The Case of the Disappearing Skewness [*]

Preliminary

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Abstract

A well-known observation about firm-level returns is that they are positively skewed. We show that this positive skewness has slowly disappeared towards the end of the 20th century. In the 21st century, the distribution of idiosyncratic returns is symmetric. Using the entire cross-section of firms, we investigate possible explanations for the source of this change. This inquiry leaves us with a puzzle: none of the standard rationales behind the asymmetry of firm returns seem able to explain this phenomenon. Instead, the disappearance of skewness is present in firms of all types.

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1 Introduction

Individual stock returns are positively skewed.

This claim has a long history in the finance literature. As early as McEnally (1974), researchers have noted the asymmetry of firm-level returns, with larger and rarer good outcomes than bad outcomes. This observation has led to a large body of work arguing that this asymmetry plays an important role in shaping investor decisions. For example, if investors desire positively skewed gambles, they might choose not to diversify and invest in high-skew stocks, a view developed, for example, in Conine Jr and Tamarkin (1981) and Barberis and Huang (2008). In equilibrium this desire for skewness should be reflected in expected return, which Boyer, Mitton, and Vorkink (2010) and Conrad, Dittmar, and Ghysels (2013) document. In this paper, we update this classic claim.

Individual stock returns are no longer positively skewed.

Specifically, the average skewness of monthly idiosyncratic stock returns has been approximately 0 since the beginning of the 2000s. Until 1980, the average firm-level skewness was indeed robustly positive. But, in the following 20 years, skewness has slowly decayed until it reached a "new normal" in which returns are symmetric. We show that this downward trend holds irrespective of how one measures skewness, constructs idiosyncratic returns or chooses the horizon at which to measure returns.

Why did skewness disappear? Our investigation of this question leaves us with a puzzle: no simple explanation rationalizes this phenomenon. First, we show that the decay of average skewness is not due to the entry of new firms with low skewness, but instead that the skewness of existing firms decreased. Second, while the literature suggests many determinants of the positive skewness in stock returns, we do not find that changes in these factors contribute significantly to the disappearance of skewness. In the spirit of this previous work, we use the cross-section of firms to measure how various firm characteristics relate to skewness. Then we ask if changes in the composition of firms along these characteristics can predict the disappearance of skewness. Associating skewness with leverage, the age of firms, their overpricing, or their investment behavior does not explain the disappearance of skewness. Besides, the dynamics of returns around earning announcements does not drive this conclusion either: excluding these dates yields an indistinguishable evolution of skewness over the last 50 years. Third, we find that the disappearance in skewness seems to have affected *all* firms. We reach this conclusion by zooming in on the evolution of skewness in various segments of the cross-section of firms. When doing so, we always find a strong trend down after 1980.

We briefly discuss two dimensions where the change in firm-level skewness can have significant implications. First, statistical models of firm dynamics are central to valuation and hedging of financial products or are useful inputs in larger economic models. We show how changes in the entire term structure of skewness can inform the design of realistic models. Second, given the vast body of work documenting investors' appetite for skewness, the disappearance of skewness is bad news. Not only will they be less willing to invest, but also the required rate of returns on stocks might increase, which could also have real effects. We find that the magnitude of the change in average skewness implies that the impact on expected returns could be sizable.

2 Firm-Level Skewness

2.1 Measuring Skewness

We are interested in characterizing the amount of asymmetry in the distribution of firm-level idiosyncratic returns. We first discuss how to measure the asymmetry of a distribution, then explain our construction of idiosyncratic returns in the context of data on stock returns.

Our main metric of skewness is the third standardized moment, also known as Pearson's moment coefficient of skewness. For a random variable X with mean μ and standard deviation σ, skewness is defined by

$$skew = \mathbb{E}\left[\left(\frac{X-\mu}{\sigma}\right)^3\right]. \tag{1}$$

Positive values indicate a distribution which is tilted towards the right. Intuitively, this situation corresponds to more extreme values above the mean rather than under the mean. Negative values indicate a distribution tilted towards the left.

Given the observation of a sample of length n of draws x_i of the random variable X, a consistent estimator of the skewness can be constructed as:

$$\widehat{skew} = \frac{\sqrt{n(n-1)}}{n-2} \frac{\frac{1}{n}\sum_{i=1}^n (x_i - \bar{x})^3}{\left(\frac{1}{n}\sum_{i=1}^n (x_i - \bar{x})^2\right)^{3/2}}. \tag{2}$$

This estimate is rather imprecise. For large n and a normal distribution, the standard deviation of the estimate is equivalent to $\sqrt{6/n}$. For example, with 5 years of monthly data, the standard deviation is about 0.3. In contrast, we will see that the average firm skewness is about an order of magnitude smaller. Constructing skewness estimates for individual firms therefore goes along with sizable measurement error. However, using the entire panel of firms will give us additional statistical power.

We entertain three different alternative measures of skewness. The Bowley and the Kelly measures of skewness are respectively defined by:

$$skew^B = \frac{(Q_{75} - Q_{50}) - (Q_{50} - Q_{25})}{Q_{75} - Q_{25}} \tag{3}$$

$$skew^K = \frac{(Q_{90} - Q_{50}) - (Q_{50} - Q_{10})}{Q_{90} - Q_{25}}, \tag{4}$$

where Q_l is the quantile of level l of the distribution. Bowley's skewness answers relatively how much more further above the median the 75% quantile is than the 25% quantile is below the median. Kelly's measure has the same interpretation, focusing on the part of the distribution between the 10th and 90th percentiles rather than between 25th and 75th percentiles. Unlike our baseline focused on the third moment, these measures capture asymmetry closer to the center of the distribution. While these measures lose potentially important information in the tails, they have more stable statistical properties. Finally, we also measure asymmetry using second moments, by comparing semi-variances. We define the semi-variance measure of skewness by:

$$skew^{sv} = \frac{\sigma(X|X > \mu) - \sigma(X|X > \mu)}{\sigma(X|X > \mu) + \sigma(X|X > \mu)}. \tag{5}$$

We compute these three measures of skewness using the standard sample counterparts of all the quantities involved.

Finally, we annualize all these measures. We multiply skewness by $\sqrt{h}$ with h the length of the return horizon in years. This scaling implies constant skewness across horizons if returns are i.i.d., an approach similar to what is used for annualizing Sharpe ratios.

2.2 Empirical Implementation

We obtain data on daily and monthly stock returns from CRSP. We consider the universe of stocks traded on NYSE, AMEX and NASDAQ between 1926 and 2018. We restrict our attention to stocks with share codes 10 and 11.

Our main analysis uses monthly returns. We decompose the sample in 5-year periods denoted by s starting in a year ending in 0 or 5 (1925, 1930, ...). For each stock i with at least 48 return observations in a given period, we first compute idiosyncratic log returns, then measure the sample skewness. That is, we run the following regression, where t indexes dates within a period s and F_{t+1} is a set of factors:

$$log(r_{i,s,t+1}) = a_{i,s} + \sum b_{i,s} F_{s,t+1} + \varepsilon_{i,s,t+1}. \tag{6}$$

Our baseline specification uses the market model: we include the log excess market return — the value-weighted CRSP portfolio — as the only factor. We also consider versions using the log excess returns of the three factors of Fama and French (1993). Including no control at all corresponds to using the raw returns series, with a skewness driven by aggregate as well as idiosyncratic forces.

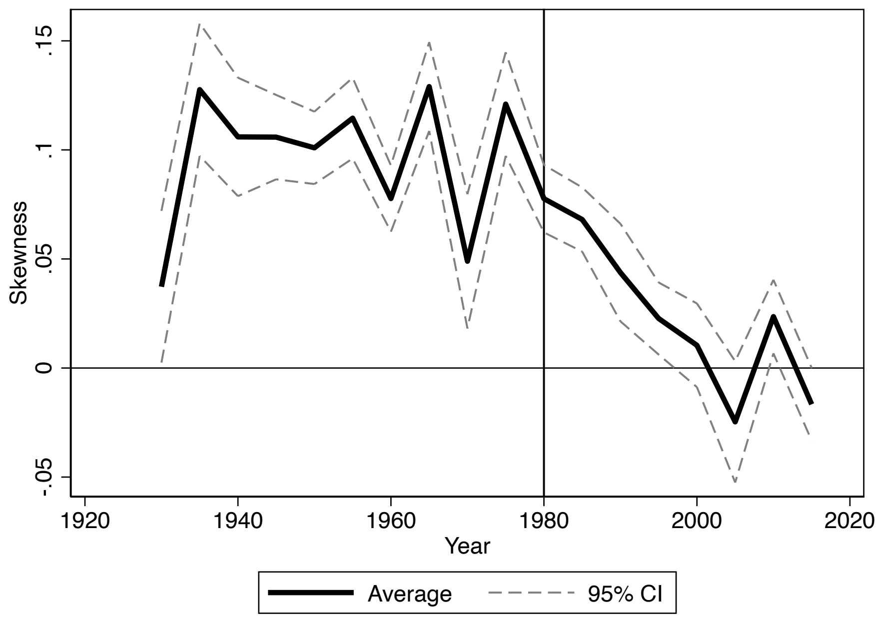
Once we have identified idiosyncratic returns, we construct our measures of skewness $skew_{i,s}$ using the sample properties of the series $\varepsilon_{i,s,t+1}$. We winsorize values of $skew_{i,s}$ larger than 5 times the interquartile range in the cross-section of firms. The construction of skewness using sample estimates implicitly assumes that returns are i.i.d. over each sample s. While this assumption is unlikely to hold exactly, it allows us to construct measures of skewness without making additional parametric assumptions on return dynamics.

An alternative approach to the measurement of skewness would be to use option prices and focus on risk-neutral moments. For example, Bakshi, Kapadia, and Madan (2015) implement measures of skewness for a panel of firms. While this approach offers the benefit of model-free real-time measures, it also comes with challenges that are particularly problematic for our exercises. The limited amount of stocks with liquid options hinders the ability to conduct a cross-sectional analysis, and imposes a focus on the largest firms. In addition the main sample available, from OptionMetrics, only starts in the late 90s, after the bulk of the changes we see in sample moments occur. Finally, it is more challenging to separate systematic and idiosyncratic risk using option prices.

2.3 Long-Term Evolution of Firm-Level Skewness

How did firm-level skewness evolve over the 20th century and the beginning of the 21st century? Figure 1 reports the mean of firm-level skewness for each 5-year period. Until about 1980, the pervasive observation of the literature holds strongly: idiosyncratic returns are positively skewed. In this early part of the sample, skewness is positive in all periods, fluctuating between about 5% and 13%. However, after that, this observation does not hold anymore. Skewness drifts down, getting to values very close to 0 in 2000 and subsequently remains at these low levels. The disappearance of firm-level skewness is the main fact of this paper. This observation constitutes a

Figure 1: **Firm-Level Skewness**. We report the mean firm-level skewness in monthly returns for the cross-section of stocks. For each 5-year interval, we first compute idiosyncratic returns by estimating the market model. Then, we compute the sample skewness for each firm. The dotted line are a 95% confidence interval computed using the bootstrap procedure described in the text.



sharp change in the behavior of asset prices.

We assess if this disappearance is statistically significant. As we have discussed, the skewness of each individual firm is estimated with sizable error. However the panel allows us a more precise estimation of the mean level of skewness. To assess the properties of this moment, we cannot just assume that each observation of skewness is independent from each other. Indeed, while our construction of idiosyncratic returns removes market-level shocks, some patterns of correlation might remain across stocks. For example two close competitors will tend to have correlated idiosyncratic returns. We account for this structure by conducting a bootstrap exercise maintaining cross-asset correlations. For each five-year period, we construct 1,000 simulated samples by *drawing with replacement* 60 random draws of dates. Then we repeat the computation of firm-level skewness and their cross-sectional mean and median for each simulated samples. The simulated distribution gives error bands for our estimates. The dotted lines on Figure 1 report 95% confidence intervals for the mean. We conclude that average skewness in and after 2000 is not distinguishable from zero, and significantly smaller than at any point before 1980.

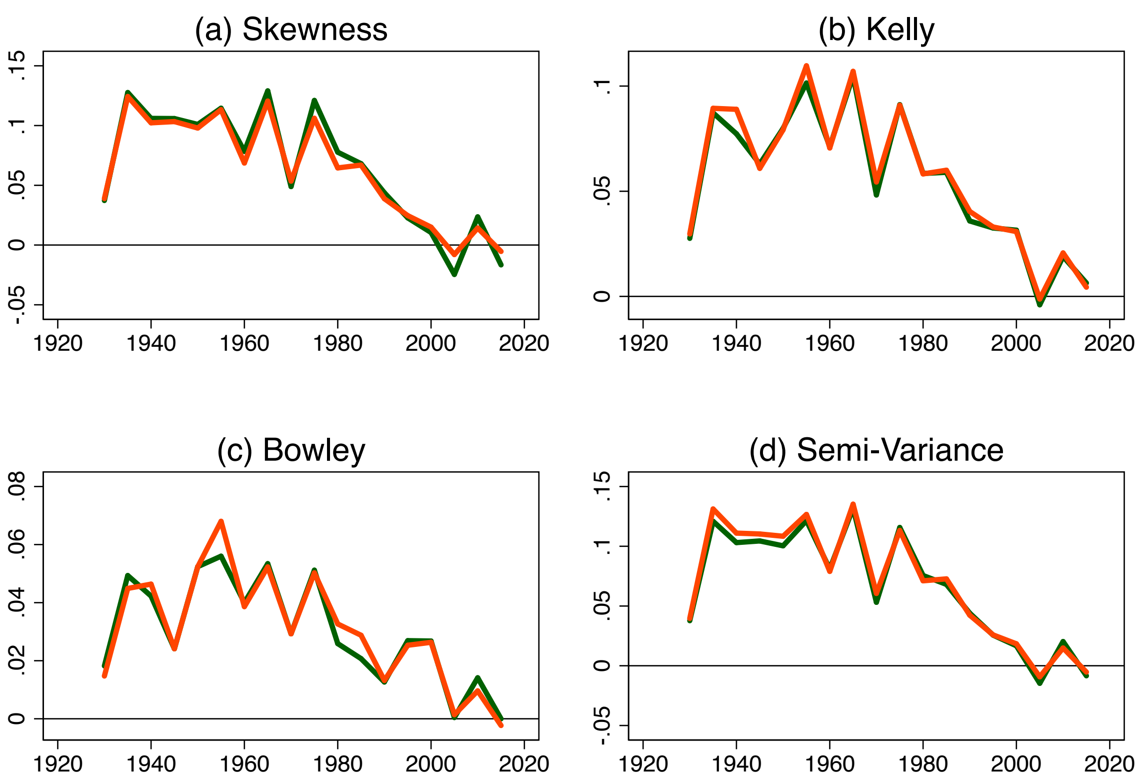
We devote the remainder of this section to assess the robustness and pervasiveness of this result.

2.4 Robustness

First, we ask whether the choice of a particular measure of skewness affects our conclusions. Figure 2 repeats our baseline analysis using the Kelly, Bowley and semi-variance measures. The pattern of disappearing skewness is striking for all these measures. Both the Kelly and semi-variance skewness measures have remarkably high and stable values before 1980: around 8% for Kelly and 10% for the semi-variance. The two measures drop to values close to 0 or 1% after 2000. The same pattern occurs for the Bowley measure, albeit less pronounced. Across all these measures, we find that the positive outcomes (over the mean or median) are about 5 to 10 percentage point larger than negative outcomes before 1980. The distribution of returns becomes approximately symmetric by the 2000s.

Second, we assess the role of aggregate risk. Panel (a) of Figure 3 reports the evolution of skewness with raw excess returns rather than controlling for the market.

Figure 2: **Alternative Skewness Measures**. We report the mean (orange) and median (green) firm-level skewness in monthly returns for the cross-section of stocks across 4 different measures of skewness. For each 5-year interval, we first compute idiosyncratic returns by estimating the market model. Then, we compute the sample skewness for each firm. Panel (a) reports Pearson's skewness, panel (b) and (c) report the Kelly and Bowley skewness, and panel (d) is the estimate based on semi-variances.



Because of the well-known negative skewness of market returns, firm-level skewness is lower than in our benchmark. The series is also more volatile. This occurs for two reasons: the panel does not provide any additional precision in the estimation of aggregate skweness, and aggregate skewness might exhibit cyclical variations. Despite these two features, the decay to lower values is still visible after 1980. We can also ask whether other common sources of risks explain the disappearance of firm-level skewness. Panel (b) of Figure 3 reproduces our analysis after controlling for the three factors of Fama and French (1993) in the construction of returns. We find that these additional factors are not relevant for firm-level skewness: average skewness is virtually identical to our baseline. Another potential concern is estimation error in the construction of idiosyncratic returns. When studying volatility, Campbell, Lettau, Malkiel, and Xu (2001) propose going around this issue by simply subtracting market returns from firm-level returns. This corresponds to assuming that beta is equal to 1. Panel (c) reports the dynamics of skewness when following this approach and strongly confirms our basic result.

Third, we verify that our result is a specific change in skewness, rather than a change in risk overall. Indeed, remember that volatility, or the size of various interquartile ranges is an input to the calculation of skewness. Specifically, we ask if the observation of a post-1980 decay and stabilization at a lower level after 2000 occurs for volatility as well. Figure 4 reports the average and median firm-level volatility over time. Consistent with Campbell et al. (2001), idiosyncratic volatility increases steadily from about 20% to 45% between the early 60s and early 2000s. However, volatility subsequently experiences a sharpe reversal, back down to approximately the level of the early 80s. While these changes are substantial and interesting in

Figure 3: **Alternative Controls for Aggregate Risks**. We report the mean and median firm-level skewness in monthly returns for the cross-section of stocks for two constructions of monthly returns. Panel (a) uses log excess returns to compute the sample skewness for each firm. Panel (b) uses residuals from a regression on the log excess returns of the three factors (market, HML, and SMB) of Fama and French (1993)). Panel (c) subtracts the log market excess returns from firm-level returns.

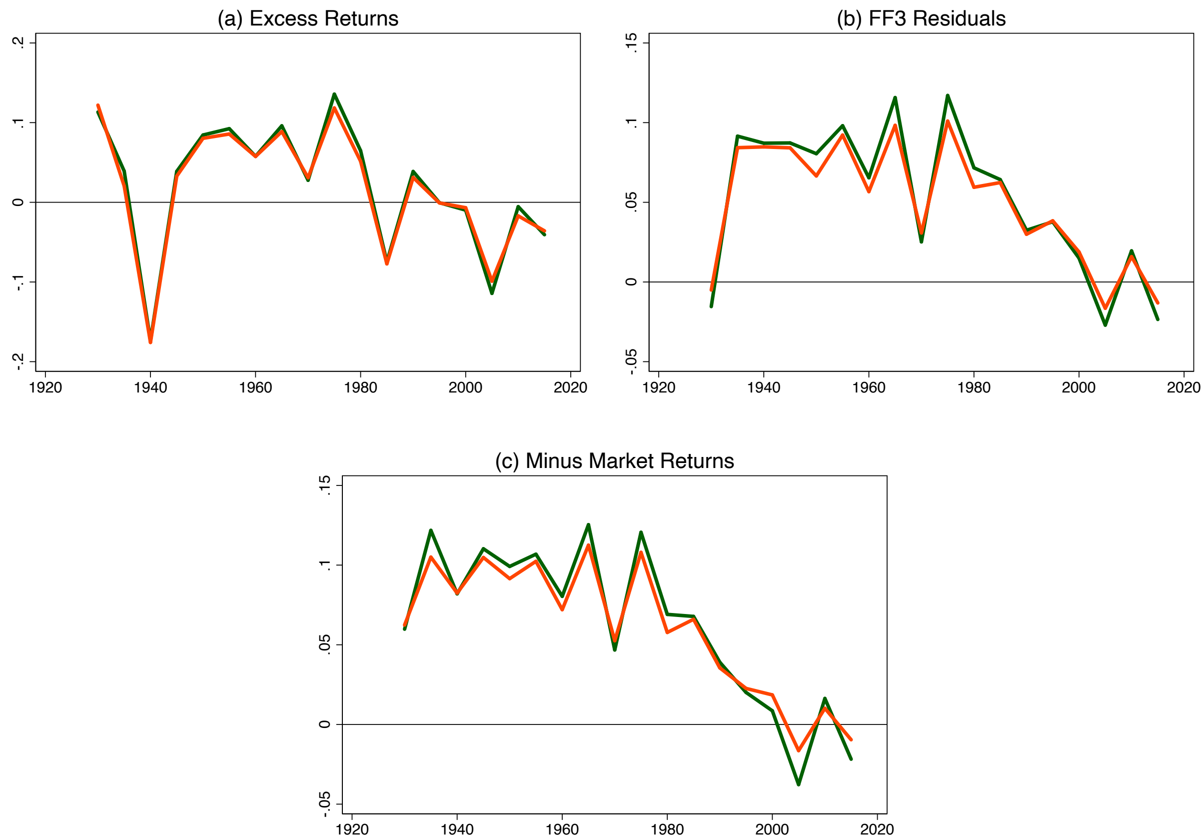
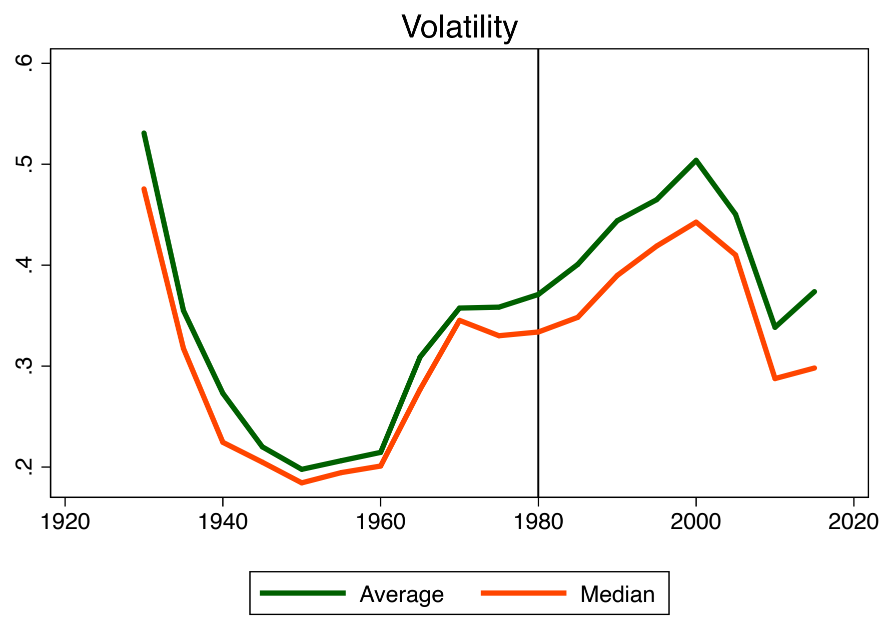


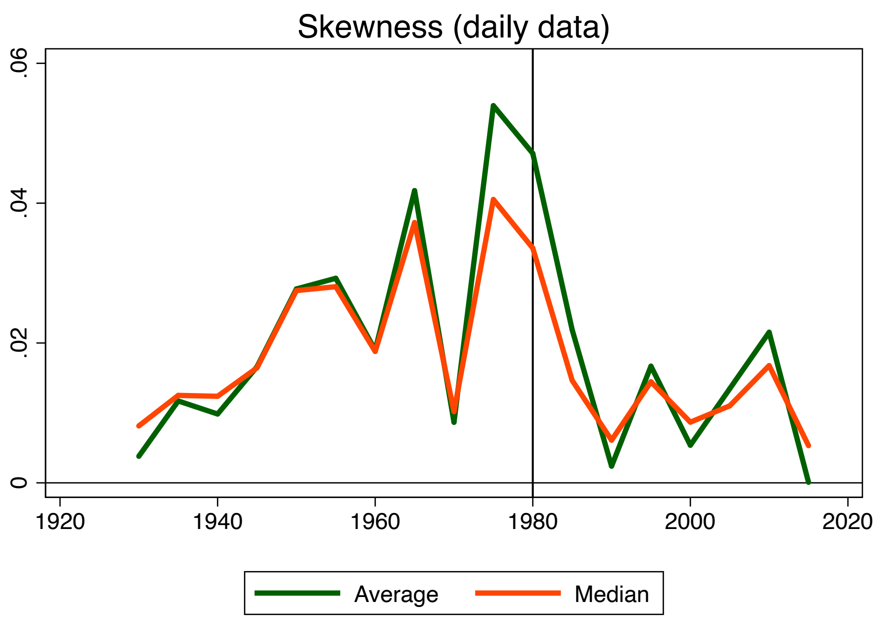
Figure 4: **Firm-level Volatility**. We report the mean and median firm-level volatility in monthly returns for the cross-section of stocks. For each 5-year interval, we first compute idiosyncratic returns by estimating the market model. Then, we compute the sample standard deviation for each firm.



their own right, they differ strongly from the evolution of firm-level skewness we document. In other words, secular trends in the overall quantity and the asymmetry of firm-level risk appear unrelated.

Finally, Figure 5 uses daily returns instead of monthly returns. For each 5-year period, we compute the sample skewness of idiosyncratic returns for any firm with more than 200 observations. In the early part of the sample, these higher frequency returns do not exhibit strong skewness, but between and 1950 and 1980, positive skewness is strong, at about 3%. After 1980. the level drops sharply and stabilizes around 1%. In Section 4, we come back to the evolution of the entire term structure

Figure 5: **Firm-level Skewness in Daily Returns**. We report the mean and median firm-level skewness in daily returns for the cross-section of stocks. For each year, we first compute idiosyncratic returns by estimating the market model. Then, we compute the sample skewness for each firm.



of skewness, and what we can learn from the more subtle differences across horizons. At this stage, we just conclude that the disappearance of skewness is also present in daily returns.

3 Why Did Firm-Level Skewness Disappear?

We now turn to potential causes for the disappearance of firm-level skewness. The literature suggests some usual suspects. Indeed, previous work has put forward a number of sources of firm-level skewness. We assess if these determinants can explain our phenomenon. The investigation is not fruitful: the simple explanations we consider account for less than half of the long-term change in skewness. Most of the trend down in skewness instead appears to be a pervasive phenomenon hitting all types of firms.

3.1 Entry and Exit

A first question is whether the disappearance of skewness is a phenomenon that can be understood by tracking changes at the firm level, or whether it originates in the comparison of different firms across time periods. After all, only 36 firms appear in our sample both in 1926 and 2018.

We answer this question by decomposing changes in skewness across five-year periods due to entry, exit, and changes within existing firms. To do so, we follow the dynamic Olley and Pakes (1996) approach of Melitz and Polanec (2015). Given two periods s and s', we split firms into the sets $\mathcal{S}$ of survivors present at both dates, $\mathcal{E}$ of firms entering at s' but not present at s, and $\mathcal{X}$ of firms which exit at s and are not present at s'. We define $s_{\mathcal{X}}$ the share of firms which exit among date-s firms. Similarly, $s_{\mathcal{E}}$ the share of firms which enter among date-s' firms. The change in average skewness between date s and s' can then be written as:

$$\overline{skew}_{s'} - \overline{skew}_{s} = \underbrace{\overline{skew}_{\mathcal{S},s'} - \overline{skew}_{\mathcal{S},s}}_{\text{within}} + \underbrace{s_{\mathcal{E}}\left(\overline{skew}_{\mathcal{E},s'} - \overline{skew}_{\mathcal{S},s'}\right)}_{\text{entry}} + \underbrace{s_{\mathcal{X}}\left(\overline{skew}_{\mathcal{S},s} - \overline{skew}_{\mathcal{X},s}\right)}_{\text{exit}}. \tag{7}$$

Average skewness can change for three reasons. The skewness of surviving firms can change, the within term. Or, if entering firms and exiting firms differ from surviving firms, average skewness will change, which corresponds to the entry and exit terms. Of note, the notions of entry and exit here are simply the first and last occurence of a firm in our sample. For example, the first time a firm is in our sample does not necessarily coincide with its IPO: firms traded on AMEX and NASDAQ were included in CRSP only in 1962 and 1972, respectively. That said, to the extent that these firms were created at some point in the long interval we consider, their inclusion in the decomposition is reasonable.

We implement this decomposition across each pair of consecutive five-year periods in our sample. Then, we accumulate each of the terms — the decomposition is additive — for the stable subsample of 1935-1975 and the subsample where skewness disappeared, 1975-2019. Table 1 reports the results. As exposed above, skewness has declined by 14 percentage points since 1975, while it had been stable (showing a one p.p. decline) in the period before.

The contribution of entry to changes in skewness decreased from five p.p. to two only p.p., accounting for less than a quarter of the decrease in skewness. The contribution of entry is positive in both periods because, on average, the skewness of new firms is 0.01 larger than that of average firms. So what has changed? It could be a drop in the average rate of entry. This is not what occured, with the entry rate barely going from 5% to 6% per year. What remains is a lower correlation of the entry rate and the excess skewness of new firms. In words, after 1975, in periods with more entry, new firms are relatively less skewed compared to other firms than in periods with less entry.

Changes in the behavior of exit do not explain any of the change in skewness. The contribution of exits to the change in skewness stays constant at -4 percentage points. This stability is the result of two opposite forces. The share of exits $s_{\mathcal{X}}$ has seen an increase in the latter period, from 2% to 7% per year. However the skewness of firms that exit is closer to the average firm than in the latter period. Both of these effects, composition and share, counteract each other and we conclude that the contribution of exit to skewness has not changed across the two periods.

Table 1: **Decomposition of Change in Skewness**

	Change in skewness	Within	Entry			Exit		
			Total	$s_{\mathcal{E}}$	$\overline{skew}_{\mathcal{E},s'} - \overline{skew}_{\mathcal{S},s'}$	Total	$s_{\mathcal{X}}$	$\overline{skew}_{\mathcal{S},s} - \overline{skew}_{\mathcal{X},s}$
1935-1975	-0.01	-0.02	0.05	0.05	0.01	-0.04	0.02	-0.04
1975-2019	-0.14	-0.12	0.02	0.06	0.01	-0.04	0.07	-0.02

Note: The table reports the decomposition of Equation (7) for each pair of consecutive 5-year periods and accumulate the terms for the 1935-1975 and 1975-2019. The first column reports the total change in skewness. Within, entry (total), and exit (total) correspond to each of the three terms. We also report the average entry and exit rate per year, $s_{\mathcal{E}}$ and $s_{\mathcal{X}}$, and the average difference in skewness and entrants or exiters and suriving firms, $\overline{skew}_{\mathcal{E},s'} - \overline{skew}_{\mathcal{S},s'}$ and $\overline{skew}_{\mathcal{S},s} - \overline{skew}_{\mathcal{X},s}$.

In summary, changes in the characteristics and quantity of entry and exit do not explain much of the disappearing skewness. Rather, we find that the lion's share of the 14p.p. decline in skewness in the 1975-2019 period can be attributed to the *within* term of the decomposition. The decreasing skewness of existing firms rationalizes a 12p.p. overall decline in average skewness. We continue our empirical investigation by looking into explanations for changes of the skewness of existing firms over the last 40 years.

3.2 Changes in Firm Characteristics.

Why would skewness change for a given firm? Simply because the nature of a firm's activities changes over time. For example, a firm experiences significant changes throughout its lifecycle; starting as a young firm with significant growth options, it can mature towards a more stable position in the economy. And, if the characteristics of a firm influence its skewness, it is possible that changes in the landscape of firms over the last 40 years explain the disappearing skewness. We ask if this is the case through the lens of characteristics that previous work has highlighted as important for skewness.

Methodology. Formally, we construct a counterfactual change in average skewness under the assumption that the only determinant of skewness is a given firm characteristic C. That is, we first estimate how skewness relates to the firm characteristic. Then we feed changes in the cross-sectional distribution of the characteristic across periods into this estimated relation to construct the counterfactual change in skewness.

We need to assess how skewness is related to firm characteristics. We augment our sample with information from the FUNDA file from COMPUSTAT. Appendix Figure IA.1 confirms that average skewness behaves similarly in this smaller sample to our baseline sample. Then, we estimate the following regression specification in the panel:

$$skew_{i,s} = a_s + \beta^C C_{i,s} + \varepsilon_{i,s}, \tag{8}$$

where the dependent variable $skew_{i,t}$ is the firm level skewness of monthly returns evaluated over a five-year period, s; a_s correspond to fixed effects for each five-year period. The inclusion of time fixed effects allows us to avoid mechanically fitting the trend, and rather identifies the effect out of cross-sectional variation in the characteristic. Specifically, the coefficient of interest, β^C, represents the cross-sectional correlation between the characteristic and firm-level skewness.

Because this estimated relation is linear, it is straightforward to create counterfactual changes in average skewness. The counterfactual change is just the coefficient β^C multiplied by the change in the average level of the characteristic within a period. Table 2 reports the estimated coefficient, change in average characteristic and counterfactual change in skewness for a number of characteristics: leverage, size, age, book-to-market, investment, profitability and industries. We also report the average level of skewness across quartiles of the characteristic. Below, we discuss why these specific characteristics are interesting, and the results they yield.

The Leverage Effect. When a firm's stock price declines, its market leverage increases. If the firm's equity volatility rises in response to this increase in leverage, this leads to a negative correlation between realized return and volatility, an empirical regularity put forward in Black (1976) and Christie (1982). This leverage effect should mechanically lead to negative skewness in stock returns, due to the correlation between returns and volatility. Indeed, remember we can write the third moment: $\mathbb{E}[X^3] = \mathbb{E}[X \times X^2]$. While potentially important, this force appears at odds with the observation of positive firm-level skewness. Empirically, Duffee (1995) documents that firm-level returns do not exhibit a negative relation between returns and volatil-

Table 2: **Change in Skewness due to Changes in Firm Composition**

	Obs.	Average Skewness by Characteristic Quartile				Coef.	Composition Effect 1975-2015	
		1	2	3	4	β	ΔC	$\beta \times \Delta C$
Skewness	28349	-0.21	-0.02	0.09	0.28	1.00	-0.14	-0.14
Leverage	28349	0.04	0.03	0.02	0.04	0.00	0.43	0.00
Age since IPO	28349	0.04	0.03	0.04	0.01	-0.01^{***}	0.81	-0.01
Size	28349	0.12	0.04	-0.00	-0.02	-0.05^{***}	0.11	-0.01
Book-to-market	28349	0.02	0.02	0.03	0.07	0.02^{***}	-1.58	-0.04
Investment / Asset	28349	0.06	0.04	0.02	0.02	-0.01^{***}	-0.51	0.00
Profitability	28348	0.07	0.04	0.02	0.01	-0.02^{***}	-0.53	0.01
Days to Cover	16477	0.04	-0.00	-0.01	-0.00	-0.01^{***}	0.84	-0.01
Industry	28331	-0.00	0.03	0.04	0.07			-0.00
All								-0.01

The table reports the counterfactual evolution of skewness due to various firm characteristics. For each characteristic, we first estimate a panel regression with the characteristic and time fixed effects, and report the coefficient β where stars denote significance using standard errors two-way clustered at the stock and period level. We also report the change in average level of each characteristic between 1975 and 2015. Multiplying these two numbers provides the counterfactual change in skewness due to the characteristics. For industries, we use dummies for each of the 12 Fama-French industries. The "All" row is the counterfactual using a multivariate regression with all characteristics and time fixed effects.

ity, but actually a positive one. In addition, he shows that variation in the covariance of returns and volatility does not appear related to firm leverage.

We revisit the role of market leverage for skewness in the longer 1975-2019 period, and ask if it can account for the decline of individual firm skewness. The results in Table 2 are consistent with Duffee (1995). In our sample, skewness does not covary with leverage at the firm level (regression coefficient $\beta^{leverage} = 0$). Therefore we conclude that the increase in leverage of firms from 1975 to 2019 cannot explain movements in skewness over the period.

The Lifecycle of Firms. Small firms or younger firms, i.e. firms that are early in their lifecycle, tend to have returns that are positively skewed. Barberis and Huang (2008) argue that such firms have positively skewed returns because a larger fraction of their value is in form of growth options. Green and Hwang (2012) show that IPOs tend to exhibit strong positive skewness. Similarly, on private markets, Moskowitz and Vissing-Jørgensen (2002) find that returns to entrepreneurship are also positively skewed.

We confirm this intuition in Table 2. We construct measures of size, the log of firm market capitalization scaled by total market capitalization, and age, the number of years since a firm's IPO. For both characteristics of size and age, smaller and younger firms tend to have higher skewness than larger and older firms. Skewness for firms in the lowest quartile of size is 12% while it is -2% for firms in the highest quartile. Similarly firms in the lowest quartile of age (since their IPO) is 4% and only 1% for the oldest group of firms. Relative to 1975, the composition of public firms in 2019 leans towards older, larger firms. Firm size has increased by 0.11 log points and their time since IPO has risen by 0.8 years over the 1975-2019 period. These changes place both characteristics as hopeful candidates for the disappearing skewness. However this hypothesis falls short quantitatively. Each channel can only account for about 1p.p. decline in skewness short of the baseline overall decline of 14p.p..

Book-to-Market. The link between skewness and book-to-market is more ambiguous. Our discussion above suggests that firms with low book-to-market, growth firms, should have positive skewness due to their fraction of growth options in their total valuation. But an alternative view, articulated for example by Hong and Stein

(1999), suggests that firms with a high valuation ratio are over-valued and might lose this advantage suddenly, creating negative skewness. The results in Table 2 appear to refute the growth options effect and support the view of a fragile high valuation. The skewness of firms in the lowest quartile of book-to-market is 2%, significantly lower than 12%, the average skewness of firms with high book-to-market. We also confirm that characteristics which are usually associated with the valuation ratio, investment and profitability, also give the same message. Firms that invest more, or are more profitable, tend to exhibit lower skewness.

Over the length of our sample, book-to-market ratios have somewhat decreased, by 1.58. Viewed through the length of the cross-sectional relationship between book-to-market ratio and skewness, this implies a four p.p. decrease in skewness. The other two characteristics imply more modest decreases. While explaining a little less than a third of the disappearing skewness is promising, this result still leaves much unexplained.

Short-Sell Constraints. Hong and Stein (1999) argue that overpricing can give rise to negative skewness. While this view does not explain the average positive level of firm skewness, it is consistent with the pattern we have seen with book-to-market. They tie down this effect to the presence of binding short-sell constraints. We investigate this possibility by using data on short interest at the stock level, which is widely accepted as a proxy for binding short-sell constraints. We scale short interest by overall trading volume in the stock to make it comparable across periods, thereby constructing days-to-cover. For example, Hong, Li, Ni, Scheinkman, and Yan (2015) argue that this scaling captures the marginal cost of shorting a stock. We find that stocks with more binding short-sell constraints (higher values of days-to-cover) have lower skewness: 0 in the highest quartile versus 4% in the lowest. However, the trend up in days-to-cover is much too mild to explain the drop in skewness.

Industries. Finally, it is natural to expect firms in different industries to exhibit different levels of skewness. While we have less guidance on which industries should be more or less skewed, changes in industry composition could play a sizable role. The last 40 years have seen a large decline in manufacturing industries towards the service sector. To investigate the role of changing industry composition we adjust our empirical specification of equation 8 and estimate:

$$skew_{i,s} = a_s + \sum_J f_J 1_{i \in J,s} + \varepsilon_{i,s}, \tag{9}$$

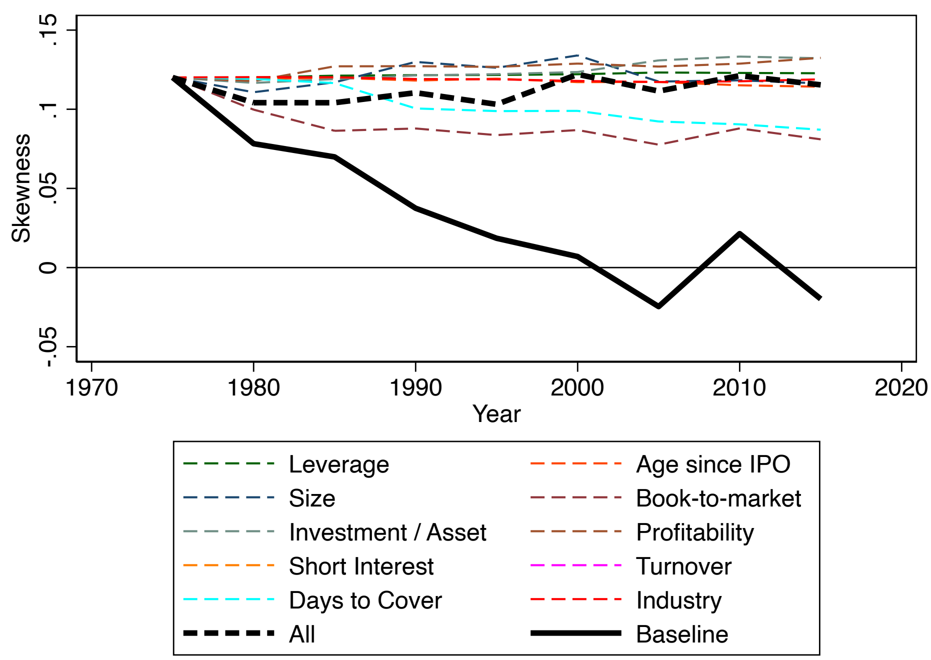
where f_J are industry-specific coefficients and $1_{i \in J,s}$ a dummy for firm i being in industry J in period s . We use the classification of 48 industries defined by Fama and French (1997) based on groups of SIC codes. The industry coefficients capture the average skewness by industry group. We construct our counterfactual skewness change by weighting these coefficients by the changes in industry shares — the cross-sectional average of the industry dummies. In Table 2 we find no role for the change in skewness due to the composition change in industries over the 1975-2019 period.

The Case for Characteristics. Taking stock of all these results, changes in firm characteristics only appear to explain a small fraction of the disappearance of firm-level skewness. Of course, we cannot consider every possible characteristic. Still, we have focused on the determinants of firm-level skewness most strongly put forward by previous research. As such, our results suggest that these existing explanations do not explain the large trend we document. This conclusion poses an important challenge for these theories: the magnitude of the trend of disappearing skewness is at least as large as (and often much larger than) the interquartile changes in skewness related to these characteristics. We extend our exercise in a couple of directions to further strengthen the assessment that changes in composition do not explain the bulk of the change in firm-level skewness.

A first remark is that while none of the characteristics we considered individually accounts for the result, combining them could go a longer way. Note that such a conclusion is not necessarily true in our approach: the various characteristics are likely somewhat correlated and could push skewness in different directions. In the last row of Table 2, we implement a counterfactual including changes in all the characteristics we considered. To do so, we replace Equation (8) by its multivariate counterpart. This combined version only predicts a 1 percentage point drop in skewness.

Second, it could be that the relation between the characteristics we consider and skewness is not linear. We entertain this possibility by replacing the linear regression Equation (8) with a regression on dummies for each characteristic decile. We report the corresponding counterfactual evolution of skewness in Figure 6. The solid black line is the actual drop in average skewness. Each of the colored dashed line corresponds to a characteristic. In line with our previous results, most of them predict virtually no change in skewness. The only meaningful changes, of about a quarter of the total drop would be attributed to book-to-market or days-to-cover. The combined prediction in the dotted black line does not explain the trend down in skewness.

Figure 6: **Skewness Dynamics due to Changing Firm Composition**. We report the counterfactual evolution of skewness due to various firm characteristics in each of the colored dashed line. We first estimate a panel regression with dummies for each decile of the characteristic and time fixed effects. We then multiply the coefficients on the dummies with the change in shares of firms across the deciles to construct the counterfactual change in skewness. For industries, we include a dummy for each of the 12 Fama-French industries. The dashed black line repeats the exercise with including dummies for all the characteristics. The solid black line is the actual change in average skewness.



3.3 Firm Communication

Another strand of literature relates the positive skewness of firms to the structure of information revelation by firms. In the model of Albuquerque (2012), the link between volatility and expected returns around firm announcements can explain positive firm skewness, as well as negative aggregate skewness. Hong and Stein (1999) mention that positive skewness could be related to firm strategically timing how they release bad and good news. Dubinsky, Johannes, Kaeck, and Seeger (2018) confirm the empirical importance of earning announcements and the associated uncertainty they bring by using option prices.

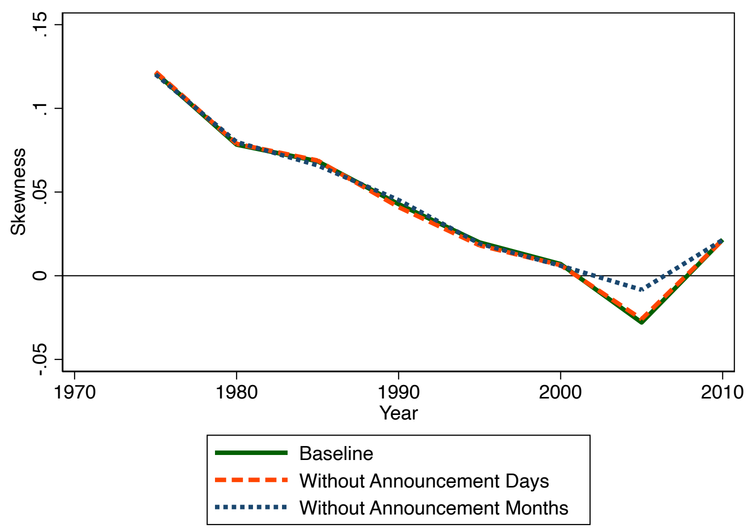
We explore the potential role of firm releases in a simple way. We collect information about the dates of quarterly earning announcements for each firm from COMPUSTAT. Then we repeat our construction of average skewness, excluding these announcements. We do so in two ways, which we report in Figure 7. First, we exclude only the day of the announcement in the construction of monthly returns. That is, for a firm-month with an earning announcement, we reconstruct the monthly return from daily return data, setting the return to 0 on the day of the announcement. The corresponding skewness is the dashed red line, which is virtually indistinguishable from our baseline calculation, the green line. Second, we take a more broad view on earning releases affecting return dynamics and exclude the whole month of the earning release from the construction of skewness. This approach again results in a very similar skewness evolution, with only a mild deviation in 2005.

Overall, we conclude that earning announcements and the month around them do not contribute to our calculation of firm skewness in five-year periods. Therefore, potential changes in behavior around these announcements cannot explain why skewness disappeared.

3.4 Heterogenous Disappearance

Our investigation so far has not pointed towards any clear explanation of the disappearance of skewness. Because the decrease in skewness is not due to changes in the composition of characteristics, it must come from changes in skewness *given* the level

Figure 7: **Firm-Level Skewness without Earning Announcements.** The solid green line reports average skewness in monthly idiosyncratic returns for each five-year periods. The dashed red line exclude earning announcement days from the construction of monthly returns. The dotted blue line exclue earning announcement months.

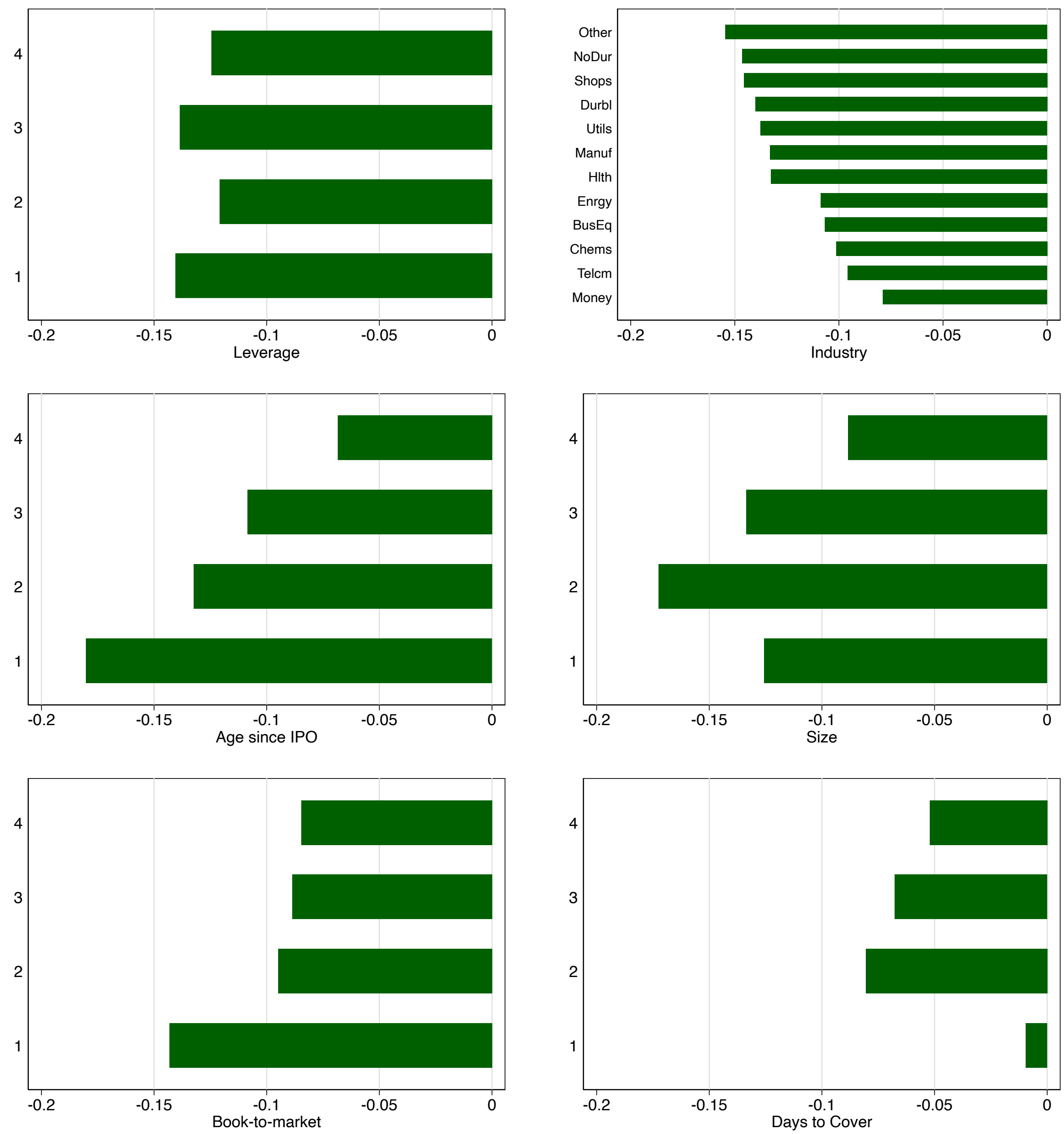


of the characteristics. By opening up the intensity of this change for each characteristic, we can potentially glean more information about the determinants behind the overall phenomenon. For example, it could be that the disappearance of skewness is located among a particular subset of firms.

To assess if it is the case, we compute the change in average skewness within each level of the characteristics. We first estimate breakpoints of characteristics in the full sample. Then, we compute the average skewness in 1975 and 2015 conditional on belonging to each quartile. Figure 8 reports the results for leverage, industries, firm age, size, book-to-market, and days-to-cover. Across almost all the groups of firms, skewness has experienced a sizable decay. Except for within the first quartile of days-to-cover, the decrease in skewness is larger than five percentage points, and often close to 10 percentage point or more. The only noticeable patterns of heterogeneity are a slightly larger drop among young firms and low book-to-market firms.

Taken together, these results suggest that the disappearance of skewness starting in the 1980s is not specific to certain types of firm. Rather it is a pervasive phenomenon throughout the economy. This observation reinforces the importance of understanding the drivers of this phenomenon, but also further deepens the mystery of what those drivers are.

Figure 8: **Long-Run Changes in Skewness across Characteristic Quartiles**. We report the change in skewness over the 1975-2019 period within levels of six characteristics. Leverage is market leverage, industries are 12 industries based on SIC codes from Fama and French, age since IPO is the period since a firm first appeared on public markets, size is the log of market capitalization over total market capitalization, book-to-market is defined as in Fama and French (1993), and days-to-cover is the ratio of short-interest to trading volume.



4 Implications of Disappearing Skewness

We now briefly discuss two areas where the disappearance of firm-level skewness can have important consequences. First, the observation that the distribution of returns is not asymmetric anymore implies a need to revisit statistical models of return dynamics. These models are important as they are often incorporated in richer economic models, into calculations of optimal portfolio choice, or for the pricing and hedging of financial products. Second, if skewness is an input to investor decisions and matters for expected returns, the secular decrease in skewness will be reflected in changes in cost of capital and potentially have real consequences for firms.

4.1 Firm Dynamics

The asymmetry of the return distribution motivates the use of models including jumps or volatility risk for the dynamics of firm returns. We ask what our findings imply for this type of models. To go beyond the simple information in monthly returns, we compute an entire term structure of skewness. For each five-year period, we start from daily idiosyncratic log returns. We aggregate these returns in a non-overlapping way for various horizons: over intervals of 5 trading days (a week), 25 trading days (a month), etc. For each of these levels of aggregation we compute a corresponding notion of sample skewness. We then average these values across the cross-section.

Figure 9 reports these results. Panel (a) corresponds to the pre-1975 sample while panel (b) corresponds to the post-1975 sample. Each line corresponds to the term structure for a specific 5-year period. The shape and evolution of these curves is informative. All these curves share a similar shape, initially increasing then reaching a plateau. Interestingly, our main horizon, one month, appears to be the shortest value at which skewness stabilizes. This suggests that our choice of focusing on this horizon is useful: it uses as much data as possible while capturing properties of skewness across longer horizons. As the convergence to the regime of low skewness occured after 1975, the entire term structure shifted down to new values approximately flat close to 0.

These patterns have implications for firm dynamics. A natural way to generate skewness is to assume that returns are i.i.d., and that short-term returns have a skewed distribution. In continuous time models, this is accomplished by the inclusion of jumps. However, i.i.d. models cannot match the term structure of skewness of the early period. Indeed, as we have already discussed, these models would generate an exactly flat term structure of skewness. The upward sloping patterns of skewness can instead be obtained in models of time-varying volatility (or time-varying jump intensity). One example of such a model is the Heston model:

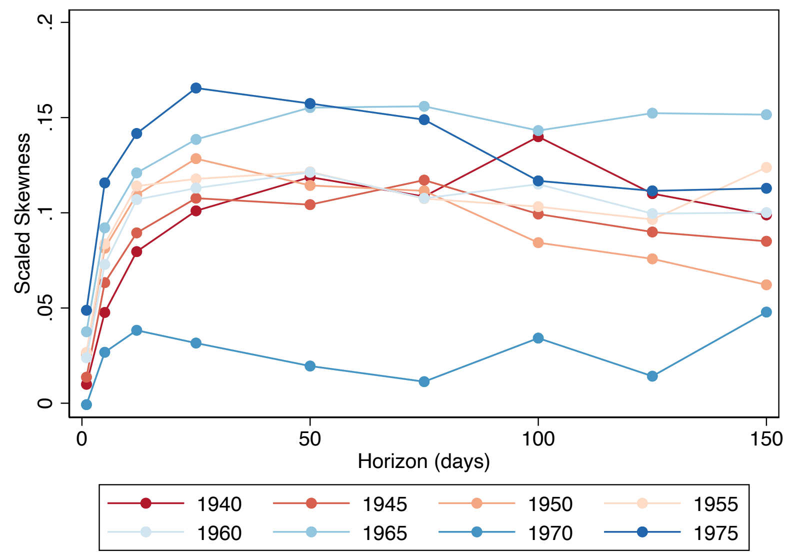
$$dR_t = \mu dt + \sqrt{\nu_t} dW_{R,t} \tag{10}$$

$$d\nu_t = -\kappa(\nu_t - \theta)dt + \xi\sqrt{\nu_t} dW_{\nu,t}, \tag{11}$$

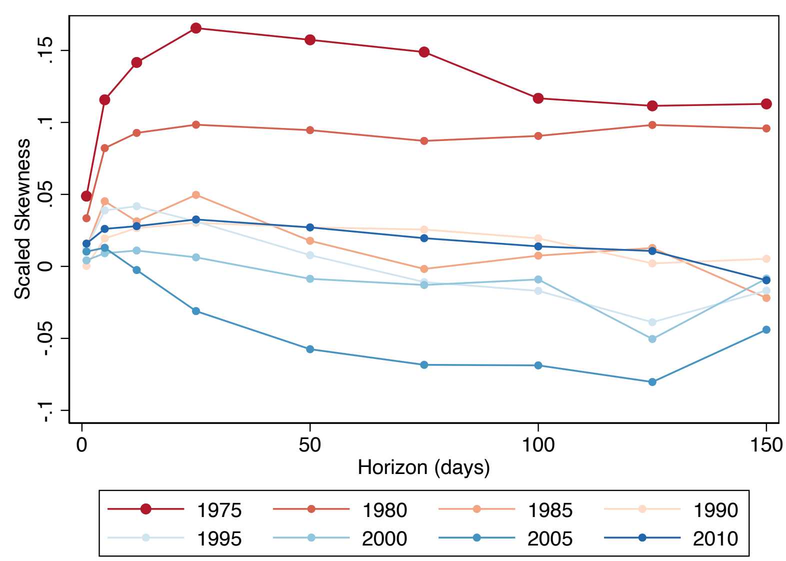
where $dW_{R,t}$ and $dW_{\nu,t}$ have instantaneous correlation ρ. For this model to generate an upward sloping term structure of skewness, a positive correlation ρ between shocks to returns and shocks to volatility is necessary. This is the flipside of the so-called leverage effect. Then, the natural next question is whether the disappearance of skewness is driven by a stabilization of volatility dynamics (a larger κ or lower ξ), or by their disconnection to return shocks (ρ going to zero).

Figure 9: **Term Structure of Skewness**. We report the average firm-level skewness for the cross-section of stocks at various horizons. Each line corresponds to a five-year period, panel (a) containing dates in 1975 and before, and panel (b) dates in 1980 and after.

(a) Pre 1975



(b) Post 1975



4.2 Skewness and Expected Returns

Skewness captures an aspect of risk beyond the common measure of variance. As such, its role for investor decisions has long been discussed, be it aggregate (see for example Rubinstein (1973) or Kraus and Litzenberger (1976)) or idiosyncratic (Conine Jr and Tamarkin (1981)). This latter dimension is the focus of this paper. In general, models of the appetite for firm-level skewness predict that investors are attracted by the positive skewness of stocks. For example, Barberis and Huang (2008) show that the prospect theory of Kahneman and Tversky (1979) yields an appetite for firm-level skewness. Investors overweight the tails of distributions when valuing risk, leading to preferences for lottery-like payoffs, i.e. positively skewed. Thus stocks with positive skewness tend to gather higher demand, leading to higher prices and lower average returns. Empirical studies find support for this prediction. Boyer, Mitton, and Vorkink (2010) predict idiosyncratic skewness using lagged realized daily skewness and other return characteristics and find that stocks with high predicted skewness have lower average returns than stocks with low predicted skewness. Using option prices to measure risk-neutral idiosyncratic skewness, Conrad, Dittmar, and Ghysels (2013) also find low average returns for stocks with high skewness. In a quantitative application of prospect theory, Barberis, Jin, and Wang (2019) find that firm-level skewness is an important determinant of anomaly returns.

This evidence suggests that the significant downward trend in skewness that we document could have dramatic consequences for asset prices. While the cross-section of stocks has been a fertile ground for testing the role of skewness for returns, a change in the skewness of all firms is not irrelevant. The disappearance of firm-level skewness makes all stocks less attractive to investors, and could therefore lead to an increase in the cost of capital. Naturally, one would need a complete model to quantify this aggregate impact. But abstracting from many forces, we can conduct a back-of-the-envelope calculation of the impact of this change using existing cross-sectional results. The magnitudes of the return spread in Boyer, Mitton, and Vorkink (2010), when applied to our observed trend down of 14 percentage point in skewnes, would predict 1% higher expected returns per month. While this number should not be taken at face value, it suggests that the disappearance of firm-level skewness can have sizable implications for valuations in the economy.

5 Conclusion

The positive skewness of individual stock returns has disappeared. Idiosyncratic stock returns where positively skewed throughout most of the 20th century. But this asymmetry has slowly decayed in the last quarter of the century and is absent since the early 2000s. This disappearance is puzzling: none of the standard rationales for firm skewness seem able to explain it. Rather, it is a phenomenon sweeping the entire cross-section of firm. We suspect that solving this puzzle will need future research to develop new theories of where skewness originates, or at least update existing theories. It is also important to quantify the implications of this pervasive change in the distribution of returns for investors and the economy more broadly.

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Internet Appendix

Figure IA.1: **Firm-level skewness: Compustat vs Full Sample.** The figure reports the average firm-level skewness in monthly returns for each 5-year period. The green line is the baseline sample of all publicly traded firms from CRSP. The orange line is the subsample matched to COMPUSTAT, in which we can compute firm characteristics.