Causal Inference for Asset Pricing

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  How to interpret estimates? Implicit assumptions on spillovers?

- Quantitative demand systems
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  Which results are robust outside of these models and which are specific to these structures?

How do those approaches account for substitution and spillovers across assets?

- Traditional methods: "everything is connected," Euler equation tests, factor models, ...

Why is it important?

- Learning from asset quantity data
- Learning from natural experiments
  - Counterpart to growth of micro-empirical methods in macro (e.g. Nakamura Steinsson 2018, Sraer Thesmar 2022)
- Classic models far from the data
- Many important questions are about quantities:
  - Quantitative easing policies (e.g. Haddad Moreira Muir 2025)
  - International capital flows, China and US Treasuries (e.g. Jansen Li Schmid 2025)
  - Rise of passive investing (Haddad, Huebner, Loualiche)

Our Framework

- **Simple portable assumption**: *homogeneous substitution conditional on observables*
  - Diagnostics, empirical design, ...

- **Flexible but parsimonious**: captures the forces of many demand structures, particularly specific to finance
  - Key missing piece of existing models: elasticity to price of factors/characteristics = substitution depends on characteristics

- **Easy estimation**: set of IV/diff-in-diff regression
  - "Separable:" map from different types of natural experiment to different counterfactual
  - Precisely define what is a valid instrument
  - Lots of work on finding instruments, not the focus here

The Problem

Question

How do an investor's portfolio decisions respond to prices?

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Elasticity matrix: sensitivity of demand to prices

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Elasticity matrix: sensitivity of demand to prices

- Defined in any theory
  - mean-variance: $D = \frac{1}{\gamma}\Sigma^{-1}(M-P)$, $\boldsymbol{\mathcal{E}} = -\frac{1}{\gamma}\Sigma^{-1}$
- Could be log, levels, shares, changes or not, ...
- Flipside: price impact $\boldsymbol{\mathcal{E}}^{-1}$, how do shifts in demand affect prices?

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- *How does CalPERS adjusts its position in 10-year corporate bonds of Ford and GM when their spread changes?*
- *How does AQR move across value and momentum based on their risk premia?*

$\Rightarrow$ **Answer to such questions about different parts of $\mathcal{E}$**

An Example: CalPERS and Corporate Bonds

- Prices have moved **and no other news**. CalPERS adjusts its bond portfolio:

	Price change	Change in position
1. 10-yr Ford	+ 5%	sell 200
2. 10-yr GM	+ 2%	sell 100
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$$\Delta D_1 = \underbrace{\mathcal{E}_{11}\Delta P_1}_{\text{became more expensive}} + \underbrace{\mathcal{E}_{12}\Delta P_2}_{\text{substitutes from GM}} + \underbrace{\sum_{k\geq 3}\mathcal{E}_{1k}\Delta P_k}_{\text{substitutes from First Solar, ...}}$$

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$$\Delta D_1 = \mathcal{E}_{11} \Delta P_1 + \mathcal{E}_{12} \Delta P_2 + \sum_{k \geq 3} \mathcal{E}_{1k} \Delta P_k$$

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$\rightarrow$ **Stuck without additional assumptions**

Two Paths

- **Causal inference:** impose elementary restriction keeping as much flexibility on mechanism as possible while letting the data speak

- **Structural approach:** choose a microfoundation and estimate the corresponding model

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Two Paths

- **Causal inference:** impose elementary restriction keeping as much flexibility on mechanism as possible while letting the data speak
  - Canonical assumption (SUTVA): when I give you medication, it affects your health but not the control group's health
  - Does not work here: demand for each asset only depends on its own price $D_i(P_i) \Rightarrow$ diagonal $\mathcal{E}$

- **Structural approach:** choose a microfoundation and estimate the corresponding model

Learning from Standard Finance Models

- Assume returns follow a factor structure: exposures β_i and idiosyncratic risk $\sigma^2_{\epsilon,i}$
- *Force 1:* factor management, if expected return only depends on exposures β
  - only buy portfolios replicating the factors (mutual fund theorem)
  - choose exposure to the factors based on the expected returns of those factors
- *Force 2:* "arbitrage": if expected returns deviate from factor pricing
  - buy more of cheap (= high alpha) assets, less of expensive ones

- A demand formula from Koijen Yogo 2019:
  - Factor model for returns: $R = \beta F + \epsilon$
  - Variance covariance matrix: $\Sigma = \beta\beta' + \sigma^2_\epsilon \mathbf{I}$
  - Demand elasticity follows factor structure (Sherman-Morrison):

$$\boldsymbol{\mathcal{E}} = -\frac{1}{\gamma}\Sigma^{-1} = \underbrace{-\frac{1}{\gamma\sigma^2_\epsilon}\mathbf{I}}_{\text{diagonal}} + \underbrace{c\boldsymbol{\beta}\boldsymbol{\beta}'}_{\text{substitution matrix}} \qquad \underbrace{c}_{\text{scalar}} = \frac{1}{\gamma\sigma^2_\epsilon}\frac{1}{\sigma^2_\epsilon + \boldsymbol{\beta}'\boldsymbol{\beta}}$$

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- **What next?**
  - KY 2019: add additional restrictions, get to logit or similar forms
  - This paper: generalize, keeping only the basic structure of diagonal + substitution driven by observables

A practical example: Simulation

- **Simulation:** Start from factor model demand, increase equally the supply of all assets $\rightarrow$ equilibrium is that price of high beta assets drops more
  - Mean-variance model: $\log D_t = \gamma^{-1}\left(\beta\beta' + \sigma_\epsilon^2 \mathbf{I}\right)^{-1}(\mu_t - \log P_t)$
  - Effect of supply shock δ:

$$\Delta \log P_t = \gamma\left(\beta\beta' + \sigma_\epsilon^2 \mathbf{I}\right)\delta$$

  - Simple one factor model (CAPM)

$$(\Delta \log P_t)_i = \gamma\sigma_\epsilon^2\delta_i + \gamma\beta_i\sum_k \beta_k\delta_k$$

  - Uniform shock $\delta = \delta\mathbf{1}$ has heterogeneous effects:

$$\frac{(\Delta \log P_t)_i}{\delta} = \gamma\sigma_\epsilon^2 + \gamma\beta_i\sum_k \beta_k$$

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A practical example: Estimation

Can we back out from prices the structure of the supply shock?

- KY 2019: demand and predict what the change in supply was: *erroneously* predict larger increase of supply for high beta assets
  - Adding a macro elasticity (Gabaix Koijen 2025) does not fix it
- Logit estimation (as in KY 2019)

$$\log D_{i,t} = b_0 + \hat{\mathcal{E}} \log P_{i,t} + \theta_t \beta_i + e_{it}$$

  - Econometrician recovers demand shocks that are linear in the price change.
  - Misses the differential preferences for assets with different β

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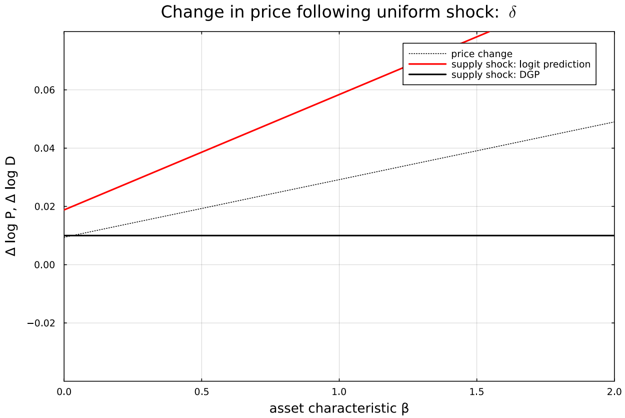

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  - Adding a macro elasticity (Gabaix Koijen 2025) does not fix it
- Logit estimation with Macro elasticity (as in Gabaix-Koijen)
  - Separate logit (cross-section) from aggregate (time-series)
  - Construct aggregate and idiosyncratic variables:

$$D_{t,agg} = \sum_i D_{i,t}, \qquad D_{i,t}^{idio} = D_{i,t} - D_{t,agg}.$$

  - Fixes the mean of the estimated supply shock
  - Still misses the differential preferences for assets with different β

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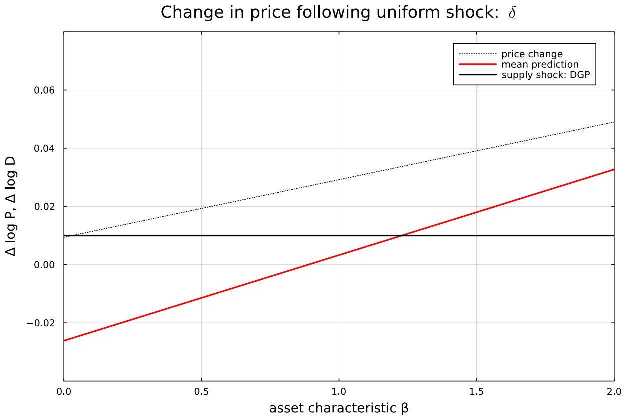

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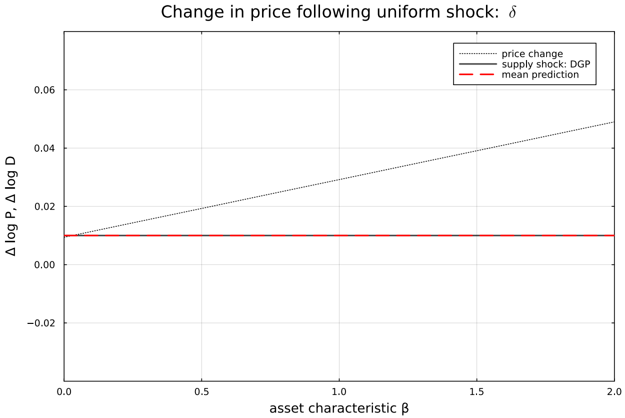
- Our methodology: account for factor substitution:



Framework

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A simple assumption:

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$$\textit{GM:} \quad \Delta D_2 = \mathcal{E}_{22}\Delta P_2 + \mathcal{E}_{21}\Delta P_1 + \sum_{k\geq 3} \underbrace{\mathcal{E}_{2k}}_{=\mathcal{E}_{1k}} \Delta P_k$$

- **Compare bonds with same observables:** Ford vs. GM
  - E.g.: CalPERS adjusts Ford and GM equally in response to price of First Solar $\mathcal{E}_{13} = \mathcal{E}_{23}$

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$$\textit{Diff-in-diff:} \quad \Delta D_1 - \Delta D_2 = \widehat{\mathcal{E}}(\Delta P_1 - \Delta P_2) \text{ if same relative elasticity}$$

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  - E.g.: CalPERS adjusts Ford and GM equally in response to price of First Solar $\mathcal{E}_{13} = \mathcal{E}_{23}$

$\rightarrow$ *comparing assets with same observables differences out substitution*

Formal Setup

- **Homogeneous substitution conditional on observables X**

$$\boxed{\mathcal{E}_{il} = \mathcal{E}_{jl} \text{ if } X_i = X_j} \quad \text{for all } i, j \in \mathcal{S}, \text{ and } l \neq i, j,$$

  – If price of 3rd asset move, response of demand for 2 assets with same observables is the same
  – Parametrize linearly: $\mathcal{E}_{il} = \mathcal{E}_{\text{cross}}(X_i, X_l) = X_i' \boldsymbol{\mathcal{E}}_X X_l$

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$$\boldsymbol{\mathcal{E}} = \text{relative elasticity} + \text{substitution}$$
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$$= \underbrace{\widehat{\mathcal{E}}}_{\text{scalar}} I + X \underbrace{\boldsymbol{\mathcal{E}}_X}_{K \times K} X'$$

  - Assume constant relative elasticity $\widehat{\mathcal{E}}$ for simplicity, relax in the paper

Questions Revisited

$$\mathcal{E} = \text{relative elasticity} + \text{substitution}$$
$$= \widehat{\mathcal{E}} I + X \boldsymbol{\mathcal{E}}_X X'$$

Different questions are about different parts of $\mathcal{E}$

- *How does CalPERS adjusts its position in 10-year corporate bonds of Ford and GM when their spread changes?*

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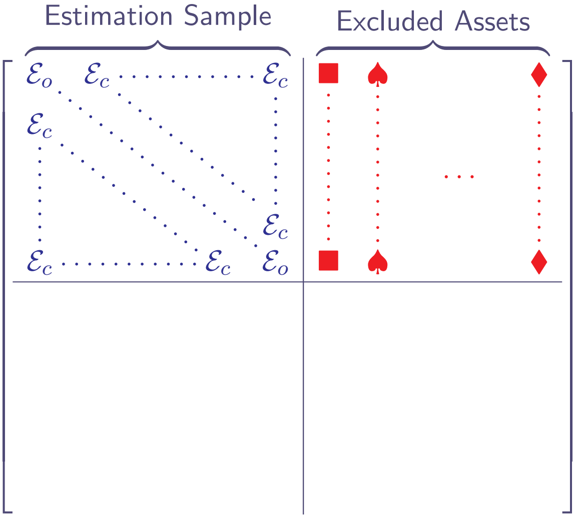
Different questions are about different parts of $\mathcal{E}$

- *How does CalPERS adjusts its position in 10-year corporate bonds of Ford and GM when their spread changes?*
  - Asset-specific behavior characterized by the relative elasticity $\widehat{\mathcal{E}}$
- *How does AQR move across factors based on factor risk premia?*
  - Question about substitution characterized by $\boldsymbol{\mathcal{E}}_X$

Local Experiments

- With few close assets: ignore observables & assume full homogeneity
  - Same own- and cross-price elasticity for every pair of assets in $\mathcal{S}$
  - Demand for all assets in $\mathcal{S}$ responds in same way to price of 5-year First Solar bond (outside $\mathcal{S}$)



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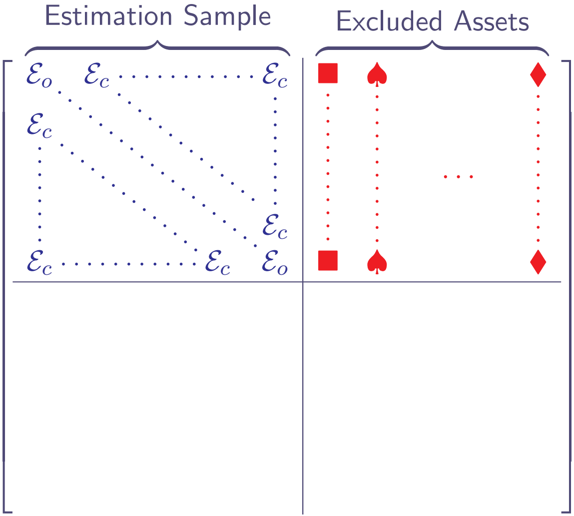

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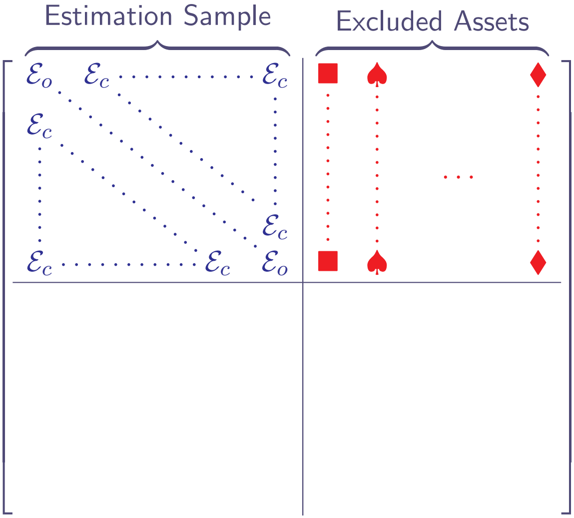
- Risk-based models: assets have common variance & covariance + *identical covariance with outside assets*
- Diagnostic: balance between treated (high Z_i) and control (low Z_i) on covariance with broad factors



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- *Risk based motives*: care about portfolio-level factor exposure, so X_i are factor loadings or characteristics that proxy for them
  - Markowitz: $D = \frac{1}{\gamma}\Sigma^{-1}(\mu - P) \Rightarrow \boldsymbol{\mathcal{E}} = -\frac{1}{\gamma}\Sigma^{-1}$
  - If Σ has factor structure: idio risk drives $\hat{\mathcal{E}}$, factor risk drives $\boldsymbol{\mathcal{E}}_X$

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- *Risk based motives*: care about portfolio-level factor exposure, so X_i are factor loadings or characteristics that proxy for them
- *Non-risk motives*: X_i is asset weight in this objective

$$\max_D \quad D'(\mu - P) - \frac{\gamma}{2} D'\Sigma D - \frac{\kappa}{2}\left(D'X^{(1)}\right)^2$$
$$\text{such that} \quad D'X^{(2)} \leq \Theta$$

  – Binding constraints (leverage), regulatory score (capital ratio), or stakeholders pressure (greenness)

Cross-Sectional Identification

- **Data-Generating-Process:** Elasticity matrix $\mathcal{E}$ + *homogeneous substitution conditional on observable* X

$$\Delta \mathbf{D} = \mathcal{E} \Delta \mathbf{P} + \epsilon$$

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- **Proposition 1** Under our assumption, and the *usual exclusion and relevance restrictions*, the IV estimator identifies the **relative elasticity** $\widehat{\mathcal{E}} = \boldsymbol{\mathcal{E}}_{ii} - \boldsymbol{\mathcal{E}}_{ji}$ for $X_i = X_j$

$$\Delta D_i = \widehat{\mathcal{E}} \Delta P_i + \theta' X_i + e_i$$
$$\Delta P_i = \lambda Z_i + \eta' X_i + u_i$$

  with Z_i instrument for prices ($Z_i \perp \epsilon_i | X_i$)

  – E.g.: Fed buys some bonds but not others

Absorbing Substitution

- Key step: coefficient on observables θ absorbs substitution from other assets

$$\begin{aligned}\Delta D_i &= \mathcal{E}_{ii}\Delta P_i + \sum_{j\neq i} X_i'\mathcal{E}_X X_j \Delta P_j + \epsilon_i \\ &= \left(\mathcal{E}_{ii} - X_i'\mathcal{E}_X X_i\right)\Delta P_i + \sum_j X_i'\mathcal{E}_X X_j \Delta P_j + \epsilon_i \\ &= \underbrace{\left(\mathcal{E}_{ii} - X_i'\mathcal{E}_X X_i\right)}_{\text{relative elasticity}}\Delta P_i + X_i' \underbrace{\sum_j \mathcal{E}_X X_j \Delta P_j}_{\text{constant across assets, absorbed in } \theta} + \epsilon_i\end{aligned}$$

- **Relative elasticity:** difference between own-price and cross-price elasticity for assets with same observables
  - How does the relative demand for Ford and GM respond to their relative price?
  - Useful to answer relative Qs and construct relative counterfactuals
  - In large cross-sections with substantial idiosyncratic risk $\approx$ own-price elasticity
  - What GE theorists call the Morishima elasticity Gabaix Koijen 2025 the micro-elasticity

FAQ

1. *"I exclude some assets from my sample because I don't have data on them. Is this a problem?"* **No**, as long as you assume the **same structure** applies to **excluded assets**. Conditional on X_i, the omitted asset affects others symmetrically, which differences out.

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2. *"What if I have a repeated cross-section?"* Control for observables X_i **interacted with time fixed effects**.

3. *"What do I estimate in a price impact regression of exogenous demand shocks on prices?"* Under **our assumption**, the identified coefficient is the **relative multiplier** $-1/\widehat{\mathcal{E}}$.

FAQ

1. *"I exclude some assets from my sample because I don't have data on them. Is this a problem?"* **No**, as long as you assume the **same structure** applies to **excluded assets**. Conditional on X_i, the omitted asset affects others symmetrically, which differences out.

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5. *"Can I recover the own-price elasticity from my cross-sectional regression?"* In general, **no** because the **own-price elasticity** combines **both** the relative elasticity and substitution. Cross-sectional regressions only identify part of $\boldsymbol{\mathcal{E}}$.

Substitution and Its Estimation

Estimating substitution $\mathcal{E}_X$ crucial for many questions:

- How does CalPERS adjust its portfolio when the price of all bonds drops?
- Will CalPERS maintain its green tilt if green bonds become very expensive relative to brown bonds?

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$\boxed{\textbf{Proposition 2}}$ **Impossible to identify substitution with the cross-section alone**

$$\Delta D_i = \widehat{\mathcal{E}} \Delta P_i + \underbrace{X_i' \sum_j \mathcal{E}_X X_j \Delta P_j + \epsilon_i}_{\textbf{BOTH absorbed in } \theta}$$

- Coefficient on X_i measures both substitution and shift in demand for observable
  - Does CalPERS reduce its green tilt because of expensive green bonds or weaker environmental priorities?
  - This is a **missing coefficients** problem

Demand–Price Decomposition

Classic strategy: construct portfolios sorted on observables, and measure their price and demand (= portfolio tilt)

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$$\Delta D_{agg} = \frac{1}{N}\sum_i \Delta D_i, \qquad \Delta P_{agg} = \frac{1}{N}\sum_i \Delta P_i$$

$$\Delta D_X = \frac{1}{N}\sum_i X_i \Delta D_i \qquad \Delta P_X = \frac{1}{N}\sum_i X_i \Delta P_i$$

$$\Delta D_{idio,i} = \Delta D_i - \Delta D_{agg} - X_i \Delta D_X \qquad \Delta P_{idio,i} = \Delta P_i - \Delta P_{agg} - X_i \Delta P_X$$

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- *Separate the response of demand to prices into three univariate components:*

Relative: $\Delta D_{idio,i} = \widehat{\mathcal{E}} \Delta P_{idio,i}$

Meso: $\Delta D_X = \tilde{\mathcal{E}}_{agg} \Delta P_{agg} + \tilde{\mathcal{E}}_X \Delta P_X$

Macro: $\Delta D_{agg} = \bar{\mathcal{E}}_{agg} \Delta P_{agg} + \bar{\mathcal{E}}_X \Delta P_X$

Estimating Substitution with the Time Series

- $\boxed{\textbf{Proposition 3}}$ Regressing portfolio tilts on portfolio prices with **time series instruments** identifies substitution $\mathcal{E}_X$

$$\Delta D_{X,t} = \tilde{\mathcal{E}}_{agg} \Delta P_{agg,t} + \tilde{\mathcal{E}}_X \Delta P_{X,t} + \epsilon_{X,t}$$
$$\Delta D_{agg,t} = \bar{\mathcal{E}}_{agg} \Delta P_{agg,t} + \bar{\mathcal{E}}_X \Delta P_{X,t} + \epsilon_{agg,t}$$

  - Effectively only K assets = portfolios
  - E.g. Fed does more or less QE and operation twist over time

Summary

Homogeneous substitution conditional on observables X:

$$\boldsymbol{\mathcal{E}} = \text{relative elasticity} + \text{substitution}$$
$$= \widehat{\mathcal{E}} I + X \boldsymbol{\mathcal{E}}_X X'$$

Consistent with many motives: risk, constraints, non-pecuniary preferences, irrational, ...

Identification:

- Relative elasticity: compare similar assets = cross-sectional IV controlling for X
- Substitution: demand for portfolios based on X = time-series portfolio level instruments

What About Logit?

- Koijen Yogo (2019), Koijen Richmond Yogo (2024):
  - $\exists$ factor models where volatility and expected returns depend non-linearly of prices which yield asset demand in the logit form
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- Logit satisfies our assumption, and its parameter can be robustly interpreted as relative elasticity
- Logit strongly restricts substitution: **an arbitrary factor model is not equivalent to logit**
  - Logit: when the price of any bond $\uparrow$, CalPERS replaces it proportionally to its existing portfolio
  - Factor model: CalPERS replaces it disproportionately with bonds loading on similar factors

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- Logit demand is a very special case of factor demand:
  - Single factor elasticity: substitution based on factor loadings (characteristics β)

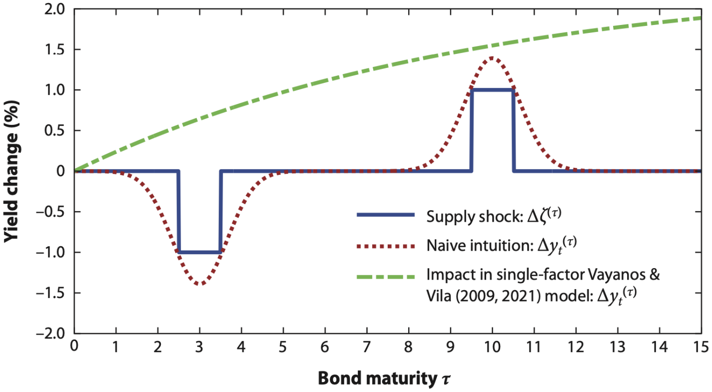
$$\gamma^{-1}\,\mathrm{diag}(\sigma_\epsilon^{-2}) + c\beta\beta'$$

  - Logit elasticity: substitution based on shares (ω):

$$\alpha\,\mathrm{diag}(\omega) + \alpha\omega\omega'$$

Group-based substitution vs Factor Models

- Nested logit (Fang 2023, Koijen Yogo 2024): symmetric groups based on values of observables → can use the cross-section of groups to estimate substitution
  - Predict strong local effect and diffuse effect across all other groups
  - Sharply different from factor model with exposure depending on observable (see Cochrane 2008, Vayanos Vila 2021)



Example: Corporate Bond Relative Multiplier

- U.S. investment-grade corporate bonds (following Chaudhary Fu Li, 2024)

- Steps to go through when conducting causal inference in asset pricing:
  1. choose a source of variation
  2. assess exogeneity
  3. assess assumptions A1 and A2 and select observables + units
  4. implement the regression analysis

- Step 1: flow-induced demand shock Z_{it}: fund flow in mutual funds $\times$ portfolio composition (Coval Stafford 2007, Lou 2012)

- Step 2: assess exogeneity, i.e., $Z_{it} \perp \epsilon_{it} | X_{it}$
  - example threat to identification: another investor type (e.g., insurance companies) buying the same bonds held by mutual funds that receive a lot of inflows

Step 3: Diagnostic for Homogeneous Substitution – Balance on covariances

Do treated & control bonds comove the same way with broad portfolios?

1. At each date t, form a long-short portfolio based on treatment status
2. Compute the β of the long-short return on broad indices in a window around t (here: 2y)
3. β different from zero $\Rightarrow$ substitution likely not homogeneous

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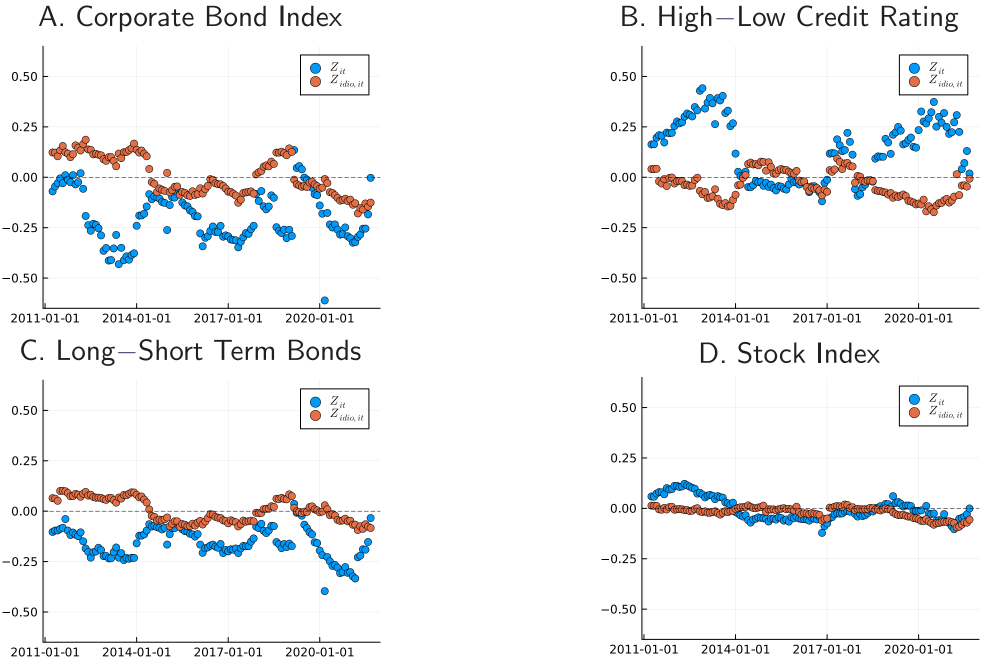
- Alternative unit to bond returns: yield changes ▸ A1 yield changes ▸ Multiplier yield changes
- Similar diagnostic for constant relative elasticity: balance on idiosyncratic volatility ▸ A2 diagnostic


A. Corporate Bond Index
B. High—Low Credit Rating
C. Long—Short Term Bonds
D. Stock Index
Z_{it}
$Z_{idio,\,it}$
0.50
0.25
0.00
−0.25
−0.50
2011-01-01
2014-01-01
2017-01-01
2020-01-01

Step 4: Implement the regression

Relative multiplier $\widehat{\mathcal{M}} \approx 0$

	Return $\Delta P_{it}/P_{i,t-1}$				
	(1)	(2)	(3)	(4)	(5)
Demand shock:					
Z_{it}	1.541*	-0.254	0.019		
	(0.637)	(0.229)	(0.065)		
$Z_{idio,it}$				0.019	0.019
				(0.065)	(0.065)
Date Fixed Effects		Yes	Yes	Yes	Yes
Duration $\times$ Date Fixed Effects			Yes	Yes	
Credit Rating $\times$ Date Fixed Effects			Yes	Yes	
N	646,335	646,335	646,335	646,335	646,335
R^2	0.010	0.415	0.632	0.632	0.415

Takeaways

- To draw causal inference about demand elasticity, need:
  - A simple assumption: **homogeneous substitution conditional on observables**
    - CalPERS substitutes based on duration and greenness
  - (Standard) source of exogenous variation
    - Fed randomly buys more of some bonds than others, Fed surprisingly engages in QE

- Relative elasticity for similar assets: cross-sectional IV
  - Ford vs GM?
- Substitution = demand for portfolios: time-series IV
  - Green vs brown? Aggregate price?

- Standard structural models of demand rule out most factor-style substitution

Why Causal Inference in Asset Pricing?

- Causal inference particularly valuable when:
  - existing theories are far from the data
  - it is challenging to understand all sources of variations simultaneously
- First step towards better economic theory

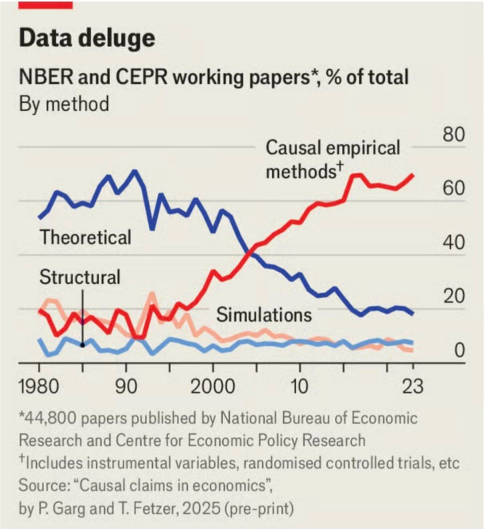
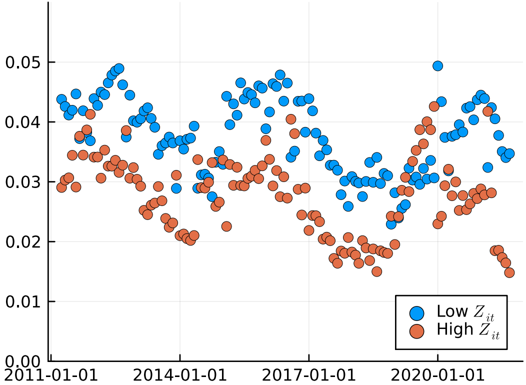


CHART: THE ECONOMIST

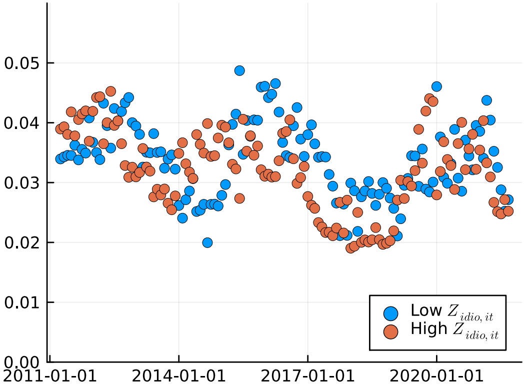
Diagnostic for Constant Relative Elasticity

- **Balance on idiosyncratic volatility**

A. Idiosyncratic Volatility (Z_{it})



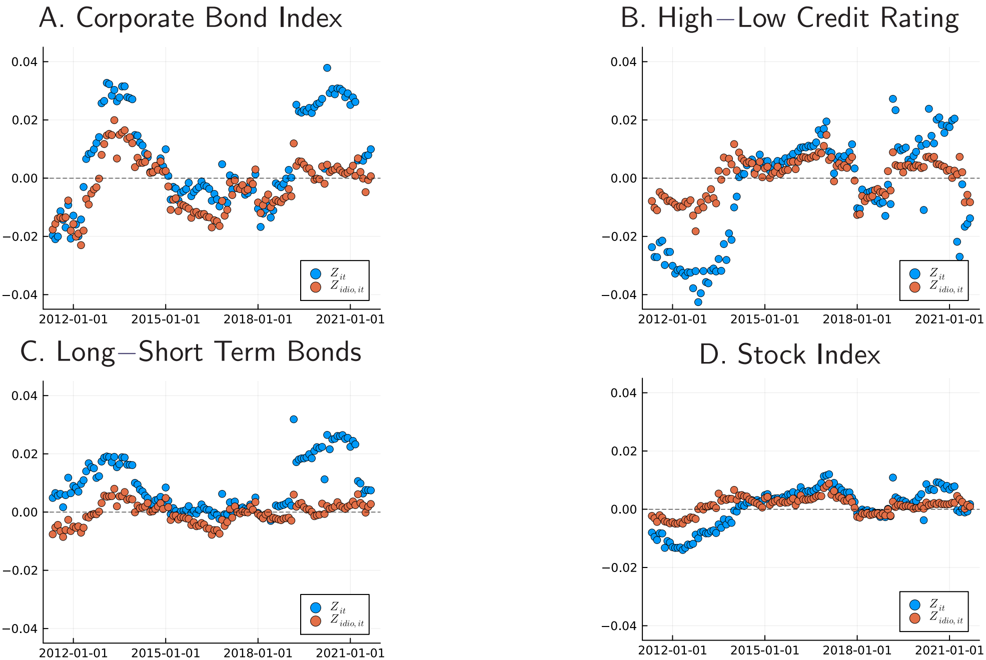
B. Idiosyncratic Volatility ($Z_{idio,it}$)



- Average idiosyncratic volatility among treated versus control bonds

Back

Back


A. Corporate Bond Index
B. High−Low Credit Rating
C. Long−Short Term Bonds
D. Stock Index
0.04
0.02
0.00
−0.02
−0.04
2012-01-01
2015-01-01
2018-01-01
2021-01-01
Z_{it}
$Z_{idio,it}$

Back

Relative multiplier $\widehat{\mathcal{M}} = -0.072$

	Yield change ΔY_{it}				
	(1)	(2)	(3)	(4)	(5)
Demand shock:					
Z_{it}	-0.384*	-0.104*	-0.072**		
	(0.166)	(0.047)	(0.027)		
$Z_{idio,it}$				-0.072**	-0.072**
				(0.027)	(0.027)
Date Fixed Effects		Yes	Yes	Yes	Yes
Duration $\times$ Date Fixed Effects			Yes	Yes	
Credit Rating $\times$ Date Fixed Effects			Yes	Yes	
N	630,255	630,255	630,255	630,255	630,255
R^2	0.004	0.071	0.089	0.089	0.070