
D'où : $T_{(x)} - T_{ext} = A e^{-x/a}$

Condition limite : $T(x = 0) = T_A$

$T_A - T_{ext} = A$

Donc $\boxed{T(x) = T_{ext} + (T_A - T_{ext}) e^{-x/a}}$

turbine



ou

$P_{\text{méca reçu}} > 0$ $\qquad$ $P_{\text{méca reçu}} < 0$

$D_m \, \Delta(\cancel{h} + e_c + e_p) = P_{\text{méca}} + \cancel{P_{th}}$

transfert d'énergie potentielle → travail utile ...

. Interprétation du th de Bernoulli

Rappel : $\boxed{dU = TdS - PdV}$

$$dS = \underbrace{\delta S_{ech}}_{\frac{\delta Q}{T}} + \underbrace{\delta S_{créé}}_{\geq 0}$$

$\delta W = -p_{ext} dV$

$\delta W_{rev} = -p\,dV$

Bernoulli CHIPS : $\dfrac{v^2}{2} + \dfrac{p}{\rho} + e_{p\,mas} = cst\,(ldc)$

$e_{c\,mas}$

ldc

P. tube de courant élémentaire.

$D_m \, \Delta(h + e_c + e_p) = \cancel{P_{méca}} + \cancel{P_{th}}$

↑ pas de machine $\qquad$ ↑ pas de viscosité

$\Delta(h + e_c + e_p) = 0 \Rightarrow h + e_c + e_p = cst\,(ldc)$

avec $h = u + pV_m$ ← volume massiq $= \dfrac{1}{\rho}$

$= u + \dfrac{p}{\rho}$

on a donc $u + \dfrac{p}{\rho} + e_c + e_p = \text{const}$ (~ Bernoulli)

$u = T\,ds - p\,dv_m = T\,ds - p\,\underbrace{d\left(\frac{1}{\rho}\right)}$ $= 0$ car incompressible

donc $u = 0$ si $ds = 0$ donc Bernoulli valable si $s = cst$ donc si le fluide subit une transf isentropique en plus de CHIPS

Bilan d'entropie

P. sys fermé : S enthalpie f° d'état !

$\Delta S = S_{ech} + S_{créée}$

$$dS = \underbrace{\delta S_{éch}}_{\frac{\delta Q_{reçu}}{T_{ext}}} + \underbrace{\delta S_{créé}}_{\geq 0}$$

T_{ext} ← T° de la paroi d'échange

Transf lente ⇒ transf reversible

→ pour un écoulement permanent :

$D_m\,dt\,(s_2 - s_1) = \delta S_{éch} + \delta S_{créée}$