### A Q-Theory of Inequality

Matthieu Gomez and Émilien Gouin-Bonenfant

Remarks by Nicolae Gârleanu UC Berkeley-Haas

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## Summary

- ▶ Theoretical model:
  - Observation: a lower interest rate increases the growth rate of the value of net-investment projects
  - Implications for inequality, focus on Pareto exponent
- Empirical exercise:
  - Estimate Pareto exponent from Forbes data
  - Estimate model-implied interest-rate effect, cet par
  - Significant proportion (> 0.5) of increase in exponent could be due to discount-rate drop

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- ▶ Paper: investment-driven growth over potentially long time

#### Model

- Investment i per unit of capital; capital grows at rate g, becomes consumption good at Poisson  $(\delta)$
- ▶ Value of a share grows at expected rate r:

$$r = -\frac{i}{q} + g + \delta \left(\frac{1}{q} - 1\right)$$

- ▶ Conditional on continued growth, value-growth rate > r (q > 1)
- ▶ In fact, g i/q decreases in r (b/c increases in q)
- ▶ Given consumption at rate  $\rho$ , wealth growth is  $\gamma = g i/q \rho$ , and depend on r given by

$$\partial_r \gamma = \underbrace{-\frac{i}{q}}_{\text{payout}} \times \underbrace{\partial_r \log(q)}_{\text{duration}}$$

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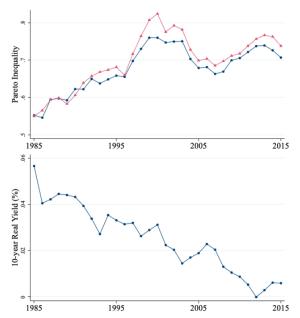
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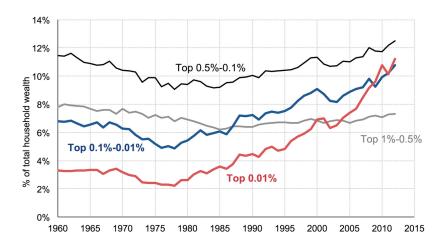
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- ► How long does it take for tail to look different? Transition exercise could seek to characterize tail after 30 years



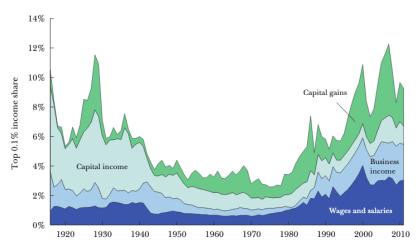
# Pareto Inequality vs Interest Rate



## Saez-Zucman (2016) Wealth Shares



## Top 0.1 Percent Income Share



 ${\it Source:} \ These \ data \ are \ taken \ from \ the \ "data-Fig4B" \ tab \ of \ the \ September \ 2013 \ update \ of \ the \ spreadsheet \ appendix \ to \ Piketty \ and \ Saez \ (2003).$