

Growth-Indexed Securities

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Once upon a time

The Author

- *Displacement Risk and Asset Returns* (Gârleanu, Kogan, Panageas; JFE, 2012)
- Aggregate consumption and aggregate dividend cointegrated?
 - Is the value of the stock market a claim to consumption stream
 - Market incompleteness: stock market does not include the value yet-to-be invented ventures

Other People

- Sargent-Wallace, Sims, etc...
- Government Debt is a claim on fiscal surpluses

$$D = P(\text{taxes}) - P(\text{expenditures})$$

- What if $P(\text{expenditures}) \rightarrow +\infty$...
- ... do not worry taxes and expenditures are *cointegrated*

Other People

- By the way: $r < g$

This paper

Cointegrated has a specific meaning

- If C and X are cointegrated, there is a linear combination of both that is *stationary*
- A process is stationary if its distribution is invariant over time

Finance

- Laws of statistics in finance usually translate but not necessarily
- Stationarity under \mathbf{P} measure does not imply stationarity under a different measure \mathbf{Q}
- What if that other measure is the pricing measure?

Quickly

Simple cointegration

- Simplest consumption process: constant $c_t = C_0$
- Dividend process is mean-reverting $dx_t = -\kappa(x_t - x_0)dt + \sigma dW$
- Define stochastic process as the ratio

$$z_t = \frac{x_t}{c_t} = \frac{1}{c_0} x_t$$

- z_t is stationary!

Pricing measure

- Take some pricing measure \mathbf{Q} such that R-N derivative introduces some constant drift
 - $d\mathbf{Q}/d\mathbf{P} = \exp(aW_t - \frac{1}{2}a^2t)$
- Change in measure introduces a constant drift ... z_t is not stationary under \mathbf{Q}

$$dz_t = -\kappa\left(z_t - \frac{x_0}{c_0}\right)dt + \sigma \frac{a}{c_0}dt + \sigma d\hat{W}_t$$

Complete version

- Things move around: c_t is stochastic, so are dividends x_t
- Moreover x_t depends on c_t (dividend policy is state contingent)
- Under a general dividend distribution rule
 - Wealth consumption ratio is stationary
 - So is the dividend to wealth ratio
- There is a dividend policy that is cointegrated with consumption

Complete version

The dividend claim

- Price of strips go to 0
 - The value of all the strips is bounded: it is the value of the asset

$$P_t(X_T) \xrightarrow{T \rightarrow +\infty} 0$$

The consumption claim

- Standard pricing with a fixed risk-premium

$$P_t\left(\frac{C_T}{C_t}\right) = \exp(-(r + \kappa\sigma - g)(T - t))$$

- Not implausible case ... $r + \kappa\sigma < g$

$$P_t(C_T) \xrightarrow{T \rightarrow +\infty} +\infty$$

Complete version

What happened to cointegration of C and X ?

$$P_t(C_T) \xrightarrow{T \rightarrow +\infty} +\infty; \quad P_t(X_T) \xrightarrow{T \rightarrow +\infty} 0$$

- Pricing operator is a change of measure: distorts the probabilities
- The ratio $\zeta = \log(X/C)$ is stationary
- The ratio of prices is not $p(\zeta) = P(X)/P(C)$
- Actual proof is slightly more complicated
- It is constructive: build a portfolio strategy

Who cares or who should care?

Asset pricing in a world where $r < g_c$

- Under some conditions, this is actually ok

Sustainability of debt

- Jiang, Lustig, Van Nieuwerburgh, and Xiaolan present value equality

$$D = P(\text{taxes}) - P(\text{expenditures})$$

- ▶ Maybe prices of taxes and expenditures are not cointegrated
- ▶ The price of a claim to the difference does not goes to zero at large horizon
- ▶ Bubble, specialness of U.S. treasury ...
- Matters for the sustainability of debt trajectory given path of fiscal surpluses

Some practical applications

Actual GDP-indexed bonds

- France 1956-1975: coupon is $\bar{r} + \alpha \Delta_{0,t} \text{GDP}$
- Useful to evaluate whether $r > g$
 - Back out risk-adjusted growth g^Q from prices and risk-premium

$$\underbrace{g^Q}_{2.6\%} > \underbrace{r}_{2.1\%}$$

- Suggesting ideas around bubbles and transversality conditions are not completely outside of the empirical realm
- Usual caveat on the specific and novel bonds used to estimate g^Q
 - Liquidity, size, risk, market participants

Should government issue GDP-linked bonds

- What could go wrong?
- Debt payments are now a function of GDP
 - Automatic stabilizer: no more drift in debt/gdp ratio (if sole portfolio)
- If there is a “bubble” component in the price of government bonds
 - Isn't it optimal to use it?
- What shape would monetary policy take in a world of GDP-indexed bonds?

Final Thoughts

Very interesting Paper!

Take away

- Clean explanation of some confusion in the literature
- Flesh out the difference between cointegration and the pricing of two processes

Great Paper!