Quaternion feedback regulator for spacecraft eigenaxis rotation

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Quaternion Feedback Regulator for Spacecraft Eigenaxis Rotations

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A quaternion feedback regulator is developed for spacecraft eigenaxis rotational maneuvers. The Euler's eigenaxis rotation that provides the shortest angular path between two orientations is considered as an "optimal" maneuver. The control algorithm basically consists of linear feedback of error quaternions and body rates, and includes decoupling control torque that counteracts the natural gyroscopic coupling torque. But, in some cases with small angular rates, the gyroscopic decoupling control is not necessary for eigenaxis rotations. It is shown that large-angle, rest-to-rest maneuver about the Euler's eigenaxis can be simply achieved by a proper selection of feedback gain matrices of the quaternion feedback regulator. Furthermore, previous results in quaternion feedback stability analysis based on the Lyapunov method are significantly extended. Robustness of the globally stable, quaternion feedback regulator to spacecraft inertia matrix uncertainty is also discussed. Simulation results show that a proper selection of the quaternion feedback regulator gains provides near-eigenaxis rotation, even in the presence of initial body rate and inertia matrix uncertainty.

I. Introduction

SOME future spacecraft will need an attitude control system that provides rapid multitarget acquisition, pointing, and tracking capabilities. Many spacecraft control systems are currently based on a sequence of rotational maneuvers about each control axis. The maneuver time of such successive rotations is longer (by a factor of 2 or 3) than that of a single maneuver about the eigenaxis. Because the overall cost of a space-based laser system is greatly affected by the average retargeting time, the development of control algorithms for rapid retargeting is crucial. It also may be necessary to maintain rotation about an inertially fixed axis during an acquisition mode so that a particular sensor will pick up a particular target.

In this paper, various quaternion feedback control algorithms are investigated for large-angle retargeting maneuvers (possibly about the eigenaxis). Different gain matrices for the quaternion feedback regulator are studied from both mathematical and practical viewpoints. Previous results in the use of quaternions for large-angle maneuver controls are significantly extended. The concept and mathematical results presented in this paper do not appear to have been published previously in the open literature. In order to distinguish the new results of this paper from the previous studies by many other researchers, we briefly review the literature in three-axis, large-angle maneuver control of a rigid spacecraft.

Many open-loop control schemes have been studied for large-angle maneuvers (Refs. 1 and 2). The open-loop schemes, however, are sensitive to spacecraft parameter uncertainty, unexpected disturbances, and initial attitude rates. In general, a combination of feedforward (open-loop) and feedback (closed-loop) controls is desirable. The eigenaxis rotation via feedforward command has been used in maneuvering control of Apollo, Skylab, and Shuttle. It has been considered as a natural approach to a rapid, rotational maneuver.[3–5] From a fuel-optimal control viewpoint, however, Redding and Adams[6] have developed a noneigenaxis rotational maneuver for the Shuttle with significant cross-axis jet couplings. Nonlinear optimal feedback control schemes,[7,8] a sliding mode concept,[9] and a general nonlinear feedback control theory[10] have also been applied to the spacecraft attitude control problems.

A simple concept using Cayley-Rodrigues parameters (Gibbs' vector) or quaternions as attitude feedback signals was first studied by Mortensen[11,12] in mid-1960. Meyer[13] has studied similar three-axis control using direction cosine matrices. Hrastar[14] has applied Mortensen's concept[11] to the large-angle slew control of the OAO spacecraft. The use of quaternions as a measure of attitude errors was also suggested by Ickes.[15] Recently, Wie and Barba[16] have extended the quaternion feedback concept to a spacecraft equipped with pulse-width pulse-frequency modulated reaction jets. A similar quaternion feedback scheme was also studied by Vadali and Junkins[7] for a spacecraft equipped with reaction wheels.

Mortensen[12] has chosen the quaternion feedback gain in each axis to be inversely proportional to its principal moment of inertia. Thus, his scheme requires an exact knowledge of the inertia matrix in order to be globally stable. In Refs. 7 and 16, the quaternion gains are restricted to be identical in each axis. The reason behind such special selections of quaternion feedback gains is to facilitate determination of a Lyapunov function for the proof of global stability.

Expanding on these previous studies, we present new results, which are summarized as follows:

1) For the ideal case of an exactly known inertia matrix, a rest-to-rest maneuver about the eigenaxis can be simply achieved by use of the quaternion feedback regulator with a proper selection of the feedback gain matrices.

2) The quaternion feedback controllers studied by Vadali and Junkins[7] and Wie and Barba[16] are shown to be globally stable regardless of spacecraft inertial property uncertainty; hence, they are globally stable and robust. It is also shown that the gain selection in Refs. 7 and 16, however, does not result in the eigenaxis rotation, even with perfect gyroscopic decoupling control.

3) A Lyapunov function that provides a sufficient condition for the global stability of the proposed controller is derived. The new result includes all previous results[7,12,16] as special cases. Furthermore, the new result allows a proper gain selec-

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tion, which provides near-eigenaxis rotation with guaranteed global stability.

4) For the first time, the q_i vs $q_j (i \neq j)$ plot is introduced, where q_i is the ith quaternion element. This plot clearly illustrates an "optimal" maneuver with the shortest angular path (a straight line between two points). Using digital simulation, performance and stability of four different gain matrices are compared for a 160-deg (eigenangle) rotational maneuver control of a rigid spacecraft with initial body rate and 10% inertia matrix uncertainty.

II. Eigenaxis Rotation via Quaternion Feedback

In this section, the general case of a rigid spacecraft rotating, under the influence of body-fixed torquing devices is considered. For simplicity, an ideal control torquer is assumed; however, as can be found in Refs. 5, 7, 14, and 16, reaction wheels, control moment gyros, or pulse-modulated jets must be properly accommodated for more detailed analysis.

Euler's Equations of Motion

Euler's equations describe the rotational motion of a rigid body about body-fixed axes with origin at the center of mass. The equations that follow are associated with the general case in which the body-fixed control axes do not coincide with the principal axes of inertia:

$$J\dot{\omega} = \Omega J\omega + u \tag{1}$$

where $\omega = [\omega_1, \omega_2, \omega_3]^T$ is the angular velocity vector, $u = [u_1, u_2, u_3]^T$ the control torque vector, J the inertia matrix, and Ω a skew-symmetric matrix defined by

$$\Omega \triangleq -\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{2}$$

The subscripts 1, 2, and 3 denote the body-fixed control axes.

It is assumed that the angular velocity components along the body-fixed control axes are measured by rate gyros and used to calculate the orientation. Since quaternions are well suited for onboard real-time computation, spacecraft orientation is now commonly described in terms of quaternions (e.g., HEAO, Space Shuttle, and Galileo[17]).

Quaternion Kinematics

Euler's rotational theorem states that a rigid-body attitude can be changed from any given orientation to any other orientation by rotating the body about an axis called Euler axis or eigenaxis. A simple kinematic relation between the eigenaxis rotation and conventional body-axis and space-axis rotations was studied by Wie.[18] The quaternion defines the rigid-body attitude as a Euler-axis rotation. The vector part of the quaternion (the first three components) indicates the direction of the Euler axis. The scalar part of the quaternion (the fourth component) is related to the rotation angle about the Euler axis.

The four elements of the quaternion are defined as

$$q_i = c_i \cos(\phi/2), \qquad i = 1,2,3 \tag{3a}$$

$$q_4 = \cos(\phi/2) \tag{3b}$$

where ϕ is the magnitude of the Euler axis rotation, and (c_1, c_2, c_3) are the direction cosines of the Euler axis relative to a reference frame.

The quaternion kinematic differential equation is described by

$$\dot{q} = \tfrac{1}{2}\Omega q + \tfrac{1}{2}q_4\omega \tag{4a}$$

$$\dot{q}_4 = -\tfrac{1}{2}\omega^T q \tag{4b}$$

where $q = [q_1, q_2, q_3]^T$ and Ω is defined by Eq. (2). Using Eqs. (3), one can show that the quaternion satisfies the relation

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \tag{5}$$

Equations (4) were first published by Robinson[19] in 1958 and derived independently by Mortensen,[11] Margulies,[20] and Harding,[21] in mid-1960. Similar equations, where the angular rates are expressed in terms of quaternions and quaternion rates, can be found in Whittaker.[22] It is interesting to note that Eqs. (4) were not derived by Hamilton, Euler, or Whittaker. Various strapdown attitude determination algorithms based on Eqs. (4) can be found in Refs. 23–25. A semianalytical solution of the quaternion kinematical equation can be found in Ref. 26.

Quaternion as a Measure of Attitude Errors

The initial quaternion $[q_1(0), q_2(0), q_3(0), q_4(0)]$ defines the initial orientation of the spacecraft body axis at $t = 0$. The commanded quaternion $[q_{1c}, q_{2c}, q_{3c}, q_{4c}]$ defines the desired orientation. The error quaternion that represents the attitude error between the current orientation and the desired one is then given by

$$\begin{bmatrix} q_{1e} \\ q_{2e} \\ q_{3e} \\ q_{4e} \end{bmatrix} = \begin{bmatrix} q_{4c} & q_{3c} & -q_{2c} & -q_{1c} \\ -q_{3c} & q_{4c} & q_{1c} & -q_{2c} \\ q_{2c} & -q_{1c} & q_{4c} & -q_{3c} \\ q_{1c} & q_{2c} & q_{3c} & q_{4c} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \tag{6}$$

The equation is the result of successive quaternion rotations using the quaternion multiplication and inversion rules.[22]

For the special case of attitude regulation with respect to the reference frame specified at $t = 0$, the commanded quaternion is $[0,0,0,1]$. In this case, the error quaternion coincides with the current attitude quaternion; i.e., $q_e = q$ and $q_{4e} = q_4$.

For the case with $[q_{1c}, q_{2c}, q_{3c}, q_{4c}] = [0,0,0,1]$ and small attitude changes from the reference frame, we can approximate the error quaternion by

$$2q_{ie} = 2q_i = \theta_i, \qquad i = 1,2,3 \tag{7}$$

where the θ_i are the conventional Euler angles. For this case, the error quaternion rate also can be approximated as

$$2\dot{q}_{ie} = 2\dot{q}_i = \omega_i, \qquad i = 1,2,3 \tag{8}$$

Quaternion Feedback Regulator

The proposed feedback controller for eigenaxis rotations consists of linear error-quaternion feedback, linear body-rate feedback, and a nonlinear body-rate feedback term that simply counteracts the gyroscopic coupling torque. Following Eq. (1), the control torque vector u is, in general, represented as

$$u = -\Omega J\omega - D\omega - Kq_e \tag{9}$$

where D and K are 3×3 constant gain matrices to be properly determined. For simplicity, we shall consider the case of $q_e = q$. The gyroscopic decoupling feedback control is not necessary for slow rotational maneuvers. But, in some cases (e.g., see Refs. 27 and 28), it may be desirable to counteract the natural gyroscopic coupling by control torque.

Eigenaxis Rotation

In this section, we show that a large-angle, rest-to-rest reorientation maneuver about the eigenaxis can be achieved by a proper selection of the gain matrices of the quaternion feedback regulator. An ideal case of $\omega(0) = 0$ is considered here.

According to Euler's rotation theorem, the angle ϕ of Eqs. (3) is always smaller than the algebraic sum of three successive Euler angles and represents the shortest angular path between two orientations. For certain cases, it may be desirable to rotate the spacecraft about the Euler axis to perform a minimum angular path maneuver. This can be achieved by using a quaternion feedback of the form kJq, where k is a scalar, J the inertia matrix, and q the vector part of the quaternion. Since the vector q coincides with the spacecraft eigenaxis, the control torque kJq causes the Euler axis rotation to occur. Notice that, unless the principal inertias are all equal, the eigenaxis rotation cannot be accomplished if the control torque vector lies along the Euler axis or eigenaxis.

Consider a gain selection $D = dJ$ and $K = kJ$ for the eigenaxis rotation (d and k are scalars). The closed-loop equations of motion then become

$$\dot{\omega} = -d\omega - kq \tag{10a}$$

$$\dot{q} = \tfrac{1}{2}\Omega q + \tfrac{1}{2}q_4\omega \tag{10b}$$

$$\dot{q}_4 = -\tfrac{1}{2}\omega^T q \tag{10c}$$

Instead of solving the preceding equations directly, we assume that the solution is the eigenaxis rotation

$$q(t) = c_q(t)q(0) \tag{11}$$

where $c_q(t)$ is a scalar function of time with $c_q(0) = 1$ and $q(0)$ is the initial quaternion vector.

Substituting Eq. (11) into Eq. (10a) gives

$$\dot{\omega} = -d\omega - kc_q(t)q(0) \qquad [\omega(0) = 0]$$

which has the solution

$$\omega(t) = c_\omega(t)q(0) \tag{12}$$

where

$$c_\omega(t) = -k\int_0^t e^{-d(t-\tau)}c_q(\tau)d\tau$$

Substituting Eq. (12) into Eq. (10b) gives

$$\dot{q} = \tfrac{1}{2}\Omega q + \tfrac{1}{2}q_4\omega = \tfrac{1}{2}q_4(t)c_\omega(t)q(0) \tag{13}$$

where $\Omega q \equiv 0$ since both q and ω have the same direction as $q(0)$. Equation (13) has the solution

$$q(t) = c_q(t)q(0)$$

where

$$c_q(t) \triangleq \left[1 + \tfrac{1}{2}\int_0^t q_4(\tau)c_\omega(\tau)d\tau\right]$$

This shows that if the applied control torque vector is along the direction of $J\lambda$, where λ is a unit vector along the eigenaxis, the eigenaxis rotation can be achieved for the ideal case of $\omega(0) = 0$. A more rigorous proof can be done by showing that, if and only if $\Omega q \equiv 0$, ω is collinear with q and the rotation is about the eigenaxis.

III. Stability Analysis

In this section, we discuss the stability of the closed-loop system with general D and K matrices

$$J\dot{\omega} = \Omega J\omega - \mu\Omega J\omega - D\omega - Kq \tag{14a}$$

$$\dot{q} = \tfrac{1}{2}\Omega q + \tfrac{1}{2}q_4\omega \tag{14b}$$

$$\dot{q}_4 = -\tfrac{1}{2}\omega^T q \tag{14c}$$

where $\mu = 1$ means that the control torque exactly counteracts the gyroscopic coupling torque, and $\mu = 0$ means that only quaternion feedback and linear rate feedback are used. For simplicity, we assume that the commanded quaternion is [0,0,0,1] and, therefore, the error quaternion can be replaced by the current attitude quaternion.

Assuming that K^{-1} exists and that $K^{-1}J$ is positive definite, we define the following Lyapunov function

$$\begin{aligned} V &= \tfrac{1}{2}\omega^T K^{-1}J\omega + q_1^2 + q_2^2 + q_3^2 + (q_4 - 1)^2 \\ &= \tfrac{1}{2}\omega^T K^{-1}J\omega + 2(1 - q_4) \end{aligned} \tag{15}$$

Note that V is positive definite and asymptotically unbounded in ω. This particular form of Lyapunov function is a generalization of Lyapunov functions used in Refs. 7, 12, and 16.

The time derivative of V is given by

$$\dot{V} = \tfrac{1}{2}\dot{\omega}^T K^{-1}J\omega + \tfrac{1}{2}\omega^T K^{-1}J\dot{\omega} - 2\dot{q}_4$$

Assuming that $K^{-1}J = (K^{-1}J)^T$, we can calculate $\dot{V}$ along the system trajectories as

$$\begin{aligned} \dot{V} &= \omega^T K^{-1}J\dot{\omega} - 2\dot{q}_4 \\ &= -\omega^T K^{-1}D\omega + (1-\mu)\omega^T K^{-1}\Omega J\omega \end{aligned} \tag{16}$$

The second term in Eq. (16) is identically zero under the following conditions: 1) precise cancellation of the gyroscopic coupling torque, i.e. $\mu = 1$, or 2) no cancellation of the gyroscopic coupling torque and selection of the quaternion feedback gain matrix K such that

$$K^{-1} = \alpha J + \beta I \tag{17}$$

where α and β are nonnegative scalars and I is a 3×3 unit matrix.

Using Eq. (17), we obtain

$$\begin{aligned} \omega^T K^{-1}\Omega J\omega &= \omega^T(\alpha J + \beta I)\Omega J\omega \\ &= \alpha(J\omega)^T\Omega(J\omega) + \beta\omega^T\Omega J\omega \end{aligned} \tag{18}$$

Since $\Omega = -\Omega^T$, the first term in Eq. (18) is identically zero. Since Ω is defined by Eq. (2), $\Omega\omega = \omega^T\Omega \equiv 0$, and the second term of Eq. (18) is identically zero. Equation (17) guarantees that K^{-1} exists and that $K^{-1}J$ is symmetric and positive definite.

Under condition 1 or 2, we find that

$$\dot{V} = -\omega^T K^{-1}D\omega \tag{19}$$

Global stability is guaranteed if $K^{-1}D > 0$.[29,30] A natural selection D that guarantees this is

$$D = dJ \tag{20}$$

where d is a positive scalar.

Remark 1

In the case where the body-fixed axes coincide with the principal axes, the inertia matrix is given as

$$J = \mathrm{diag}[J_1, J_2, J_3] \tag{21}$$

For this case, Eq. (17) can be reduced to

$$K = \mathrm{diag}\left[\frac{1}{\alpha J_1 + \beta}, \frac{1}{\alpha J_2 + \beta}, \frac{1}{\alpha J_3 + \beta}\right] \tag{22}$$

and Eq. (20) can be relaxed to

$$D = \mathrm{diag}[d_1, d_2, d_3] > 0 \tag{23}$$

Observe that $\alpha = 0$ implies $\boldsymbol{K} = (1/\beta)\boldsymbol{I}$, which indicates an identical gain in each axis. The stability result for the case was derived in Refs. 7 and 16. Also, observe that $\beta = 0$ implies $\boldsymbol{K} = (1/\alpha)\boldsymbol{J}^{-1}$. This case was studied by Mortensen.[12] Hence, previous results for the feedback gains can be derived as special cases of the matrix $\boldsymbol{K}$ defined in this paper.

Remark 2

The following constraint for the global stability can be obtained from Eq. (22):

$$\frac{J_2 - J_3}{K_1} + \frac{J_3 - J_1}{K_2} + \frac{J_1 - J_2}{K_3} = 0 \tag{24}$$

This equation indicates that only two gains can be selected independently. The third gain is related to the first and second gains via the principal moments of inertia.

Remark 3

Because we want an eigenaxis rotation, and because this can be achieved only by a quaternion gain matrix proportional to the inertia matrix, the selection of α and β in Eq. (17) should minimize some measure of the distance between the gain matrix and the inertia matrix. A natural selection of a performance index is

$$PI = \sum_{i=1}^{3} (\alpha J_i + \beta - 1/J_i)^2 \tag{25}$$

Minimization of Eq. (25) with respect to α and β leads to the following results:

$$\alpha = \left[9 - \left(\sum_{i=1}^{3} 1/J_i\right)\left(\sum_{i=1}^{3} J_i\right)\right] \Big/ \left[3\left(\sum_{i=1}^{3} J_i^2\right) - \left(\sum_{i=1}^{3} J_i\right)^2\right] \tag{26a}$$

$$\beta = \left[\left(\sum_{i=1}^{3} 1/J_i\right)\left(\sum_{i=1}^{3} J_i^2\right) - 3\left(\sum_{i=1}^{3} J_i\right)\right] \Big/ \left[3\left(\sum_{i=1}^{3} J_i^2\right) - \left(\sum_{i=1}^{3} J_i\right)^2\right] \tag{26b}$$

Remark 4

The equilibrium points associated with the system described by Eqs. (14) are $\boldsymbol{\omega} = 0$, $\boldsymbol{q} = 0$, $q_4 = 1$ or $\boldsymbol{\omega} = 0$, $\boldsymbol{q} = 0$, $q_4 = -1$. The selection of the quaternion gain matrix sign determines the convergent equilibrium point. The Lyapunov function defined in Eq. (15) considers the *negative* quaternion feedback gain and the equilibrium point $\boldsymbol{\omega} = 0$, $\boldsymbol{q} = 0$, $q_4 = 1$. The Lyapunov function associated with the *positive* quaternion feedback gain and the equilibrium point $\boldsymbol{\omega} = 0$, $\boldsymbol{q} = 0$, $q_4 = -1$ is described by

$$\begin{aligned} V &= \tfrac{1}{2}\boldsymbol{\omega}^T\boldsymbol{K}^{-1}\boldsymbol{J}\boldsymbol{\omega} + q_1^2 + q_2^2 + q_3^2 + (q_4 + 1)^2 \\ &= \tfrac{1}{2}\boldsymbol{\omega}^T\boldsymbol{K}^{-1}\boldsymbol{J}\boldsymbol{\omega} + 2(1 + q_4) \end{aligned} \tag{27}$$

In this case, Eq. (14a) is replaced by

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} = (1 - \mu)\boldsymbol{\Omega}\boldsymbol{J}\boldsymbol{\omega} - \mathbf{D}\boldsymbol{\omega} + \boldsymbol{K}\boldsymbol{q} \tag{28}$$

Using Eqs. (27), the stability analysis is identical to the one discussed previously.

Remark 5

In order to guarantee the shortest angular path, the sign of the quaternion feedback gain is defined by the initial value of q_4. The corresponding control torque is then

$$\boldsymbol{u} = -\mu\boldsymbol{\Omega}\boldsymbol{J}\boldsymbol{\omega} - \boldsymbol{D}\boldsymbol{\omega} - \mathrm{sign}[q_4(0)]\boldsymbol{K}\boldsymbol{q} \tag{29}$$

where μ may take the values 0 or 1 and $\boldsymbol{K} > 0$.

Remark 6

Selection of $\boldsymbol{K}$ and $\boldsymbol{D}$ as $\boldsymbol{K}^{-1} = \alpha\boldsymbol{J} + \beta\boldsymbol{I}$, $\boldsymbol{D} = d\boldsymbol{J}$ leads to the following Lyapunov function and its decay rate

$$V = \tfrac{1}{2}\boldsymbol{\omega}^T(\alpha\boldsymbol{J} + \beta\boldsymbol{I})\boldsymbol{J}\boldsymbol{\omega} + 2(1 \mp q_4) \tag{30a}$$

$$\dot{V} = -d\boldsymbol{\omega}^T(\alpha\boldsymbol{J} + \beta\boldsymbol{I})\boldsymbol{J}\boldsymbol{\omega} \tag{30b}$$

where $1 - q_4$ indicates negative quaternion feedback and $1 + q_4$ indicates positive quaternion feedback.

Since

$$-d\boldsymbol{\omega}^T(\alpha\boldsymbol{J} + \beta\boldsymbol{I})\boldsymbol{J}\boldsymbol{\omega} \geq -2d[\tfrac{1}{2}\boldsymbol{\omega}^T(\alpha\boldsymbol{J} + \beta\boldsymbol{I})\boldsymbol{J}\boldsymbol{\omega} + 2(1 \mp q_4)]$$

then

$$\dot{V} \geq -2dV$$

and therefore

$$V(t) \geq V(t_0)\exp[-2d(t - to)] \tag{31}$$

Equation (31) defines the maximal decay rate of the selected Lyapunov function. Observe that the maximal decay rate is independent of the quaternion feedback gain.

Remark 7

The constraints on the quaternion feedback gain matrix are imposed only if the gyroscopic coupling torque is not canceled by the control torque. In the case of precise cancellation of the gyroscopic torque, the quaternion feedback gain matrix is unconstrained. Simulation results show that the closed-loop system is still (globally) stable even when the quaternion feedback gain is proportional to the inertia matrix, and the control torque *does not* counteract the gyroscopic torque via the gyroscopic decoupling.

IV. Selection of the Quaternion Feedback Gain and the Damping Gain Matrices

As discussed in Sec. II, the quaternion feedback gain matrix $\boldsymbol{K}$ should satisfy $\boldsymbol{K} = k\boldsymbol{J}$ and $\boldsymbol{D} = d\boldsymbol{J}$, where k and d are positive scalars, in order to achieve eigenaxis rotation.

Let $\boldsymbol{\lambda}$ be a unit vector along the eigenaxis; we then have $\boldsymbol{q} = \sin(\phi/2)\boldsymbol{\lambda}$. Assuming that the angular rate $\boldsymbol{\omega}$ is small enough to allow the gyroscopic term to be neglected (or that the gyroscopic torque can be counteracted if its effect is significant), Eq. (14a) can be approximated by

$$(\ddot{\phi} + d\dot{\phi} + k\ \sin\phi/2)\boldsymbol{J}\boldsymbol{\lambda} = 0 \tag{32}$$

As we assume the eigenaxis rotation, the angular rate satisfies $\boldsymbol{\omega} = \dot{\phi}\boldsymbol{\lambda}$.

With $\boldsymbol{J}\boldsymbol{\lambda} \neq 0$, Eq. (32) is reduced to

$$\ddot{\phi} + d\dot{\phi} + k\ \sin\phi/2 = 0 \tag{33}$$

For the purpose of selecting gains, we may approximate $\sin\phi/2$ by $\phi/2$, for $\phi \leq 90$ deg. We then have the well-known linear second-order equation

$$\ddot{\phi} + d\dot{\phi} + k\phi/2 = 0 \tag{34}$$

where the damping ratio ζ and the natural frequency ω_n satisfy

$$d = 2\zeta\omega_n; \qquad k/2 = \omega_n^2$$

Proper selection of ζ and ω_n defines d and k. For $\phi(0) \cong 180$ deg, however, a modified settling time relation of $8/\zeta\omega_n$ should be used (instead of the standard $4/\zeta\omega_n$ relation) to account for the nonlinear effect of $\sin(\phi/2)$.

V. Global Stability with Robustness to Inertia Uncertainty

Let J_n denote the nominal value of the inertia matrix and ΔJ the uncertainty. Then, Eq. (14a) has the following form:

$$(J_n + \Delta J)\dot{\omega} = \Omega(J_n + \Delta J)\omega - \Omega J_n\omega - D\omega - Kq \quad (35)$$

where the gyroscopic torque is not precisely canceled because of inertial property uncertainty.

Equation (35) may be rewritten as

$$J\dot{\omega} = \Omega \Delta J\omega - D\omega - Kq \quad (36)$$

where $J = J_n + \Delta J$.

Using the results of Sec. III, we realize that $K^{-1} = \beta I$ (or, equivalently, $K = kI$) can guarantee global stability even though we do not know the value of ΔJ. The price we pay for robustness is that the rotation is not performed about the eigenaxis.

In Refs. 7 and 16, the stability result was derived for the case of identical gains for all axes; however, robustness issues were not considered.

Remark 1

Consider the case in which we have a perfect cancellation of the gyroscopic torque. For this case, minimization of the following performance index

$$PI = \frac{1}{2}\int_0^{\infty} [a_1^2 q^T J^{-1} q + 2a_1 a_2 q^T J^{-1}\omega + a_2^2 \omega^T J^{-1}\omega - a_1 q_4 \omega^T\omega + u^T J^{-1} u]\, dt \quad (37)$$

with respect to u and subject to

$$J\dot{\omega} = u \quad (38a)$$

$$\dot{q} = \tfrac{1}{2}\Omega q + \tfrac{1}{2}q_4\omega \quad (38b)$$

$$\dot{q}_4 = -\tfrac{1}{2}\omega^T q \quad (38c)$$

leads to the optimal control torque

$$u^0 = -a_1 q - a_2\omega \quad (39)$$

The preceding result is derived when we assume that the adjoint vector has the form $\lambda = [\lambda_\omega^T, \lambda_q^T, \lambda_{q4}]^T$, where $\lambda_\omega = a_1 q + a_2\omega$, $\lambda_q = a_1\omega$, and $\lambda_{q4} = \text{const.}$[31]

Equation (39) indicates that the optimal control associated with Eq. (37) consists of identical quaternion feedback gain for all the axes. This globally stable, optimal control is also *robust* to inertia matrix uncertainty.

VI. Design Example

In this section, we present a control design example and simulation results. An asymmetric rigid spacecraft with the following inertia matrix is considered:

$$J = \begin{bmatrix} 1200 & 100 & -200 \\ 100 & 2200 & 300 \\ -200 & 300 & 3100 \end{bmatrix} \text{Kg} \cdot \text{m}^2$$

The nominal values of the principal moments of inertia are assumed as $J_1 = 1200$, $J_2 = 2200$, and $J_3 = 3100$ for controller design. The products of inertia or the control-axis misalignment relative to the principal axes are assumed to be quite uncertain; hence, they are not used in controller gain selection.

The initial quaternion elements at $t = 0$ are assumed as

$$[q_1, q_2, q_3, q_4] = [0.57, 0.57, 0.57, 0.159]$$

which corresponds to an initial eigenangle-to-go of 161.7 deg. The desired reorientation time or settling time is assumed as 50 s. For a critically damped response, we have $\omega_n = 0.158$ rad/s, which results in $k = 0.05$ and $d = 0.316$.

Four different cases are considered here, but each case has the same rate gain matrix of $D = 0.316\,\text{diag}(1200,2200,3100)$. All of the quaternion feedback gain matrices are normalized with respect to $K_2 = 110$.

Case 1

$K_i = k/J_i$ by Mortensen:[12]

$$K = \text{diag}(201,110,78)$$

Case 2

$K = kI$ by Vadali/Junkins[7] and Wie/Barba:[16]

$$K = \text{diag}(110,110,110)$$

Case 3

$K = (\alpha J + \beta I)^{-1}$ of Eqs. (26):

$$K = \text{diag}(72,110,204)$$

Case 4

$K = kJ$ of Sec. IV:

$$K = \text{diag}(60,110,155)$$

In simulation, we assume 10% mismatching of J_i ($i = 1,2,3$) and an initial body rate of 0.01 rad/s in each axis; hence, $\mu = 0.9$ for the cancellation of the gyroscopic coupling term in Eq. (14a). Figure 1 shows time histories of the quaternions and control torques. We notice that all four cases have similar q_4 histories and that cases 3 and 4 are nearly identical in all the

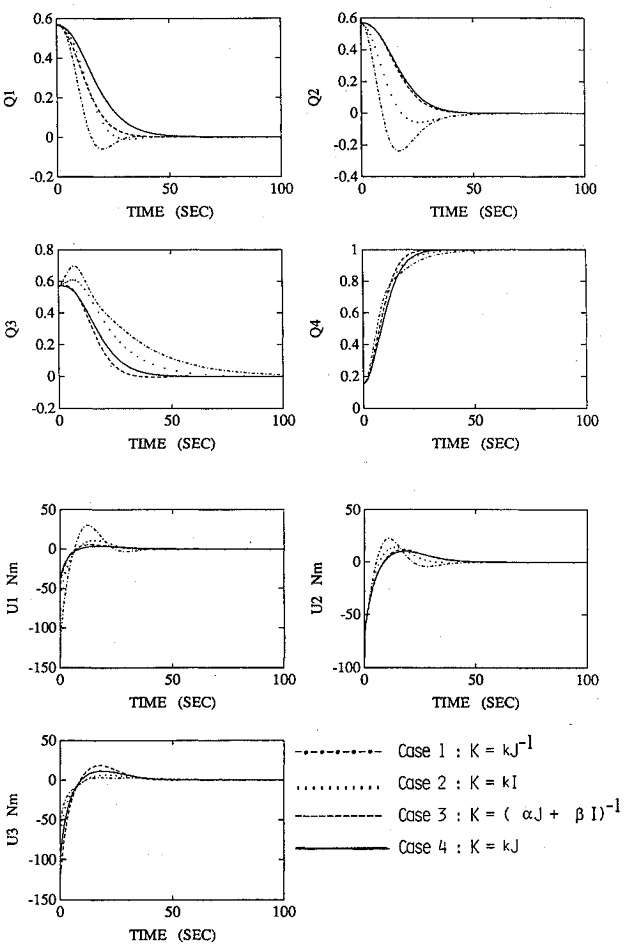

Fig. 1 Time histories of quaternions and control torques.

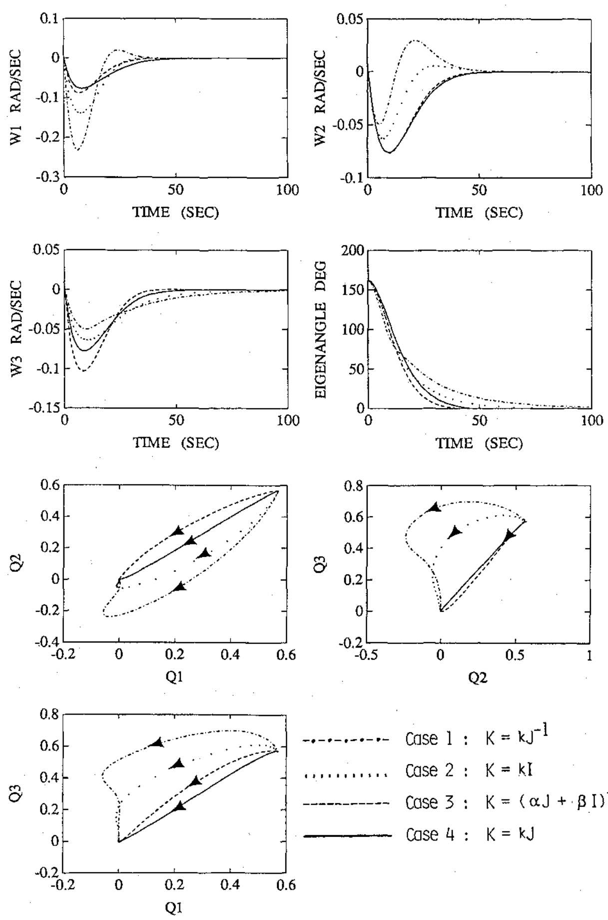

Fig. 2 Time histories of angular rates, and q_i vs q_j plots.

quaternion histories. A similar observation can be made for the angular velocity time histories of cases 3 and 4 shown in Fig. 2. The time-history plots of quaternions or angular velocities, however, do not indicate the deviation of the angular path from the eigenaxis rotation. The q_i vs q_j plots in Fig. 2 clearly indicate the angular deviation of the instantaneous rotational axis with respect to the initial eigenaxis. The perfect eigenaxis rotation becomes a straight line in the q_i vs q_j plot. The q_i vs q_j plots in Fig. 2 show that case 4 has a stable, near-eigenaxis rotation, even in the presence of 10% mismatching of the inertia matrix. It can also be seen that case 1 and case 2 ($\boldsymbol{k} = k\boldsymbol{J}$) have a noneigenaxis rotation.

VII. Conclusions

We have shown that, for the ideal case, the eigenaxis rotation can be simply achieved by a proper selection of the feedback gain matrices of the proposed quaternion feedback regulator. We have also derived a more general Lyapunov function for the quaternion feedback control, which includes all of the previous results as special cases. Since the eigenaxis rotation provides the shortest angular path, the proposed controller may provide a simple solution for the large-angle reorientation of future spacecraft. The mathematical results of this paper will also be of interest to control researchers who use the Lyapunov method for stability analysis of highly coupled, nonlinear, dynamical systems.

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