Effects of Filter Parameters

For proper tuning of a Kalman filter, the noise parameters v and w must be properly estimated and tuned. In this section, we compare the effects of proper and improper tuning using a single pendulum system with natural dynamics as an example. We define the true values of process noise and measurement noise and observe what happens when the filter overestimates or underestimates the values. For the sake of brevity, the full derivation and form of the system model is omitted here, and will be discussed in further detail in the Single-Link Robot Joint section. Here, we treat the system as a black box and compare errors for the natural dynamics example to find which give the best performance. In all cases, the system is simulated over an interval of 10 seconds, with Kalman updating every .005 seconds and a fourth order Runge-Kutta integration timestep of .0005 seconds.

Effects of R in the absence of process noise

First, we assume that the model exhibits no process noise and that its true measurement noise has a standard deviation of 30 degrees, leading to the form

$$Q_k = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

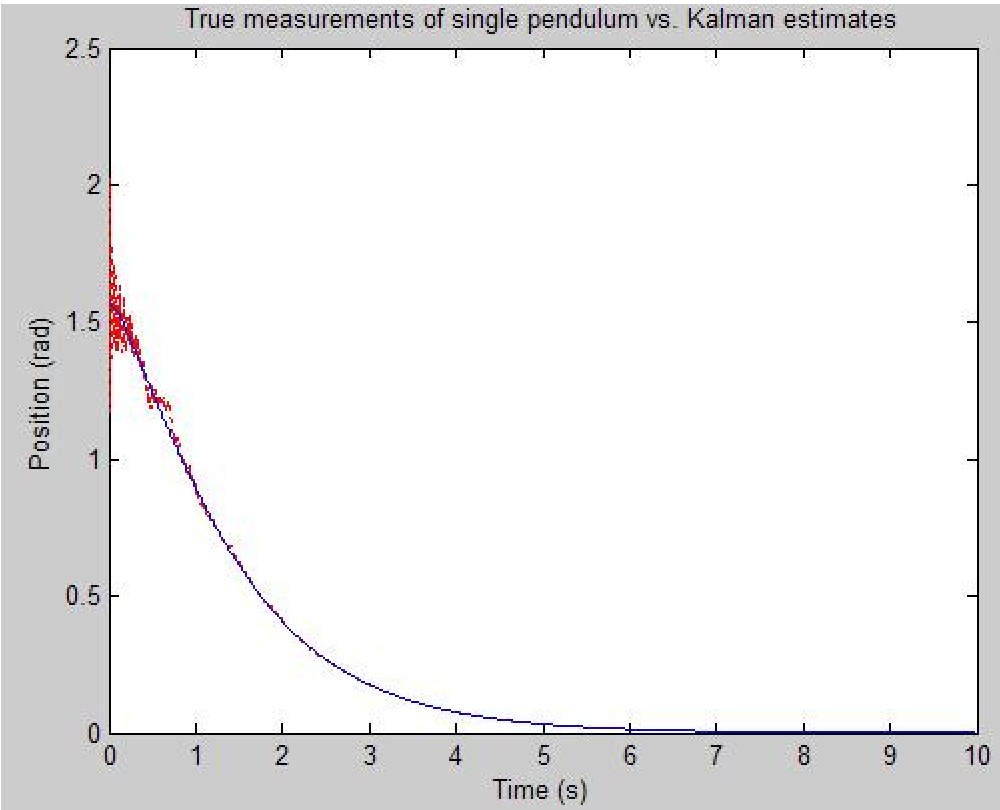
$$R_k = \sigma^2$$

where, in radians,

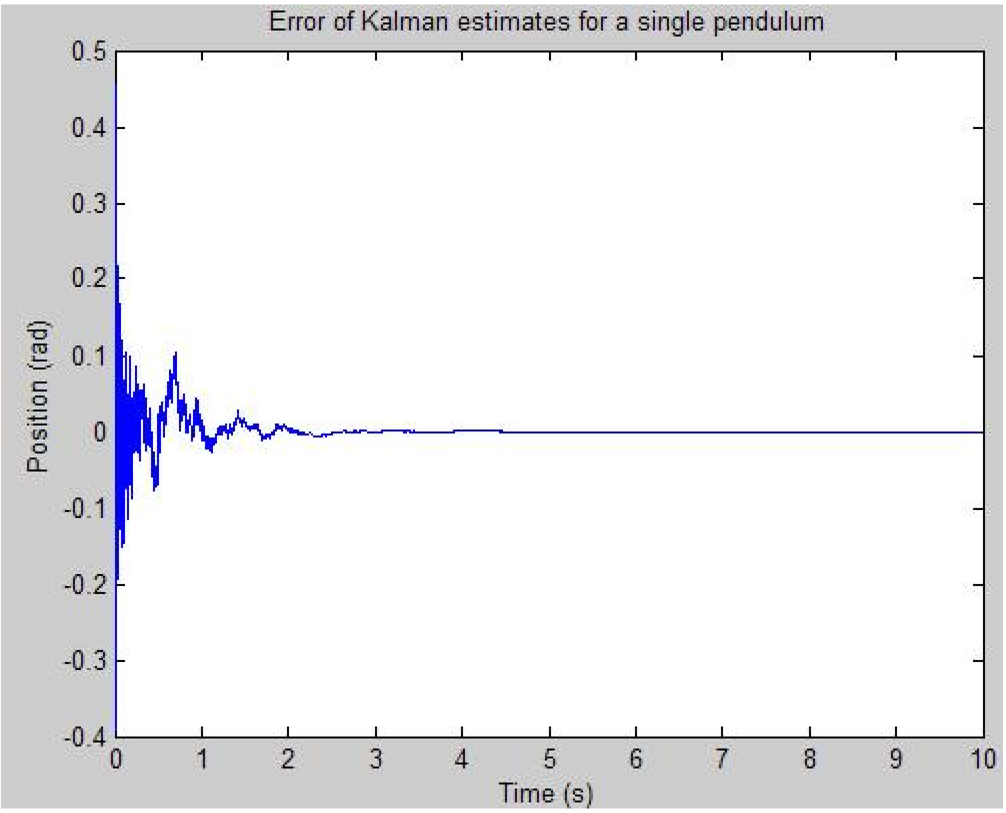
$$\sigma^2 = .2742$$

Equal Measurement Noise

In this case, the estimated noise variance is equal to the true noise variance encountered, and the initial error estimate is .2 radians in angular position and .05 radians/sec in angular velocity. We simulate the system to observe its rate of convergence and long-term behavior.

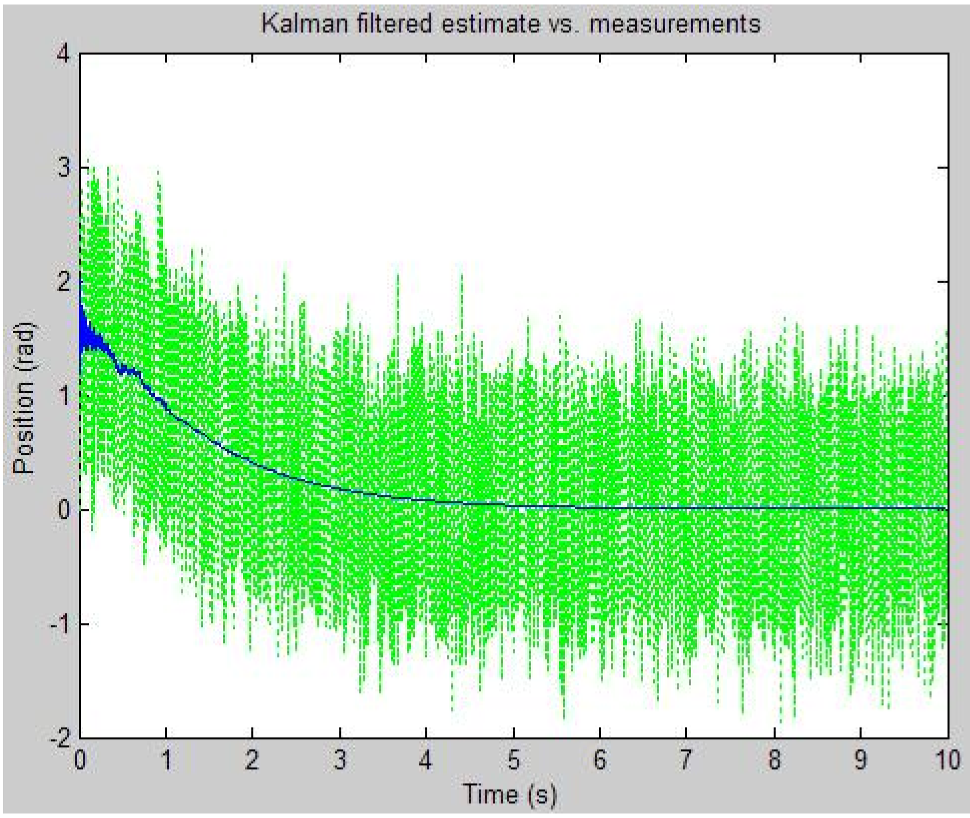


Estimate (red) vs. actual angular displacement (blue) over time



Error in position over time

It is also apparent that when compared to the unfiltered measurement estimates, the filter provides a far more accurate measurement, despite using the same errors.

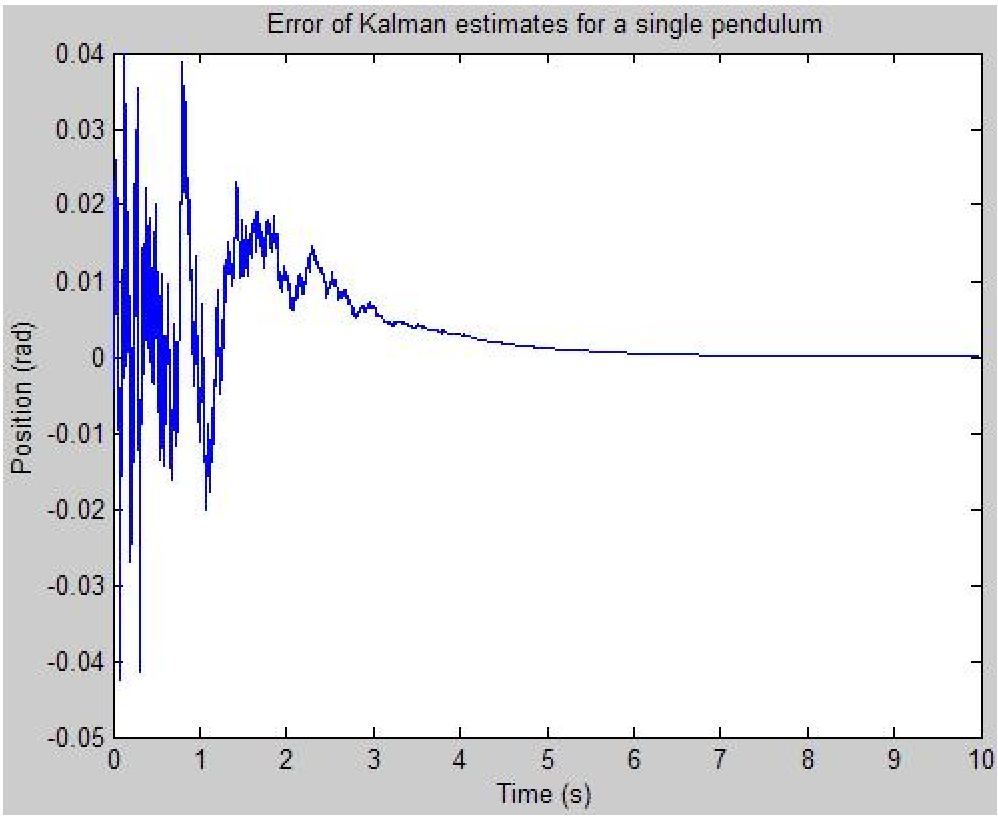


Actual measurements (green) vs. Kalman filtered estimates (blue)

We see that the system rapidly converges to its true values within three seconds and exhibits stable performance in the absence of process noise. This long-term stability, however, is due to the absence of any error being introduced in the dynamic equations. In effect, the model presented is perfect, and only the estimated measurement is imperfect. Once an estimate of the true state is obtained over time, it can be iterated to obtain the next state without any dynamic error. We can compare this to results with process noise later.

Overestimated Measurement Noise

In this case, we increase the estimated noise's variance by a factor of 100 (thus increasing its standard deviation to 300 degrees) and simulate using otherwise identical parameters.

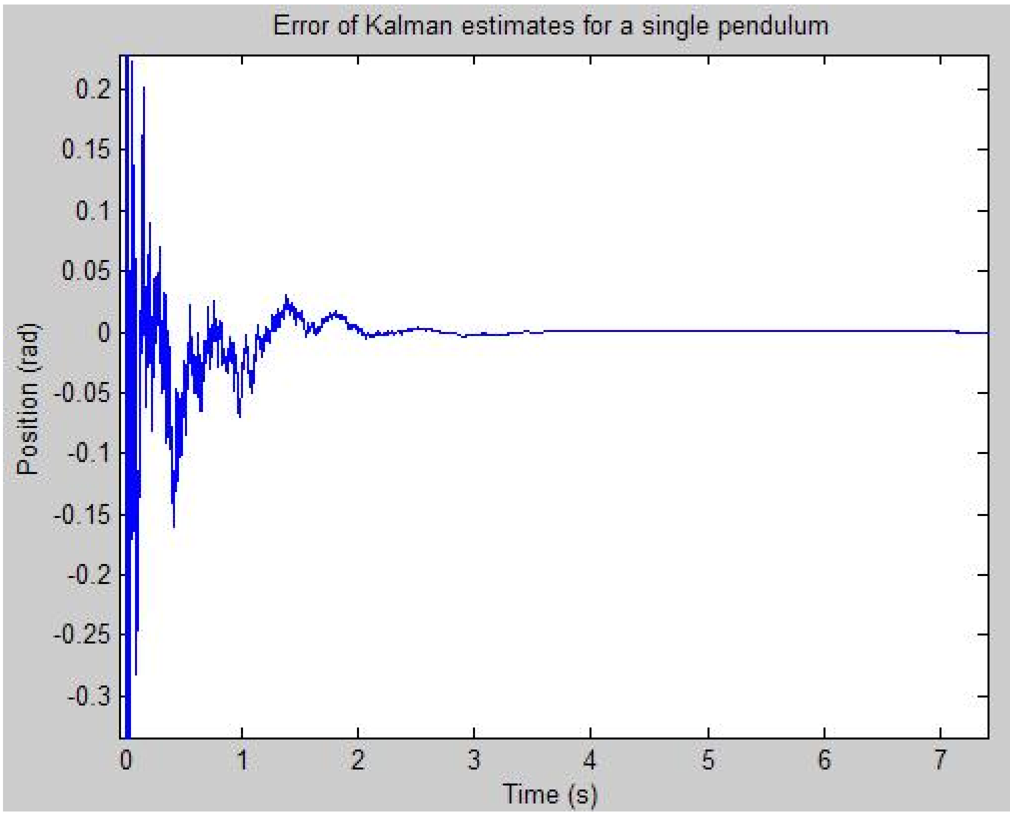


Error in position over time - overestimated measurement noise

Given a larger error estimate, the filter is slower to respond to the comparatively small errors that the system actually encounters. It takes longer to converge (about eight seconds in this example) and exhibits exponential behavior on approaching steady state.

Underestimated Measurement Noise

In this case, we decrease the estimated noise's variance by a factor of 100 (decreasing its standard deviation to 3 degrees) and determine the effects of errors that are large compared to the estimate.



Error in position over time - underestimated measurement noise

The filter here responds much more quickly and dramatically to errors in measurement. In the case where there is no process noise, the filter converges to an accurate measurement quickly. However, **in the presence of process noise**, underestimating the measurement noise leads to rapid deviations in the filter. It attempts to correct for comparatively large noise (in this case, 10 times what was predicted) and change its measurements quickly, leading to more erratic behavior over time.

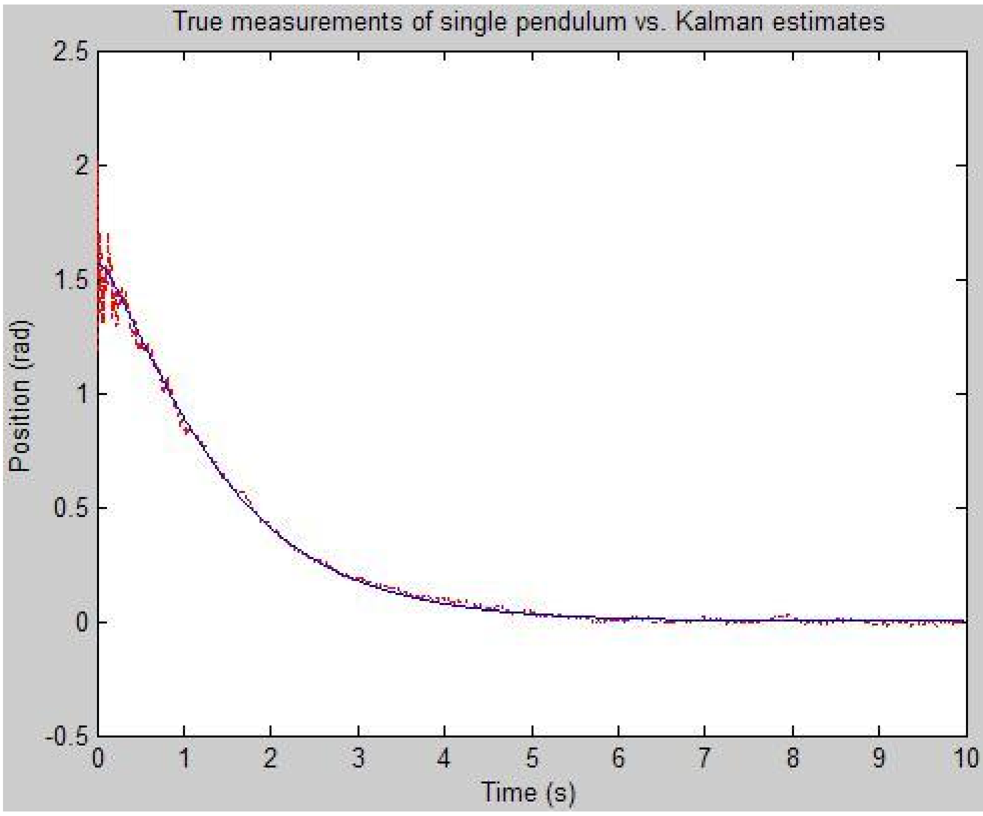
Effects of Process Noise Estimates

In few cases is the dynamic model of a system ever truly accurate. The previous example chose a process noise of zero in an effort to only show the effects of measurement noise. This is equivalent to saying that the system model that we have derived is completely perfect and free from any outside influence. This is seldom the case, even under nearly ideal conditions. By accounting for process noise, the system can more readily account for any deviations and ensure that the filter will converge over time.

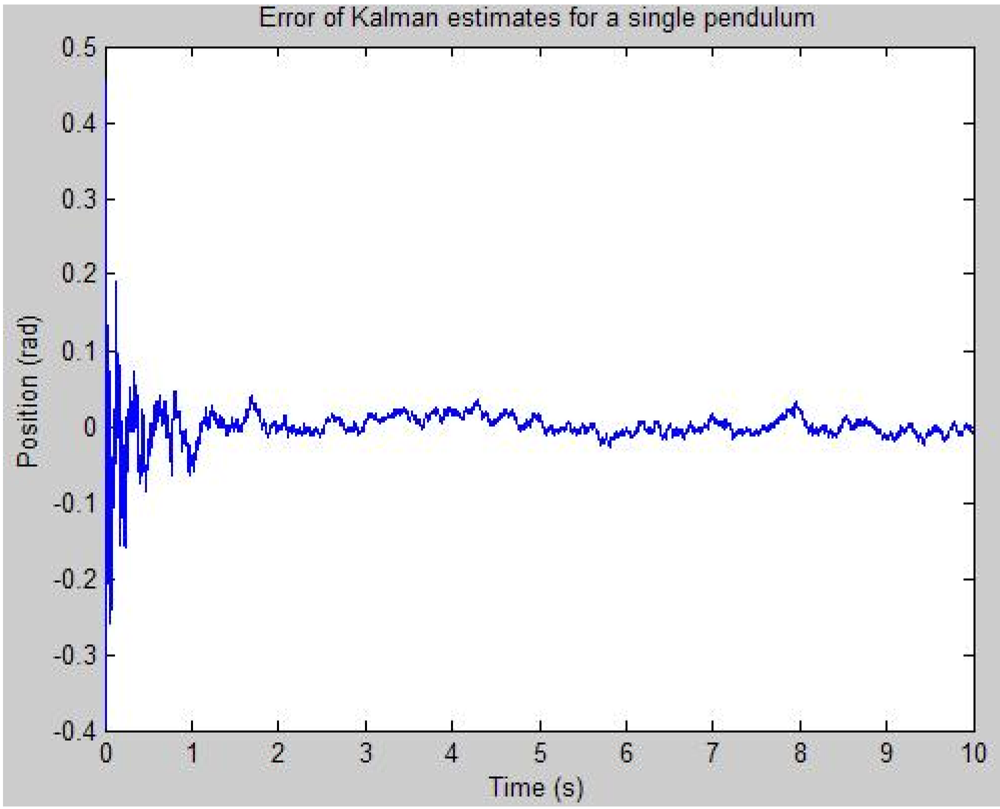
For this example, we use the same parameters defined above, with our estimated measurement noise equal to the true measurement noise (standard deviation of 30 degrees). We also add random white noise angular accelerations with actual spectral density of .1, and compare how overestimating and underestimating process noise can produce varying results. For a more complete derivation of the discrete process noise matrix, see the Single-Link Robot Joint page.

Equal Process Noise

With the addition of process noise, the system's filter continues to perform adequately, though not as well as in the previous example. Here, there are two separate sources of noise, one of which influences measurements and the other of which influences the dynamic processes upon which the measurements are based. The error converges quickly and maintains a mean of zero over time, within about +/- .02 radians (1.15 degrees). Nonzero mean error is a desired trait of any Kalman filter because small errors notwithstanding, the estimates do not diverge and the residual remains zero.



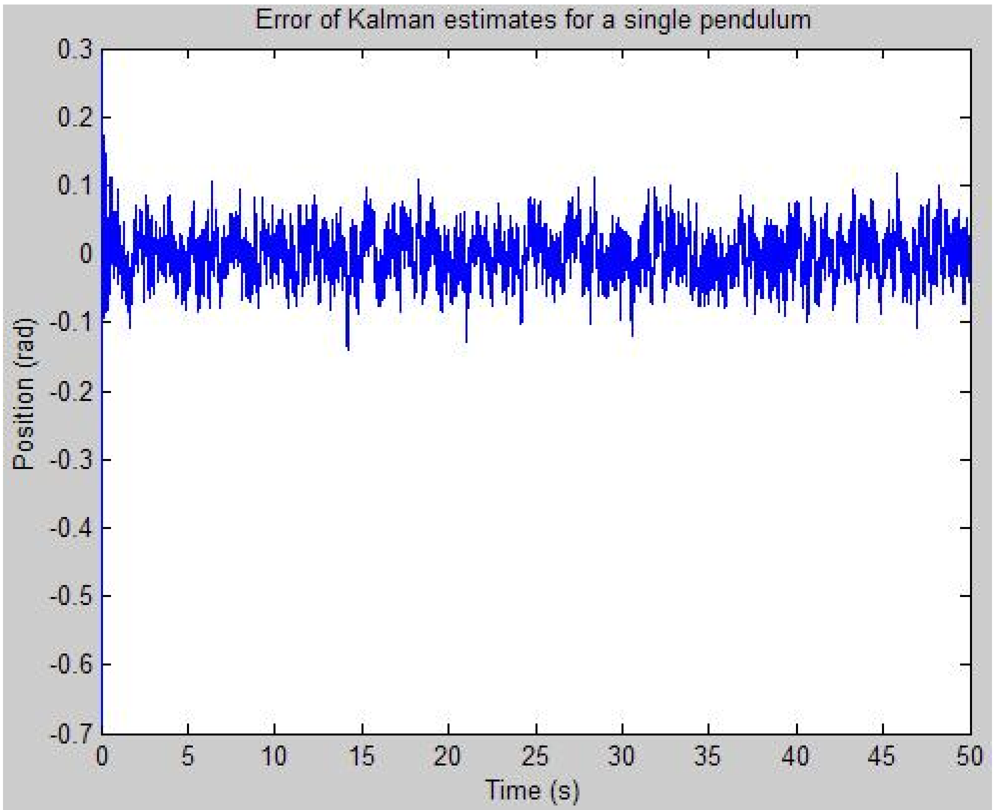
Comparison of estimated and true states with perfectly estimated process noise



Error in position over time with perfectly estimated process noise

Overestimated Process Noise

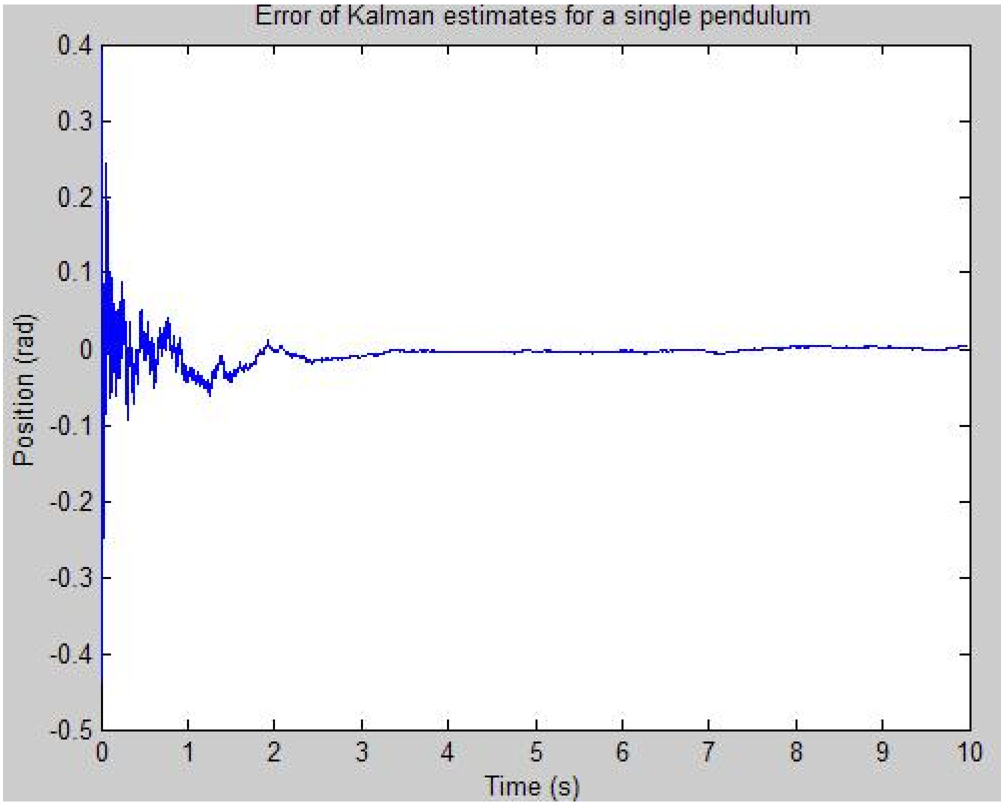
In the next case, we overestimate the process noise by a factor of 10 (spectral density = 1) and observe the effects. Any error arising from the process noise is relatively small compared to the values that the filter estimates should be occuring. It is slower to respond to these "small" errors, and the filter reaches a final standard deviation greater than that for the perfectly estimated error, and seems to stay within the spectral density of the accelerations.



Error with overestimated process noise

Underestimated Process Noise

Finally, consider the effect of underestimating the process noise. Any change in the noise is comparatively large when compared to what the filter considers significant, whilch leads to greater changes in the Kalman gain matrix. We underestimate the spectral density by a factor of 10 to observe the effects.



Error with underestimated process noise

The filter converges to within a very high tolerance as the system's process noise is much larger than what the filter predicted. In this case, the error residual converges to zero. However, in a more complicated system, if the error is underestimated, this may not be the case. Compensating too little for process noise may lead to the residual shifting from zero and diverging, rendering the filter worthless.

In actuality, while the process noise should be estimated as best as possible, detemining an optimum for any given problem is more difficult. The addition of process noise in the Kalman filtering equations can help to make a filter more stable over time and compensate for any residual drift, but such values must be carefully tuned. Predicting too much process noise can lead to poor estimates, while predicting too little may drive a system unstable beyond the point of any useful estimate.

Effect of initial uncertainty estimates

During the initial tuning of any Kalman filter, the final parameter that must be esimates is the initial uncertainty in state estimates. The value chosen is iterated at each Kalman filter update, but it is still crucial to choose a value that will converge. In practice, choosing nonzero gains on the diagonal of this matrix is almost always adequate to tune prevent the system from diverging. The exercise of estimating initial uncertainty is left to the specific problem being determined to ensure that the filter does not diverge or exhibit poor behavior.

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Created by Nicholas Gerasimowicz, Spring 2005