An Approach to Magnetic Torque Attitude Control of Satellites via 'H_∞' Control for LTV Systems

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***Abstract*— A new approach to the three-axis attitude control of satellites using magnetic torque rods is detailed in this paper. The system is under-actuated as the magnetic torque rods cannot apply torque in a direction parallel to the Earths magnetic field. However for orbits inclined to the Earth's magnetic equator, the direction of the field vector changes and this fact can be used to design controllers for the system. An extension of the H_∞ control technique to LTV systems is used to design periodic controllers for the periodic system. The resulting controllers are simulated on the full non-linear system. The stability and accuracy of the closed-loop system, to less than a degree of pointing error, is demonstrated in the presence of disturbances including gravity-gradient torques, drag torques, and solar radiation pressure torques.**

I. Introduction

The problem of attitude control of satellites is not new and has been addressed by several researchers using many different approaches. Momentum-wheels have been typically used [1] to achieve high pointing accuracy. Passive attitude control of satellites has also been suggested via gravity-gradient control [2] or spin-control [3]. The approach employed for attitude control in this paper is however different from the work cited above in that it relies on the use of three orthogonal magnetic torque rods and the Earth's magnetic field to generate stabilizing control torques. This approach was first suggested by [4] and has been subsequently followed by many other works that employ torque rods to provide control actuation, for example [5] and [6]. A recent work [7] that proposes the use of magnetic torque rods tackles the control problem by designing a periodic controller by extending the classical linear quadratic regulator technique for periodic systems. This paper closely follows the work done in [7] and also considers the same satellite configurations considered. It differs from the previous approach in the control tools used to solve the problem. Magnetic attitude control is an attractive alternative to the attitude control of small light-weight satellites as these are often subject to stringent cost and weight constraints.

Attitude control of a satellite using the Earth's magnetic field vector b and a current carrying coil of dipole strength m is a particularly challenging problem and as can be seen by inspecting the equation that gives the torque n_m generated,

$$n_m = m \times b. \tag{1}$$

The torque generated is at all times orthogonal to the Earth's magnetic field vector, and as such cannot provide a control torque in a direction parallel to the vector. However for orbits which are inclined to the Earth's magnetic equator the direction of the field vector changes and it is possible to use this changing direction to stabilize motion over the entire orbit.

The present work utilizes the periodic nature of the system to design a periodic controller for the system. The non-linear equations of motion are linearized and discretized and the problem is converted to the problem of control of a periodic LTV system. In order to stabilize this periodic LTV system, tools developed for LTV systems that extend the H_∞ synthesis problem, which has in the past been formulated only for LTI systems, are used.

The application of these tools results in a controller that stabilizes the nonlinear system in the presence of disturbance which include gravity-gradient effects, aerodynamic drag torques and solar radiation pressure torques. The thrust required by the control law is seen to be reasonable. The main contributions of the paper essentially hinge on the control methodology adopted. The Earth's magnetic field is not completely known and while reasonably accurate models exist it is still subject to modeling errors. In this light a control tool that uses the Earth's magnetic field for actuation should perform robustly in the presence of uncertainty in the field. The current approach is ideal for further investigations of the effect of parametric uncertainty on the robust performance and stability of the designed closed-loop system [8].

II. Magnetic Attitude Control Problem

A. Reference Frames

The three coordinate frames relevant to the attitude control problem are:

- Earth-Centered-Earth-Fixed (ECEF): This reference frame is fixed to the center of the Earth and assumes that the Earth is fixed in inertial space.
- Local Level (LL): The origin of this coordinate frame is at the center of mass of the satellite, with its z axis pointing towards the Earth (nadir) and its x axis pointing along the instantaneous orbital velocity vector. The y axis is given by the right-hand rule.
- Body Fixed (BF): This frame is fixed to the body of the satellite. For the current discussion this frame is

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assumed to be aligned along the principal axis of the satellite.

For a nadir-pointing satellite the pointing error will be zero when the LL coordinate frame is aligned with the BF frame. The deviations of the BF frame from the LL frame are parameterized by the quaternion $q(t)$. The notation $A_{\zeta/\eta}$ will be used to denote the transformation matrix that takes vectors from coordinate from η to ζ. For an introduction to the quaternion and its use in parametrization of satellite attitude refer to [9]. The equations of motion are expressed in the BF frame as this frame is typically the most convenient to capture rigid-body dynamics due to the fact that the inertia tensor is constant in this frame. The ECEF frame is used to calculate the position $r_{ECEF}(t)$, and the velocity $v_{ECEF}(t)$ of the satellite, the absolute angular velocity of the LL frame $\omega_{LL}(t)$ and the transformation matrix $A_{LL/ECEF}(t)$

B. Nonlinear Equations of Motion

The nonlinear equations of motion are used to validate the controller via numerical simulation. The equations are given by

$$\dot{q} = \begin{bmatrix} 0 & \omega_{BF/LL3} & -\omega_{BF/LL2} & \omega_{BF/LL1} \\ -\omega_{BF/LL3} & 0 & \omega_{BF/LL1} & \omega_{BF/LL2} \\ \omega_{BF/LL2} & -\omega_{BF/LL1} & 0 & \omega_{BF/LL3} \\ -\omega_{BF/LL1} & -\omega_{BF/LL2} & -\omega_{BF/LL3} & 0 \end{bmatrix} q, \tag{2}$$

$$\mathbf{I} \cdot \dot{\omega} + \omega \times \mathbf{I} \cdot \omega = m \times b + n_{gg} + n_d. \tag{3}$$

$\mathbf{I}$ is the moment of inertia tensor and is assumed to be diagonal for the present study. ω is the absolute angular velocity of the satellite expressed in the BF frame. $\omega_{BF/LL}$ is the angular velocity of the BF frame with respect to the LL frame expressed in BF coordinates. n_{gg} and n_d are the gravity gradient and other disturbance torques respectively. Assuming a spherical model for the Earth the gravity-gradient torques are inversely proportional to the cube of the distance from the center of the Earth and vanish for a perfectly spherical satellite. The non-linear simulations employed in this paper also account for J_2 terms. The total drag torque about the center of mass is calculated by determining the sum of the vector product of the individual center of pressure of the surfaces of the satellite from the center of mass of the satellite with the force acting on the corresponding surface. The drag force is proportional to the product of the atmospheric air-density and the square of the satellites translational velocity. For a more detailed discussion on the various terms and equations used to determine them refer [7] and [9].

The magnetic field vector b is a function of both time and the orbital position of the satellite. A part of the time dependence of the magnetic field comes from an observed drift in the mean value of the field which has been accounted for in the nonlinear simulation. A dipole approximation of the Earth's magnetic field [9] in the absence of the Earth's rotation in LL frame is given by,

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \frac{\mu_f}{a^3} \begin{bmatrix} \cos\omega_0 t \sin i_m \\ \cos i_m \\ 2\sin\omega_0 t \cos i_m \end{bmatrix} \tag{4}$$

where $\mu = 7.9 \times 10^{15} Wb \cdot m$, a is the radius of the satellites orbits, ω_0 is the absolute angular velocity of the LL frame expressed in satellite coordinates and i_m is the inclination of the orbit with respect to the Earth's magnetic equator.

C. Linearized Equations of Motion

The kinematic and dynamic equations (2) and (3) are too complex for control synthesis. To circumvent this problem the equations are linearized following the standard approach outlined in [9]. The equations are linearized about deviations from the nadir-pointing attitude assuming a circular orbit. The linearized equations which accounts for gravity-gradient effects, aerodynamic drag torques and control torques are given by

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{\omega}_{BF/LL1} \\ \dot{\omega}_{BF/LL2} \\ \dot{\omega}_{BF/LL3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -4\omega_0^2\sigma_1 & 0 & 0 & 0 & 0 & \omega_0(1-\sigma_1) \\ 0 & 3\omega_0^2\sigma_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_0^2\sigma_3 & -\omega_0(1+\sigma_3) & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi \\ \theta \\ \psi \\ \omega_{BF/LL1} \\ \omega_{BF/LL2} \\ \omega_{BF/LL3} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_3(t)/I_{11} & -b_2(t)/I_{11} \\ -b_3(t)/I_{22} & 0 & b_1(t)/I_{22} \\ b_2(t)/I_{33} & -b_1(t)/I_{33} & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/I_{11} & 0 & 0 \\ 0 & 1/I_{22} & 0 \\ 0 & 0 & 1/I_{33} \end{bmatrix} n_d \tag{5}$$

where ϕ, θ and ψ are the roll, pitch and yaw deviations about the equilibrium nadir-pointing configuration. For small deviations [10] the quaternion $q \approx [\phi/2, \theta/2, \psi/2, 1]'$. $\sigma_i = (I_{jj} - I_{kk})/I_{ii}$ for cyclic variants of the set $\{1,2,3\}$. The dipole moment of the current carrying coils is denoted by m_i and is expressed in BF coordinates.

III. 'H_∞ SYNTHESIS': PERIODIC LTV SYSTEMS

Having obtained, from literature, the nonlinear and linear equations of motion for the rigid body motion of the satellite, attention is now turned to the problem of stabilizing the satellite. Techniques recently developed by Dullerud and Lall, introduced in [8] for the analysis and control of periodic LTV systems, were used to design a controller for the system. The usual discrete time state-space description of an LTV system is given by

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k u_k \\ y_k &= C_k x_k + D_k u_k, \end{aligned} \tag{6}$$

where x_k is the state of the system, u_k is the control input, y_k is the sensor measurement, and A_k, B_k, C_k and D_k are time-varying matrices that capture the dynamics of the system. Consider an operator $\mathcal{A}$, defined in the following block-diagonal form,

$$\begin{bmatrix} A_0 & & & 0 \\ & A_1 & & \\ & & A_2 & \\ 0 & & & \ddots \end{bmatrix}.$$

Similar definitions can be made for $\mathcal{B}$,$\mathcal{C}$ and $\mathcal{D}$. Also define $x = (x_0, x_1, x_2, \ldots)$ and make similar definitions for u and y. Now with the introduction of a shift-operator defined by,

$$(\mathcal{Z}x) = (0, x_0, x_1, x_2 \cdots), \tag{7}$$

it can be seen that an equivalent representation of the system (6) can be made in terms of block-diagonal operators.

$$\begin{aligned} x =& \mathcal{Z}\mathcal{A}x + \mathcal{Z}\mathcal{B}u \\ z =& \mathcal{C}x + \mathcal{D}u \end{aligned} \tag{8}$$

This formulations leads to an operator-based description of the system and a function, called the *system function*, which has many properties analogous to those of transfer functions for LTI systems, like the induced norm being the maximum of a matrix norm over frequency. This framework thus allows us to apply techniques formerly restricted to LTI systems.

In particular, the traditional H_∞ analysis and synthesis problem for LTI systems can now be formulated for LTV systems. The basic set-up for control design is depicted in Fig. 1. $\mathbf{G}$ is the system to be controlled, while $\mathbf{K}$ is the controller that needs to be designed. Variables w are the exogenous signals which consist of disturbances and tracking signals. Variables z are the error signals that must be kept small. Variables y are the sensor signals, while variables u are the control signals.

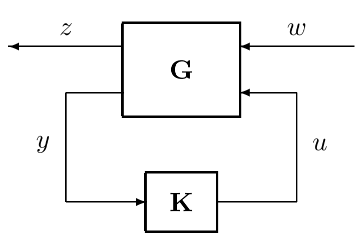

Fig. 1. Feedback interconnection

Let the system $\mathbf{G}$ be defined by the following state space equations:

$$\begin{aligned} x_{k+1} =& A_k x_k + B_{1k} w_k + B_{2k} u_k, \\ z_k =& C_{1k} x_k + D_{11k} w_k + D_{12k} u_k, \\ y_k =& C_{2k} x_k + D_{21k} w_k. \end{aligned} \tag{9}$$

A controller $\mathbf{K}$ (a relation between the sensor signals y and the control variables u) is to be designed, of the form

$$\begin{aligned} x^K_{k+1} =& A^K_k x^K_k + B^K_k y_k, \\ u_k =& C^K_k x^K_k + D^K_k y_k, \end{aligned} \tag{10}$$

such that the closed loop system is stable and the map from w to z is minimized. A closed-loop realization of the system can be written as

$$\begin{aligned} x^L_{k+1} =& A^L_k x^L_k + B^L_k w_k, \\ z_k =& C^L_k x^L_k + D^L_k w_k, \end{aligned} \tag{11}$$

where x^L_k has the states of both G and K, and all the matrices are of appropriate dimensions. This equation is in a form similar to (6), and once again using the shift-operator the system can be rewritten in an operator framework. Now following the standard approach outlined in textbooks on robust control [11], the problem of determining a stabilizing controller K that minimizes the input to output norm can be cast in terms of a feasibility condition for linear operator inequalities.

The important difference to note here with respect to the standard formulation is that while the H_∞ control problem for LTI systems is stated using linear matrix inequalities, for LTV systems the results are in the form of linear operator inequalities. The problem therefore is typically infinite-dimensional. However, for the case of periodic LTV systems the repetitive structure of the block diagonal operators leads to a solution in the form of linear matrix inequalities, which can be solved by traditional means.

IV. Control Problem Formulation

To obtain the equations of motion in a form suitable for controller synthesis, the linear equations of motion (5), which can be expressed as

$$\dot{x} = Ax + B_u(t)u + B_w w \tag{12}$$

have to be discretized, where w is the disturbing torque.

As the system is subjected on the average to constant disturbing torques their effect can be mitigated by minimizing an integral of the angular deviations, $z = \int_0^t x_i(\tau)d\tau$ for $i = 1, 2, 3$ in addition to minimizing the state. Specifically the equations of motion are augmented as in [7] by the introduction of additional states,

$$\dot{x}_{aug} = \begin{bmatrix} A & 0 \\ [I_{(3,3)}, 0] & 0 \end{bmatrix} x_{aug} + \begin{bmatrix} B(t) \\ 0 \end{bmatrix} u + \begin{bmatrix} B_w(t) \\ 0 \end{bmatrix} w \tag{13}$$

The above linear system is discretized using a zero-order hold with the sampling step, ΔT. The discrete-time equation is given by

$$x_{k+1} = \phi x_k + \Gamma_u u_k + \Gamma_w w_k, \tag{14}$$

where k is the time index. Also,

$$\phi = e^{\Delta T \bar{A}}, \tag{15}$$

$$\Gamma_u = \int_0^{\Delta T} e^{\tau \bar{A}} d\tau \bar{B}_u, \tag{16}$$

$$\Gamma_w = \int_0^{\Delta T} e^{\tau \bar{A}} d\tau \bar{B}_w. \tag{17}$$

where $\bar{A}$,$\bar{B}_u$ and $\bar{B}_w$ refer to the augmented matrices in (13).

Having obtained the equation of motion in a form suitable for controller synthesis ($\phi = A_k, \Gamma_u = B_{1k}$ and $\Gamma_w = B_{2k}$) as required by (9) attention is now turned to the other matrices that need to be determined to formulate the problem completely. As the state of the plant is to be minimized so as to be 'close' to the nominal Halo orbit, while at the same time penalizing control effort, the following choices are made for C_{1k}, D_{11k} and D_{12k},

$$C_{11k} = \begin{bmatrix} I_{9,9} \\ 0_{3,9} \end{bmatrix} \qquad D_{11k} = \begin{bmatrix} 0_{9,3} \\ I_{3,3} \end{bmatrix} \qquad D_{12k} = \begin{bmatrix} 0_{12,3} \end{bmatrix}. \tag{18}$$

Full-state feedback of the original systems without augmentation is considered, therefore C_{2k} is chosen to be the matrix, $\begin{bmatrix} I_{(6,6)} \\ 0_{(3,6)} \end{bmatrix}$ with D_{21k} set to zero. The periodic orbit was discretized at twenty-five points along the orbit.

V. Controller Synthesis and Non-Linear Simulation Results

The LTV Toolbox developed by Dullerud and Lall [8] was used for control design and analysis. Controllers for two different space-crafts, as considered in [7] were designed and the closed-loop response of the nonlinear systems subject to disturbance was obtained. The closed-loop system for both satellite was found to maintain a pointing accuracy of less than a degree.

Satellite Y is a cube of dimensions 0.7, 0.6, and 0.9 along the roll, pitch, and yaw axes. The inertia tensor I=diag[8.7,10,6.5] $kg{\cdot}m^2$. From (5) one can see that gravity-gradient stability of the open-loop system depends on the sign of σ_i $i = 1, 2, 3$ and that Satellite Y is stable along all three-axes. It is placed in a circular orbit at an altitude of 600 km from the surface and with an inclination of 90 deg. Fig. (2) shows a plot of the pointing error with an initial condition of 10 deg deviation along all three axes. The nonlinear simulation is carried out while the satellite is subjected to a mean disturbance torque of about $1 \times 10^{-7} Nm$ about the roll and yaw axes and $5 \times 10^{-8} Nm$ about the pitch axis. In the face of this disturbance the satellite maintains a pointing accuracy of about a degree along the yaw axis and better than half a degree along the pitch and roll axes. Fig. (3) shows a plot of the magnetic dipole moment required by this controller. This is well within the actuation limit typically encountered in magnetic coils used for these purposes [7]. The open-loop response of this configuration would be 0, 0.5 and 7.8 deg along the roll, pitch and yaw axis. It can be seen that at a small cost of accuracy along the roll and pitch axes substantial accuracy is obtained along the yaw axis. Given the disturbance torques that the system is subject to the periodic deviations observed are to be expected . Also the system stabilizes about the desired configuration in the order of about two periods which is a reasonable time given the penalty on the control effort.

While Satellite Y was gravity-gradient stable along all three axes Satellite Z is chosen so that it is neutrally stable

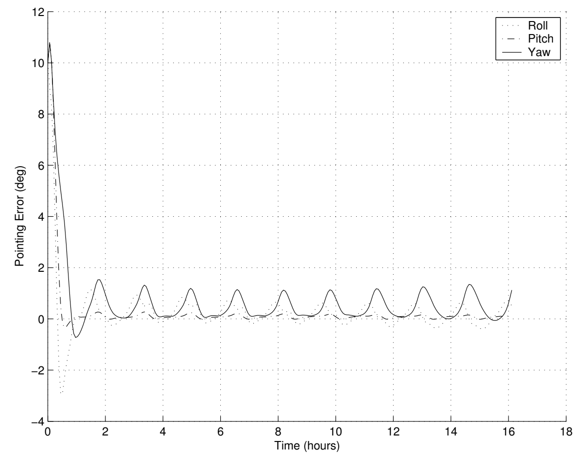

Fig. 2. Pointing error of the simulated controlled nonlinear system, Satellite Y.

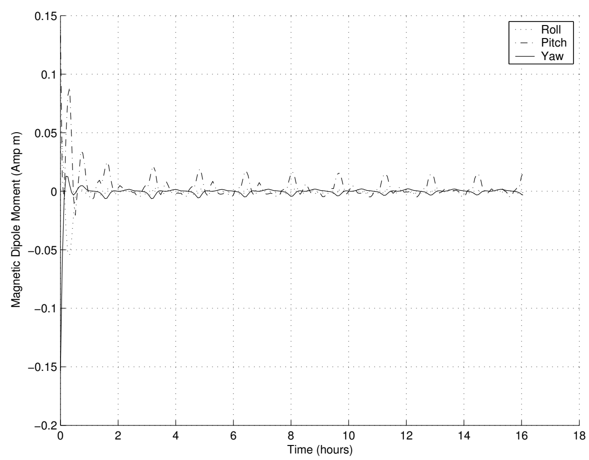

Fig. 3. Control effort required for Satellite Y.

along the yaw axis. The inertia tensor for this satellite is give by I=diag[250,250,10]$kg \cdot m^2$. It is placed in an orbit with an altitude above the surface of the Earth of 657 km and an inclination of 57 deg. Fig. (4) and Fig. (5) show similar plots as for Satellite Y. The initial deviation is again chosen to be 10 deg along all three axes. The average disturbing torques were about $6 \times 10^{-6} Nm$ along all three axes. In the presence of this disturbance the satellite maintains a pointing accuracy of less than 0.2 deg along all the three axes and achieves this in about four periods. This performance is quite exceptional especially when the open-loop system is neutrally stable along the yaw axis.

The robust stability and performance of both the closed-loop systems was tested numerically by simulating the closed-loop system in the presence of errors which included but were not limited to a changed semi-major axis of the orbit, the strength of the magnetic field, and orbital inclinations. The closed-loop systems were found to perform reasonably satisfactorily even in the presence of such errors which is a testament to the robustness of controllers

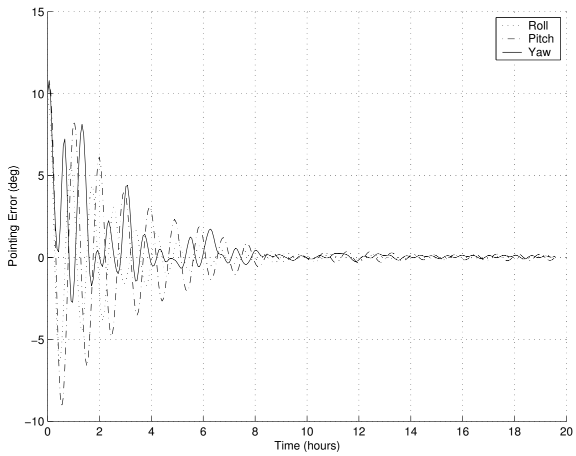

Fig. 4. Pointing error of the simulated controlled nonlinear system, Satellite Z.

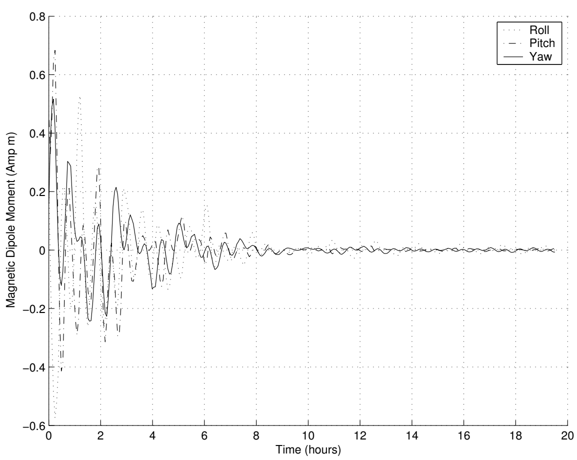

Fig. 5. Control effort required for Satellite Z.

synthesized by H_∞ control techniques. Naturally it is to be noted that such tests are not a substitute for a rigorous proof of robust stability and performance of the closed-loop system.

VI. Conclusions and Directions for Future Work

In this paper a method, utilizing the extension of the typical H_∞ control to an LTV system, for the three-axis attitude control of a satellite is presented. The controller obtained is shown to do exceptionally well when it is simulated on the non-linear system even in the presence of reasonably large disturbing torques. One point that has to be made is that this method requires an internal clock to be maintained. This clock should be reset to zero every time the satellite crosses the magnetic equator. This crossing can be determined by the onboard magnetometers when the third component of the Earth's magnetic field vector goes to zero.

The advantage of this formulation over other perviously considered formulations is the fact that other results from traditional H_∞ control can be brought over to the analysis of LTV systems. For the current problem this should be a promising extension as the Earth's magnetic field is not accurately known and guaranteed results on robust stability of the controller would be most welcome. This avenue of research is currently being explored by the authors.

Acknowledgements

As can be seen the structure and flow of this paper closely follows a paper on the same subject by Prof. Mark Psiaki in the Department of Mechanical and Aerospace Engineering at Cornell University. Prof. Psiaki was also extremely generous by providing the authors access to his extensive high-fidelity non-linear simulation code for satellite attitude control for which we are very thankful. We would also like to thank Prof. Raffaello D'Andrea, again in the Department of Mechanical and Aerospace Engineering at Cornell University for suggesting this approach to the problem of attitude control.

References

[1] P. Tsiotras, H. Shen, and C. Hall, "Satellite attitude control and power tracking with energy/momentum wheels," *Journal of Guidance, Control, and Dynamics*, vol. 24, pp. 23–34, Jan. 2001.

[2] M. Ovchinnikov and A. Dyachenko, "Gravity-gradient attitude control systems for a satellite on a high-elliptical orbits," in *Proc. 2nd International Conference of ICSC*, 1996, pp. 87–93.

[3] D. Childs, B. Tapley, and W. Fowler, "Suboptimal attitude control of a spin-stabilized axisymmetric spacecraft," *Automatica*, vol. 14, no. 6, pp. 736–740, 1969.

[4] F. Martel, P. K. Pal, and M. L. Psiaki, "Active magnetic control system for gravity gradient stabilized spacecraft," in *Proc. 2nd Annual AIAA/USU Conference on Small Satellites*, Utah State University, USA, 1988.

[5] C. Arduini and P. Baiocco, "Active magnetic damping attitude control for gravity gradient stabilized spacecraft," *Journal of Guidance, Control, and Dynamics*, vol. 20, pp. 117–122, Jan. 1997.

[6] R. Wisniewski and M. Blanke, "Fully magnetic attitude control for spacecraft subject to gravity gradient," *Automatica*, vol. 35, no. 7, pp. 1201–1214, 1999.

[7] M. L. Psiaki, "Magnetic torquer attitude control via asymptotic periodic linear quadratic regulation," *Journal of Guidance, Control, and Dynamics*, vol. 24, pp. 386–393, Mar. 2001.

[8] G. E. Dullerud and S. Lall, "A new approach for analysis synthesis of time-varying systems," *IEEE Transactions on Automatic Control*, vol. 44, no. 8, pp. 1486–1497, 1999.

[9] J. R. Wertz, Ed., *Spacecraft attitude Determination and Control*. Holland: Kluwer Academic Publishers, 1978.

[10] D. T. Greenwood, Ed., *Principles of Dynamics*. New Jersey, USA: Prentice Hall, 1988.

[11] G. E. Dullerud and F. Paganini, *A Course in Robust Control Theory*. New York: Springer, 1999.