

Array & Matrices

What is Array :- Array is linear collection of homogeneous data items. This means that an array can store either all integers, all floating numbers or all character but all of same type.

Linear Array is One Dimensional Array :-

A linear array is a list of a finite number n of homogeneous data elements (i.e. data elements of same type) such that:-

- (a) the elements of the Array are ~~ref~~ referenced respectively by an index set consisting of n consecutive numbers.
- (b) The elements of the array are stored respectively in successive memory location.

the number n of elements is called the length or size of the array.

	A[0]	A[1]	A[2]	A[3]	A[4]	A[5]
Values	10	20	30	40	50	60
	1000	1002	1004	1006	1008	1010

Representation of linear Array in Memory

$$\text{length} = \text{UB} - \text{LB} + 1$$

UB = Upper Bound (largest Index)

LB = Lower Bound (Smallest Index)

In the above example

$$\begin{aligned} \text{Length} &= 5 - 0 + 1 \\ &= 6 \end{aligned}$$

$$\text{Size in bytes} = \text{length of Array} * \text{Size of data type}$$

In above example

$$\text{Size in bytes} = 6 * 2 = 12 \text{ bytes}$$

Calculation of Address for 1-D Array :-

$$\text{Address of } n^{\text{th}} \text{ element} = \text{Base Address} + \text{Size of data type} * (n-1)$$

Let in the above example we want to calculate the 3rd element Address
so

$$\begin{aligned} \text{Address of 3}^{\text{rd}} \text{ element} &= 1000 + 2(3-1) \\ &= 1000 + 4 \\ &= 1004 \end{aligned}$$

for example, let we have an array $A[4][4]$.

	Col 1	Col 2	Col 3	Col 4
Row 1	11	12	13	14
Row 2	21	22	23	24
Row 3	31	32	33	34
Row 4	41	42	43	44

4×4
 $m \times n$

This Array is stored in memory as follows:-

1000	02	04	06	08	10	12	14	16	18	20	22	24	26	28	30
11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
← Row 1 →				← Row 2 →				← Row 3 →				← Row 4 →			

Row Major order

Calculation of Memory Address for any element in i^{th} Row and j^{th} column of $m \times n$ array.

let we have a 2-D Array $A[m][n]$ and we want to calculate the Address of $A[i][j]$. then the formula for Row major order is:-

$$A[i][j] = \text{Base Address} + [(i-1)*n + (j-1)] * (\text{size of data type})$$

m = total No. of Rows
 n = total No. of columns.

hul

let in above Example we want to calculate
the Address of $A[3,3]$

then $m=4$, $n=4$, $i=3$, $j=3$

$$\begin{aligned} A[3,3] &= 1000 + [(3-1)4 + (3-1)] \times 2 \\ &= 1000 + [4 + 2] \times 2 \\ &= 1000 + 12 = 1012 \end{aligned}$$

Column Major Order :- In column major the elements are stored column-by-column i.e. first elements of the first column is stored then stored second column and so on.

Let we have a Matrix $A[4][4]$

	Col 1	Col 2	Col 3	Col 4
row 1	11	12	13	14
row 2	21	22	23	24
row 3	31	32	33	34
row 4	41	42	43	44

then this Array is stored in Memory as follows:-

1000	02	04	06	08	10	12	14	16	18	20	22	24	26	28	30
11	21	31	41	12	22	32	42	13	23	33	43	14	24	34	44
← Column 1 →				← Column 2 →				← Column 3 →				← Column 4 →			

Column Major Order

Calculation of Memory Address for any elements in i^{th} row and j^{th} column of $m \times n$ array by column major order.

Address of element

$$A[i, j] = \text{Base address} + [(i-1) + (j-1) * m] * \text{size of data type}$$

Let in above example we want to calculate the Address of $A[2, 3]$

then $m = 4$, $i = 2$
 $n = 4$, $j = 3$

$$\begin{aligned} A[2, 3] &= 1000 + [(2-1) + (3-1) * 4] * 2 \\ &= 1000 + [1 + 8] * 2 \\ &= 1000 + 18 \\ &= 1018 \end{aligned}$$

Numerical Questions

Q1. Consider a linear Array $A[10]$. Find the Address of $A[5]$ ^{5th element}. Consider the Array is float type and index is started with 0 and the base address is 2000.

Sol. Given $A[10]$,
 index = 0
 base Address is = 2000

$$\begin{aligned}
 \text{Address of } n^{\text{th}} \text{ element} &= \text{Base Address} + \text{Size of data type} \times (n-1) \\
 &= 2000 + 4(5-1) \\
 &= 2000 + 16 = 2016
 \end{aligned}$$

Q2. Consider the linear Array $A[5:50]$, and $B[-5:10]$

- a) Find the No. of elements in each Array.
- b) Suppose base Address is 300 and size of data type is 4 then find the Address of $A[15]$, $A[35]$, $A[50]$.

Ans (a)

A [5 : 50]

LB

UB

$$\text{Length } [A] = 50 - 5 + 1 = 46$$

$$\text{Length } [B] = 10 - (-5) + 1 = 16$$

(b)

$$\text{LOC } [A[K]] = \text{Base Address} + W(K - LB)$$

$$\text{LOC } [A[15]] = 300 + 4(15 - 5) = 340$$

$$\text{LOC } [A[35]] = 300 + 4(35 - 5) = 420$$

A[60] is not an element of Array.

Q3 Consider the 25×4 matrix array SCORE. Suppose $\text{Base}(\text{SCORE}) = 200$ and $W = 4$ words per memory cell. Furthermore, suppose the programming language stores two dimensional Arrays using column major order. then find the Address of ~~SCORE~~ SCORE(12,3).

$$\underline{\text{Sol}} \quad \text{LOC } [A(i, j)] = \text{Base}(A) + W((i-1) + M(j-1))$$

$$\text{SCORE}(12,3) = 200 + 4((12-1) + (3-1) \times 5)$$

$$= 200 + 4(11 + 50)$$

$$= 200 + (61 \times 4)$$

$$= 444$$

Q.4 Suppose Multidimensional Arrays A and B are declared using

$$A[-2:2, 2:22]$$

$$B[1:8, -5:5, -10:5]$$

find the length of each dimension and the number of elements in A and B.

Sol

$$A \begin{matrix} -2:2 & 2:22 \\ \swarrow & \downarrow & \swarrow & \downarrow \\ L_1 & U_1 & L_2 & U_2 \end{matrix}$$

$$\text{Length} = \text{Upper Bound} - \text{Lower Bound} + 1$$

$$\text{Length}_1 = U_1 - L_1 + 1$$

$$\text{Length}_2 = 2 - (-2) + 1 = 5$$

$$\text{Length}_2 = 22 - 2 + 1 = 21$$

So No of elements is $2 \times 5 = 105$ elements.

$$B \begin{matrix} 1:8 & -5:5 & -10:5 \\ \swarrow & \downarrow & \swarrow & \downarrow & \swarrow & \downarrow \\ L_1 & U_1 & L_2 & U_2 & L_3 & U_3 \end{matrix}$$

$$\text{Length}_1 = 8 - 1 + 1 = 8$$

$$\text{Length}_2 = 5 - (-5) + 1 = 11$$

$$\text{Length}_3 = 5 - (-10) + 1 = 16$$

So total No. of elements $8 \times 11 \times 16 = 1408$ elements

Q 5 Consider a two dimensional Array given below

$$A[-55:100, -100:55]$$

Base Address is = 1010

and $w = 4$

Calculate the Address of $A[0,0]$ using Row major and Column major order.

Ans

Row major order

$$A[i,j] = BA + [(i - LB_1) * \text{No. of Col.} + (j - LB_2)] * w$$

$$\text{No. of Rows} = UB_1 - LB_1 + 1$$

$$\text{No. of Col} = UB_2 - LB_2 + 1$$

$$A \begin{matrix} [-55:100, & -100:55] \\ \begin{matrix} / & | & / & | \\ LB_1 & UB_1 & LB_2 & UB_2 \end{matrix} \end{matrix}$$

$$\begin{aligned} \text{No. of Rows} &= UB_1 - LB_1 + 1 \\ &= 100 - (-55) + 1 \\ &= 156 \end{aligned}$$

$$\begin{aligned} \text{No. of Col} &= UB_2 - LB_2 + 1 \\ &= 55 - (-100) + 1 \\ &= 156 \end{aligned}$$

$$\text{Total No. of elements} = 156 \times 156$$

$$\begin{aligned} A[0,0] &= 1010 + [0 - (-55) * 156 + (0 - (-100))] * 4 \\ &= 1010 + [34720] \\ &= 35730 \end{aligned}$$

Column Major Order

$$A[i,j] = BA + [(i - lb_1) + (j - lb_2) \text{ No. of rows}] * w$$

$$= 1010$$

$$= 1010 + [55 + (100 \times 156)] * 4$$

$$= 1010 + (55 + 15600) * 4$$

$$= 1010 + (15655) * 4$$

$$= 1010 + 62620 = 63630$$