Causal Inference for Asset Pricing

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Causal inference methods

- e.g. use IV/diff-in-diff to learn about investors' portfolio choice or equilibrium asset prices
  - If the stock price of Tesla drops by 1%, how do you change your position?
  - If a group of investors starts buying GameStop, how does its price change?
- *natural experiments + inference without taking a strong stand on mechanism*

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"This is not how we do asset pricing"

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Traditional asset pricing empirical methods

- Euler equations, factor models, Epstein-Zin preferences, ...
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→ **This paper: a causal inference framework that is compatible with finance ideas**

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- OLS is a bad idea:
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  - CalPERS (and many others) demand affect prices
- **Natural experiment:** the Fed decides to do a one-off intervention buying random corporate bonds

Laurent's Dilemma

Canonical causal inference: IV with $Z_i =$ Fed purchases of bond i

$$\Delta D_i = \widehat{\mathcal{E}} \Delta P_i + \theta' X_i + e_i$$
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Finance: holdings decided as a portfolio

- When price of a green bond increases, CalPERS sells some of it ... and replace by investing disproportionately more in other green bonds than brown bonds

$$\Delta D = \underset{\text{matrix}}{\mathcal{E}} \Delta P + \epsilon$$

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$\rightarrow$ Challenge of demand estimation with many goods

- SUTVA is violated
- All other prices are omitted variables ... too many to instrument them all

This Paper: Assumption

1. An elementary assumption about demand: homogeneous substitution conditional on observables

- When CalPERS substitutes, does so differentially for bonds with different observables (e.g. greenness, duration), but does not distinguish between bonds with the same observables

- Markowitz finance: factor structure of covariance matrix
- Many others: targeting of portfolio level targets (e.g. regulatory scores), logit, ...

- Empirical design, supporting evidence, ...

This Paper: Identification

2. **Cross-sectional causal inference** identifies the **relative elasticity** ($\widehat{\mathcal{E}}$):
   - How does the relative demand for two bonds with the same greenness and duration respond to a change in their relative price?
   - Difference between own-price and cross-price elasticity

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3. **A small set of time series regressions** identifies **substitution**
   - Substitution = meso and macro elasticity
     - Meso: How does the demand for green bonds relative to brown bonds respond to the price of the green-minus-brown portfolio and the market portfolio?
     - Macro: How does the demand for bonds responds to the price of the market portfolio and the green-minus-brown portfolio?
   - Needs simultaneous instruments over time for the price of all portfolios

Related literature

- **Asset pricing using causal inference methods**
  - Shleifer (1986); Harris, Gurel (1986); Chang, Hong, Liskovich (2014); Pavlova, Sikorskaya (2023); Greenwood, Sammon (2024); Gompers, Metrick (2001); Coval, Stafford (2007); Lou (2012); Ben-David, Li, Rossi, Song (2022); Hartzmark, Solomon (2022); Krishnamurthy, Vissing-Jorgensen (2011); Haddad, Moreira, Muir (2021, 2025); Selgrad (2024); Du, Tepper, Verdelhan (2018); Greenwood, Vissing-Jorgensen (2018); Haddad, Muir (2021); Chen, Chen, He, Liu, Xie (2023); ...
- **Structural approach and demand systems**
  - *Koijen, Yogo (2019, 2024); Koijen, Richmond, Yogo (2024)*; Haddad, Huebner, Loualiche (2024); van der Beck (2024); Lu, Wu (2023); Gabaix, Koijen (2024); Bretscher, Schmid, Sen, Sharma (2024); Jansen, Li, Schmid (2024); Fang (2023); Fang, Xiao (2024); ...
  - Li, Lin (2024); Chaudhary, Fu, Li (2023); Aghaee (2024); An, Huber (2025); Peng, Wang (2023); Fuchs, Fukuda, Neuhann (2024); ...
- **What we don't do:**
  - Strategic responses (Haddad, Huebner, Loualiche, 2024)
  - Dynamics (Greenwood, Hanson, Liao, 2018; Huebner, 2024; Gabaix, Koijen, 2024; He, Kondor, Li)
  - State-contingent shocks (Haddad, Moreira, Muir, 2025)
- **Spillovers/substitution outside asset pricing:**
  - *Berry, Levinsohn, Pakes (1995)*, Berg, Reisinger, Streitz (2021); Chodorow-Reich, Nenov, Simsek (2021); Guren, McKay, Nakamura, Steinsson (2021), Huber (2023); Wolf (2023); ...

Outline

1. Homogeneous substitution conditional on observables
2. Cross-Sectional Causal Inference
3. Estimating substitution
4. Estimating multipliers

An Assumption for Demand in Asset Pricing

The Demand Function for Assets

- Investor chooses portfolio ... taking prices are given: $D(P, ...)$
  - Similar structure with market power or learning from prices: post a demand curve

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This talk: what if you want to figure $\boldsymbol{\mathcal{E}}$ out from data without assuming much?

An Elementary Assumption

A1. **Homogeneous substitution conditional on observables**

→ *Any pair of assets in the estimation sample $\mathcal{S}$ with the same observables shares the same cross-price elasticity with respect to any third asset, within or outside the estimation sample:*

$$\boxed{\mathcal{E}_{il} = \mathcal{E}_{jl} \text{ if } X_i = X_j} \quad \text{for all } i, j \in \mathcal{S}, \text{ and } l \neq i, j,$$

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- When substituting, the investor differentiates with respect to X only
- X_i: $K \times 1$ vector of observables for asset i.
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- Can apply everywhere, or just to a sample of assets $\mathcal{S}$

Regularizing a Bit More

A2. **Constant relative elasticity**

→ *Assets in the estimation sample have the same value of relative elasticity* $\mathcal{E}_{relative}$ *with respect to other assets with the same characteristics:*

$$\boxed{\boldsymbol{\mathcal{E}}_{ii} - \boldsymbol{\mathcal{E}}_{ji} = \mathcal{E}_{relative} \text{ if } X_i = X_j} \quad \text{for all } i, j \in \mathcal{S}$$

- How does the relative demand for two assets with the same observables respond to a change in their relative price?

- Similar local behavior across assets → homogeneous treatment effect
- Can relax a lot for cross-sectional results (function of characteristics, LATE)

Using the Assumptions: Local Experiments

- With few close assets, could ignore observables and assume full homogeneity
  - Demand for 10-yr bonds of Ford and GM responds in same way to price of 5-year First Solar bond

$$\begin{array}{cc} & \begin{array}{cc} \overbrace{\hphantom{\mathcal{E}_o\ \mathcal{E}_c\ \cdots\ \mathcal{E}_c}}^{\text{Estimation Sample}} & \overbrace{\hphantom{\blacksquare\ \spadesuit\ \cdots\ \blacklozenge}}^{\text{Excluded Assets}} \end{array} \\ \mathcal{E} = & \left[\begin{array}{cccc|cccc} \mathcal{E}_o & \mathcal{E}_c & \cdots & \mathcal{E}_c & \blacksquare & \spadesuit & & \blacklozenge \\ \mathcal{E}_c & \ddots & \ddots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \ddots & \ddots & \mathcal{E}_c & \vdots & \vdots & & \vdots \\ \mathcal{E}_c & \cdots & \mathcal{E}_c & \mathcal{E}_o & \blacksquare & \spadesuit & & \blacklozenge \\ \hline & & & & & & & \\ & & & & & & & \end{array}\right] \end{array}$$

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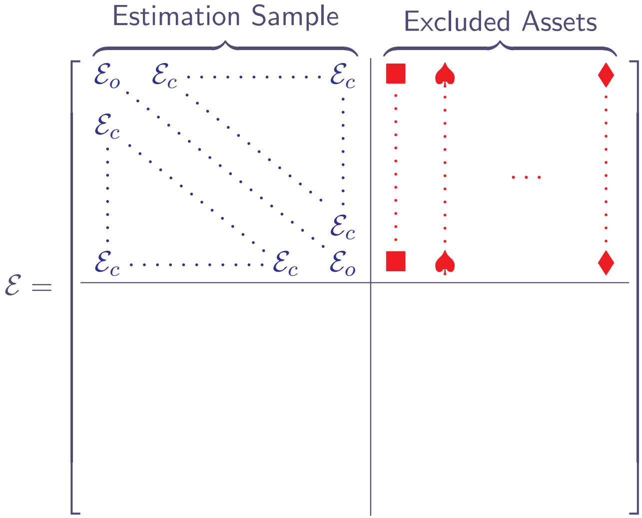

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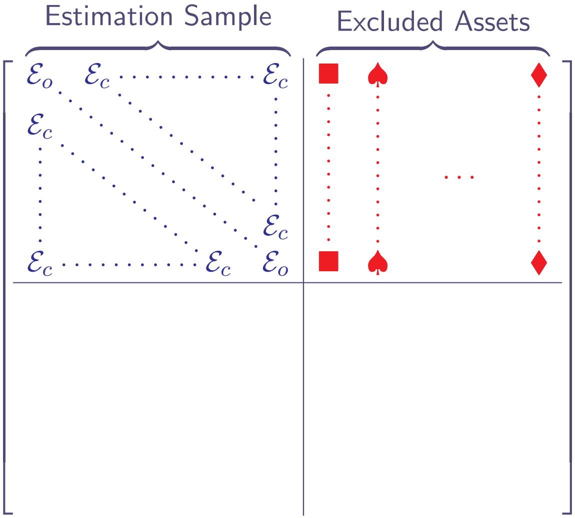
- Diagnostic: balance between treated (high Z_i) and control (low Z_i) on covariance with broad factors

$\mathcal{E} =$ 

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Key question: What does the investor consider when substituting between assets?

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- *Non-risk motives*: X_i is asset weight in this objective
  - Binding constraints (e.g. leverage)
  - Manages a regulatory score (e.g. capital ratio,...)
  - Stakeholders pressure (greenness, ...)

$$\max_{D} \quad D'(M-P) - \frac{\gamma}{2} D'\Sigma D - \frac{\kappa}{2}\left(D'X^{(1)}\right)^2$$

$$\text{such that} \quad D'X^{(2)} \leq \Theta$$

Cross-Sectional Causal Inference

Cross-Sectional Identification

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Mechanics for the simple case

- Take 2 assets with same characteristics, $X_1 = X_2$

First difference

$$\Delta D_1 = \mathcal{E}_{11} \Delta P_1 + \mathcal{E}_{12} \Delta P_2 + \sum_{k \geq 3} \mathcal{E}_{1k} \Delta P_k$$

$$\Delta D_2 = \mathcal{E}_{22} \Delta P_2 + \mathcal{E}_{21} \Delta P_1 + \sum_{k \geq 3} \mathcal{E}_{2k} \Delta P_k$$

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  - $Z_i \perp \epsilon_i | X_i$
- If equilibrium is such that the two prices cannot deviate *at all* from each other, relevance might fail
  - You can assess this empirically!

The Missing Piece: Substitution

- Key step: control for characteristics θ absorbs substitution from other assets

$$\sum_{k\geq 3} \mathcal{E}_{\text{cross}}(X_1, X_k)\Delta P_k = X_1' \underbrace{\mathcal{E}_X X \Delta P}_{\text{constant in data}}$$

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Absorbing substitution $\neq$ Estimating substitution

- "Missing intercept and coefficients" problem: doesn't know how θ would change with different prices

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- Both models satisfy our assumptions and hence can have relative elasticity estimated from the cross-section
  - Assuming logit-specific structure makes it enough to back out substitution
  - Analogy: if you assume no substitution at all, you would also get all $\mathcal{E}$ from the cross-section

Estimating Substitution with the Time Series

Why Substitution Matters

Laurent wants to know:

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Laurent wants to know:

- Will CalPers maintain its green tilt if the price of green bonds become very expensive relative red bonds?
- How much will CalPers size down its bond positions if all bond prices increase?
- Answer to these questions relies on knowing substitution!

Simplifying Substitution

Under assumptions A1 and A2, can replace the asset-level problem of substitution with a portfolio-level problem:

- Re-package prices and demand:

$$\Delta P_{agg} = \frac{1}{N}\sum_i \Delta P_i, \qquad \Delta D_{agg} = \frac{1}{N}\sum_i \Delta D_i$$

$$\Delta P_X = \frac{1}{N}\sum_i X_i \Delta P_i \qquad \Delta D_X = \frac{1}{N}\sum_i X_i \Delta D_i$$

$$\Delta P_{idio,i} = \Delta P_i - \Delta P_{agg} - X_i \Delta P_X \qquad \Delta D_{idio,i} = \Delta D_i - \Delta D_{agg} - X_i \Delta D_X$$

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- *Decompose the response of demand to prices into three univariate components:*

Relative: $\Delta D_{idio,i} = \widehat{\mathcal{E}} \Delta P_{idio,i}$

Meso: $\Delta D_X = \tilde{\mathcal{E}}_{agg} \Delta P_{agg} + \tilde{\mathcal{E}}_X \Delta P_X$

Macro: $\Delta D_{agg} = \bar{\mathcal{E}}_{agg} \Delta P_{agg} + \bar{\mathcal{E}}_X \Delta P_X$

Aggregation Intuition

- Under assumptions A1 and A2:

$$\mathcal{E} = \widehat{\mathcal{E}}\mathbf{I} + X\mathcal{E}_X X'$$

  - $\widehat{\mathcal{E}}$ is a scalar
  - $\mathcal{E}_X$ is only $K \times K$ (factor model)
- Project the change in price along the factor X direction:

$$\begin{aligned}
\Delta P_X &= (X'X)^{-1}X'\Delta P \\
&= \left(\widehat{\mathcal{E}}(X'X)^{-1}X'\mathbf{I}_N + (X'X)^{-1}X'X\mathcal{E}_X X'\right)\Delta D \\
&= \left(\widehat{\mathcal{E}}\mathbf{I}_K + \mathcal{E}_X(X'X)\right)\Delta D_X
\end{aligned}$$

Aggregation Intuition

- Two easy cases:
  - Only one characteristic ($K = 1$):

$$\check{\mathcal{E}} = \begin{pmatrix} \widehat{\mathcal{E}} + N(\mathcal{E}_X)_{11} & N(\mathcal{E}_X)_{12} \\ N(\mathcal{E}_X)_{21} & \widehat{\mathcal{E}} + N(\mathcal{E}_X)_{22} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{E}}_{agg} & \bar{\mathcal{E}}_X \\ \widetilde{\mathcal{E}}_{agg} & \widetilde{\mathcal{E}}_X \end{pmatrix}$$

  - Observables are group dummies: consider the case when the observables are dummy variables for disjoint groups.

$$\Delta P_{X,k} = \frac{1}{N_k} \sum_{i \in k} \Delta P_i$$

Estimating the Meso and Macro Elasticities

Meso: $$\Delta D_X = \tilde{\mathcal{E}}_{agg} \Delta P_{agg} + \tilde{\mathcal{E}}_X \Delta P_X$$

Macro: $$\Delta D_{agg} = \bar{\mathcal{E}}_{agg} \Delta P_{agg} + \bar{\mathcal{E}}_X \Delta P_X$$

- Substitution boils down to relation between aggregate and observable based portfolios
  - Response of overall demand and green portfolio tilt to aggregate bond price and price of green-minus-brown portfolio
  - Low dimensional
- Need joint instruments for prices **in time series**:
  - To estimate macro elasticity, need to account for simultaneous change in price of green-minus-brown
  - Only controlling for the price is generally a bad control (in particular if demand shocks are correlated)

Estimating Multipliers: an Empirical Example

Example: Corporate Bond Relative Multiplier

- U.S. investment-grade corporate bonds (following Chaudhary, Fu, Li, 2024)

- Steps to go through when conducting causal inference in asset pricing:
  1. choose a source of variation
  2. assess exogeneity
  3. assess assumptions A1 and A2 and select observables + units
  4. implement the regression analysis

- Step 1: flow-induced demand shock Z_{it}: fund flow in mutual funds $\times$ portfolio composition (Coval Stafford 2007, Lou 2012)

- Step 2: assess exogeneity, i.e., $Z_{it} \perp \epsilon_{it} | X_{it}$
  - example threat to identification: another investor type (e.g., insurance companies) buying the same bonds held by mutual funds that receive a lot of inflows

Step 3: Diagnostic for A1 – Balance on covariances

Do treated bonds comove the same way with broad portfolios as the control bonds?

1. At each date t, form a long-short portfolio based on whether Z_{it} is above ("treated") or below ("control") the median
2. Compute the β of the long-short return on broad indices in a window around t (here: 2y)
3. β different from zero $\Rightarrow$ substitution likely not homogeneous

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- Treated and control bonds may differ systematically based on the observables, which may drive differences in β
  $\rightarrow$ natural if investors choose their flows along dimensions like duration and credit risk
- Do the treated and control comove the same way *conditional on observables*?
- $Z_{idio,it}$: residual of instrument regressed on a date fixed effect, **duration** $\times$ date fixed effects and **credit rating** $\times$ date fixed effects

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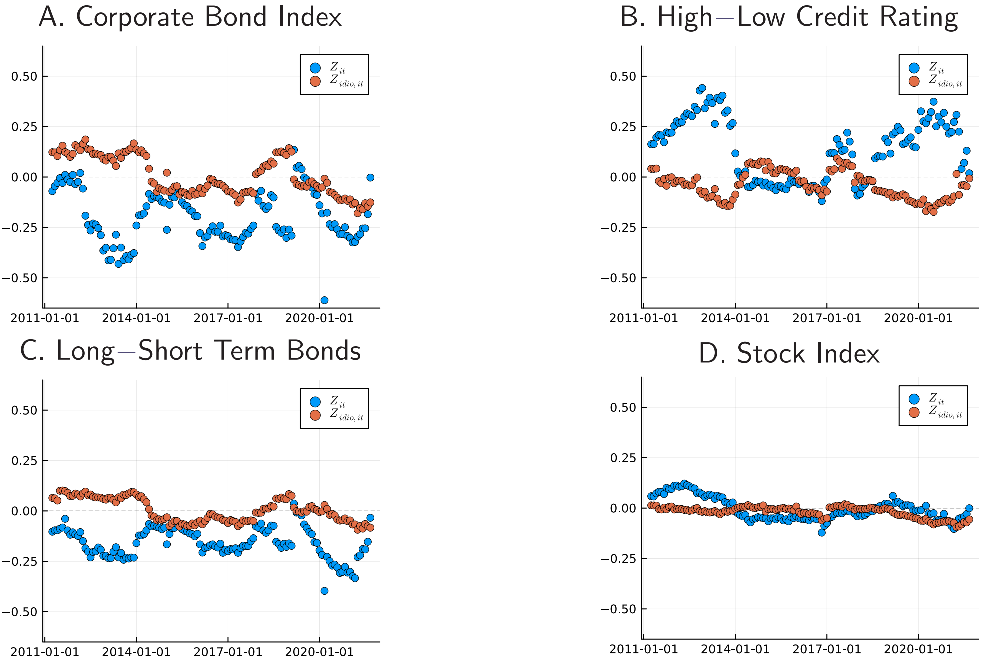
- Alternative unit to bond returns: yield changes ▸ A1 yield changes ▸ Multiplier yield changes
- Similar diagnostic for A2: balance on idiosyncratic volatility ▸ A2 diagnostic


A. Corporate Bond Index
B. High—Low Credit Rating
C. Long—Short Term Bonds
D. Stock Index
Z_{it}
$Z_{idio,it}$
0.50
0.25
0.00
−0.25
−0.50
2011-01-01
2014-01-01
2017-01-01
2020-01-01

Step 4: Implement the regression

Relative multiplier $\widehat{\mathcal{M}} \approx 0$

	Return $\Delta P_{it}/P_{i,t-1}$				
	(1)	(2)	(3)	(4)	(5)
Demand shock:					
Z_{it}	1.541*	-0.254	0.019		
	(0.637)	(0.229)	(0.065)		
$Z_{idio,it}$				0.019	0.019
				(0.065)	(0.065)
Date Fixed Effects		Yes	Yes	Yes	Yes
Duration $\times$ Date Fixed Effects			Yes	Yes	
Credit Rating $\times$ Date Fixed Effects			Yes	Yes	
N	646,335	646,335	646,335	646,335	646,335
R^2	0.010	0.415	0.632	0.632	0.415

Conclusion

- I hope Laurent is happy now

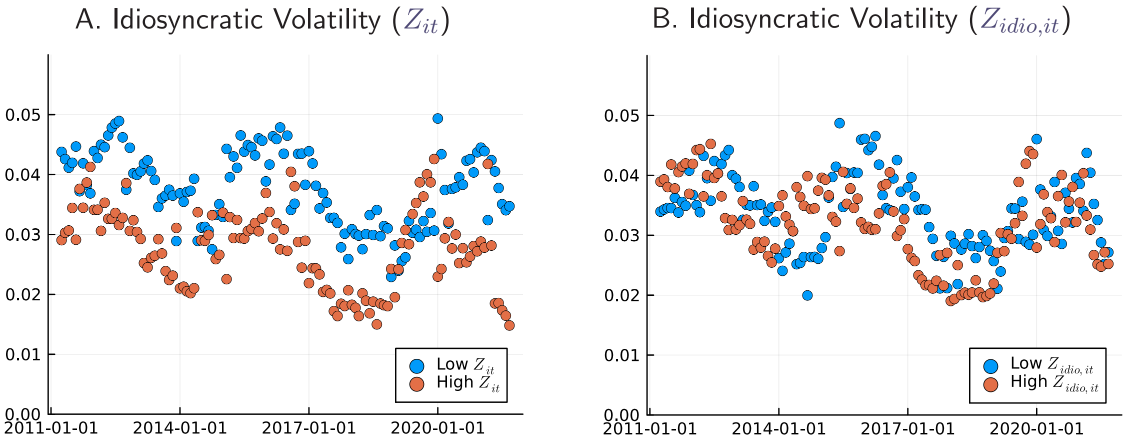
Conclusion

- Key challenge for causal inference in asset pricing: substitution across assets

Conclusion

- Key challenge for causal inference in asset pricing: substitution across assets
- **An elementary condition for valid inference**: homogenous substitution conditional on observables
  - difference in substitution driven by a known set of observables
- **Standard cross-sectional causal inference** identifies relative elasticity or its inverse, relative multiplier
  - Guidance on designing settings such that assumptions are plausible
  - Compatible with usual covariance matrix assumptions
- **Time series identification with observable-based portfolios** reveals substitution
  - Need to consider all dimensions of substitution jointly

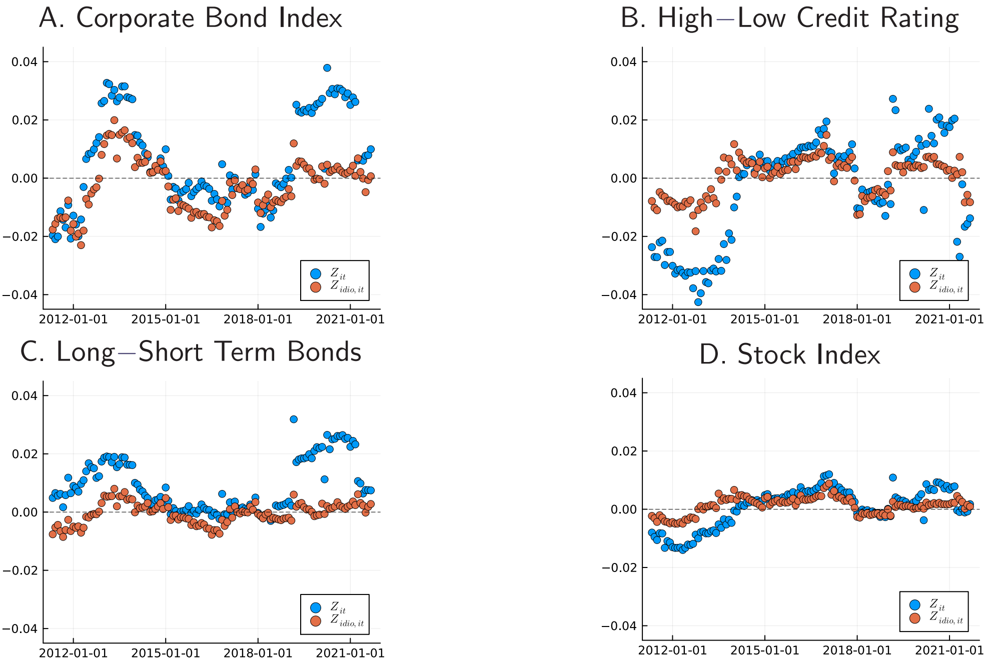
Diagnostics for A2 – Balance on idiosyncratic volatility



- Average idiosyncratic volatility among treated versus control bonds

Back

Back


A. Corporate Bond Index
B. High—Low Credit Rating
C. Long—Short Term Bonds
D. Stock Index
0.04
0.02
0.00
−0.02
−0.04
2012-01-01
2015-01-01
2018-01-01
2021-01-01
Z_{it}
$Z_{idio,it}$

Back

Relative multiplier $\widehat{\mathcal{M}} = -0.072$

	Yield change ΔY_{it}				
	(1)	(2)	(3)	(4)	(5)
Demand shock:					
Z_{it}	-0.384*	-0.104*	-0.072**		
	(0.166)	(0.047)	(0.027)		
$Z_{idio,it}$				-0.072**	-0.072**
				(0.027)	(0.027)
Date Fixed Effects		Yes	Yes	Yes	Yes
Duration $\times$ Date Fixed Effects			Yes	Yes	
Credit Rating $\times$ Date Fixed Effects			Yes	Yes	
N	630,255	630,255	630,255	630,255	630,255
R^2	0.004	0.071	0.089	0.089	0.070