Causal Inference for Asset Pricing

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How do investors make financial decisions?

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The causal inference approach

- Estimates of causal micro-estimates of the demand for financial assets
  - If the price of Tesla stock drops by 1%, how do you change your position?
  - If a group of investors start buying Game Stop, how does the price change?
- Relies on sources of exogenous variation (*instruments*)
- Aggregate demand through demand systems

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The concern: incompatible with asset pricing

- Key insights (Samuelson, Markowitz): general equilibrium, diversification
  - Modern versions CAPM, APT, Long Run Risk
- Consider buying a specific stock within context of whole portfolio
  - Violates standard causal inference assumption (Stable Unit Treatment Value)

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How do we design causal inference framework to fit finance?

This Paper

- How to design causal regressions that are well specified
  - Simple conditions on treatment, control, *and other assets*
  - Natural interpretation in standard asset pricing: restriction on covariance matrix

- What do the estimates give us?
  - Relative elasticity between treatment and control
  - Difference between own-price elasticity and cross-price elasticity
- What counterfactuals do they inform?
  - Relative price response to local change in demand
  - Macro counterfactual with either a model (logit, mean-variance, ...) or aggregate instruments

Basic Demand Estimation

Alice decides how many apples to purchase at the State Fair

$$D = \underline{D} - \mathcal{E}P$$

- $\mathcal{E}$, demand elasticity: how much more do I buy when the price decreases?
- $\underline{D}$, other components of demand: how much do I like apples? are apples good this year?

Identification

- Cannot directly regress quantity on prices
  - Maybe prices are high because everybody really likes apples
- Ideal experiment: two parallel worlds where Alice faces different prices at the State fair
- In practice: use an instrument to vary the price
  - exogenous variation in costs of producing apples
  - exogenous variation in the taste of other apple consumers

Price Impact $\mathcal{M}$

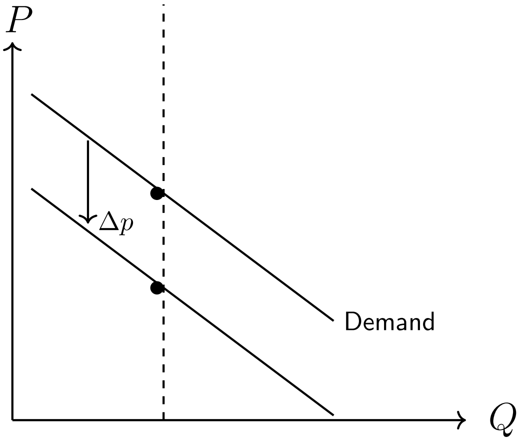
What happens to the price of apples when Bob unexpectedly goes to the Fair and buys one apple?

$$\text{Supply} = \underline{D}_{\text{agg}} - \mathcal{E}_{\text{agg}} P + D_{\text{Bob}}$$
$$\Delta P = \mathcal{M} \cdot d_{\text{Bob}}$$
$$\mathcal{M} = \frac{1}{\mathcal{E}_{\text{agg}}}$$

Identification

- Exogenous shock to supply
- Known exogenous shift in the demand of

Price Impact $\mathcal{M}$



What is different about Asset Pricing?

Portfolio choice over many comparable assets

- Example: choose among 5,000 stocks, bonds, treasuries, ...
- Benefit
  - Identification from the cross-section of assets

$$\Delta D_i = \widehat{\mathcal{E}} \Delta P_i + \theta' X_i + \epsilon_i$$
$$\Delta P_i = \lambda Z_i + \eta' X_i + u_i$$

  - Find two comparable assets that are hit by different shocks

Examples

- What is the price impact of the Fed buying some specific assets?
  - Fed corporate bond purchase of 2020: focused only on maturity under 5 years (Haddad et al.)
- Do demand curves for stocks slope down?
  - Index inclusion: passive funds investment flows in "just included stocks" vs. "just excluded stocks" leading to differential price pressure
  - Active funds change their stock portfolio in response to the induced change in price

The Asset Pricing Challenge:

Portfolio choice over many comparable assets

- The substitution challenge in asset pricing
  - Price of all assets matter for my individual stock demand

$$D_i = \underline{D}_i + \mathcal{E}_{ii} P_i + \sum_{k \neq j} \mathcal{E}_{ik} P_k$$

- Matrix of elasticities $\mathcal{E}_{ij}$
  - own-price elasticities $\mathcal{E}_{ii}$
  - cross-price elasticities $\mathcal{E}_{ij}, j \neq i$
- Matrix of multipliers
  - Flow in asset i lead to price pressure in asset j $\mathcal{M}_{ij}$

$$\mathcal{M} = \mathcal{E}^{-1}$$

  - "Own-flow" multiplier is different from "own-price" elasticity

$$\mathcal{M}_{ii} \neq \frac{1}{\mathcal{E}_{ii}}$$

Markowitz's Insight

- Assets are close substitutes
  - Alternative means of transferring money across states of the world
- Substitution summarized by the variance-covariance matrix
- Markowitz mean-variance demand

$$D_i = \frac{1}{\gamma}\Sigma^{-1}(\mu - P)$$

- Complete matrix of demand elasticities $\mathcal{E}_{ij} = -[\Sigma^{-1}]_{ij}/\gamma$

Fed Asset Purchase

- Fed treasury purchases affect multiple bonds at the same time under a maturity cutoff rule
- These assets are closely related to one another
- Estimation of price impact
  - Markowitz highlights the role of substitution
  - Markowitz provides structure on the substitution (covariance matrix)

Making Causal Inference Work

- DGP: matrix of elasticities $\mathcal{E}$
- Empirical estimation with IV for some sample of assets $\mathcal{S}$

$$\Delta D_i = \widehat{\mathcal{E}} \Delta P_i + \theta' X_i + \epsilon_i$$
$$\Delta P_i = \lambda Z_i + \eta' X_i + u_i$$

- Identification assumption $Z_i \perp u_i$

Restrictions on the elasticity matrix $\mathcal{E}$ such that we can identify something with $\widehat{\mathcal{E}}$

Making Causal Inference Work

We make the following two assumptions on the matrix of elasticities $\mathcal{E}$:

A1. **Homogeneity in the estimation sample $\mathcal{S}$**

- For all assets i in $\mathcal{S}$, the own-elasticity is $\mathcal{E}_{ii} = \mathcal{E}_{\text{own}}$
- For two assets i, j in $\mathcal{S}$, the cross-elasticity is $\mathcal{E}_{ij} = \mathcal{E}_{\text{cross}}$
- *Assets need to be comparable*

A2. **Equal dependence of irrelevant alternatives**

- Assets in the sample have the same elasticities with respect to any asset outside:
- For $i \in \mathcal{S}$ and $k \notin \mathcal{S}$, $\mathcal{E}_{ik} = \mathcal{E}_{jk}$
- *Outside assets (unobserved interactions) can be differenced out*

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Proposition 1.

- The two-stage least square estimator of $\widehat{\mathcal{E}}$ identifies the **relative elasticity**:

$$\widehat{\mathcal{E}} = \mathcal{E}_{\text{own}} - \mathcal{E}_{\text{cross}}$$

Mechanics of identification

First difference

$$\gamma \Delta D_1 = \mathcal{E}_{11} \Delta P_1 - \mathcal{E}_{12} \Delta P_2 - \sum_{k \geq 3} \mathcal{E}_{1k} \Delta P_k$$

$$\gamma \Delta D_2 = \mathcal{E}_{22} \Delta P_2 - \mathcal{E}_{21} \Delta P_1 - \sum_{k \geq 3} \mathcal{E}_{2k} \Delta P_k$$

Second difference

$$\Delta D_1 - \Delta D_2 = \underbrace{\left(\mathcal{E}_{\text{own}} - \mathcal{E}_{\text{cross}}\right)}_{\widehat{\mathcal{E}}} \left(\Delta P_1 - \Delta P_2\right).$$

- Conditions on $\mathcal{E}$ equivalent to conditions on Σ where $\mathcal{E} = \gamma\Sigma^{-1}$
- Example with Σ from factor model
- Extension with subgroup
- Extension with heterogeneous factor exposure ... controlling for beta
- Unobserved heterogeneity ... local relative elasticity
- Link between multipliers and elasticities

Why are A1 and A2 reasonable assumptions

- Standard goods (IO): hard to fully justify assumptions A1 and A2
- Markowitz: assets are special type of goods
- Restrictions on $\mathcal{E}$ are equivalent to restrictions on Σ

A1'. **Homogeneity in the estimation sample $\mathcal{S}$**

- For i in $\mathcal{S}$, $\mathrm{Var}(R_i) = \Sigma_{ii} = \Sigma_{\text{own}}$
- For i, j in $\mathcal{S}$, $\mathrm{Cov}(R_i, R_j) = \Sigma_{ij} = \Sigma_{\text{cross}}$

A2'. **Equal dependence of irrelevant alternatives**

- For $i \in \mathcal{S}$ and $k \notin \mathcal{S}$, $\mathrm{Cov}(R_i, R_k) = \mathrm{Cov}(R_j, R_k)$

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Example: the estimation sample $\mathcal{S}$ constitute the whole set of assets.

- Then Σ exhibits a strong symmetry $\Sigma = \sigma^2(I + \rho\mathbf{1}\mathbf{1}')$

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{pmatrix}$$

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Example: Factor model with homogeneous exposure.

$$R_{i,t} = \beta F_t + \epsilon_{i,t},$$
$$\mathrm{Var}(\beta F_t) = \rho\sigma^2, \quad \mathrm{Var}(\epsilon_{it}) = (1-\rho)\sigma^2, \quad \epsilon_i \perp \epsilon_j, \text{ for } i \neq j.$$

$$\Sigma = \left(\beta^2\sigma_F^2 + \sigma_\epsilon^2\right)\mathbf{1}\mathbf{1}' - \sigma_\epsilon^2\mathbf{I}$$
$$= \begin{pmatrix} \beta^2\sigma_F^2 + \sigma_\epsilon^2 & \beta^2\sigma_F^2 & \cdots & \beta^2\sigma_F^2 \\ \beta^2\sigma_F^2 & \beta^2\sigma_F^2 + \sigma_\epsilon^2 & \cdots & \beta^2\sigma_F^2 \\ \vdots & \vdots & \ddots & \vdots \\ \beta^2\sigma_F^2 & \beta^2\sigma_F^2 & \cdots & \beta^2\sigma_F^2 + \sigma_\epsilon^2 \end{pmatrix}$$

Relaxing the assumptions: homogeneity within subgroups

- Reframe assumptions A1 and A2 to hold within groups
- A1 and A2 hold within a narrow industry but not across industries

  A1g Homogeneity within a group for i, j in the same group $\mathcal{S}_g$, $\mathcal{E}_{ii} = \mathcal{E}_{own}$ and $\mathcal{E}_{ij} = \mathcal{E}_{cross}$

  A2g Equal dependence of irrelevant alternatives to each group for i, j in the same group $\mathcal{E}_{ik} = \mathcal{E}_{jk}$

Identify $\mathcal{E}_{own} - \mathcal{E}_{cross}$.

- Same baseline regressions with group fixed effects

$$\Delta D_i = \mathcal{E}\Delta P_i + \theta_g + \epsilon_i,$$
$$\Delta P_i = \lambda Z_i + \eta_g + u_i,$$

Relaxing the assumptions

Heterogenous factor exposure

- Standard factor model

$$R_{i,t} = \beta_i' F_t + \epsilon_{i,t},$$
$$\epsilon_i \perp \epsilon_j, \text{ for } i \neq j.$$

- Covariance matrix under homoscedastic idiosyncratic risk

$$\Sigma = \beta \Sigma_F \beta' + \sigma_\epsilon^2 \mathbf{I}$$

- Recover the relative elasticity while "controlling for β"

$$\Delta D_i = \mathcal{E} \Delta P_i + \theta' \beta_i + \epsilon_i,$$
$$\Delta P_i = \lambda Z_i + \eta' \beta_i + u_i,$$

Relaxing the assumptions

Heterogenous factor exposure

- Mechanics of identification through "hedging the factor"

$$\frac{dD_j}{dS} = -\frac{1}{\gamma\sigma_\epsilon^2}\frac{dP_j}{dS} + \frac{\beta_j}{\gamma\sigma_\epsilon^2}\frac{1}{\tau}\sum_k \frac{\beta_k}{\sigma_\epsilon^2}\frac{dP_k}{dS}$$

Relaxing the assumptions

Unobserved heterogeneity

A0.h. The data generating process of the first stage follows

$$\Delta P_i = \lambda_i Z_i + u_i, \quad \text{with } Z_i \perp\!\!\!\perp (u_i, \lambda_i).$$

A1.h. Homogeneity of the elasticity with respect to the instrument: $(\mathcal{E}_{ii}, \mathcal{E}_{ij})|Z_i \sim (\mathcal{E}_{ii}, \mathcal{E}_{ij}), \forall i \in \mathcal{S}$

A2 Equal dependence of irrelevant alternatives:

$$\mathcal{E}_{ik} = \mathcal{E}_{jk}, \forall i, j \in \mathcal{S}, k \notin \mathcal{S}.$$

The two-stage least square estimation identifies the **local relative elasticity**

$$\widehat{\mathcal{E}} = \frac{\mathbf{E}_i \left\{ \lambda_i (\mathcal{E}_{ii} - \mathbf{E}_j(\mathcal{E}_{ji})) \right\}}{\mathbf{E}_i(\lambda_i)}$$

Multiplier

- Same results apply to price impact regressions: multiplier estimation

$$\Delta P_i = \mathcal{M}_{ii} D_i + v_i$$

- Regression only identifies the relative multiplier: $\widehat{\mathcal{M}} = \mathcal{M}_{own} - \mathcal{M}_{cross}$

Connection between regressions

- The matrix of multiplier ties in to the matrix of elasticities: $\mathcal{E}^{-1} = \mathcal{M}$
- But not the individual coefficients: $\mathcal{E}_{own} \neq \mathcal{M}_{own}$
- Under our assumptions we have:

$$\mathcal{M}_{own} - \mathcal{M}_{cross} = \frac{1}{\mathcal{E}_{own} - \mathcal{E}_{cross}}$$

Counterfactuals

Careful around counterfactual analysis

- We can only recover relative elasticities $\mathcal{E}_{own} - \mathcal{E}_{cross}$
- Change the supply of one asset: how much does the relative price of the asset changes

Moving to aggregate effects

- The Fed decides to purchase all corporate bonds ...
- Change the supply of all assets: we need to figure out individually $\mathcal{E}_{own}$ and $\mathcal{E}_{cross}$
- Need some more structural assumptions (missing intercept) (logit)