Discussion of

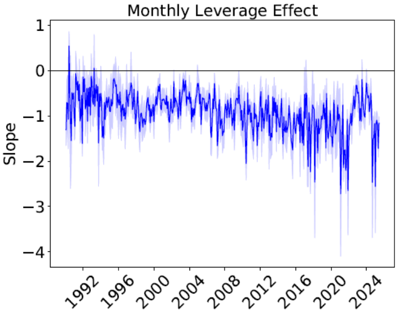
Learning and the emergence of nonlinearity in financial markets

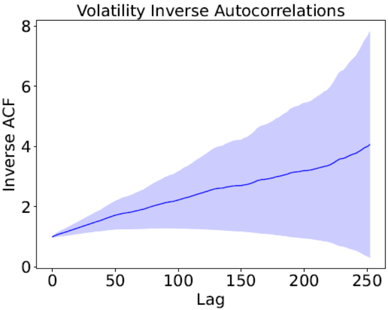
by Dew-Becker, Giglio, and Molavi

Lars A. Lochstoer
UCLA Anderson School of Management

NBER AP Fall 2025

Motivating stylized facts





1. $Corr\left(r_t, vol_t - E_{t-1}\left(vol_t\right)\right) < 0$ very robust fact!
2. Volatility does not have exponential decay

Papers main claim: Learning naturally gives rise to these effects

Outline of discussion

How paper fits in literature (brief and subjective)

Novel predictions/insights

Comments/suggestions

Linear Gaussian learning

True dynamics:

$$\begin{aligned} y_t &= x_t + \sigma_y \varepsilon_t, \\ x_t &= \rho x_{t-1} + \sigma_x \eta_t \end{aligned}$$

Filtered dynamics (investors' beliefs, steady state):

$$\begin{aligned} y_t &= \hat{x}_t + \tilde{\sigma}_y \tilde{\varepsilon}_t, \\ \hat{x}_t &= \rho \hat{x}_{t-1} + \tilde{\sigma}_x \tilde{\eta}_t, \end{aligned}$$

where $\tilde{\sigma}_y \approx \sigma_y$ and $\tilde{\sigma}_x < \sigma_x$

- In sum: learning not very exciting as overall dynamics unchanged

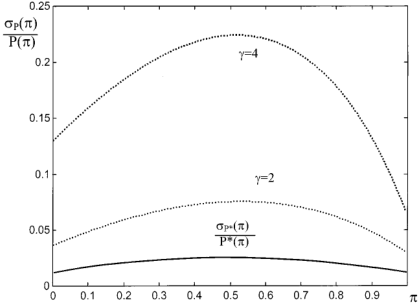
Detemple (1986), Dothan and Feldman (1986), Gennotte (1986)

- Incomplete information: stocks, bonds, portfolio choice

Nonlinear learning dynamics

Veronesi (1999): "this model ... explain(s) features of stock returns, including volatility clustering, "leverage effects," excess volatility..."

- Leverage effect only on right side of plot, so does depend on calibration



David and Veronesi (2013) match lots of moments with regime learning

- "The learning dynamics generate strong nonlinearities ..."

Parameter/model learning

Collin-Dufresne, Johannes, and Lochstoer (2016)

- Leverage effect arise and faster mean-reversion of vol in bad state
  - Higher prior uncertainty for less frequented states
  - Much higher excess vol in those states (80%), which don't last as long as normal states
- Learning about severity and persistence of bad states particularly important

Johannes, Lochstoer, and Mou (2016): Confounded learning

- Learning about multiple uncertainty parameters/states at same time is empirically critical
  - Strong empirical correlation with real-time updates in beliefs and stock returns
- Also, Farmer, Nakamura, and Steinsson (2024)

Learning about slowly unfolding disasters

Ghaderi, Kilic, and Seo (2022)

- Disaster risk (e.g., Barro, Gabaix, Wachter) but with unobserved disaster risk
- Achieves gradual revelation of disasters as in data

Plus many other papers on learning and dynamics of asset return volatility and risk premia

- My apologies for omissions, I blame myself and time constraints

In sum: Many existing insights on how learning is important for understanding nonlinear asset price dynamics

Paper setup

Main idea: learning about fundamental value X_t, price under full information θ_t (really, just more information):

$$\begin{aligned} X_t &= E_t\left(\sum_{j=1}^{\infty} M_{t,t+j} D_{t+j} | \theta_t\right), \\ P_t &= E\left(X_t | Y^t\right) \end{aligned}$$

- Interesting point of view, gives analytical flexibility!
- But: requires SDF and cash flows stream to be exogenous relative to information set
  - Rules out EZ, changing consumption/dividend decisions due to learning

Signal

$$dY_t = x_t dt + \sigma_{Y,t} dW_t$$

Solve nonlinear filtering problem by applying result in Lipster and Shiryaev (2013) and Bain and Crisan (2009)

Main theoretical result

Market return volatility dynamics:

$$d\left(vol_t\right) = \frac{1}{\sigma_{Y,t}} \frac{\kappa_{3,t}}{\kappa_{2,t}} \left(dp_t - E_t\left(dp_t\right)\right) - \frac{vol_t^2}{\sigma_{Y,t}} dt + \text{less interesting stuff}$$

1. Negative skewness of beliefs key for generating leverage effect, $\kappa_{3,t} < 0$
2. Time-varying volatility inherent, unless homoscedastic Gaussian system
3. Non-linear dynamics in vol, quadratic mean-reversion: faster mean reversion when vol is high
    - Assumes $\sigma_{Y,t}$ doesn't undo the effect

General result, insightful equations

- Learning can, but does not have to, deliver the effects that motivate the paper

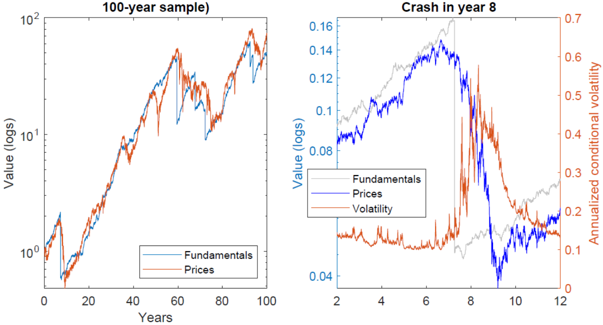
Simulation-based example

iid changes in fundamental value:

$$dx_t = (\phi\lambda + g)\, dt - J_t dN_t + \sigma_x dB_t$$

Calibration close to Barro and Jin (2011), $\sigma_{Y,t}$ free parameter

- Delivers time-varying vol, leverage effect, skewness increasing in return horizon, volatility ACF non-exponential



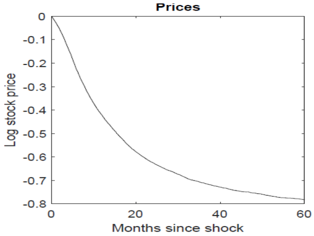
Clarify nature of learning

Signal is unbiased signal about fundamental value:

$$dY_t = x_t dt + \sigma_Y dW_t$$

- What is a real-world counterpart to this?
- Typical signals dY in literature: consumption/dividend/earnings growth
  - Forces calibration and learning effects to be reasonable relative to data

Example: Jumps in fundamental value not picked up quickly



- Constraints imposed: Can't be observed cash flow jump or similar
- What if signal is $dY_t = dx_t + \sigma_Y dW_t$?

Example of mapping to typical setup

Assume risk-neutral investors, risk-free rate $r_f > 0$, and dividends:

$$dD_t = \rho\left(\kappa e^{x_t} - D_t\right)dt + \sigma_D dW_t^D.$$

If x_t is unobserved and $\mathcal{F}_t = \sigma\left(D_s : 0 \leq s \leq t\right)$

$$P_t = E\left(\int_t^\infty e^{-r_f(s-t)} D_s ds \mid \mathcal{F}_t\right) = aD_t + bE\left(e^{x_t} | \mathcal{F}_t\right)$$

Thus, this economy is effectively the same as authors' example with signal

$$dY_t = \frac{1}{\rho\kappa}\left(dD_t - \rho D_t\right) = e^{x_t} dt + \frac{\sigma_D}{\rho\kappa} dW_t^D$$

- How do we know if the calibration is reasonable? If you take stand on the channel, calibration has to be reasonable relative to observed data (dividends)
- Make link to existing learning settings in litearture clearer?

Volatility results

$$d\left(vol_t\right) = 1 \times \left(\frac{1}{3}\Delta t^{-1/2}\right) skew_t\left(p_{t+\Delta t}\right) dp_t - \frac{1}{\sigma_{Y,t}} vol_t^2 dt + E_t\left(d\left\langle x\right\rangle_t\right)$$

- Test if first coefficient is 1, second coefficient can be used to calibrate $\frac{1}{\sigma_{Y,t}}$
- Assuming $E_t\left(d\left\langle x\right\rangle_t\right)$ uncorrelated with other RHS terms. Is it?

Find support for 1 in the data, on S&P500 and Natural Gas market

Coefficient on second term is negative, consistent with non-linear mean-reversion

- Very cool implication:

$$\kappa_{2,t} = vol_t \sigma_{Y,t} \Delta t^{-1/2}$$

- Uncertainty about market level betwen $\pm 20\%$ and $\pm 33\%$

Q: what happens if you run this regression in Campbell and Cochrane (1999), Wachter (2013)? To what extent are these predictions unique to learning?

Minor comments

- Use options for conditional skew in regression?
- Are dynamics consistent with option prices?
- Estimate conditional vol by projecting onto VIX, only gets rid of unconditional vol risk premium
- Currently, signal and prices are continuous – is it possible to extend to a case with learning and jumps in prices, more consistent with the data?
- *Learning and emergent nonlinearity:* This does require some underlying nonlinearity/nonnormality. But, given that, learning adds to it.

Conclusion

Very nice paper, illuminating and general equations!

Learning naturally generates nonlinear dynamics important to understand asset prices

- Leverage effects, time-varying degree of mean reversion in volatility

Really interesting results, but we do still want to know the underlying learning channel

More discussion on information environment and nature of signal

- Jumps in prices, multi-factor learning?