Causal Inference for Asset Pricing

Valentin Haddad Zhiguo He Paul Huebner Peter Kondor Erik Loualiche

UCLA, Stanford, SSE, LSE, Minnesota

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This paper: a causal inference framework that is compatible with finance ideas

Outline

An Example: CalPERS and Corporate Bonds

- Prices have moved (no other news) and CalPERS adjusts its bond portfolio:

	Initial position	Price change	New position
1. 10-yr Ford	1,000	$+$ 5%	$=$ 1,000
2. 10-yr GM	1,000	- 5%	$\uparrow$ 1,100
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- Naive approach: to get $D_i(P_i)$, relate position changes ΔD_i to price changes ΔP_i
- **Portfolio choice:** holdings decided as a portfolio $\mathbf{D}(\mathbf{P})$
  - When price of First Solar increases, CalPERS sells some of it ... and likely, replaces by investing disproportionately more in other green bonds than brown bonds
  - **Asset demand system**: How does demand i depends on price j:

$$\boldsymbol{\mathcal{E}} = \left[\frac{\partial D_i}{\partial P_j}\right]_{ij}$$

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$$\Delta D_1 = \underbrace{\mathcal{E}_{11}\Delta P_1}_{\text{became more expensive}} + \underbrace{\mathcal{E}_{12}\Delta P_2}_{\text{substitutes from GM}} + \underbrace{\sum_{k\geq 3}\mathcal{E}_{1k}\Delta P_k}_{\text{substitutes from First Solar, ...}}$$

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- Stuck without making assumptions

Making Progress

- **Key identifying assumption**: **homogeneous substitution conditional on observables**
  - When CalPERS substitutes, does so differentially for bonds with different observables (e.g. greenness, duration), but does not distinguish between bonds with the same observables
  - E.g.: portfolio replacing First Solar has equal quantity of Ford and GM: $\mathcal{E}_{13} = \mathcal{E}_{23}$

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Diff-in-diff: $$\Delta D_1 - \Delta D_2 = (\mathcal{E}_{11} - \mathcal{E}_{21})\,\Delta P_1 - (\mathcal{E}_{22} - \mathcal{E}_{12})\,\Delta P_2$$

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$= \hat{\mathcal{E}}(\Delta P_1 - \Delta P_2)$ if same **relative elasticity**

Relative Elasticity

	Initial position	Price change	New position
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How does the relative demand for 10-yr Ford and 10-yr GM respond to a change in their relative price?

$$\hat{\mathcal{E}} = \frac{\Delta D_1 - \Delta D_2}{\Delta P_2 - \Delta P_1} = \frac{0 - 100}{+5 - -5} = -10$$

Relative Elasticity and Exogenous Variation (IV)

- Recover **relative elasticity**: How does the relative demand for two bonds with the same greenness and duration respond to a change in their relative price?

Relative Elasticity and Exogenous Variation (IV)

- Recover **relative elasticity**: How does the relative demand for two bonds with the same greenness and duration respond to a change in their relative price?
- In practice, there are always other news
  - Shifts in demand curve ϵ: news about the assets, change in CalPERS financial health, ...

$$\Delta \mathbf{D} = \mathcal{E} \Delta \mathbf{P} + \epsilon$$

  - Might correlate with prices (e.g. Ford price up because the new F150 is amazing)
  - Classic demand estimation challenge
- Solution: **instrument** that moves prices in uncorrelated way to ϵ
  - E.g.: Fed randomly buys more Ford than GM, ...
  - IV regression: compare changes in demand and changes in prices for all such pairs

What is Missing?

Under our assumption:

1. **Cross-sectional causal inference** identifies the *relative elasticity* between assets with the same observables
2. Impossible to recover **substitution** between different observables with cross-section alone
3. **A small set of time series regressions** identifies **substitution** across observables
   - Intuition: look at portfolios based on observables, then focus on meso and macro elasticity
     - Meso: How does the demand for green bonds relative to brown bonds respond to the price of the green-minus-brown portfolio and the market portfolio?
     - Macro: How does the demand for bonds responds to the price of the market portfolio and the green-minus-brown portfolio?
   - Need simultaneous instruments over time for the price of all portfolios

Related literature

- Asset pricing using causal inference methods
  - Shleifer (1986); Harris, Gurel (1986); Chang, Hong, Liskovich (2014); Pavlova, Sikorskaya (2023); Greenwood, Sammon (2024); Gompers, Metrick (2001); Coval, Stafford (2007); Lou (2012); Ben-David, Li, Rossi, Song (2022); Hartzmark, Solomon (2022); Krishnamurthy, Vissing-Jorgensen (2011); Haddad, Moreira, Muir (2021, 2025); Selgrad (2024); Du, Tepper, Verdelhan (2018); Greenwood, Vissing-Jorgensen (2018); Haddad, Muir (2021); Chen, Chen, He, Liu, Xie (2023); ...
- Structural approach and demand systems
  - *Koijen, Yogo (2019, 2024); Koijen, Richmond, Yogo (2024)*; Haddad, Huebner, Loualiche (2024); van der Beck (2024); Lu, Wu (2023); Gabaix, Koijen (2024); Bretscher, Schmid, Sen, Sharma (2024); Jansen, Li, Schmid (2024); Fang (2023); Fang, Xiao (2024); ...
  - Li, Lin (2024); Chaudhary, Fu, Li (2023); Aghaee (2024); An, Huber (2025); Peng, Wang (2023); Fuchs, Fukuda, Neuhann (2025); ...
- **What we don't do**:
  - Strategic responses (Haddad, Huebner, Loualiche, 2024)
  - **Dynamics** (Greenwood, Hanson, Liao, 2018; An, 2024, Huebner, 2024; Gabaix, Koijen, 2024; Davis, Kargar, Li, 2025; **He, Kondor, Li, 2025**)
  - State-contingent shocks (Haddad, Moreira, Muir, 2025)
- Spillovers/substitution outside asset pricing:
  - *Berry, Levinsohn, Pakes (1995)*, Berry, Haile (2014) ...

Outline

An Assumption for Demand in Asset Pricing

The Demand Function for Assets

- Investor chooses portfolio ... taking prices are given: $D(P, ...)$
  - Similar structure with market power or learning from prices: post a demand curve
- In changes, linearize:

$$\Delta D = \underbrace{\mathcal{E}}_{\text{elasticity}} \Delta P + \underbrace{\epsilon}_{\text{shifts}}$$

  - could be logs, levels, portfolio weights, yields: flexibly chosen to give regularity to $\mathcal{E}$
  - Markowitz: $D = \frac{1}{\gamma}\Sigma^{-1}(\mu - P) \Rightarrow \mathcal{E} = \frac{1}{\gamma}\Sigma^{-1}$
- *Multipliers* or price impact: effect of demand shifts on equilibrium prices $\mathcal{M} = -\mathcal{E}_{agg}^{-1}$

Framework: what if you want to figure $\mathcal{E}$ out from data without assuming much?

An Elementary Assumption

A1. **Homogeneous substitution conditional on observables**

→ *Any pair of assets in the estimation sample $\mathcal{S}$ with the same observables shares the same cross-price elasticity with respect to any third asset, within or outside the estimation sample:*

$$\boxed{\mathcal{E}_{il} = \mathcal{E}_{jl} \text{ if } X_i = X_j} \quad \text{for all } i, j \in \mathcal{S}, \text{ and } l \neq i, j,$$

- When substituting, the investor differentiates with respect to X only

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- When substituting, the investor differentiates with respect to X only
- X_i: $K \times 1$ vector of observables for asset i (e,g. greenness, duration)
- And, add **linearity** to handle continuous observables

$$\boxed{\mathcal{E}_{il} = \mathcal{E}_{\text{cross}}(X_i, X_l) = X_i' \underbrace{\mathcal{E}_X}_{K \times K} X_l}$$

– Can apply everywhere, or just to a sample of assets $\mathcal{S}$

Regularizing a Bit More

A2. **Constant relative elasticity**

→ *Assets in the estimation sample have the same value of relative elasticity $\mathcal{E}_{relative}$ with respect to other assets with the same characteristics:*

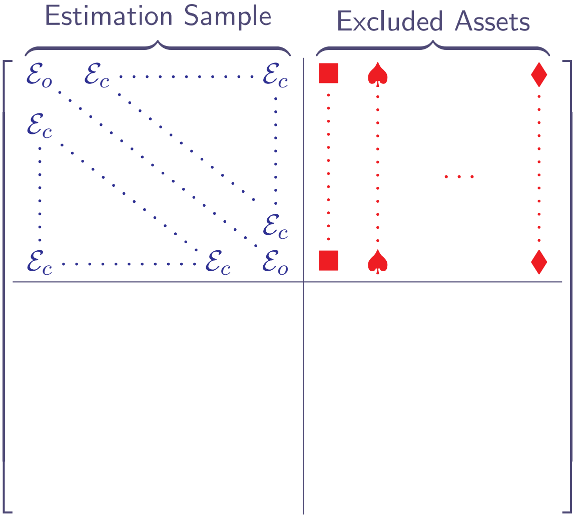
$$\boxed{\boldsymbol{\mathcal{E}}_{ii} - \boldsymbol{\mathcal{E}}_{ji} = \mathcal{E}_{relative} \text{ if } X_i = X_j} \quad \text{for all } i, j \in \mathcal{S}$$

- How does the relative demand for two assets with the same observables respond to a change in their relative price?
  - ⋆ In our example, GM 10-year bonds to the price of Ford 10-year bonds

- Similar local behavior across assets → homogeneous treatment effect
- Can relax a lot for cross-sectional results (function of characteristics, LATE)

Using the Assumptions: Local Experiments

- With few close assets: ignore observables & assume full homogeneity
  - Demand for 10-yr bonds of Ford and GM responds in same way to price of 5-year First Solar bond



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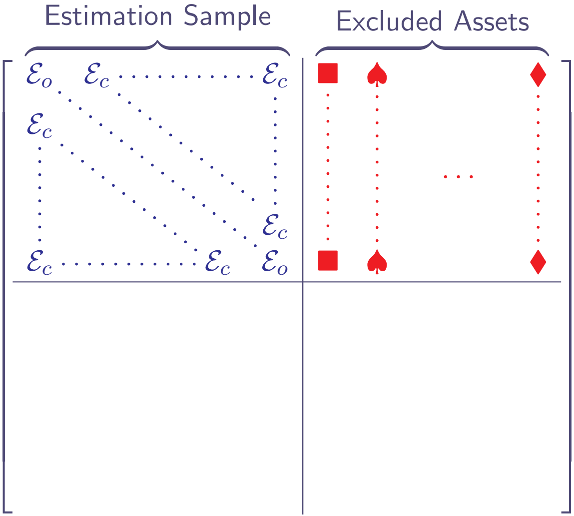

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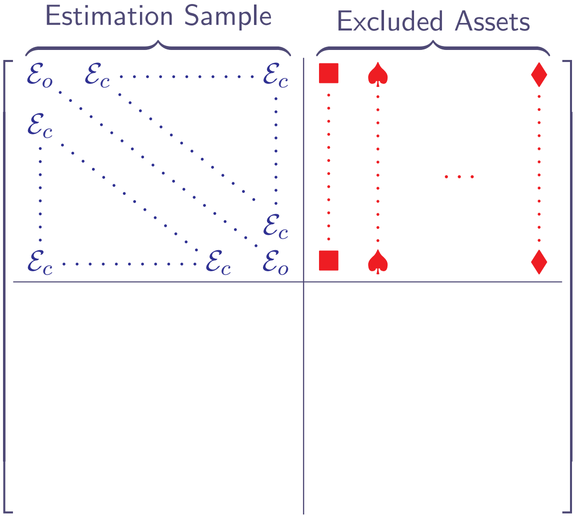
- Diagnostic: balance between treated (high Z_i) and control (low Z_i) on covariance with broad factors



Using the Assumptions: Rich Cross-Sections

Key question: What do investors consider when substituting between assets?

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This matters for what observables X_i to include

- *Broad categories*: X_i are group dummies say on durations or industries
- *Risk based motives*: care about portfolio-level factor exposure, so X_i are factor loadings or characteristics that proxy for them
- *Non-risk motives*: X_i is asset weight in this objective

$$\max_D \quad D'(M-P) - \frac{\gamma}{2} D'\Sigma D - \frac{\kappa}{2}\left(D'X^{(1)}\right)^2$$
$$\text{such that} \quad D'X^{(2)} \leq \Theta$$

  - Binding constraints (e.g. leverage)
  - Manages a regulatory score (e.g. capital ratio,...)
  - Stakeholders pressure (e.g. greenness, ...)

Cross-Sectional Causal Inference

Cross-Sectional Identification

- Data-Generating-Process: Elasticity matrix $\mathcal{E}$ + CalPERS cares about greenness (X)

$$\Delta \mathbf{D} = \boldsymbol{\mathcal{E}} \Delta \mathbf{P} + \epsilon$$

- Run an IV regression, with Z_i instrument for prices ($Z_i \perp \epsilon_i | X_i$)

$$\Delta D_i = \widehat{\mathcal{E}} \Delta P_i + \theta' X_i + e_i$$
$$\Delta P_i = \lambda Z_i + \eta' X_i + u_i$$

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- **Proposition 1.** Under A1 & A2, and the usual exclusion and relevance restrictions, the IV estimator identifies the **relative elasticity** $\widehat{\mathcal{E}} = \mathcal{E}_{relative} = \boldsymbol{\mathcal{E}}_{ii} - \boldsymbol{\mathcal{E}}_{ji}$ for $X_i = X_j$

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- In literature, some researchers switch the sides of P-D in regression, and find IV for ΔD
  - Commonly used in empirical asset pricing (exploiting fund flows)
  - Under A1 & A2 the identified coefficient is $-1/\widehat{\mathcal{E}}$

Getting Rid of Substitution

- Key step: coefficient on observables θ absorbs substitution from other assets

$$
\begin{aligned}
\Delta D_i &= \mathcal{E}_{ii} \Delta P_i + \sum_{j \neq i} X_i' \mathcal{E}_X X_j \Delta P_j + \epsilon_i \\
&= \left(\mathcal{E}_{ii} - X_i' \mathcal{E}_X X_i \right) \Delta P_i + \sum_j X_i' \mathcal{E}_X X_j \Delta P_j + \epsilon_i \\
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Absorbing substitution $\neq$ **Estimating substitution**

- In cross-section, some aggregate variables do not change across assets
- "Missing intercept" (here, **coefficient** actually) problem: don't know how θ would change when prices change

What About Equilibrium Price Adjustment?

In the data, there are always changes in price

- Just like in our model so there is no notion of "separately moving each price"
  - If the Fed doesn't buy a bond, its price might move as Fed bought its substitutes
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Two important issues for applying IV methodology in our framework

- Equilibrium price adjustment per se is not an issue for exogeneity
  - **Exclusion restriction**: $Z_i \perp \epsilon_i | X_i$, that is, "Fed buying a bond or not" uncorrelated to *demand shifts* of CalPERS
  - In a dynamic setting, this could fail (Haddad, Moreira, Muir 2024; He Kondor, Li 2025)
- If equilibrium is such that the two prices cannot deviate *at all* from each other (so law of one price holds strictly), **Relevance** condition might fail
  - You can assess this empirically!

What About Logit?

- Koijen Yogo (2019), Koijen Richmond Yogo (2024):
  - $\exists$ factor models where volatility and expected returns depend non-linearly of prices which yield asset demand in the logit form
  - Logit has non-zero substitution and can be estimated from the cross-section alone

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  - Logit: when the price of any bond increases, CalPERS replaces it proportionally to its existing portfolio
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- All those models (with proper units) satisfy our assumptions and hence can have relative elasticity estimated from the cross-section
  - Imposing logit structure ($\mathcal{E}_{ij} \propto D_i \cdot D_j$) allows researchers back out substitution ▸ LogitvsOurs
  - To the extreme, if assuming no substitution, cross-section is enough for $\mathcal{E}$

Estimating Substitution with the Time Series

Substitution and Its Estimation

More interesting questions about how portfolio responds to prices:

- Will CalPERS maintain its green tilt if the price of green bonds become very expensive relative to red bonds? How much to size down?
- Answer to these questions relies on knowing substitution (across red and green assets)!

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High-level idea:

- $\mathcal{E} = \hat{\mathcal{E}} \boldsymbol{I}_N + X' \mathcal{E}_X X$ so we need to estimate $\mathcal{E}_X$ which is $K \times K$
- Recall that

$$\Delta D_i = \underbrace{\left(\mathcal{E}_{ii} - X_i' \mathcal{E}_X X_i\right)}_{\text{relative elasticity, } \hat{\mathcal{E}}} \Delta P_i + X_i' \underbrace{\sum_j \boldsymbol{\mathcal{E}_X} X_j \Delta P_j}_{\text{constant across assets but time-varying}} + \epsilon_i$$

- To estimate $\mathcal{E}_X$, construct X-based portfolio $X_j \Delta P_j$ and finding IV for it

Simplifying Substitution

Under A1 & A2, replace the asset-level problem of substitution with a portfolio-level problem:

- Say X_i (which is normalized) captures greenness
- Re-package prices and demand:

$$\Delta P_{agg} = \frac{1}{N}\sum_i \Delta P_i, \qquad \Delta D_{agg} = \frac{1}{N}\sum_i \Delta D_i$$

$$\Delta P_X = \frac{1}{N}\sum_i X_i \Delta P_i \qquad \Delta D_X = \frac{1}{N}\sum_i X_i \Delta D_i$$

$$\Delta P_{idio,i} = \Delta P_i - \Delta P_{agg} - X_i \Delta P_X \qquad \Delta D_{idio,i} = \Delta D_i - \Delta D_{agg} - X_i \Delta D_X$$

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- *Decompose the response of demand to prices into three univariate components:*

Relative: $\Delta D_{idio,i} = \widehat{\mathcal{E}} \Delta P_{idio,i}$

Meso: $\Delta D_X = \tilde{\mathcal{E}}_{agg} \Delta P_{agg} + \tilde{\mathcal{E}}_X \Delta P_X$

Macro: $\Delta D_{agg} = \bar{\mathcal{E}}_{agg} \Delta P_{agg} + \bar{\mathcal{E}}_X \Delta P_X$

Estimating the Meso and Macro Elasticities

$$\textbf{Meso:} \qquad \Delta D_X = \tilde{\mathcal{E}}_{agg} \Delta P_{agg} + \tilde{\mathcal{E}}_X \Delta P_X$$

$$\textbf{Macro:} \qquad \Delta D_{agg} = \bar{\mathcal{E}}_{agg} \Delta P_{agg} + \bar{\mathcal{E}}_X \Delta P_X$$

- Substitution boils down to relation between aggregate and observable based portfolios
  - Response of overall demand and green portfolio tilt to aggregate bond price and price of green-minus-brown portfolio (but low dimensional)
- Need joint instruments for prices **in time series**:
  - To estimate macro elasticity, need to account for simultaneous change in price of green-minus-brown
  - Only controlling for portfolio prices is generally a bad control (in particular if demand shocks are correlated)

(Attempt) of Estimating Multipliers: an Empirical Example

Example: Corporate Bond Relative Multiplier

- U.S. investment-grade corporate bonds (following Chaudhary Fu Li, 2024)
- Steps to go through when conducting causal inference in asset pricing:
  1. choose a source of variation
  2. assess exogeneity
  3. assess assumptions A1 and A2 and select observables + units
  4. implement the regression analysis
- Step 1: flow-induced demand shock Z_{it}: fund flow in mutual funds $\times$ portfolio composition (Coval Stafford 2007, Lou 2012)
- Step 2: assess exogeneity, i.e., $Z_{it} \perp \epsilon_{it} | X_{it}$
  – example threat to identification: another investor type (e.g., insurance companies) buying the same bonds held by mutual funds that receive a lot of inflows

Step 3: Diagnostic for A1 – Balance on covariances

Do treated bonds comove the same way with broad portfolios as the control bonds?

1. At each date t, form a long-short portfolio based on whether Z_{it} is above ("treated") or below ("control") the median
2. Compute the β of the long-short return on broad indices in a window around t (here: 2y)
3. β different from zero $\Rightarrow$ substitution likely not homogeneous

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  $\rightarrow$ natural if investors choose their flows along dimensions like duration and credit risk
- Do the treated and control comove the same way *conditional on observables*?
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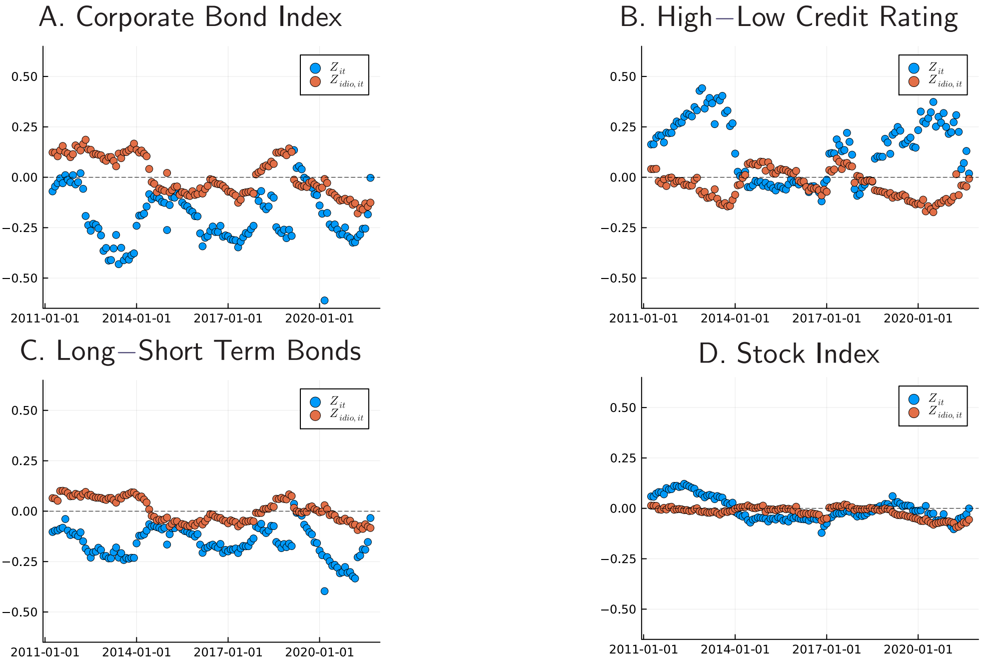
- Alternative unit to bond returns: yield changes A1 yield changes | Multiplier yield changes
- Similar diagnostic for A2: balance on idiosyncratic volatility A2 diagnostic



A. Corporate Bond Index
B. High−Low Credit Rating
C. Long−Short Term Bonds
D. Stock Index
Z_{it}
$Z_{idio,it}$
0.50
0.25
0.00
−0.25
−0.50
2011-01-01
2014-01-01
2017-01-01
2020-01-01

Step 4: Implement the regression

Relative multiplier $\widehat{\mathcal{M}} \approx 0$

	Return $\Delta P_{it}/P_{i,t-1}$				
	(1)	(2)	(3)	(4)	(5)
Demand shock:					
Z_{it}	1.541*	-0.254	0.019		
	(0.637)	(0.229)	(0.065)		
$Z_{idio,it}$				0.019	0.019
				(0.065)	(0.065)
Date Fixed Effects		Yes	Yes	Yes	Yes
Duration $\times$ Date Fixed Effects			Yes	Yes	
Credit Rating $\times$ Date Fixed Effects			Yes	Yes	
N	646,335	646,335	646,335	646,335	646,335
R^2	0.010	0.415	0.632	0.632	0.415

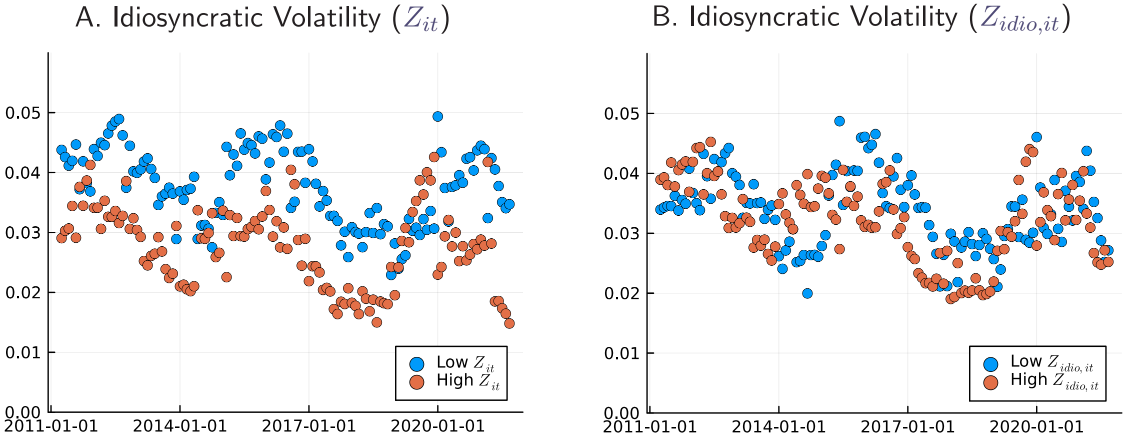
Extra Step: Meso- And Macro Multipliers

	Return $\Delta P_{agg,t}/P_{agg,t-1}$		Return $\Delta P_{X,t}/P_{X,t-1}$	Return $\Delta P_{it}/P_{i,t-1}$	
	(1)	(2)	(3)	(4)	(5)
$Z_{agg,t}$	14.231***	12.347**	7.294**	12.347**	12.347**
	(3.643)	(3.985)	(2.423)	(3.959)	(3.958)
$Z_{X,t}$		-6.170	0.817	-6.170	-6.170
		(7.810)	(4.591)	(7.757)	(7.757)
$Z_{agg,t} \times X_{it}$					7.294**
					(2.407)
$Z_{X,t} \times X_{it}$					0.817
					(4.558)
$Z_{idio,it}$				0.090	0.090
				(0.055)	(0.054)
Duration X_{it}				0.001	-0.001
				(0.001)	(0.001)
N	150	150	150	646,335	646,335
R^2	0.242	0.250	0.135	0.101	0.125

Conclusion

- Key challenge for causal inference in asset pricing: substitution across assets
- **An elementary condition for valid inference**: homogenous substitution conditional on observables
  - difference in substitution driven by a known set of observables
- **Standard cross-sectional causal inference** identifies relative elasticity or its inverse, relative multiplier
  - Guidance on designing settings such that assumptions are plausible
  - Compatible with usual covariance matrix assumptions
- **Time series identification with observable-based portfolios** reveals substitution
  - Need to consider all dimensions of substitution jointly

Diagnostics for A2 – Balance on idiosyncratic volatility



- Average idiosyncratic volatility among treated versus control bonds

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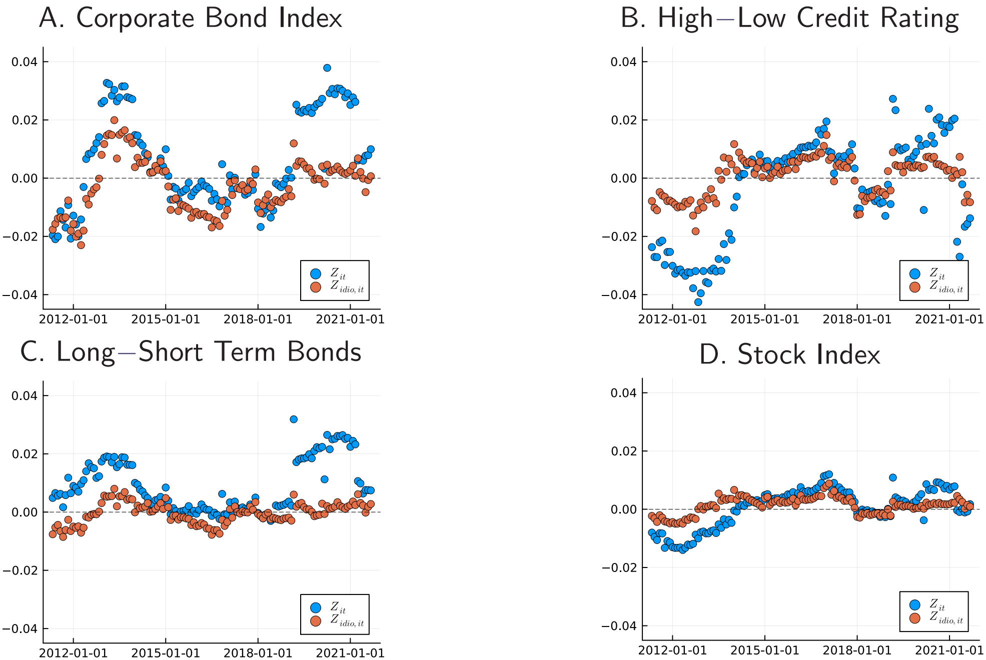
▸ Back



A. Corporate Bond Index
B. High—Low Credit Rating
C. Long—Short Term Bonds
D. Stock Index
0.04
0.02
0.00
−0.02
−0.04
2012-01-01
2015-01-01
2018-01-01
2021-01-01
Z_{it}
$Z_{idio,it}$

▸ Back

Relative multiplier $\widehat{\mathcal{M}} = -0.072$

	Yield change ΔY_{it}				
	(1)	(2)	(3)	(4)	(5)
Demand shock:					
Z_{it}	-0.384*	-0.104*	-0.072**		
	(0.166)	(0.047)	(0.027)		
$Z_{idio,it}$				-0.072**	-0.072**
				(0.027)	(0.027)
Date Fixed Effects		Yes	Yes	Yes	Yes
Duration $\times$ Date Fixed Effects			Yes	Yes	
Credit Rating $\times$ Date Fixed Effects			Yes	Yes	
N	630,255	630,255	630,255	630,255	630,255
R^2	0.004	0.071	0.089	0.089	0.070

Asset Demand System with Logit Form vs Our Paper

- In Koijen Yogo 2019, Mean-variance optimization + factor structure + **certain assumptions** (especially on the share of outside asset) $\Rightarrow$

$$D_i = w\left(p_i, x_i, \boldsymbol{p}, \boldsymbol{x}\right) = \frac{\exp\left(-\alpha p_i + \beta' x_i\right)}{1 + \sum_l \exp\left(-\alpha p_l + \beta' x_l\right)}$$

- The key substitution pattern of logit system is

$$\mathcal{E}_{ij} = \frac{dD_i}{dp_j} = \frac{\exp\left(-\alpha p_i + \beta' x_i\right) \cdot \alpha \exp\left(-\alpha p_j + \beta' x_j\right)}{\left(1 + \sum_l \exp\left(-\alpha p_l + \beta' x_l\right)\right)^2} = \alpha D_i D_j$$

- Implied substitution matrix $\mathcal{E}_{sub} = \alpha D D'$, with Rank 1
  - Any non-linear transformation of D will not affect the rank of this matrix
- In our setting, this substitution matrix has a rank K

$$\mathcal{E}_{sub} = \left[X_i' \mathcal{E}_X X_j\right] = \underbrace{X'}_{N\times K} \underbrace{\mathcal{E}_X}_{K\times K} \underbrace{X}_{K\times N}$$

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