

# A Portfolio Approach to Global Imbalances\*

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## Abstract

Recent decades are characterized by large imbalances in the U.S. net foreign asset (NFA) position. Adopting a portfolio-based approach, we estimate demand curves for international debt and equity markets at the country level, and use the estimated demand system to quantitatively attribute the long-run deterioration of the U.S. NFA position to three forces: the global savings glut, changes in official holdings, and changes in investor demand. We find that standard narratives of the global savings glut and change official holdings contribute to the global imbalances by decreasing the U.S. NFA position from  $-10\%$  to  $-94\%$  of 2002 GDP, whereas investor's demand shifts reverse this trend by  $62\%$  of 2002 GDP. Furthermore, we quantify how changes in U.S. NFA position were driven by different forces across asset classes and between assets versus liabilities. Our decomposition shows that underlying a seemingly simple trend in global imbalances lie opposing forces related to investor savings behavior, official holdings, and investor demand shifts.

*Key Words:* Global Imbalances, Low Interest Rates, Global Savings Glut, Quantitative Easing.

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# 1 Introduction

Foreign holdings of the U.S. assets have increased dramatically in the past two decades. At the end of 2016, non-U.S. private sector investors held 11.3 trillion dollars' worth of U.S. assets, which accounted for 48% of the wealth in their external portfolio. Non-U.S. central banks held another 2.5 trillion dollars' worth of U.S. long-term debt, accounting for 54% of the long-term debt in their foreign currency reserves. By contrast, U.S. investors held only 6.4 trillion dollars in foreign assets<sup>1</sup>. The sustained net capital flows into the U.S. financial markets have been referred to as the *global imbalances* (Lane and Milesi-Ferretti 2007a; Gourinchas and Rey 2014), which have led to the U.S. net foreign asset position decreasing from  $-22$  percentage points of GDP in 2002 to  $-43$  percentage points of GDP by the end of 2016.

In addition to these asymmetric capital flows into and out of the U.S., the U.S. and foreign countries' external portfolios have very different compositions. As shown in Figure 1, the U.S. holdings of foreign equity exceeds the foreign holdings of the U.S. equity, resulting in a positive net equity imbalance in the U.S. On the other hand, the foreign holdings of the U.S. debt far exceeds the U.S. holdings of foreign debt, resulting in a negative net debt imbalance in the U.S. This asymmetry in portfolio allocations and hence risk exposures are known as the *exorbitant privilege* (Gourinchas et al. 2010), which contributes to further divergence in the wealth accumulation between the U.S. and foreign countries.

The literature has provided several explanations for these global imbalances and asymmetries. One view, known as the global savings glut, argues that foreign savings in excess of their domestic investment opportunities contribute to the flows into the U.S. debt market (Bernanke 2005). One source of the savings glut is the strong saving motive of developed countries with aging populations and higher inequality (Rachel and Smith 2018; Mian et al. 2020). Another source is the emerging countries with economic growth and willingness to save in the developed markets and in particular in the U.S.

In addition to the private savings glut, both foreign central banks and the Federal Reserve have been increasing their holdings of the U.S. debt in their official reserves (Bernanke 2005;

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<sup>1</sup>These numbers are from the U.S. Bureau of Economic Analysis.

Farhi et al. 2011) and via quantitative easing. The official holdings in foreign countries also drive a large portion of capital flows.

A third view focuses on the shortage of safe assets and flight to safety (Krishnamurthy and Vissing-Jorgensen (2012); Caballero et al. (2017); Maggiori (2017); Jiang et al. (2020b)). As the supply of safe assets is concentrated in a small number of advanced economies, most prominently the U.S. Recent crises (the Global Financial Crisis, the European Debt Crisis, and the recent Covid Crisis) drive up demand for safe assets and hence flows into the U.S. debt market.

In this paper, we use a portfolio-based approach to empirically quantify the effects of these channels on the global imbalances. To begin, we construct bilateral equity and debt portfolio positions between 43 investor countries and 36 issuer countries. We also measure the portfolio holdings of the U.S. Federal Reserve and foreign central bank reserves. We model and estimate investor countries' demand for assets as a function of observed and unobserved asset characteristics, and allow for substitutions across countries and across asset classes (Kojen and Yogo 2019a,b; Kojen et al. 2019). In equilibrium, investor countries and central banks must hold the total quantity of assets available for purchase. The estimated demand curves of investor countries, along with the portfolios of central banks and the market clearing condition constitute our asset demand system.

This asset demand system allows us to evaluate how asset prices and investor portfolio holdings change in response to a set of variables that we take as exogenous: investors' net savings, asset issuances, central banks' holdings, investors demand for assets with specific characteristics, and investors' latent demand for each asset that is not described by the observed asset characteristics. On the other hand, asset prices, exchange rates, investors' wealth, and investors' portfolio allocations are endogenous. Had all of these exogenous variables remained constant from the start of our sample period to the end, the U.S. net foreign asset positions would have stayed the same. Hence, all changes in portfolio positions observed in the data can be attributed to these inputs to the demand system.

We start with the baseline case in which all exogenous variables remain constant throughout our sample period, and then sequentially introduce each component and quantify how much the U.S. net foreign asset position changes between 2002 and 2016. First, to capture

the effect of the *savings glut*, which represents savings in excess of domestic investment opportunities, we restore the actual time series of investors' net savings and asset issuances. Second, on top of the first counterfactual, we restore central banks' *official holdings* to capture the effect of central bank purchases in different countries. Third, we restore the actual time series of the investors' *demand shifts* to capture the effect of changes in the relative desirabilities of the assets. After we complete all three steps, by construction, the equilibrium asset prices and portfolio allocations are also restored to the actual values in the data.

We report three key findings. First, we find that the savings glut contributes to the widening of the U.S. external debt and equity imbalances. The savings glut channel alone raises the U.S. external portfolio imbalance from  $-9\%$  in 2002 to  $-76\%$  in 2016, in the unit of 2002 U.S. GDP. We further differentiate the contributions from the developed markets and the emerging markets, and find that the imbalance is predominantly driven by the private savings from the developed markets.

Second, official holdings also contribute to the widening of the U.S. external portfolio imbalances, leading to another 17% decline in the U.S. external portfolio imbalance from 2002 to 2016. One interesting finding is that the central bank purchases in the debt markets also spill over to the equity markets, as they raise debt valuation and encourage investors to substitute from debt to equity. As a result, both the U.S. and foreign investors also hold more equity across borders.

Third, and most surprisingly, the investors' demand shifts partially offset these trends, driving a reversal in the U.S. external portfolio imbalances from  $-94\%$  to  $-32\%$  in the unit of 2002 U.S. GDP. We do not directly observe this effect because it is overshadowed by the effects of savings glut and official holdings. So, when we combine all these channels, the U.S. external portfolio imbalances deteriorate considerably, but the deterioration would have been much greater if the investors' demand had stayed constant. Further investigation shows that the investors' demand is characterized by a market-wide shift from debt assets to equity assets, which impacts the U.S. external imbalances because the U.S. debt is widely held by foreigners.

In summary, we use a portfolio approach to evaluate the impact of savings glut, official holdings, and investor demand shifts on the dynamics of the U.S. external portfolio imbal-

ance. We find that savings glut and official holdings contribute to the external portfolio imbalances, investor demand shifts partially offset this trend. These results suggest that any theory that seeks to explain the decline in the U.S. debt imbalance with just one channel alone are unlikely to succeed quantitatively.

**Literature Review.** Our paper contributes to a large empirical literature studying the drivers of net foreign asset dynamics and the composition of global portfolios (Lane and Milesi-Ferretti 2007b; Gourinchas and Rey 2007; Curcuru et al. 2008; Froot and Ramadorai 2005; Maggiori et al. 2019; Coppola et al. 2020). Drivers of capital flows long and-term yields in the previous literature includes institutional quality (Alfaro et al. 2008), demographic factors (Lane and Milesi-Ferretti 2001; Carvalho et al. 2016), financial development (Caballero et al. 2008), information and transaction cost (Portes et al. 2001; Portes and Rey 2005), abilities to insure against idiosyncratic risk (Mendoza et al. 2009; Angeletos and Panousi 2011), and interactions between financial frictions and international trade (Antras and Caballero 2009). Our paper focuses on the imbalances in the U.S. net foreign asset position and the secular decline in long-term interest rates (Caballero et al. 2008; Gourinchas et al. 2010; Gourinchas and Rey 2014). We take a finance perspective from the portfolio allocation to analyze these phenomena, taking into account both the country-level characteristics as well as investor substitution between assets within each asset class.

The methodology we use in this paper builds upon an empirical literature that explicitly measures asset demand elasticities to understand changes in asset prices (Krishnamurthy and Vissing-Jorgensen 2012; Koijen and Yogo 2019b; Koijen et al. 2019, 2020).<sup>2</sup> Closely related to our work is Koijen and Yogo (2019b), which develops a demand system for international financial assets and provides a variance decomposition of exchange rates and asset prices globally. However, rather than focus on asset prices, our paper uses the demand system approach to study the drivers of portfolio holdings and the U.S. external imbalance. Our paper also relates to Gabaix and Koijen (2020) who show how flows into and out of asset markets can have substantial price impact when demand is inelastic.

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<sup>2</sup>A related literature on the asset price dynamics in the bond market also adopts a quantity-centric view (Vayanos and Vila 2009; Greenwood et al. 2010; Greenwood and Vayanos 2014; Malkhozov et al. 2016; Greenwood et al. 2019).

Our paper is also related to studies of the effects of central banks' unconventional monetary policy (Krishnamurthy et al. 2013; Kojen et al. 2017; Krishnamurthy et al. 2018; Acharya and Krishnamurthy 2018; Jiang et al. 2020a). We provide a structural estimate of its effects on international asset prices and capital flows.

This paper proceeds as follows: Section 2 provides a theoretical framework for estimating asset demand and decomposing changes in asset prices and portfolio positions. Section 3 reports data sources and summary statistics, discusses the estimation procedure, and presents the estimation results. Section 4 presents the results from the counterfactual exercises. Section 5 concludes.

## 2 Model

In this section, we present a structural model of international asset markets. Our specification of demand curves largely follows Kojen and Yogo (2019a), while the wealth dynamics and market clearing conditions are specific to our setting in order to study the long-run dynamics of portfolios and wealth.

Time is discrete and there are  $N$  countries in the world which issue assets. There are  $I$  investor countries which invest in the assets. Each investor country  $i = 1, \dots, I$  contains a representative investor who allocates her asset under management (AUM),  $A_{i,t}$ , across the asset space. Each investor country also contains a central bank that holds a portfolio of assets as reserves.

There are three asset classes indexed by  $\ell$ : short-term debt ( $\ell = 1$ ), long-term debt ( $\ell = 2$ ), and equity ( $\ell = 3$ ). Each asset class is comprised of  $N + 1$  assets indexed by  $j$  — one for each country and an additional “outside” asset indexed by  $j = 0$ . The outside assets allows the investors to allocate a portion of their AUM outside of the country specific assets.

The market-to-book ratio in local currency units at time  $t$  for asset class  $\ell$  in country  $n$  is given by  $P_t(n, \ell)$ . We denote the quantity supplied of asset  $n$  in asset class  $\ell$  by  $Q_t(n, \ell)$ . We assume  $Q_t(n, \ell)$  is given exogenously. The nominal exchange rate in US dollars per unit of country  $n$ 's currency is denoted  $E_t(n)$ . An increase in  $E_t(n)$  implies an appreciation of country  $n$ 's currency relative to the USD. The consumer price index of country  $n$  relative to

the US is denoted  $Z_t(n)$ . As a result, the real exchange rate is given by  $E_t(n)/Z_t(n)$ . We use lower case letters to denote logs of all of these variables denoted by capital letters. For example  $p_t(n, \ell) = \log(P_t(n, \ell))$ .

**Demand Curves.** The portfolio weight of investor  $i$  in country  $n$  and asset class  $\ell$  is

$$w_{i,t}(n, \ell) = w_{i,t}(n|\ell)w_{i,t}(\ell), \quad (1)$$

where  $w_{i,t}(n|\ell)$  is the portfolio weight on country  $n$  within asset class  $\ell$  and  $w_{i,t}(\ell)$  is the total portfolio weight on asset class  $\ell$ .

The portfolio weight for investor  $i$  at time  $t$  in country  $n$  and asset class  $\ell$  is a logistic function:

$$w_{i,t}(n|\ell) = \frac{\delta_{i,t}(n, \ell)}{1 + \sum_{k=0}^N \delta_{i,t}(k, \ell)}, \quad (2)$$

where

$$\delta_{i,t}(n, \ell) = \exp(\beta_\ell \mu_{i,t}(n, \ell) + \boldsymbol{\theta}'_\ell \mathbf{x}_{i,t}(n) + \kappa_{i,t}(n, \ell)), \quad (3)$$

with  $\mu_{i,t}(n, \ell)$  denoting the expected return at time  $t$  for country  $i$ 's investor in country  $n$ 's asset of class  $\ell$ <sup>3</sup>. To measure expected returns, we use the combination of market-to-book ratios and exchange rates that best predicts future returns — the details of this regression are in the following section. The asset characteristics  $\mathbf{x}_{i,j,t}$  can be asset-specific or bilateral in nature.  $\kappa_{i,j,t}$  is referred to as latent demand, which describes additional variation in the demand curve that is not captured by the price or observed asset characteristics.

By construction, the total sum of shares invested into each asset equals 1,  $\sum_{n=0}^N w_{i,t}(n|\ell) = 1$ . As a result, the portfolio weight in the outset asset in asset class  $\ell$  is given by

$$w_{i,t}(0|\ell) = \frac{1}{1 + \sum_{k=0}^N \delta_{i,t}(k, \ell)}. \quad (4)$$

To allow for substitution across asset classes, the asset class portfolio weight is specified

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<sup>3</sup>Koijen and Yogo (2019a) show that demand curves of this form can be derived as approximation of the portfolio allocation model in Merton (1973).

as a nested logit. The portfolio weight for investor  $i$  at time  $t$  in asset class  $\ell$  is given by

$$w_{i,t}(\ell) = \frac{(1 + \sum_{k=0}^N \delta_{i,t}(n, \ell))^{\lambda_\ell} \exp(\alpha_\ell + \xi_{i,t}(\ell))}{\sum_{m=1}^3 (1 + \sum_{k=0}^N \delta_{i,t}(k, m))^{\lambda_m} \exp(\alpha_m + \xi_{i,t}(m))}, \quad (5)$$

where  $\alpha_\ell$  are asset class fixed effects and  $\xi_{i,t}(\ell)$  are asset class latent demand. The terms  $(1 + \sum_{k=0}^N \delta_{i,t}(n, \ell))$  are referred to as inclusive values for a given asset class  $\ell$ . The inclusive value captures the relative attractiveness of each of the asset classes. For example, when average relative prices of assets within an asset class change, so too will the total allocations to these asset classes<sup>4</sup>.

**Expected Returns.** Investors care about expected returns in their own currency when forming their portfolios. To measure how expected returns relate to exchange rates and prices, we use a forecasting regression for excess returns as in [Kojien and Yogo \(2019b\)](#):

$$r_{t+1}(n, \ell) - y_t(US) = \phi_\ell p_t(n, \ell) + \psi_\ell(e_t(n) - z_t(n)) + \chi_{n,\ell} + \nu_{t+1}(n, \ell), \quad (6)$$

where  $r_{t+1}(n, \ell)$  is the continuously compounded return in USD on asset class  $\ell$  in country  $n$  from time  $t$  to  $t+1$  and  $y_t(n)$  is the continuously compounded yield on country  $n$ 's short-term debt.

The expected excess return on asset  $n$  in investor  $i$ 's currency is then given by

$$\begin{aligned} \mu_{i,t}(n, \ell) &= E_t[r_{t+1}(n, \ell) - \Delta e_{t+1}(i) - y_t(i)] = E_t[r_{t+1}(n, \ell) - r_{t+1}(i, 1)] \\ &= \phi_\ell p_t(n, \ell) + \psi_\ell(e_{k,t} - z_t(n)) + \chi_{n,\ell} - \phi_1 p_t(n, 1) - \psi_1(e_{k,t} - z_t(n)) - \chi_{n,1}. \end{aligned}$$

The first equality is simply the definition of  $\mu_{i,t}(n, \ell)$ . The second equality says that the expected excess return investor  $i$ 's currency is equal to expected excess return on asset  $n$  in USD, minus the expected excess return of investor  $i$ 's short term-debt in USD. The third equality immediately follows from Equation 6.

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<sup>4</sup>See [Kojien and Yogo \(2019b\)](#) for further discussion of this specification and for some examples of special cases



**Central Banks.** In this paper, we take asset purchases and sales by central banks as exogenous. Let  $B_{i,t}(n, \ell)$  denote the quantity of country  $n$  assets held in the portfolio of country  $i$ 's central bank. In practice, central banks can hold assets for several reasons. For example, central banks may hold foreign long-term debt as currency reserves, which can be used to buffer the exchange rate movement of domestic currency. These holdings are denoted as  $B_{i,t}(n, \ell)$  where the investor central bank  $i$  is different from the issuer country  $n$ .

In recent years, central banks have also purchased their domestic assets in attempts to lower long-term interest rates. By reducing long-term interest rates, the central bank further stimulate the economy even when the short-term interest rate reaches zero. These holdings are denoted as  $B_{n,t}(n, \ell)$  where the holding country and the issuer country coincides.

**AUM Dynamics.** In order to study variation in portfolio positions and wealth across countries, it is important in our setting to have realistic dynamics for AUM. The AUM for investor  $i$  evolves according to the following equation:

$$A_{i,t} = A_{i,t-1} \sum_{\ell=1}^3 \sum_{k=0}^N w_{i,t-1}(\ell) w_{i,t-1}(k|\ell) (1 + R_t(k, \ell)) + F_{i,t} \quad (7)$$

where  $R_t(k, \ell)$  is the return at time  $t$  on asset  $k$  in asset class  $\ell$  and  $F_{i,t}$  is the flow for investor  $i$  at time  $t$ . The flow captures any increase or decrease in assets under management that is not associated with the portfolio returns. The return is calculated to account for dividends, capital gains, and issuances:

$$R_t(k, \ell) = \frac{P_t(k, \ell) S_t(k, \ell) + D_t(k, \ell)}{P_{t-1}(k, \ell) S_{t-1}(k, \ell)}, \quad (8)$$

where  $D_t(k, \ell)$  are dividends per share, and  $S_t(k, \ell)$  is the conversion factor between book value and share number. Recall that  $P_t(k, \ell)$  denotes the price-to-book ratio. So,  $P_t(k, \ell) S_t(k, \ell)$  is the price per share. For bonds, the book value is the par value, and hence the conversion factor  $S_t(k, \ell)$  is always 1.

**Market Clearing.** At each date  $t$ , total value of asset  $j$  available for purchase must be held by the investors and the central banks:

$$P_t(n, \ell) S_t(n, \ell) E_t(n) Q_t(n, \ell) = \sum_{i=1}^I A_{i,t} w_{i,t}(\ell) w_{i,t}(n|\ell) + P_t(n, \ell) S_t(n, \ell) E_t(n) \sum_{i=1}^I B_{i,t}(n, \ell). \quad (9)$$

The left-hand side is the total market value of asset  $n$ , and the right-hand side is the sum of the dollar value of investors' portfolio holdings of asset  $n$ ,  $A_{i,t} w_{i,t}(n, \ell)$ , plus the sum of the dollar value of central banks' reserve holdings,  $P_t(n, \ell) B_{i,t}(n, \ell)$ . As shown above, portfolio weights are a function of asset prices and exchange rates. There are 3 asset class with  $N$  assets each, which leads to  $3N$  market clearing conditions. Taking short-term bond prices as given, there are  $N$  long-term bond prices,  $N$  equity prices, and  $N - 1$  exchange rates with respect to the USD. Following [Koijen and Yogo \(2019b\)](#) we assume that the Federal Reserve adjusts the supply of US short term debt to clear markets. This assumption leads to an exactly determined system in the  $N$  long-term bond prices,  $N$  equity prices, and  $N - 1$  exchange rates. We use this system in the following section to study how various components which we take as exogenous have driven variation in global imbalances.

## 3 Model Estimation

### 3.1 Data Sources

In this section, we briefly describe the data we use to decompose international capital flows and prices. We rely on two types of data: (1) cross-country holdings data and (2) the characteristics of the assets being held.

We observe cross-country asset holdings data from the Coordinated Portfolio Investment Survey (CPIS) provided by the IMF. For each country  $i$ , we observe year-end holdings of foreign financial assets in US dollars by asset class and issuer country. The asset holders included in the CPIS include government entities, corporations, and individuals. The asset classes comprise short-term debt, long-term debt and equity. We also observe central bank holdings of foreign exchange reserves by asset class through the SEFER survey. However,

central bank holdings are aggregated across holding countries for confidentiality, therefore we supplement the SEFER data with data on the US Federal Reserve Balance sheet. Hence, we treat all central banks other than the US Federal Reserve as a single investor unit.

The CPIS portfolio holdings are reported on a residency basis. As a result, portfolio flows to and from offshore financial centers can present a highly distorted view of capital flows, because these portfolio allocations are not associated with the investment decisions of their ultimate investor or issuer country (Coppola et al. 2020). In particular, Coppola et al. (2020) point out that investment by countries in the European Monetary Union are often funneled through Luxembourg, and separating this investment back to individual countries is impossible.<sup>5</sup> In order to mitigate these problems, we use the CPIS reallocation matrices provided by Coppola et al. (2020) to reattribute portfolio holdings to their investor nationality, as much as possible. We also aggregate all investment holdings by Euro Area countries into a single European Monetary Union investor.

The CPIS also does not record domestic holdings of financial assets, which we need in order to understand how investors in all asset classes substitute between domestic and international investment. Hence, we estimate domestic portfolio holding data by subtracting foreign asset holdings from total market capitalization data. We observe country-level stock market capitalization data from the World Bank, and we observe the aggregate value of outstanding short-term and long-term debt securities from the BIS.

In addition to the holdings data, we construct a panel of characteristics. We choose a set of characteristics that investors could potentially use to proxy for expected returns. These characteristics include asset-level characteristics such as the total market capitalization of equity, the book value of equity, the yields on short-term and long-term debt, and the returns from investing in each asset. We use yields on 3-month government debt to capture the yield on short-term debt, and we use the yield on 10-year government debt to capture the yield on long-term debt. We also observe country-level characteristics that may affect the risk profile for all assets in a country. These country-level characteristics include proxies for country size (GDP, GDP per capita), trade network centrality (Richmond 2016), and sovereign default risk. Finally, we include a standard set of macroeconomic characteristics:

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<sup>5</sup>As a result, in the raw CPIS data, Luxembourg is in the top 10 investor for all asset classes.

the real dollar exchange rate, inflation, bilateral export share, bilateral import share and the distance between countries.

Ultimately, our sample comprises asset issuances by 36 issuer countries and the asset holdings of 43 total investor countries. Table A.1 provides the list of countries. The 43 investor countries include the 36 issuer countries plus an additional 7 countries for which we observe holdings data, but do not observe all asset characteristics. We also observe the portfolio position of aggregated foreign reserves, and the U.S. Federal Reserve. The sample period ranges from 2002 to 2016.

### 3.2 Demand Estimation and Identification

We now turn to the estimation of the within asset class and cross-asset class demand. Our estimation procedure follows Koijen and Yogo (2019b). Dividing equation (2) by equation (4) gives:

$$\log \left( \frac{w_{i,t}(n, \ell)}{w_{i,t}(0, \ell)} \right) = \beta_\ell \mu_{i,t}(n, \ell) + \boldsymbol{\theta}'_\ell \mathbf{x}_{i,t}(n) + \kappa_{i,t}(n, \ell). \quad (10)$$

This regression is estimated separately for each asset class  $\ell$  using an instrument for expected returns, the panel of bilateral holdings data and a set of characteristics. We detail the construction of these instruments below.

The estimation equation for cross-asset demand is obtained by dividing equation (5) for short-term ( $\ell = 1$ ) and long-term debt ( $\ell = 2$ ) by the equation for equity ( $\ell = 3$ ):

$$\begin{aligned} \log \left( \frac{w_{i,t}(\ell)}{w_{i,t}(3)} \right) &= \lambda_\ell \log \left( 1 + \sum_{n=0}^N \delta_{i,t}(n, \ell) \right) - \lambda_3 \log \left( 1 + \sum_{n=0}^N \delta_{i,t}(n, 3) \right) + \alpha_\ell + \xi_{i,t}(\ell) \\ &= -\lambda_\ell \log(w_{i,t}(0|\ell)) + \lambda_3 \log(w_{i,t}(0|3)) + \alpha_\ell + \xi_{i,t}(\ell), \end{aligned} \quad (11)$$

where the second equality follows from equation (4). This regression is estimated using a panel of aggregate holdings of each asset class  $\ell$  by investors  $i$  at time  $t$  and instruments for  $w_{i,t}(0|\ell)$ .

For both sets of regressions equation (10) and equation (11), we use the panel of portfolio shares and characteristics to estimate equation. Each observation records investor  $i$ 's portfolio share of country  $j$  at the end of year  $t$ . All portfolio holdings that cannot be attributed to

the issuer countries in our sample are instead attributed to an “outside”. The issuer country characteristics on the right-hand side of equation (10) are its log nominal GDP, log GDP per capita, trade centrality, sovereign default risk, the real exchange rate and inflation. We also include bilateral import and exports exposures and distance. Finally, we include an indicator variable for domestic investment, investor country, and year fixed effects.

The main identification challenge is to consistently estimate equations (10) and (11) given that expected returns may be endogenous to the latent demand of investors. We follow the identification strategy of [Kojien and Yogo \(2019b\)](#), which we briefly summarize here. The estimation proceeds by first constructing an instrument for  $w_{i,t}(0|\ell)$  in order to consistently estimate equation (11). Then, using these estimates and market clearing, an instrument is constructed for exchange rates and prices. Using the instruments for exchange rates and prices then allows for the consistent estimation of equation (10).

The instruments for  $w_{i,t}(0|\ell)$  are constructed by calculating the predicted values from equation (10), but only using characteristics that are plausibly exogenous to the system: log GDP, bilateral distance, investor fixed effects, and the own country dummy. Using these characteristics, we are able to identify exogenous variation in portfolio weights of individual investors. Furthermore, by using these predicted weights and market clearing, we are able to construct instruments for the endogenous quantities in the model. The predicted values from equation (10) using only the exogenous characteristics are denoted by  $\hat{\delta}_{i,t}(n, \ell)$ , which allows for the construction of the instrument for  $w_{i,t}(0|\ell)$ :

$$\hat{w}_{i,t}(0|\ell) = \frac{1}{1 + \sum_{n=0}^N \hat{\delta}_{i,t}(n, \ell)}. \quad (12)$$

Using these instruments, we estimate  $\hat{\lambda}_\ell$  and  $\hat{\alpha}_\ell$  from equation (11).

Given  $\hat{\delta}_{i,t}(n, \ell)$ ,  $\hat{\lambda}_\ell$ , and  $\hat{\alpha}_\ell$  we construct an instrument for prices and exchanges rates using market clearing. To do so we first compute a predicted weight for country  $n$  in asset class  $\ell$  at time  $t$ :

$$\hat{w}_{i,t}(n, \ell) = \frac{\hat{\delta}_{i,t}(n, \ell)}{1 + \sum_{n=0}^N \hat{\delta}_{i,t}(n, \ell)} \frac{\left(1 + \sum_{n=0}^N \hat{\delta}_{i,t}(n, \ell)\right)^{\hat{\lambda}_\ell} \exp \hat{\alpha}_\ell}{\left(1 + \sum_{n=0}^N \hat{\delta}_{i,t}(n, \ell)\right)^{\hat{\lambda}_\ell} \exp \hat{\alpha}_\ell}. \quad (13)$$

The final step in constructing the instruments is to use market clearing to construct instruments for exchange rates and prices. The market clearing is done using portfolio weights that are constructed only using the exogenous characteristics. We use short-term debt markets to calculate the instrument for exchange rates. In particular, market clearing in the short-term debt market given the predicted weights implies our instrument for exchange rates:

$$\hat{E}_t(n) = \frac{1}{Q_t(n, 1)} \sum_{i=1}^I \frac{O_{i,t} \hat{w}_{i,t}(n, 1)}{1 - \sum_{m=1}^3 \sum_{k=1}^N \hat{w}_{i,t}(k, m)}.$$

The instruments for long-term bond prices and stock prices also clear their markets at the predicted weights

$$\hat{P}_t(n, \ell) = \frac{1}{\hat{E}_t(n) Q_t(n, \ell)} \sum_{i=1}^I \frac{O_{i,t} \hat{w}_{i,t}(n, \ell)}{1 - \sum_{m=1}^3 \sum_{k=1}^N \hat{w}_{i,t}(k, m)}.$$

With these instruments in hand we can estimate equation (10). For short-term debt we instrument expected returns with  $\hat{E}_t(n)$ . For long-term debt and equity we instrument expected returns with  $\hat{E}_t(n)$  and  $\hat{P}_t(n, \ell)$  for  $\ell = 2, 3$ .

The estimates for within asset class demand curves are presented in Table A.3 and the across asset parameters are presented in Table A.4 in Appendix A. For the within asset demand curves, the coefficients on expected returns are all positive, which implies that conditional on a set of asset characteristics, assets with higher expected returns are preferred by investors. The coefficients on asset characteristics are all intuitive. Investors prefer assets that provide better hedges against systematic risks. These are assets of larger countries and countries with higher trade centrality. Investors also prefer assets of countries that are closer and with whom they have a stronger trade relationship. Finally, the last row of Table A.3 shows there is strong home bias in all asset classes.

Turning to the cross-asset substitution parameters, we see that all  $\lambda_\ell$  values are between 0 and 1. This implies that there is some substitution between asset classes when the relative value of an asset class varies. This is in contrast to the case when  $\lambda_\ell = 0$  where asset level allocations are independent of the relative values of the asset classes. In contrast, when  $\lambda_\ell = 1$ , substitution between assets only depends on the individual country level prices as

discussed in [Koijsen and Yogo \(2019b\)](#).

## 4 Decomposing the U.S. External Imbalance

### 4.1 Counterfactual Setup

In the following section, we use our estimated demand system to run counterfactual exercises to decompose trends in the U.S. debt and equity imbalance. To re-iterate, our demand system can be summarized by four sets of equations: The investor demand functions within an asset class given by equation (2):

$$\begin{aligned} w_{i,t}^{cf}(n|\ell) &= \frac{\delta_{i,t}^{cf}(n, \ell)}{1 + \sum_{k=0}^N \delta_{i,t}^{cf}(k, \ell)} \\ \delta_{i,t}^{cf}(n, \ell) &= \exp(\beta_\ell \mu_{i,t}^{cf}(n, \ell) + \boldsymbol{\theta}'_\ell \mathbf{x}_{i,t}(n) + \kappa_{i,t}(n, \ell)), \end{aligned}$$

the investor demand functions across asset classes given by equation (5):

$$w_{i,t}^{cf}(\ell) = \frac{(1 + \sum_{k=0}^N \delta_{i,t}^{cf}(n, \ell))^{\lambda_\ell} \exp(\alpha_\ell + \xi_{i,t}(\ell))}{\sum_{m=1}^3 (1 + \sum_{k=0}^N \delta_{i,t}^{cf}(k, m))^{\lambda_m} \exp(\alpha_m + \xi_{i,t}(m))},$$

the dynamics of investor assets under management given by equation (7):

$$A_{i,t}^{cf} = A_{i,t-1}^{cf} \sum_{\ell=1}^3 \sum_{k=0}^N w_{i,t-1}^{cf}(\ell) w_{i,t-1}^{cf}(k|\ell) (1 + R_t^{cf}(k, \ell)) + F_{i,t},$$

and the asset market clearing conditions (9):

$$P_t^{cf}(n, \ell) S_t(n, \ell) E_t^{cf}(n) Q_t(n, \ell) = \sum_{i=1}^N A_{i,t}^{cf} w_{i,t}^{cf}(\ell) w_{i,t}^{cf}(n|\ell) + P_t^{cf}(n, \ell) S_t(n, \ell) E_t^{cf}(n) \sum_{i=1}^N B_{i,t}(n, \ell).$$

The superscripts  $cf$  denote the endogenous variables in our demand system that respond to counterfactual changes in the primitive parameters that we describe below. These endogenous variables comprise the asset prices  $P_t^{cf}(n, \ell)$ , the exchange rates  $E_t^{cf}(n)$ , the expected returns  $\mu_{i,t}^{cf}(n, \ell)$ , the cross-asset class portfolio weights  $w_{i,t}^{cf}(\ell)$ , the within-asset class portfolio

weights  $w_{i,t}^{cf}(n|\ell)$ , and investor AUM  $A_{i,t}^{cf}$ .

We now describe the primitive variables in our demand system along with our counterfactual exercises. We conduct our counterfactual exercises cumulatively such that we ultimately explain 100 percent of the changes in the US debt and equity imbalances and US asset prices. We start by setting asset quantities ( $Q_{\ell,j,t}$ ), central bank holdings ( $B_{i,\ell,j,t}$ ), and investor demand parameters ( $(\beta'_i \mathbf{x}_{i,j,t} + \kappa_{i,j,t})$ ) to their 2002 values, and setting investors' savings ( $F_{i,t}$ ) to offset the assets' dividend payouts  $D_{\ell,j,t}$ , so that the endogenous portfolio quantities and asset prices stay constant at their 2002 levels. We call this case the Baseline case.

Next, we iteratively allow each of the primitive variables to vary according to their observed time series, solving for the equilibrium asset prices, exchange rates, and portfolio allocations at each step.

**Savings Glut** We start by allowing investors' savings  $F_{i,t}$  and asset supply  $Q_t(n, \ell)$  to vary in accordance with the data. This step isolates the effects of private savings gluts, which is driven by foreign private savings in excess of domestic investment opportunities.

**Official Holdings** Next, we account for the effect of changes in central bank holdings  $B_{i,t}(n, \ell)$  by allowing the share of assets held by the central banks to vary in accordance with the data. This step evaluates theories of global imbalances that ascribe a large role to the central bank reserve policies and quantitative easing.

**Demand Shifts** Finally, we restore the asset characteristics  $\mathbf{x}_{i,t}(n)$ , the within-asset latent demand  $\kappa_{i,t}(n, \ell)$ , and the cross-asset latent demand  $\xi_{i,t}(\ell)$ . This step accounts for changes in the relative desirability of assets and asset classes over time, which are captured by changes in asset characteristics and latent demand. After this step, we have restored all variables in the counterfactual to match the data, and thus portfolio allocations and asset prices also match the data perfectly.

**Counterfactual Results** Figure 2 displays the US net portfolio imbalances in the actual data as well as in each counterfactual scenario. The net portfolio imbalances are defined as



the difference between the US holdings of foreign assets, and the foreign holdings of the U.S. assets. These foreign holdings of US assets include foreign central banks reserve assets and private investor holdings. These figures are all normalized by 2002 US GDP such that the benchmark counterfactual shows up as a flat line. The top panel always reports the aggregate imbalances, and the bottom panel reports debt and equity imbalances separately. To help the readers quantify these trends, Table 1 reports level of the US net portfolio positions at the end of our sample, as well as the changes in the positions relative to 2002.

In the data, the U.S. net external portfolio imbalance declined dramatically from  $-10\%$  of the 2002 U.S. GDP to  $-32\%$ . This decline in the overall portfolio position was primarily driven by a decline in the US debt imbalance from  $-13\%$  of the 2002 U.S. GDP to  $-43\%$ . On the other hand, the US maintained a positive equity imbalance, which partially cancelled out its debt liabilities.

## 4.2 Savings Glut

From 2002 to 2016, the total assets under management from our sample of 43 investor countries increased from 60.8 trillion dollars to 147.9 trillion dollars. Out of this 87.1 trillion dollar increase, 52.6 trillion dollars flowed into debt markets and 34.5 trillion flowed into equity markets. Both of these flows represent significant increases in the size of investor portfolios since 2002. Our demand system framework captures an overwhelming share of investor savings during this period. In 2016, the 43 investor countries allocated 96% of their wealth among the assets of the 36 issuer countries.

During this same period, shares of debt outstanding increased by 72%, and the total book value of equity increased by 393%. Both of these changes indicate significant increases in the quantity of assets available for purchase. A priori, we would expect increases in investor wealth to raise asset prices, and increases in asset supply to depress asset prices. Moreover, the relative differences in investment and asset issuances across countries could be a major driver of capital flows internationally.

Our first counterfactual restores the amount of investors' savings  $F_{i,t}$  and asset supply  $Q_{\ell,j,t}$  from the data onto the baseline. At the aggregate level, accounting for relative differences in savings and asset issuances dramatically widens the U.S. net portfolio imbalances

from  $-10\%$  of the 2002 GDP in 2002 to  $-76\%$  in 2016. This is shown by the blue line in top panel of Figure 2.

The bottom two panels of Figure 2 decompose the change in the portfolio debt and portfolio equity positions separately. The counterfactual debt and equity imbalances both widen, but the magnitude in the debt market is much greater. Table 1 shows the savings glut channel decreases the US debt imbalance from  $-13\%$  of the 2002 GDP in 2002 to  $-72\%$  in 2016. Similarly, the U.S. equity imbalance declines from  $+3\%$  of the 2002 GDP in 2002 to  $-5\%$  in 2016.

These results suggest there is strong support for the global savings glut in driving capital into the US. After restoring investors' savings, restoring asset issuances and then allow investors to rebalance their portfolio based on their 2002 demand curves, foreign investors choose to allocate a large portion of their capital in the US debt and equity markets. This increase in foreign capital flows into the US are shown more clearly in the growth in US liabilities in the blue line representing the savings glut counterfactual in Figure 3.

At the same time, increases in US savings relative to domestic asset issuances also increased US holdings of foreign assets, as demonstrated in Figure 4. However, the relative magnitude of this increase in the US foreign asset position is relatively small when compared to the flow of capital into the US.

Notably, however, this counterfactual decline in the US portfolio imbalance that can be attributed to a private savings glut is much larger than the observed change in the data. This result suggests that the savings glut channel alone overshoots the observed widening in the U.S. net imbalance, whereas some other channels must provide an offsetting force.

**Emerging Market vs. Developed Market Savings Glut** Before moving on to other counterfactual exercises, we use our framework to explore the differential effect of developed market and emerging market savings gluts on the US portfolio imbalance. Figure 5 further separates the contribution of the Emerging Markets and the Developed Markets in the Savings Glut counterfactual. In the line "EM Only", we only allow the emerging markets' investor savings and asset supply to vary according to the data, while fixing the developed markets' investor savings and asset supply, all countries' central bank holdings and asset

demand at the 2002 level.

Figure 5 and Table 2 both suggest that the private excess savings from the emerging markets have a very small impact. Instead, an excess of private savings in developed markets is primarily responsible for the large decline in the savings glut counterfactual in Figure 2. In our sample, the aggregate AUM of emerging market economies increases from 1.7 trillion dollars in 2002 to 13.7 in 2016, whereas the AUM of developed market economies increases from 57 trillion to 124.6 trillion. Thus, perhaps it is unsurprising that the effect of the developed market savings glut is significantly larger.

Nevertheless, these results do not imply that investment from emerging market economies are not important for explaining the decline in the US portfolio position. The savings glut counterfactual only accounts for private investment, and does not yet account for any accumulation of central bank reserves.

### 4.3 Official Holdings

Central banks have long played an active role in global financial markets. After the Asian Financial Crisis of 1997, emerging market central banks increased their foreign currency reserves in preparation for potential future turmoil. In more recent years, many central banks engaged in quantitative easing to control credit conditions during a period where policy rates were stuck at the zero lower bound (Kojen et al. 2017, 2019). Our data captures the aggregate portfolio of central bank reserves, as well as the portfolio of U.S. domestic assets purchased by the Federal Reserve through quantitative easing. Crucially for the U.S. portfolio position, both data series show a significant increase in reserve holdings of U.S. debt assets.

The bottom panel of Figure 2 shows that the central banks' asset holdings further amplify the effects of the private savings glut in the debt market. When we restore the actual holdings of the US FRB portfolio and foreign central banks' reserve portfolios, the U.S. debt imbalances widen to over  $-94\%$  of the 2002 GDP in 2016 from  $-76\%$  in the counterfactual with savings glut alone. A priori, the U.S. QE should raise the value of the US debt and potentially price out the foreign investors, closing the gap in the US net debt position. On the other hand, the increase in the foreign central bank reserves should raise the foreign

holdings of the US debt, leading to a greater decline in the US net debt position. The results suggest that the latter channel is much more dominant.

Central bank reserves consist of mostly debt assets rather than equity assets. However, this counterfactual shows that changes in reserve holdings also generate large effects in the US equity imbalance. This result highlights the cross-asset substitution channel allowed in our model: The central banks' reserve purchases decrease the expected returns from investing in debt, which leads investors to shift their wealth towards equity investment. Figure 4 shows that the US investors hold more foreign equity under the reserve holdings counterfactual, and Figure 3 shows that the foreign investors also hold more U.S. equity. In aggregate, as the US investors hold much more foreign equity than the foreign investors holding the U.S. equity, the net effect increases the U.S. net equity portfolio positions.

## 4.4 Demand Shifts

All remaining variation in portfolio imbalances result from shifts in investors' demand functions. These demand shifts reflect changes in observed country characteristics such as economic growth, as well as changes in unobserved characteristics captured by the latent demand. Intuitively, the changes in investor demand reflect changes in the relative desirability of assets over time, such as recent narratives describing an increased demand for safe assets (Krishnamurthy and Vissing-Jorgensen 2012).

In both debt and equity markets, the change in investor demand tilts the international debt portfolio away from the U.S. and towards foreign markets. As shown in the bottom panel of Figure 2, shifts in investor demand improve the U.S. debt and equity imbalances relative to their levels in the Central Bank counterfactual. For the debt market, restoring the changes in investor demand reduces the U.S. imbalance from  $-95\%$  of the 2002 GDP in 2016 to  $-43\%$  in the data. Figure 4 and 3 show that this increase in the US debt position is mainly driven by a decline in US debt liabilities, rather than an increase in US holdings of foreign debt.

Similarly for the equity market, restoring the changes in investor demand curves increases the U.S. imbalance from  $+2\%$  of the 2002 GDP in 2016 to  $+11\%$  in the data. Figure 4 and 3 show that the US investors increase their holdings of foreign equity much more than the

foreign investors increase their holdings of the U.S. equity.

This result suggests an intriguing hypothesis of changing imbalances: Contrary to the popular belief that the widening U.S. debt imbalances are driven by a flight-to-safety phenomenon towards the U.S. debt assets, the relative desirability of the U.S. debt is actually declining. If its demand curves for assets had stayed the same as they were in 2002, the US debt imbalances would have been even bigger as a result of private sector savings and central bank reserve accumulation. However, we do not directly observe the decline in the U.S. assets' popularity, because these effects are overshadowed by the effects of private savings and central bank purchases.

**Asset Characteristics vs. Latent Demand** Shifts in investor demand curves can arise from two sources. There can be changes to the characteristics of individual assets or there can be changes in the latent demand. To explore the role of changes in demand curves in driving the US portfolio imbalance, we further separate the contribution of the changes in asset characteristics and the changes in latent demand.

For both the debt and the equity imbalances, Figure 6 and Table 3 show that changes in asset characteristics alone do not significantly impact the US portfolio position relative to the Official Holdings counterfactual. Instead, changes in latent demand are the main drivers of the change between the Official Holdings counterfactual and the data.

Recall that the Savings Glut and Official Holdings channels produce an overshooting in the U.S. imbalances, while the Investor Demand channel partially offsets these trends. The result in Figure 6 then suggests the offset is mainly a result of the changes in latent demand that is not captured by the asset characteristics.

Figure 7 reports the time-series of the AUM-weighted average of the latent demand differential between the U.S. and the sum of foreign countries. The top panel plots the average difference between each investor's within-asset latent demand  $\kappa$  towards the U.S. asset and its latent demand towards other assets, weighted by the investor's portfolio holding and its AUM. The relative unobserved desirability of the U.S. long-term debt and equity remains stable, and that of the U.S. short-term debt has been rising slightly.

The bottom panel plots the average cross-asset latent demand  $\xi$  towards each asset class,

weighted by the investor's AUM. As we normalize the equity's cross-asset latent demand to 0, this panel shows that the unobserved desirability of long-term and short-term debt has been declining relative to the equity. As a result, the Demand Shifts channel partially offsets the widening imbalance in the U.S. debt because there is an asset class-wide shift from debt to equity, and the foreign investors hold a lot more U.S. debt than the U.S. investors hold foreign debt. This overall shift from debt to equity is potentially consistent with a reach-for-yield story.

## 5 Conclusion

This paper uses a portfolio approach to evaluate the impact of savings glut, central bank policies, and investor demand shifts on the dynamics of the US external portfolio imbalance. We highlight three key insights from the counterfactual exercises that are important moments to consider for models of global imbalances. First, global savings gluts resulting from increases in private savings are crucial for explaining the large flow of foreign capital into the US. Our counterfactual exercise reveals that the vast majority of this private capital originates from developed market economies, rather than emerging markets. Second official holdings of foreign central banks play a significant role in weakening the US debt position, and also in driving private savers towards equity markets. Finally, the counterfactuals show the effects of private savings gluts and official holdings alone would suggest an even more negative US portfolio position than the data. Instead, changes in investor demand curves, characterized by a market-wide shift from debt to equity assets, actually improved the US portfolio position and drove foreign investors away from US debt assets.

More generally, the portfolio approach to global imbalances provides a framework for analyzing many additional features of the data. First, just as we have done with the savings glut counterfactual, one could use additional data on official holdings to separate the roles of the official holdings of different central banks. This exercise would be especially useful to understand the impact of emerging market central banks, and their reserve activities after the East Asian Financial Crisis. Second, the portfolio approach allows us to break down changes in external imbalances into changes in prices and quantities separately. This feature

will allow us to better understand the role of valuation effects in net foreign asset positions. Finally, we have yet to fully explore the geographic origins of the changes in demand curves. Further analysis of the changes in demand curves of different countries could clarify the origin of capital flows. In this sense, we believe this portfolio approach to be an important tool for future research.

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# Tables and Figures

TABLE 1  
DECOMPOSING LONG-RUN VARIATION IN PORTFOLIO IMBALANCES

	(1) 2016 Level (% 2002 GDP)	(2) Change in Pos.	(3) Share of Total (%)
Panel A: Debt and Equity			
Baseline (2002)	-9.82		
Savings Glut	-76.47	-66.65	301.00
Official Holdings	-93.74	-83.93	379.00
Data / Demand Shifts	-31.96	-22.15	100.00
Panel B: Debt			
Baseline (2002)	-13.09		
Savings Glut	-71.58	-58.49	193.00
Official Holdings	-95.36	-82.26	272.00
Data / Demand Shifts	-43.38	-30.29	100.00
Panel C: Equity			
Baseline (2002)	3.28		
Savings Glut	-4.89	-8.17	-100.00
Official Holdings	1.61	-1.66	-20.00
Data / Demand Shifts	11.42	8.14	100.00

**Notes:** This table presents the changes in the U.S. portfolio imbalance under each counterfactual exercise. The counterfactual exercises are cumulative. Portfolio imbalances normalized by U.S. GDP in 2002. Column (1) provides the level of the portfolio position in each counterfactual exercise in 2016. Column (2) provides the change in portfolio position between the counterfactual exercise and the 2002 portfolio position. Column (3) provides changes in portfolio as a share of the total position change between 2002 and 2016. Panel A presents results for the aggregate U.S. portfolio position. Panel B and Panel C provide results for the aggregate debt and equity portfolio positions, respectively.

TABLE 2  
DECOMPOSING EMERGING VS DEVELOPED SAVINGS GLUTS

	(1) Level (% 2002 GDP)	(2) Change in Pos.	(3) Share of Total (%)
Panel A: Debt and Equity			
Baseline (2002)	-9.82		
EM Only	-12.60	-2.78	13.00
EM and DM	-76.47	-66.65	301.00
Data / Demand Shifts	-31.96	-22.15	100.00
Panel B: Debt			
Baseline (2002)	-13.09		
EM Only	-14.70	-1.61	5.00
EM and DM	-71.58	-58.49	193.00
Data / Demand Shifts	-43.38	-30.29	100.00
Panel C: Equity			
Baseline (2002)	3.28		
EM Only	2.10	-1.18	-14.00
EM and DM	-4.89	-8.17	-100.00
Data / Demand Shifts	11.42	8.14	100.00

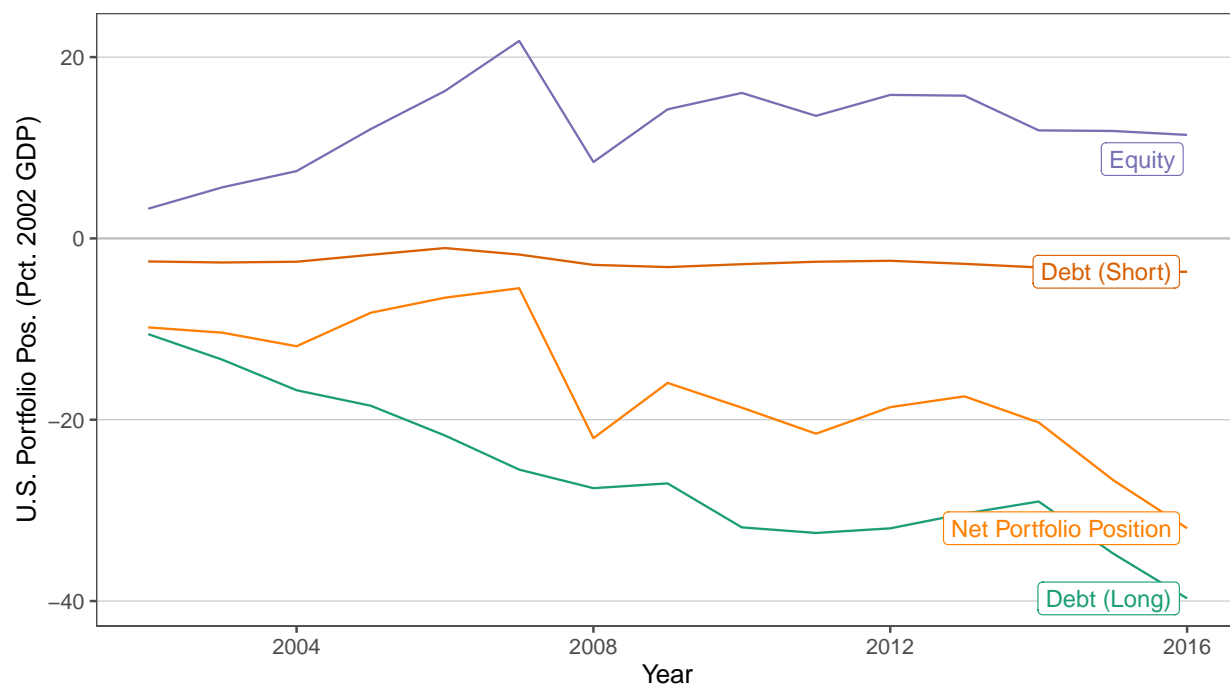
**Notes:** This table presents the changes in the U.S. portfolio imbalance under each counterfactual exercise. The counterfactual exercises are cumulative. Portfolio imbalances normalized by U.S. GDP in 2002. Column (1) provides the level of the portfolio position in each counterfactual exercise in 2016. Column (2) provides the change in portfolio position between the counterfactual exercise and the 2002 portfolio position. Column (3) provides changes in portfolio as a share of the total position change between 2002 and 2016. Panel A presents results for the aggregate U.S. portfolio position. Panel B and Panel C provide results for the aggregate debt and equity portfolio positions, respectively.

TABLE 3  
CHANGES IN CHARACTERISTICS AND LATENT DEMAND

	(1) Level (% 2002 GDP)	(2) Change in Pos.	(3) Share of Total (%)
Panel A: Debt and Equity			
Baseline (2002)	-9.82		
Official Holdings	-93.74	-83.93	379.00
Chars Only	-102.00	-92.19	416.00
Data / Latent Demand	-31.96	-22.15	100.00
Panel B: Debt			
Baseline (2002)	-13.09		
Official Holdings	-95.36	-82.26	272.00
Chars Only	-103.45	-90.36	298.00
Data / Latent Demand	-43.38	-30.29	100.00
Panel C: Equity			
Baseline (2002)	3.28		
Official Holdings	1.61	-1.66	-20.00
Chars Only	1.45	-1.82	-22.00
Data / Latent Demand	11.42	8.14	100.00

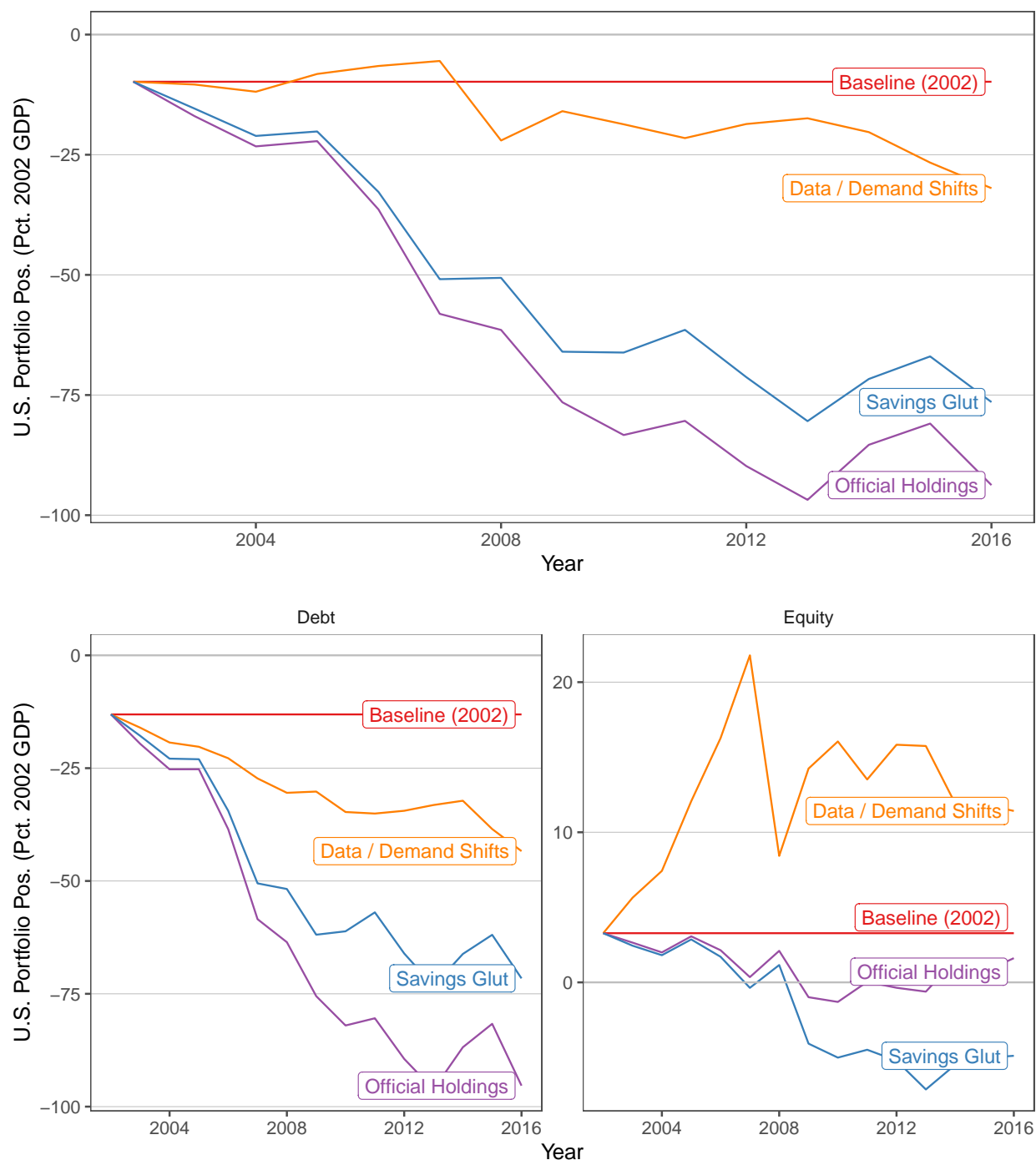
**Notes:** This table presents the changes in the U.S. portfolio imbalance under each counterfactual exercise. The counterfactual exercises are cumulative. Portfolio imbalances normalized by U.S. GDP in 2002. Column (1) provides the level of the portfolio position in each counterfactual exercise in 2016. Column (2) provides the change in portfolio position between the counterfactual exercise and the 2002 portfolio position. Column (3) provides changes in portfolio as a share of the total position change between 2002 and 2016. Panel A presents results for the aggregate U.S. portfolio position. Panel B and Panel C provide results for the aggregate debt and equity portfolio positions, respectively.

FIGURE 1. U.S. NET PORTFOLIO POSITION



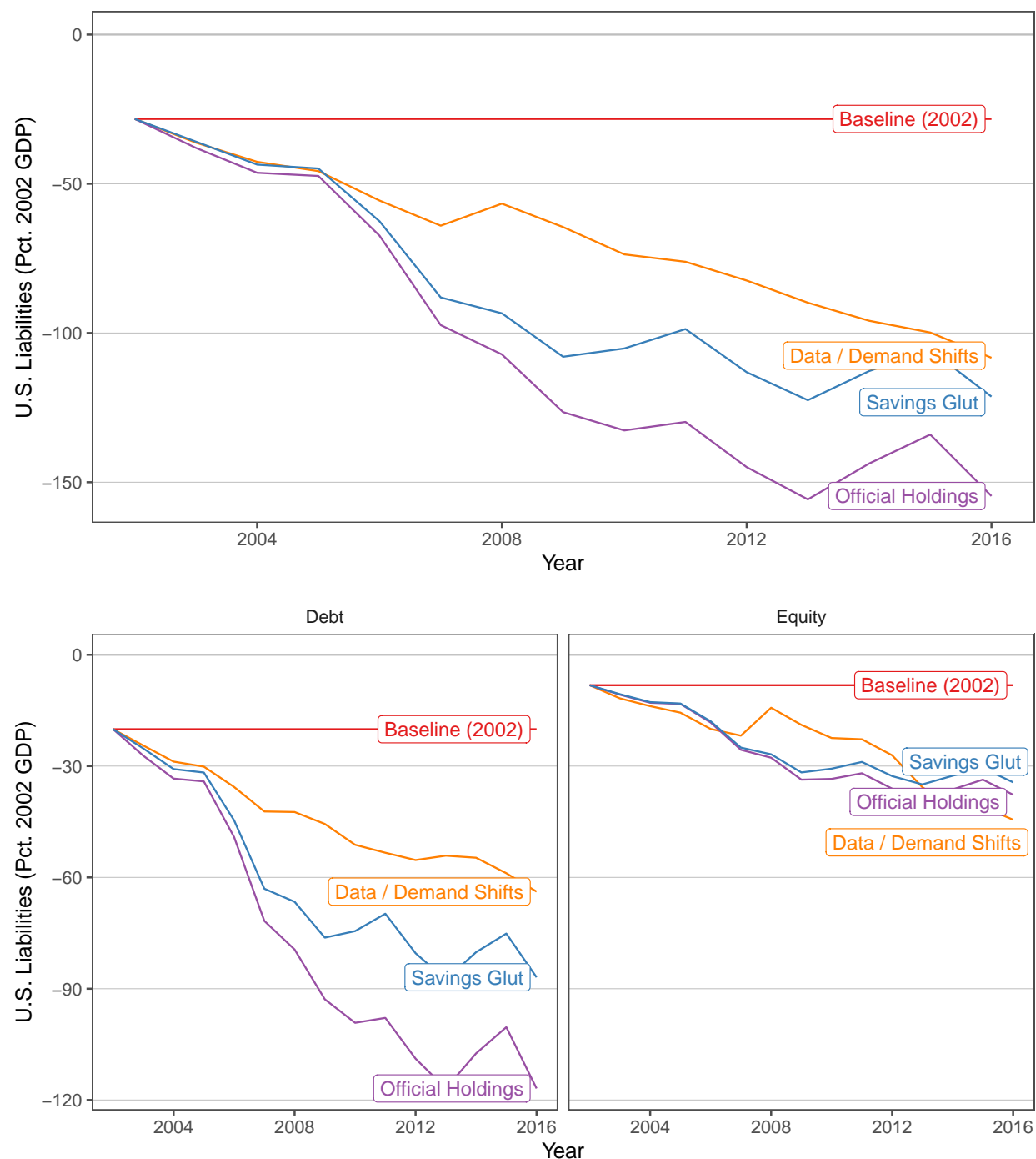
**Notes:** The figure plots the aggregate US net portfolio, as well as the positions within each asset class, with respect to the issuer and investor countries within our sample.

FIGURE 2. DECOMPOSING LONG-RUN VARIATION IN PORTFOLIO IMBALANCES  
(CUMULATIVE COUNTERFACTUALS)



**Notes:** These figures present the US portfolio position under different counterfactual exercises. The counterfactual exercises are cumulative and start with the “Baseline (2002)” position. The order of the counterfactuals continues with the “Savings Glut”, “Official Holdings”, and finally “Data / Demand Shift”. The “Data / Demand Shifts” exercise corresponds with the data, but also captures the effects of shifts in the demand curves for assets. Portfolio positions are normalized by U.S. GDP in 2002. The top panel presents the overall US portfolio position. The bottom two panels present the US aggregate debt position and US equity position separately.

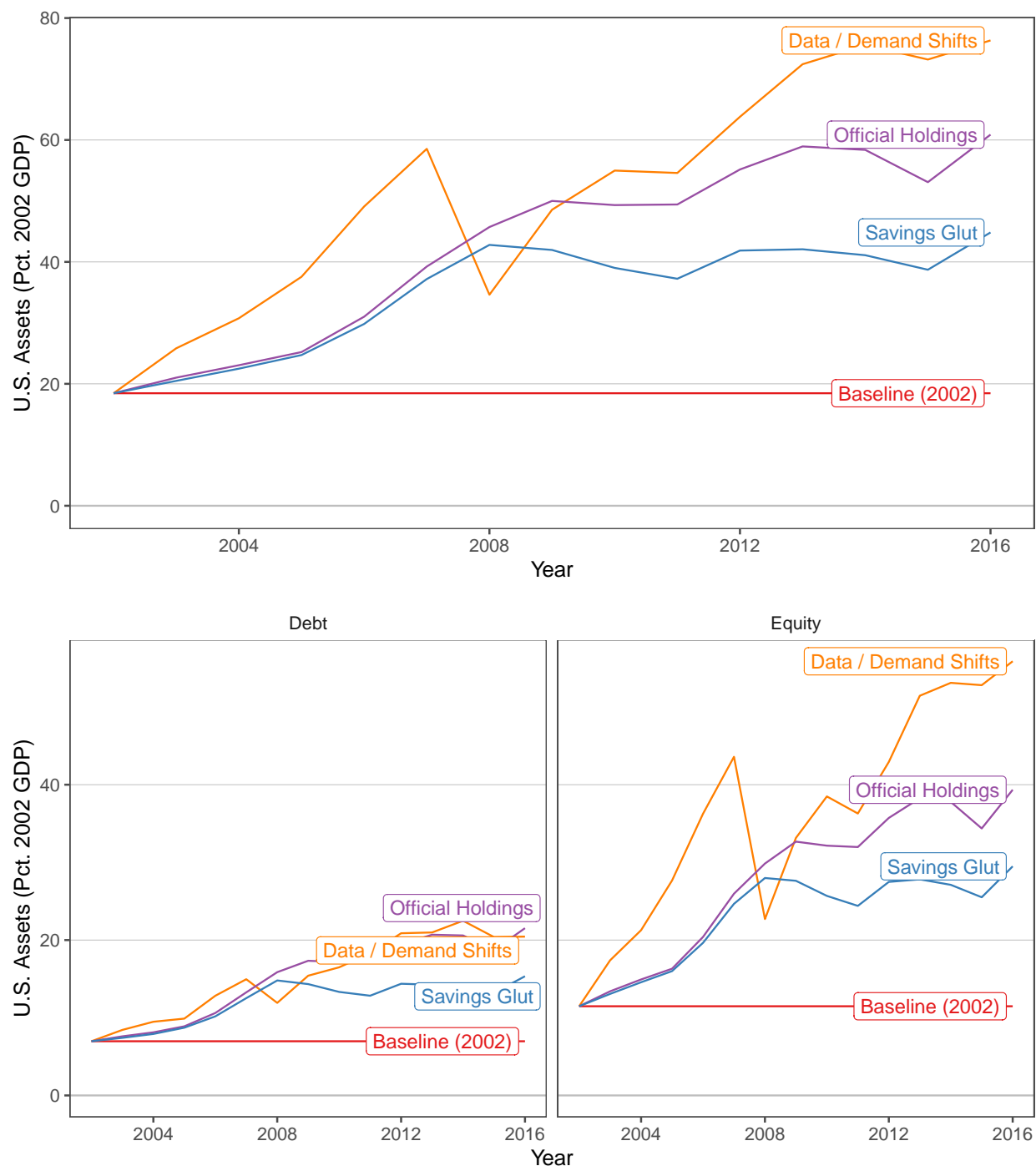
FIGURE 3. DECOMPOSING LONG-RUN VARIATION IN PORTFOLIO LIABILITIES  
(CUMULATIVE COUNTERFACTUALS)



**Notes:** These figures present the value of the US liabilities under different counterfactual exercises. The counterfactual exercises are cumulative and start with the “Baseline (2002)” position. The order of the counterfactuals continues with the “Savings Glut”, “Official Holdings”, and finally “Data / Demand Shift”. The “Data / Demand Shifts” exercise corresponds with the data, but also captures the effects of shifts in the demand curves for assets. Liability positions are normalized by U.S. GDP in 2002. The top panel presents the overall US liability position. The bottom two panels present the US aggregate debt liability position and US equity liability position, separately.



FIGURE 4. DECOMPOSING LONG-RUN VARIATION IN PORTFOLIO ASSETS  
(CUMULATIVE COUNTERFACTUALS)



**Notes:** These figures present the value of the US asset portfolio under different counterfactual exercises. The counterfactual exercises are cumulative and start with the “Baseline (2002)” position. The order of the counterfactuals continues with the “Savings Glut”, “Official Holdings”, and finally “Data / Demand Shift”. The “Data / Demand Shifts” exercise corresponds with the data, but also captures the effects of shifts in the demand curves for assets. Asset positions are normalized by U.S. GDP in 2002. The top panel presents the overall US asset position. The bottom two panels present the US aggregate debt asset position and US equity asset position, separately.

FIGURE 5. EMERGING MARKET VS. DEVELOPED MARKET SAVINGS GLUTS  
(CUMULATIVE COUNTERFACTUALS)



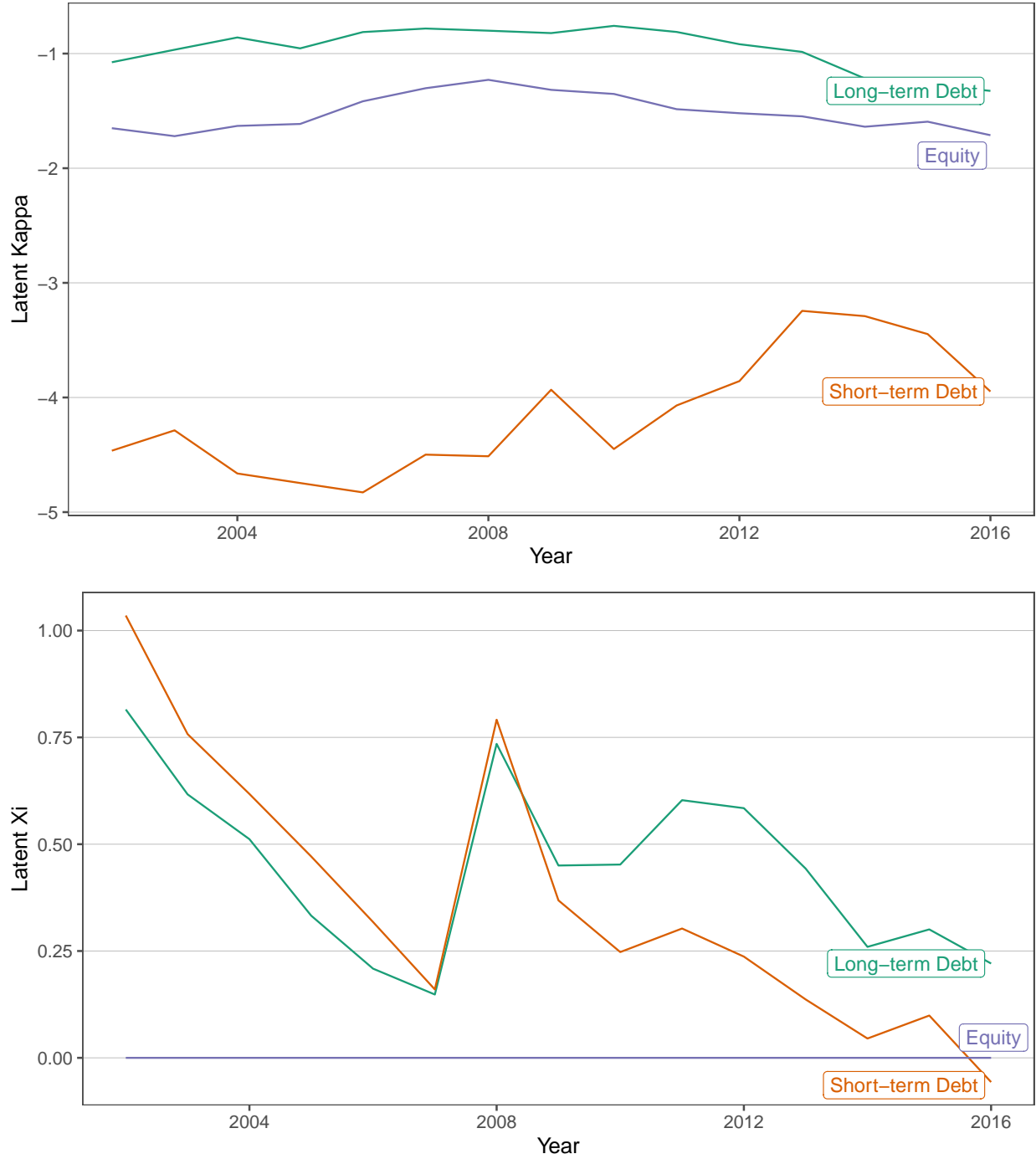
**Notes:** These figures present the US portfolio position under different counterfactual exercises, and breaks up the “Savings Glut” counterfactual into a portion attributed to emerging markets alone. The counterfactual exercises are cumulative and start with the “Baseline (2002)” position. The order of the counterfactuals continues with the “EM Only” portion of the savings glut, followed by “EM and DM”, and finally “Data / Demand Shift”. Portfolio positions are normalized by U.S. GDP in 2002. The top panel presents the overall US portfolio position. The bottom two panels present the US aggregate debt position and US equity position separately.

FIGURE 6. CHANGES IN CHARACTERISTICS VS. LATENT DEMAND (CUMULATIVE COUNTERFACTUALS)



**Notes:** These figures present the US portfolio position under different counterfactual exercises, and breaks up the “Demand Shifts” counterfactuals into changes in characteristics and changes in latent demand. The counterfactual exercises are cumulative and start with the “Baseline (2002)” position. The order of the counterfactuals continues with the “Official Holdings” counterfactual, followed by “Chars Only”, and finally “Data / Latent Demand”. Portfolio positions are normalized by U.S. GDP in 2002. The top panel presents the overall US portfolio position. The bottom two panels present the US aggregate debt position and US equity position separately.

FIGURE 7. LATENT DEMAND FOR US DEBT AND DEBT OVERALL



**Notes:** The top panel plots the AUM-weighted average of the within-asset latent demand for US assets over time by asset type. The bottom panel plots the AUM-weighted average of the cross-asset latent demand for each asset type.

$$\bar{\kappa}(USA, \ell) = \frac{\sum_i A_{i,t} w_{i,t}(\ell) \left( w_{i,t}(USA|\ell) \kappa(USA, \ell) - \sum_{n \neq USA} w_{i,t}(n|\ell) \kappa(n, \ell) \right)}{\sum_i A_{i,t} w_{i,t}(\ell)}$$

$$\bar{\xi}(\ell) = \frac{\sum_i A_{i,t} w_{i,t}(\ell) \xi_{i,t}(\ell)}{\sum_i A_{i,t} w_{i,t}(\ell)}$$

# Appendix

## A Empirical Appendix

TABLE A.1  
LIST OF ISSUER AND INVESTOR COUNTRIES

<i>Developed Markets Issuers:</i>	<i>Emerging Markets Issuers:</i>
Australia	China
Austria	Colombia
Belgium	Czechia
Canada	Greece
Denmark	Hungary
Finland	India
France	Malaysia
Germany	Mexico
Hong Kong	Philippines
Israel	Poland
Italy	Russia
Japan	South Africa
Netherlands	South Korea
New Zealand	Thailand
Norway	
Portugal	<i>Other Investor Countries:</i>
Singapore	Chile
Spain	Estonia
Sweden	Iceland
Switzerland	Latvia
United Kingdom	Lithuania
United States	Slovakia
	Turkey

**Notes:** All issuer countries are also investor countries. We aggregate European investor countries into a single European Monetary Union (EMU) investor. The EMU comprises investment from the following countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Malta, Netherlands, Portugal, Slovenia and Spain.

TABLE A.2  
PREDICTING EXPECTED EXCESS RETURNS

	Short-term Debt	Long-term Debt	Equity
Log market-to-book	−12.57*** (1.11)	−0.37*** (0.07)	−0.20*** (0.04)
Log real exchange rate	−0.18 (0.09)	−0.19 (0.11)	−0.67*** (0.08)
Num. obs.	810	749	707
R <sup>2</sup>	0.33	0.22	0.17

**Notes:** This table displays results from estimating equation (6). For debt, the log market-to-book ratio is minus the maturity times the yield. All specifications include country fixed effects. Standard errors are clustered by year.

TABLE A.3  
DEMAND ESTIMATION WITHIN ASSET CLASS

	(1) Short-term Debt	(2) Long-term Debt	(3) Equity
E[Excess Return]	9.76** (3.26)	0.83 (1.28)	0.20 (0.66)
Log GDP	1.11*** (0.05)	0.88*** (0.03)	1.09*** (0.03)
Log GDP per capita	0.18* (0.07)	0.14*** (0.03)	0.24*** (0.04)
Centrality	0.23*** (0.04)	0.11*** (0.02)	0.16*** (0.02)
Default	-0.25*** (0.06)	-0.05 (0.05)	-0.11*** (0.03)
Distance	-0.78*** (0.05)	-0.92*** (0.02)	-0.86*** (0.02)
Import Exposure	0.12*** (0.03)	0.06* (0.02)	0.12*** (0.02)
Export Exposure	0.15*** (0.03)	0.16*** (0.02)	0.25*** (0.02)
Inflation	-0.24* (0.12)	0.14*** (0.02)	0.12*** (0.02)
Volatility	0.01 (0.05)	-0.17*** (0.02)	-0.12*** (0.02)
Indicator: Own Country	5.97*** (0.21)	5.27*** (0.12)	5.55*** (0.13)
Indicator: USA Issuance	0.75*** (0.18)	1.77*** (0.11)	1.59*** (0.12)
Num. obs.	8,675	14,162	14,719
R <sup>2</sup> (full model)	0.42	0.71	0.74
R <sup>2</sup> (proj model)	0.20	0.48	0.52

**Notes:** This table estimates equation (10) separately for each asset class. The sample comprises annual data from 2002 to 2016. Default is the 5-year default probability for the sovereign debt category imputed by S&P. All specifications include investor country, year and issuer country MSCI market fixed effects. Heteroskedasticity-robust standard errors are reported in parentheses. \*\*\* $p < 0.001$ , \*\* $p < 0.01$ , \* $p < 0.05$

TABLE A.4  
DEMAND ESTIMATION ACROSS ASSET CLASSES

Variable	Symbol	Estimate
Log outside asset weight:		
Short-term Debt	$\lambda_{st}$	0.22** (0.08)
Long-term Debt	$\lambda_{lt}$	0.26** (0.09)
Equity	$\lambda_{eq}$	0.63*** (0.14)
Asset class fixed effects:		
Short-term Debt	$\alpha_{st}$	-0.84*** (0.18)
Long-term Debt	$\alpha_{lt}$	1.33*** (0.22)
Num. obs.		911
R <sup>2</sup>		0.42

**Notes:** This table estimates equation (11) \*\*\* $p < 0.001$ , \*\* $p < 0.01$ , \* $p < 0.05$

## A.1 Calculating Counterfactual Asset Prices

In the following appendix, we apply an approximation of Newton's Method to calculate the equilibrium price in the counterfactual analysis. Our algorithm closely follows [Kojien and Yogo \(2019a\)](#). For each asset  $j$  in sector  $l$  at time  $t$ , we want to find the zero of the following function:

$$H(\mathcal{P}) = p_{j,t}^l + q_{j,t} - \log \left[ \sum_{i=1}^N A_{i,t} w_{i,t}^l w_{i,j,t}^l \right],$$

where the vector of parameters:

$$\mathcal{P} = [e_{j,t}, q_{j,t}, p_{j,t}^{lt}, p_{j,t}^{eq}]$$

comprises nominal exchange rates, short-term debt quantities for issuers in fixed exchange rate regimes, prices of long-term debt, and prices of equity. To re-iterate, the share of investor  $i$  assets within asset type  $l$  that are allocated to country  $j$  at time  $t$  is:

$$w_{i,j,t}^l = \frac{\exp(\beta^l \mu_{i,j,t}^l + \Theta_{i,j,t}^l \mathbf{x}_{i,j,t} + \kappa_{i,j,t})}{1 + \sum_{n=1}^N \exp(\beta^l \mu_{i,n,t}^l + \Theta_{i,n,t}^l \mathbf{x}_{i,n,t} + \kappa_{i,n,t})}$$



The share of investor  $i$  assets allocated to asset type  $l$  is:

$$w_{i,t}^l = \frac{\left(1 + \sum_{n=1}^N \exp(\beta^l \mu_{i,n,t}^l + \Theta_{i,n,t}^l \mathbf{x}_{i,n,t} + \kappa_{i,n,t})\right)^{\lambda^l} \exp(\alpha^l + \xi_{i,t}^l)}{\sum_{m=\{st,lt,eq\}} \left[\left(1 + \sum_{n=1}^N \exp(\beta^m \mu_{i,n,t}^m + \Theta_{i,n,t}^m \mathbf{x}_{i,n,t} + \kappa_{i,n,t})\right)^{\lambda^m} \exp(\alpha^m + \xi_{i,t}^m)\right]},$$

and the expected return of asset  $j$  of type  $l$  for investor  $i$  at time  $t$  is defined:

$$\mu_{i,j,t}^l = \gamma_p^l p_{j,t}^l + \gamma_e^l (e_{j,t} - \pi_{j,t}) - (\gamma_p^{st} p_{j,t}^{st} + \gamma_e^{st} (e_{i,t} - \pi_{j,t}))$$

Given any initial parameter vector  $\mathcal{P}$ , Newton's Method would update the price vector with:

$$\mathcal{P}' = \mathcal{P} - \mathcal{J}_H^{-1} H(\mathcal{P})$$

where  $\mathcal{J}_H$  represents the Jacobian of the multivariate function  $H$ . However, rather than calculate the full Jacobian, we approximate  $\mathcal{J}_H$  with its diagonal. Let  $H_{j,t}^l$  denote the row of  $H$  that corresponds to the market clearing condition for asset  $j$  of asset type  $l$  in period  $t$ .

For an asset  $j$  in the short-term debt market with floating exchange rates, the diagonal element of  $\mathcal{J}_H$  is:

$$\frac{\partial H_{j,t}^{st}}{\partial e_{j,t}} = - \frac{\sum_{i=1}^N A_{i,t} \left( \frac{\partial w_{i,t}^{st}}{\partial e_{j,t}} \times w_{i,j,t}^{st} + \frac{\partial w_{i,j,t}^{st}}{\partial e_{j,t}} \times w_{i,t}^{st} \right)}{\sum_{i=1}^N (A_{i,t} w_{i,t}^{st} w_{i,j,t}^{st})} \quad (\text{A.1})$$

where

$$\frac{\partial w_{i,t}^{st}}{\partial e_{j,t}} = \begin{cases} \lambda^{st} \beta^{st} \gamma_e^{st} w_{i,t}^{st} w_{i,j,t}^{st} - w_{i,t}^{st} \left( \sum_{m=st,lt,eq} \lambda^m \beta^m \gamma_e^m w_{i,t}^m w_{i,j,t}^m \right) & \text{if } i \neq j \\ -\lambda^{st} \beta^{st} \gamma_e^{st} w_{i,t}^{st} \left( \sum_{k \neq i} w_{i,k,t}^{st} \right) + w_{i,t}^{st} \left( \sum_{m=st,lt,eq} \lambda^m \beta^m \gamma_e^m w_{i,t}^m \left( \sum_{k \neq i} w_{i,k,t}^m \right) \right) & \text{if } i = j \end{cases}$$

and

$$\frac{\partial w_{i,j,t}^{st}}{\partial e_{j,t}} = \begin{cases} \beta^{st} \gamma_e^{st} w_{i,j,t}^{st} (1 - w_{i,j,t}^{st}), & \text{if } i \neq j \\ -\beta^{st} \gamma_e^{st} w_{i,j,t}^{st} \left( \sum_{k \neq i} w_{i,k,t}^{st} \right), & \text{if } i = j \end{cases} \quad (\text{A.2})$$

For an asset  $j$  in the short-term debt market that is part of a currency union, the diagonal element of  $\mathcal{J}_H$  is:

$$\frac{\partial H_{j,t}^{st}}{\partial q_{j,t}} = 1, \quad (\text{A.3})$$

where we update the quantity  $q_{j,t}$  of short-term debt outstanding.

For long-term debt and equity assets, the diagonal element of  $\mathcal{J}_H$  is:

$$\frac{\partial H_{j,t}^l}{\partial p_{j,t}^l} = 1 - \frac{\sum_{i=1}^N A_{i,t} \left( \frac{\partial w_{i,t}^l}{\partial p_{j,t}^l} \times w_{i,j,t}^l + \frac{\partial w_{i,j,t}^l}{\partial p_{j,t}^l} \times w_{i,t}^l \right)}{\sum_{i=1}^N (A_{i,t} w_{i,t}^l w_{i,j,t}^l)} \quad (\text{A.4})$$

where

$$\frac{\partial w_{i,t}^l}{\partial p_{j,t}^l} = \lambda^l \beta^l \gamma_p^l w_{i,j,t}^l w_{i,t}^l (1 - w_{i,t}^l) \quad (\text{A.5})$$

and

$$\frac{\partial w_{i,j,t}^l}{\partial p_{j,t}^l} = \beta^l \gamma_p^l w_{i,j,t}^l (1 - w_{i,j,t}^l) \quad (\text{A.6})$$

We start with an initial parameter vector  $\mathcal{P}$  equal to the observed market prices and quantities, and we update the parameter vector according to:

$$\mathcal{P}' = \mathcal{P} - (\text{diag}[\mathcal{J}_H])^{-1} H(\mathcal{P}).$$

We continue to iterate until convergence.