Bubbles and the Value of Innovation†

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Abstract

Episodes of booming firm creation often coincide with intense speculation on financial markets, leading to what has been described as bubbles—increases in firm entry and private values followed by a crash. This paper provides a framework that reproduces these facts and shows how speculation changes the private and social value of firm entry. Our model predicts that based on market-based measures, speculation increases the private value of innovation and reduces negative spillovers to competing firms. However, neither of these effects occur in outcome-based measures of value. We confirm that these predictions hold empirically using over a million patents issued between 1926 and 2010. Furthermore, we show that speculation can reverse the sign and response of market-based measures of spillovers to changes in industry structure. Our results imply substantially different conclusions about the efficiency of firm entry in the presence of speculation.

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1 Introduction

Episodes of booming innovation often coincide with intense speculation in financial markets, leading to periods that have been described as bubbles: large increases in firm creation and market values followed by a crash (e.g. Scheinkman, 2014). This paper studies the implications of speculation for the private and social value of innovation, quantities that are central for our understanding of how innovation spreads through the economy and its efficiency.

We propose a model of endogenous firm entry in which investors disagree about which firms are more likely to succeed. Because of this disagreement, investors speculate in different firms and the pattern of a bubble emerges. The framework generates predictions about the private and social value of innovation during these episodes, which we test using over a million patents issued between 1926 and 2010. Most importantly, in both our model and the data, we find that during bubbles i) the increase in market value of a firm following the introduction of a new patent is larger, ii) the relative spillover to the market value of competitors is smaller, and iii) this pattern in *market-based* measures of private and social value does not occur in *outcome-based* measures focusing on patent citations or output. As a byproduct, we show that the presence of disagreement meaningfully alters standard conclusions about the efficiency of firm entry.

To understand the role of speculation during innovative episodes, we study a model in which new ideas are implemented in firms that compete with each other. Our setting allows for a rich range of firm interactions while remaining parsimonious enough to be tractable and yield clear predictions. In the first stage, firms are created by raising money on financial markets. In the second stage, competition and production occur. In our baseline, competition takes a simple form: only a fixed number of the best firms get to produce. The novel ingredient of our theory is that investors (agree to) disagree about which firms will be more productive. Each investor focuses her portfolio on her favorite firms, and therefore values her investments more than her average beliefs.[1] This mechanism increases valuations and incentivizes more firms to enter. In the second stage, not all investors can be correct: prices drop and many firms fail. This rise and fall as a result of speculation echoes broad narratives of bubble episodes, as well as some of their more subtle features. Disagreement is more natural in the face of new ideas so these episodes should be more likely following the introduction of a broad new technology (Brunnermeier and Oehmke, 2013; Scheinkman, 2014), or among younger firms (Greenwood, Shleifer, and You, 2018). In a dynamic extension of the model, we show that they are also related to more intense trading volume (Hong and Stein, 2007; Greenwood, Shleifer, and You, 2018).

Our framework for heterogeneous beliefs overcomes the challenge of analyzing the private and social value of firms in the presence of disagreement. Market participants agree on the aggregate distribution of firms—and therefore all macroeconomic outcomes—but disagree on the relative positions of specific firms in this distribution. Each firm faces the same distribution of beliefs across investors even though the identity of these investors differs, yielding a symmetric equilibrium in the first stage. This symmetry gives rise to clear notions of private and social value defined in both market-

[1] Van den Steen (2004) studies how disagreement and choice lead to optimism.

based and outcome-based ways. The market-based approach considers prices of firms in financial markets. Under this criterion, the private value of a firm is its price, while the social value of that firm is the amount by which its introduction changes the price of all firms in the economy.[2] From an outcome-based perspective, the private value is the average output of a firm, while the social value is how much the introduction of a firm changes total output in the economy.

The model makes two predictions about the market-based value of firms. Firstly, it predicts that speculation increases the market-based private value of firms. While the aggregate productivity distribution of new firms is unchanged, investors have higher valuations of their own firms because they expect that these firms will be more productive than the average firm in the economy. Secondly, it predicts that the wedge between the market-based social and private values of a firm is attenuated by speculation. Each new firm introduces a *business-stealing effect* as it displaces other firms. With speculation, investors believe they are investing in the most productive firms in the economy, making them less concerned about being displaced by a new firm. Their valuation of their own firms is thus impacted less by any additional firm entry. In contrast, the outcome-based measures of value unchanged since speculation affects neither the aggregate productivity distribution nor the competitive structure.

History offers an example of these ideas. In 1686, William Phips secured funding in England from the 2nd Duke of Albemarle and his syndicate to search for sunken Spanish ships in the Bahamas. His expedition found 34 tons of treasure, yielding large returns to his investors and creating considerable speculation around treasure hunting technology. Seventeen patents for ways to recover underwater bounty were registered between 1691 and 1693, a large spike in patenting activity for the time. Based on each of these innovations, numerous firms were introduced on equity markets and raised large amounts of capital despite competing for what was clearly a small pool of treasure. These high valuations were consistent with our notion of high market-based value that is unaffected by competitive spillovers in presence of speculation. The boom was so large that it is sometimes credited for the emergence of developed equity markets in England. However, these expeditions only succeeded in finding a few cannons, reflecting no increase in the outcome-based value of the new patents and firms.

To verify the model's predictions empirically, we use the universe of patents issued by public firms between 1926 and 2010 to measure the relationship between innovation and market values. We follow the empirical literature in constructing empirical counterparts to the private and social value in our model. Specifically, we measure the market-based private value of new innovations using the stock returns of issuing companies in the days following the patent approval, as in Kogan et al. (2017). We estimate the market-based social value relative to private value of new innovations by using the response of firm valuations to innovations by their competitors, controlling for innovation by technologically related firms, as in Bloom, Schankerman, and Van Reenen (2013). We proxy for speculation by isolating bubbles episodes at the industry-year level following Greenwood, Shleifer, and You (2018).

During a bubble, the market-based private value of innovation increases by 30% at the patent level and between 40% to 50% at the firm level, corroborating the model's

[2] In our richer specifications, we also incorporate effects on other participants in the economy.

prediction that speculation increases private value. The effect is smaller for firms that are involved in more industries. Intuitively, an investor is likely to have a range of views about a firm's different products, hence reducing the effect of speculation about a given product on firm stock prices. The change in market-based private value is not accompanied by a corresponding change in outcome-based private value. In particular, we show that the increase in number of citations is small relative to the increase in market-based private value, reflecting little change in the quality of innovation in bubbles.

The data also support the prediction that speculation dampens market-based but not outcome-based measures of spillovers. In particular, the estimated coefficients suggest that during bubbles the business-stealing effect measured by asset prices completely vanishes, which occurs in our model asymptotically as the level of speculation increases. On the other hand, we do not see any empirical effect of speculation on the business-stealing effect measured through sales.

We obtain an additional set of predictions in a general equilibrium version of our model, which incorporates a decreasing returns to scale production technology that uses an input in fixed supply. The input market introduces two new spillovers—an *appropriability externality* due to the surplus from firm revenue accruing to workers and a *general equilibrium externality* from the impact of firm entry on equilibrium input prices. While the market-based measure of the appropriability externality is dampened by speculation, the general equilibrium externality is unaffected. As a result, the effect of industry characteristics on the market-based wedge between social and private value is reversed. In addition, while these industry characteristics are sufficient statistics for the wedge under agreement, with speculation the wedge depends on the microeconomic structure of the economy. Finally, speculation can reverse the sign of the wedge.

Our results also have normative implications. The market-based measure of social value coincides with the objective of a planner under the Pareto criterion, which respects the beliefs of households. On the other hand, the outcome-based measures of value capture the paternalistic criterion, under which the planner maximizes aggregate consumption net of entry costs. Data on forecast dispersion or portfolio holdings can help discipline such policy analysis by identifying and providing a finer measure of the speculation we consider in our model.

In Section 2, we introduce our model of speculation with business stealing. In Section 3, we derive predictions for private and social value, which we verify empirically in Section 4. We extend the analysis to general equilibrium in Section 5 and consider the normative implications of our model in Section 6. Finally, Section 7 concludes.

2 Speculation and Business Stealing

In this section, we introduce our framework for speculation. Households speculate over claims to future profits of heterogeneous firms, generating patterns in asset prices and trading volume that are consistent with stylized facts about bubbles. We focus primarily on a static model to simplify the exposition, but also discuss an extension that incorporates dynamics.

We illustrate the effect of speculation on the value of innovation by embedding our framework in a partial equilibrium model that focuses on the business-stealing effect.

The predictions of the model are corroborated by the empirical evidence in Section 4. Further predictions are obtained in Section 5, which considers a wider range of externalities.

2.1 Model setup

Time is discrete. Firms are created at date $t = 0$ and produce at date $t = 1$. The economy is populated by households, firms, and firm creators. The commodities in the economy are a date-0 consumption good in fixed supply, blueprints produced by households to create firms, and a date-1 consumption good produced by firms.

2.1.1 Firm creators

There is a continuum of short-lived firm creators who operate a technology at date 0 to create firms that will produce at date 1. Each firm creator can use a unit blueprint to create a new firm, which is then sold on competitive financial markets at date 0. We will describe shortly how speculation occurs in these financial markets.

Firm creators do not have any information about the firms they create. They participate in competitive markets for blueprints and firms, taking their respective prices p_b and $p_{i,0}$ as given. The firm creator problem at time $t = 0$ is therefore:

$$\max_{c \in \{0,1\}} c \cdot (p_{i,0} - p_b). \tag{2.1}$$

Firm creators are owned by households.

2.1.2 Firms

A continuum of firms, indexed by i and with total mass M_e, is created in equilibrium. At date 1, firms enter the production stage and their productivity a_i is revealed. We assume that only the most productive mass M of firms is able to produce. This allocation of production slots is the key assumption to capture the business-stealing externality, as firms do not internalize that they might take up the slot of another firm.. We concentrate on situations where $M_e < M$, so that fewer firms produce than are created.[3]

The assumption of a fixed mass of production slots is especially plausible in industries that depend heavily on innovation. For instance, intellectual property law often provides exclusive use of a technology to its inventor. We can interpret the fixed production slots as corresponding to a fixed number of M processes to produce the homogeneous good. The first firm to discover a process gets its exclusive use, and the speed of discovery is perfectly correlated with the productivity type a. Alternatively, we can assume that to produce, a firm needs one unit of an indivisible good that has not been discovered yet, and that only M of those exists in nature. Again firms with a higher type a find the ingredient faster.[4] Nevertheless, we show in Appendix C that our results hold for a wide range of models of business stealing.

[3] The case where $M_e = M$ is straightforward to analyze.

[4] Network goods or industries facing institutional constraints can face similar frictions in the allocations of productive positions that lead to a business-stealing effect—see Borjas and Doran (2012) for evidence in the context of scientific research.

Given the cumulative distribution function (cdf) of productivities in the population F, only firms above a cutoff $\underline{a}$ are able to produce, with:

$$\underline{a} := F^{-1}\left(1 - \frac{M}{M_e}\right). \tag{2.2}$$

The profit function of a firm with productivity a is given by:

$$\pi(a) = a^{\eta} \cdot \mathbf{1}\left\{a \geq \underline{a}\right\}, \tag{2.3}$$

where η determines how differences in productivities translate into differences in generated profits, and the indicator function captures whether the firm produces or not.[5] We assume the population distribution F follows a Pareto distribution: $F(a) = 1 - a^{-\gamma}$ for $a \geq 1$.

2.1.3 Households

There is a unit mass of households indexed by j. At date 0, household j is endowed with a fixed unit of consumption good c_0 and her share of firm creators. In addition, each household decides how many blueprints to supply, b_j. Blueprints are produced at a convex cost $W(b_j) = f_e b_j^{\theta+1} M^{-\theta}/(\theta+1)$, where the parameter θ controls the elasticity of supply of blueprints and f_e is the level of production costs. Finally, the household also decides on the number of shares to invest in each firm on financial market, $\{s_i^j\}_i$. Households have heterogeneous beliefs about the distribution of productivity a_i for each firm i, which we describe in detail shortly. We assume that they can only take long positions in claims to firms. Households behave competitively and take prices as given. Hence, household j solves the problem:

$$\max_{c_0, s_i^j \geq 0, b_j} \; c_0 + \mathbf{E}^j\left\{\int s_i^j \pi_i di\right\} - W(b_j) \tag{2.4}$$

$$\text{s.t.} \;\; c_0 + \int s_i^j \; p_i \; di \leq 1 + p_b b_j + \Pi, \tag{2.5}$$

where $\mathbf{E}^j$ is the household j expectation and Π denotes firm creators' aggregate profits.

2.1.4 Beliefs

To capture the notion of speculation, we assume that households have heterogeneous beliefs about which firms will be successful even though they agree on the population distribution of firm productivity.

We model the disagreement by assuming that each household organizes firms into a continuum of packets containing n firms each, and believes she knows the exact ranking of productivity draws within each packet. We assume that the composition of packets and the order of firms within packets is drawn in an i.i.d. equiprobable fashion across agents and firms, and that each firm can only be in one packet. The parameter n controls

[5]The fact that the marginal active firm collects positive profits improves tractability. We show in Appendix A that our results also hold in a variant of the model where the marginal firm earns zero profits.

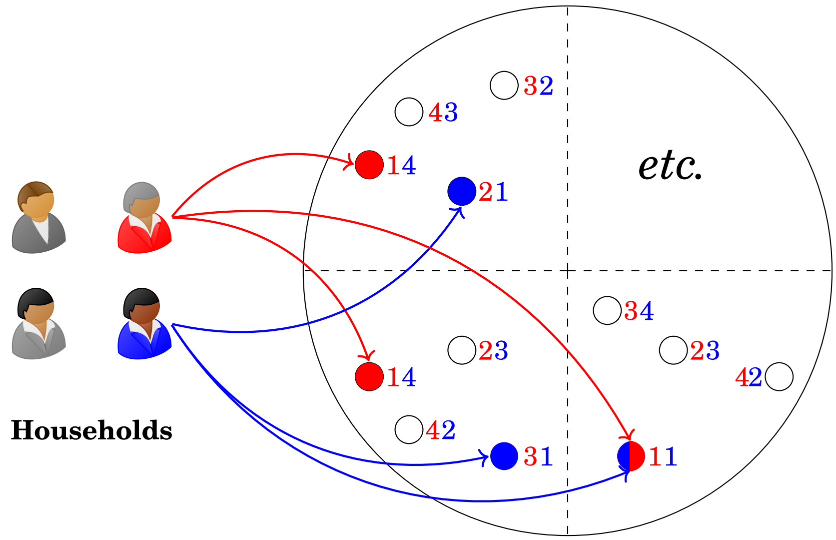

Figure 1
Firms and households beliefs for $n = 4$.[6]

the intensity of disagreement. When $n = 1$, households consider all firms to be the same, with productivity drawn from F. As n increases, households can compare more firms in each packet, and thus have a stronger prior that the best firm in each packet will have a high productivity.

In equilibrium, household j only invests in the subset of firms that she considers to be the most productive in their respective packets. These firms are perceived by household j to have productivity drawn from F_n, the distribution of the maximum of n independent draws from F. Since households rank firms differently, they have different beliefs about the productivity distribution of any given firm and invest in different sets of firms, as illustrated in Figure 1.

Two features of the framework facilitate our analysis. Firstly, while households disagree on which firms will succeed, they agree on the population distribution of firms. Hence, they agree on aggregate outcomes: both the threshold $\underline{a}$ and market conditions, which play a role in the richer settings in Section 5. Secondly, beliefs are symmetric across households. This alleviates the typical issue of having to keep track of the entire distribution of beliefs and allocations. Qualitatively, our results do not depend on the specific functional forms here or the description of households organizing firms in packets. In particular, the assumptions that F follows a Pareto distribution and that households perceive firms they invest in to have productivity distribution F^n are imposed primarily for mathematical tractability. Instead, the economic mechanisms rely on households believing that the firms they invest in are, in expectation, more productive than the average firm in the economy.

Our model for disagreement is motivated by the empirical observation that high firm

[6] The composition of each packet is identical across agents for exposition purposes and does not affect results.

entry in a sector often follows disruptive innovation, either through a large technological change or the introduction of new products. New firms then conduct micro innovations to take advantage of a macro innovation (Mokyr (1992)). With limited information ex ante about these breakthroughs, investors rely heavily on their priors. These episodes of high firm entry also tend to coincide with effervescent financial markets. The heterogeneous beliefs in our model capture the lack of information in such episodes. In particular, higher disagreement n leads to a higher level of firm entry M_e and higher asset prices.

Our model of speculation captures the salient features of asset price bubbles associated with such high entry episodes, where a "bubble" can be defined as a situation in which asset prices exceed an asset's fundamental value (see, for example, Brunnermeier (2017)). The presence of asset price bubbles with the arrival of new technologies is well-documented. Examples include railroads, electricity, automobiles, radio, micro-electronics, personal computers, bio-technology, and the Internet. Scheinkman (2014) and Brunnermeier and Oehmke (2013) provide surveys. Janeway (2012) gives a first-person account of the relationship between innovation and speculation. Finally, Greenwood, Shleifer, and You (2018) find that when the price run-up in an industry occurs disproportionately among the younger firms, crashes and low future returns are more likely. These are situations in which one would expect greater disagreement about which firms will have high profits in the future.

The current state of the Tech sector is representative of such high entry episodes. The development of large-scale social networks and new forms of communication technologies has fostered the creation of a slew of creative firms. These firms are financed at unusually high valuations in a decentralized manner through venture capital. It is difficult to predict which of the ideas will be successful in the long run, and different financiers bet on different firms.

2.1.5 Competitive equilibrium

The competitive equilibrium of this economy is defined as follows. Firm creators maximize profits from selling their firms taking wages as given. Households maximize their perceived expected utility by choosing their optimal blueprint discovery effort and a dynamic optimal portfolio allocation, taking the price of blueprints and of firms as given. Firms maximize profits given their production status. Market clear for blueprints and claims to firms profits (the stock market) respectively:

$$\int b_j dj = M_e,$$

$$\forall i \in [0, M_e], \qquad \int s_i^j dj = 1.$$

Combining the equilibrium conditions yields the following equilibrium entry condition:

$$W'(M_e) = V^{(n)}(M_e) = \int_{\underline{a}}^{\infty} \pi(a) dF^n(a). \tag{2.6}$$

The marginal cost of creating an additional firm, $W'(M_e)$, is equal to the expected profits to an investor who favors it, $V^{(n)}(M_e)$. With our simple specification of profits, we have $V^{(n)}(M_e) = \mathcal{I}_n(M_e, \eta)$, where we define:

$$\mathcal{I}_n(M_e, \eta) := \int_{F^{-1}(1-M/M_e)}^{\infty} a^{\eta} dF^n(a), \tag{2.7}$$

which will play a crucial role in our analysis.

2.1.6 Dynamics

We can incorporate dynamics by assuming that, instead of producing at date 1, firms produce at some stochastic date T. At each date t, the economy enters the production stage with probability ψ^{-1}, and stays in the development stage with probability $1 - \psi^{-1}$. At each date $t < T$ before production, firms engage into some development activity with productivity $a_{i,t}$ that is unknown. With some probability ϑ, they change their activity and receive a new productivity draw independent from their past productivity.[7] With probability $1 - \vartheta$, firms' productivity stays the same as last period and $a_{i,t} = a_{i,t-1}$.

This extension has implications for asset price dynamics and trading volume, which we discuss in Section **??**. However, introducing dynamics does not change our main results on speculation and the value of innovation. We thus focus on the static model for simplicity.

3 The Value of Innovation

In this section, we introduce various notions of the private and social value of innovation and analyze how they are affected by speculation. The model shows the distinction between *market-based* measures of value that use asset prices and *outcome-based* measure of value that use ex-post citations or output. The two measures are identical under agreement but differ in the presence of speculation.

3.1 Private value

The private value of a firm is the value of that firm to its investors. In the model, the private value in date 0 is captured by the price p_i of the firm, while the private value in date 1 is the realized profit $\pi(a)$. Empirically, asset prices measure the date-0 value, and ex-post outcomes such as profits and output measure the date-1 value. Therefore, measuring the value of innovation using stock prices, as in Kogan et al. (2017), yields the empirical counterpart of $\mathcal{I}_n$. In contrast, the average ex-post outcome in the data is a measure of $\mathcal{I}_1$.

An increase in disagreement n raises the market-based private value $\mathcal{I}_n$ relative to outcome-based private value $\mathcal{I}_1$. This generates a bubble as the total value of traded assets $\mathcal{I}_n$ exceeds the realized total output of the economy $\mathcal{I}_1$ even though all investors agree that the market portfolio is overpriced. The overvaluation manifests through a

[7] The change of firms' activity before production captures, for instance, the pivot of startups in the early stages of their development.

crash in the value of firms when productivities are realized. Such a situation arises from households' heterogeneous beliefs. Different households view different firms as the most valuable, and specialize their portfolios in those firms, increasing firm prices. The short-sale constraint prevents each household from shorting firms she views as overpriced.[8]

The empirical literature has documented several facts that are consistent with these results. Using investor survey data to quantify disagreement, Diether, Malloy, and Scherbina (2002) find that stocks with dispersed analysts forecasts experience low subsequent returns. Yu (2011) aggregates this measure to portfolios such as the market and finds a similar result. Using stock market positions to measure disagreement, Chen, Hong, and Stein (2002) find that the fraction of the mutual fund population investing in a given stock, a measure of the breadth of ownership for individual stocks, They find that this measure predicts low stock returns, again in line with our model of disagreement. In Section 4, we provide further empirical evidence by studying the private value of innovations during bubble episodes.

The dynamic extension introduces another source of overvaluation: the price of each firm exceeds the maximum valuation of its cash-flow by any specific investor in the economy. At any point in time, an investor ranking the firm first in a packet attains this maximum valuation, which we denote $p_{i,t}^{max}$. The difference between the two valuations is

$$p_{i,t} - p_{i,t}^{max} = (\mathcal{I}_n - \mathcal{I}_1) \cdot \psi \, \frac{\vartheta(\psi - 1)}{1 + \vartheta(\psi - 1)} > 0.$$

This difference comes from the fact that when firms change their activities, the current investor will typically not favor firm i anymore. Each time households exchange firms signals a change in who values them most, a mechanism reminiscent of the models of Harrison and Kreps (1978) and Scheinkman and Xiong (2003). On average, a household investing in firm i values it as a typical firm in the economy, with $\mathcal{I}_1$, rather than a favorite with $\mathcal{I}_n$.

This dynamic overvaluation is increasing in the volume per period ϑ and the length of the bubble ψ.[9] Historically, abnormally high trading volume is seen as hallmark of bubbles—see Scheinkman (2014) for a survey. For example, during the Roaring Twenties, daily records of share trading volume were reached ten times in 1928 and three times in 1929, with no new record set until 1968 (Hong and Stein (2007)). More recently, during the DotCom bubble. Internet stocks had three times the turnover of similar cases. Greenwood, Shleifer, and You (2018) also document that large stock price increases are more likely to end in a crash when they are accompanied by increased trading volume.

3.2 Social value

The social value of a firm measures the spillovers from that firm to the rest of the economy. The entry of a firm i in the model affects the date-0 price $p_{i' \neq i}$ of other firms through

[8] Van den Steen (2004) describes how disagreement combined with optimal choice leads to overvaluation and Miller (1977) first pointed out the importance of short-sale constraints in financial markets.

[9] For instance, when $\vartheta \ll \psi^{-1} \ll 1$, the overvaluation is proportional to $\vartheta\psi^2$.

the business-stealing effect, as investors account for the possible displacement of their firms by firm i. These market-based *value spillovers* are measured empirically by the change in a given firm's market value in response to the entry of an additional competing firm. In date 1 of the model, the spillovers are instead captured by the change in the average profit of firms $\mathcal{I}_1$ from the additional firm entry. To measure these outcome-based *real spillovers* in the data, one would estimate the effect of firm entry on variables such as sales or output.

In the model with agreement, there is no distinction between real and value spillovers, since the price of claims to a firm in period 0 is equal to the mean firm profits in period 1. Speculation introduces a gap between the two spillover measures. In what follows, we derive theoretical results about each type of spillover.

3.2.1 The entry wedge

The real and value spillovers in the model can be captured through the problem of a planner maximizing household's perceived utility (under expectations $\mathbf{E}^j[\cdot]$) by choosing the level of entry. This corresponds to maximizing $M_e V^{(n)}(M_e) - W(M_e)$, the perceived total expected profits minus the cost of effort for firm creation. The *entry wedge*, defined as the tax that implements this optimal level of entry, captures the social value of firm entry. In particular, we will show how the entry wedge with and without disagreement maps to real and value spillovers with and without speculation.

The first-order condition of the planner's problem characterizes optimal entry:

$$W'(M_e) = V^{(n)}(M_e) + M_e V^{(n)'}(M_e) \tag{3.1}$$

The planner equalizes the marginal cost of an additional firm, to its value $V^{(n)}(M_e)$ plus the effect it has on the value of the existing firms $M_e V^{(n)'}(M_e)$.

A simple way to implement this level of entry is to use an entry tax. Suppose firm creators pay out a fraction τ of their revenue if they sell a firm, and the total equilibrium proceeds $\mathcal{T}$ from the tax are rebated lump-sum to households. The equilibrium entry condition with such a tax becomes $W'(M_e) = (1-\tau)V^{(n)}(M_e)$, which leads to the *entry wedge* that reconciles the competitive allocation with optimal entry:

$$\tau = \tau_n(M_e) = \frac{-M_e V^{(n)'}(M_e)}{V^{(n)}(M_e)}. \tag{3.2}$$

The wedge is the negative of the elasticity of firm value to firm entry, $-\mathcal{E}_{\mathcal{I}_n} > 0$.[10] It exactly offsets the negative business-stealing externality imposed on incumbents by new entrants. As new firms enter, existing firms are displaced as the cutoff for production increases, which lowers the value of existing firms. The wedge is the value of this change applied to all firms relative to the value of one firm, which is captured by the elasticity.

Without speculation, both the real and value spillovers are captured by τ_1. We will show that with speculation, τ_1 continues to capture real spillovers as it does not depend on entry M_e. In contrast, τ_n captures value spillovers in the presence of speculation,

[10] Throughout the paper, we denote the elasticity of quantity X to firm entry M_e by $\mathcal{E}_X = d\log(X)/d\log(M_e)$.

as households value the firms they invest in under the distribution F_n instead of the population distribution F. The entry wedge τ thus allows us to study the properties of real and value spillovers. We follow the common practice of evaluating the tax at the competitive equilibrium outcome to simplify the analysis and leave all proofs to Appendix A.

3.2.2 Real spillovers

The wedge under agreement, which captures the real spillovers with and without speculation, is

$$\tau_1(M_e) = \frac{\gamma - \eta}{\gamma}. \tag{3.3}$$

The wedge captures the business-stealing effect. It is increasing in γ and decreasing in η. For intuition, recall that the wedge comes from the value of displaced profits relative to expected profits. The former is determined by the quality of marginal firms, while the latter comes from the full distribution of productivity above the cutoff $\underline{a}$. As γ falls, the tail of the distribution becomes fatter, and the wedge decreases because the value of the marginal firm falls relative to the average firm. Similarly, when η is large, small differences in productivity translate into large differences in profits, increasing the difference between the marginal and average firms.

The wedge does not depend on entry M_e because under a Pareto distribution, the ratio between marginal and average productivity is independent of the lower cutoff. In Section 4, we verify that the presence of speculation does not affect the level of real spillovers τ_1 in the data.

3.2.3 Value spillovers

The following proposition states that the value spillovers are smaller with disagreement ($n > 1$) than with agreement ($n = 1$). As a result, the presence of speculation reduces value spillovers relative to real spillovers. In Appendix A, we show that the result holds more generally with minimal assumptions on the productivity distribution $F(\cdot)$ and profit function $\pi(\cdot)$. Section 4 shows corroborating empirical evidence that value spillovers τ_n decrease during bubbles.

Proposition 1 (Disagreement lowers business stealing)**.** *With speculation, value spillovers are smaller than real spillovers:*

$$\tau_n \leq \tau_1. \tag{3.4}$$

Under disagreement, value spillovers are smaller despite the level of firm entry being higher. This seeming contradiction arises because the beliefs of households impact their valuation of equilibrium allocations. Under disagreement, households only invest in their favorite firms. Therefore, each household places a lower probability on being displaced by new entrants, thereby reducing the effect of the business-stealing externality on the market prices that determine the value spillovers.

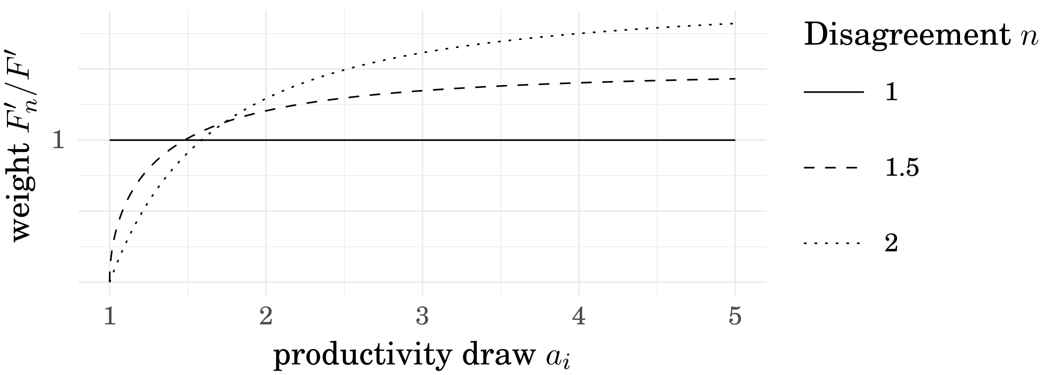

Figure 2
Distortion of productivity weights F_n'/F'

More formally, we can rewrite the wedge from (3.2) in its integral form:

$$\tau_n(M_e) = \frac{-M_e V^{(n)\prime}(M_e)}{V^{(n)}(M_e)} = \frac{\displaystyle\int_{\underline{a}}^{\infty} \pi(\underline{a}) \frac{F_n'}{F'}(\underline{a})\, dF(a)}{\displaystyle\int_{\underline{a}}^{\infty} \pi(a) \frac{F_n'}{F'}(a)\, dF(a)}.$$

Holding M_e and thus $\underline{a}$ constant, consider how this ratio changes with n. The numerator, which is the expected value of displaced firms, is affected by the change in probability weights F_n'/F' at the threshold $\underline{a}$. In contrast, the denominator, which is the expected profits of a firm, is affected by changes in the probability weights throughout the distribution above the threshold. Because increasing n corresponds to shifting the perceived distribution of productivities to the right, the change in probability weights is increasing as we move to higher productivities (see Figure 2). Such an increase affects expected profits more strongly than the value of displaced firms, decreasing the wedge.

High speculation limit. We now describe the behavior of value spillovers in an economy with high speculation by considering the limit as disagreement increases, $n \to \infty$. The proposition derives a condition under which value spillovers vanish asymptotically. Under this condition, although prices and firm entry increase with speculation, the value spillovers vanish. When the condition is not satisfied, the wedge converges instead to the wedge under agreement. Figure 3 illustrates these cases. Appendix A shows that the results hold in more general models of business stealing.

Proposition 2 (Value spillovers with high speculation)**.** *In the high disagreement limit ($n \to \infty$), the value spillovers converge to a finite limit that depends on the sign of $\gamma\theta - \eta$:*

- *If $\gamma\theta > \eta$, the value spillovers vanish:*

$$\lim_{n\to\infty} \tau_n = 0 \tag{3.5}$$

- *If $\gamma\theta < \eta$, then τ converges to the wedge in the agreement case ($n = 1$):*

$$\lim_{n\to\infty} \tau_n = \frac{\gamma - \eta}{\gamma} \tag{3.6}$$

- *In the knife-edge case of $\gamma\theta = \eta$,*

$$\lim_{n\to\infty} \tau_n = \check{\tau} < \frac{\gamma - \eta}{\gamma}, \tag{3.7}$$

  where $\check{\tau}$ is defined in appendix equation (A.15).

Two forces determine the asymptotic behavior of the value spillovers. Firstly, with more disagreement, investors increasingly believe that the firms they invest in are in the right tail of the productivity distribution. They are therefore less concerned about the risk of being displaced by new entrants, as they expect that a smaller mass of firms in their portfolio will fail to meet the entry threshold. For a given level of entry M_e, this mass converges to 0 as n goes to infinity. This is an extreme case of the result in Proposition 1.

Secondly, disagreement increases firm entry. For a given level of disagreement, n, as M_e converges to infinity, the M producing firms end up in the tail of both the population distribution, F, and the favorite-firm distribution, F_n. The tails of these two distributions have the same shape since $\lim_{x\to\infty} F_n'(x)/F'(x) = n$. Therefore disagreement does not affect the relative position of the marginal and average valuation in the tail. This force brings the value spillovers back towards the level of real spillovers.

The relative strength of the forces depends of how fast firm creation increases with speculation. If $\gamma\theta > \eta$, the first force dominates. When θ is large, the marginal cost of firm creation rises more rapidly, which reduces equilibrium entry M_e and weakens the second force. As γ increases, we have a thinner tailed firm productivity distribution. The size of the tail becomes less important than the relative ordering of firms, which weakens the second force. As η decreases, profits increase less with productivity, which again diminishes the importance of the second force. In the data, we find that the value spillovers vanish during bubbles, consistent with the $\gamma\theta > \eta$ case.

4 Empirical Evidence

We now provide empirical evidence for three predictions from our model. Firstly, market-based private value $V^{(n)}$ increases with speculation. Secondly, value spillovers decrease in the presence of speculation. Finally, these increases are not observed in outcome-based measures of both private and social value.

4.1 Data

We use the economic value of innovation from Kogan et al. (2017) and the industry spillover measure of Bloom, Schankerman, and Van Reenen (2013) as our measures of private and social value respectively. Summary statistics of the main regression variables are contained in Table 1.

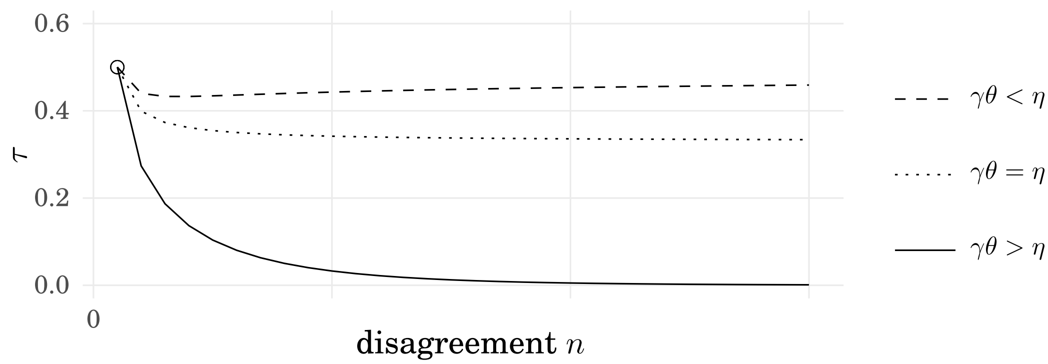

Figure 3
Entry wedge with increasing disagreement.

Speculation. We proxy for the presence of speculation using the classification of bubbles from Greenwood, Shleifer, and You (2018). Bubbles are defined as episodes in which an industry experienced a 100% of more increase in returns over two years. Consistent with our model, Greenwood, Shleifer, and You (2018) find that "price run-ups...involving younger firms (and) having higher relative returns among the younger firms...(are) more likely to crash". These industries with young and innovative firms are likely to have scarce information, making them more susceptible to the disagreement that we model. We also confirm the validity of the bubble classification empirically and show that it predicts innovation like the model. In Appendix Table E.1, we show that there are 1.4 more patents issued within a USPTO technology class during bubbles, which translates to a 15% increase in the number of patents created in a patent class during a bubble.

Private value of innovation. Kogan et al. (2017) combine stock market and patent data for U.S. firms for the period from 1926 to 2010. They measure the stock market response in the three-day window after a firm has a new patent issued, controlling for the return on the market portfolio during that period. This corresponds to the *private economic value* of a new innovation in our model. They also aggregate the stock market value from all the patents of a given firm every year to measure the level of innovation at the firm level. To capture the quality of each patent, Kogan et al. (2017) use the forward looking number of citations generated by a patent —or the number of citations generated by all the patents produced by a firm in a given year.

The private economic value from Kogan et al. (2017) corresponds to the private value of a new firm in our model, since the private value of $V^{(n)}$ of a new firm i is captured by the stock market price $p_{i,t}$ of that firm in equilibrium. In our model, each firm is produced by a single blueprint and each firm corresponds to one innovation. We thus examine how both the patent and firm level measures of innovation depend on whether there is a bubble or not.

Social value of innovation. Bloom, Schankerman, and Van Reenen (2013) measure the different sources of spillovers that affect firms. The authors quantify each firm's exposure to knowledge spillovers, coming from firms issuing patents in the same technology USPTO class, and to product market spillovers, arising from firms in neighboring industries.

Bloom, Schankerman, and Van Reenen (2013) measure the exposure of a firm to spillovers by interacting the innovation of other firms with the proximity in the product market and in knowledge spaces. These exposures are denoted *spillsic* for product markets and *spilltech* for knowledge. Proximity between two firms in the product market space is the correlation of the firms' distribution of sales across their industry segments. The business stealing effect, captured by η in our model, increases with this product market proximity. Technology proximity is analogously defined as the correlation of patent USPTO technology classes between firms, following earlier work by Jaffe (1986). This measure of technology proximity corresponds to knowledge spillovers in a standard growth model, which is captured by the parameter α of our general model below in Section 5. Both measures are extended using the Mahalanobis distance to allow for flexible weighting of the correlation between firms across different technology classes or product market classes. Further details on the measures of spillovers can be found in Appendix E and in Bloom, Schankerman, and Van Reenen (2013).

The entry wedge in our model measures the role of both product market and knowledge proximity for spillovers. Bloom, Schankerman, and Van Reenen (2013) estimate these spillovers by regressing firm level real or price outcomes on both *spillsic* and *spilltech*. Using Tobin's q and firm sales as the dependent variable yields quantitative measures of value and real spillovers, respectively.

4.2 Results

Market-based private value increases in bubbles. To study the effect of speculation on the private value of innovation at the patent level, we consider the following regression specification:

$$\log \xi_j = \alpha + \beta B_j + \gamma Z_j + \varepsilon_j, \tag{4.1}$$

where ξ_j is the private value of the patent and B_j is an indicator for whether the corresponding firm was in an industry experiencing a bubble when the patent was issued. As in Kogan et al. (2017), the controls Z_j include log of the number of citations, market capitalization and year dummies. We take the market capitalization lagged by one year because the value of a firm's stocks tends to rise during a bubble. The lagged market capitalization variable controls for the size of the firm without contaminating our estimate of how private value varies with speculation. To ensure that our results are not driven by the type of firms or industries that go through bubbles, we include firm fixed effects. We run a similar regression at the firm level, replacing the private value and citations variables with their firm level analogs. We no longer control for market capitalization because the firm level measure of private innovation value is normalized by book value. Following Kogan et al. (2017), we include year fixed effects.

Table 2 shows that speculation indeed increases the private value of innovation. In particular, the presence of a bubble increases the private value of innovation by approx-

Table 1
Summary Statistics

	N	Mean	Std. Dev.	25th pct.	Median	75th pct.
Bubble						
Bubble periods dummy	2,734	0.0271	0.162	0	0	0
Value of Innovation (KPSS)						
Patent level value						
Stock Market ($ Mn)	1,171,806	14	37.8	2.32	5.46	12.7
Citations (fwd. looking)	1,171,806	12	22.6	2	5	13
Firm level value						
Stock Market ($ Mn)	47,887	232	1880	0.519	3.06	24.5
Citations (fwd. looking)	47,887	52.9	236	2.88	7.8	26.7
Firm level statistics						
Mkt. Cap. ($ Mn)	47,887	2854	14667	47.2	200	949
Segments (# NAICS-4)	53,066	1.33	0.646	1	1	2
Segments (# NAICS-6)	53,066	1.41	0.729	1	1	2
Measuring Spillovers (BSvR)						
Firm Outcomes (real or market valued)						
Sales ($ Mn)	9,382	3563	12626	135	509	2037
Tobin's q	9,382	2.46	3.09	0.86	1.49	2.71
Measures of Spillovers (Jaffe)						
Technology	9,382	9.8	1.02	9.34	9.97	10.5
Competition	9,382	7.32	2.35	6.33	7.64	9.01
Measures of Spillover (Mahalonobis)						
Technology	9,382	11.4	0.821	10.9	11.5	11.9
Competition	9,382	8.53	1.73	7.87	8.77	9.74

Note: Table 1 presents summary statistics of the main variables included in the regression specifications. The bubble dummy corresponds to bubble detected across Fama-French 49 industries according to the methodology outlined in Greenwood, Shleifer, and You (2018). The value of innovation both at the firm and patent level is directly taken from Kogan et al. (2017). The stock market value of innovation at the patent level corresponds to the appreciation in the value of a firm issuing a patent around the patent issuance date. The stock market value of innovation at the firm level corresponds to an annual aggregation of the total value of all patents issued by a firm in a given year. Both the patent and firm level citation value of a patent corresponds to its forward looking number of citation (until the end of the sample in 2010). Other firm level statistics correspond to the CRSP-Compustat merged file (for market capitalization, sales and Tobin's q) and to the Compustat segment file. Tobin's q is measured from Bloom, Schankerman, and Van Reenen (2013) as the market value of equity plus debt divided by the stock of fixed capital. We obtain measures of technological and competition spillovers from Bloom, Schankerman, and Van Reenen (2013), corresponding to the distance between the technological class of patents issued by a firm with other public firms and to the distance between the set of product market of a firm and other public competitors.

imately 30% at the patent level and 40% to 50% at the firm level. These effects are both economically and statistically significant.

We stress that an increase in the private value of innovation in a bubble is not mechanical. We measure bubbles as times of high stock market valuations, which do not

Table 2
Private value of innovation in bubbles

	Patent Level			Firm Level		
	(1)	(2)	(3)	(4)	(5)	(6)
Bubble dummy	0.317***	0.315***	0.306***	0.514***	0.427***	0.427***
	(0.094)	(0.094)	(0.096)	(0.114)	(0.123)	(0.080)
Log Citations		0.016***	0.023***		0.823***	0.716***
(forward looking)		(0.004)	(0.005)		(0.011)	(0.009)
Log Market Cap			0.562***			0.626***
(lagged)			(0.028)			(0.020)
Fixed Effects	Y, F	Y, F	Y, F	Y, F	Y, F	Y, F
Observations	1,171,806	1,171,806	1,169,860	47,886	47,886	47,484
R^2	0.68	0.68	0.74	0.89	0.94	0.96

Note: Table 2 presents panel regressions of the value of innovation, as measured in Kogan et al. (2017) at the patent and firm levels, on a dummy from Greenwood, Shleifer, and You (2018) that captures whether the firm is an industry that is in a bubble or not. We control for the lagged market capitalization of the firm and include fixed effects for firm F and patent grant year Y. Standard errors clustered at the grant year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

necessarily coincide with a strong positive response to the news of issuing a patent. This positive response is a direct implication of our model where bubbles arise in times of high private value of innovations.

Diversity dampens the effect of speculation. Our model predicts that the effect of a bubble should be smaller for firms that have more diversified activities. Narrow firms, as opposed to firms that span multiple product lines, cater to specific investors which tend to have strong beliefs in their success. Thus, we expect a "conglomerate discount" on the value of innovation in times of bubbles: in the presence of disagreement or bubbles, multiple product firms experience a smaller increase in the value of their innovation than narrow firms.

To test this prediction empirically, we measure diversity with the variable $segments$, which is the number of 4-digit NAICS industries that a firm is in, and augment regression (4.1) with $B_j \times segments_j$.[11] As predicted, we find in Table 3 that the greater the number of segments, the less the private value of innovation increases in a bubble, as indicated by the significantly negative coefficient on the $B_j \times segments_j$ regressor. Moreover, we confirm that the effect of a bubble on the value of a patent for a firm with more than one segment is not statistically significant. Qualitatively, we find the same results whether we run the regressions at the patent or the firm level.

[11]Appendix Table E.3 show that the results are robust to using a definition of different segments using 6-digit NAICS industries.

Table 3
Diversity and private value of innovation in bubbles

	Patent Level			Firm Level		
	(1)	(2)	(3)	(4)	(5)	(6)
Bubble x Segments	-0.612^{***}	-0.600^{***}	-0.533^{***}	-0.473^{***}	-0.402^{***}	-0.307^{***}
(NAICS 4 digits)	(0.157)	(0.155)	(0.148)	(0.109)	(0.080)	(0.069)
Bubble	1.471^{***}	1.441^{***}	1.335^{***}	1.576^{***}	1.431^{***}	1.180^{***}
	(0.198)	(0.192)	(0.205)	(0.267)	(0.323)	(0.288)
Segments	0.309^{***}	0.305^{***}	0.296^{***}	0.062	0.015	0.014
(NAICS 4 digits)	(0.097)	(0.096)	(0.100)	(0.043)	(0.045)	(0.037)
Log Citations		0.047^{***}	0.044^{***}		0.044^{***}	0.044^{***}
(forward looking)		(0.010)	(0.009)		(0.009)	(0.009)
Log Market Cap			0.154^{***}			0.286^{***}
(lagged)			(0.040)			(0.046)
Fixed Effects	Y, F	Y, F	Y, F	Y, F	Y, F	Y, F
Observations	180,636	180,636	177,911	10,426	10,426	10,256
R^2	0.72	0.72	0.72	0.88	0.93	0.94

Note: Table E.3 presents panel regressions of the value of innovation, as measured in Kogan et al. (2017) at the patent and firm levels, on a bubble dummy interacted with the number of industries spanned by the different segments of the parent firm. The bubble dummy is from Greenwood, Shleifer, and You (2018) and captures whether the firm is an industry that is in a bubble or not. Compustat segments are measured at the four digit NAICS code level from the Compustat segment file. We control for the forward looking number of citations generated by a patent (or firm) from Kogan et al. (2017), and the lagged market capitalization of the firm. We include fixed effects for firm F and patent grant year Y. Standard errors clustered at the grant year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Innovation quality does not increase in bubbles. In our model, firm productivity is drawn from the same distribution F in date 1 regardless of the level of disagreement, thus innovation quality does not depend on the bubble. In Table 2 we find that the high private value of innovation during bubbles is not related to changes in innovation quality, as the estimated value of the coefficient on bubbles was not materially affected by including the number citations.

We now show that there is in fact no significant increase in innovation quality during bubbles. In particular, we consider:

$$\log\left(1 + C_j\right) = \alpha + \beta B_j + \gamma Z_j + \varepsilon_j, \tag{4.2}$$

where C_j is the number of citations received by a patent. The controls Z_j now includes log private value $\log \xi_j$, and is otherwise identical to equation (4.1) without log citations. As before, we also run the firm level analog.

In Table 4 shows that the quality of innovation see a slight increase during bubbles. The number of citations is 10% higher in times of bubbles. This increase arises partly from the higher number of patents issued during bubbles.

To show that the increase in citations in small relative to the increase in the stock market value of innovation in bubbles from Table 2, we reproduce estimates from Kogan

Table 4
Number of Citations for Innovation in Bubbles

	Patent Level		Firm Level	
	(1)	(2)	(3)	(4)
Bubble dummy	0.112***	0.113***	0.106***	0.107***
	(0.028)	(0.027)	(0.034)	(0.041)
Log Market Cap		−0.034***		0.234***
(lagged)		(0.008)		(0.008)
Fixed Effects	Y, F	Y, F	Y, F	Y, F
Observations	1,171,806	1,169,860	47,887	47,484
R^2	0.35	0.35	0.78	0.79

Note: Table 4 presents panel regressions of the number of citations (forward looking until the end of the sample), as measured in Kogan et al. (2017) at the patent and firm levels, on a bubble dummy interacted with the number of industries spanned by the different segments of the parent firm. The bubble dummy is measured from Greenwood, Shleifer, and You (2018) and captures whether the firm is an industry that is in a bubble state or not. We control for the lagged market capitalization of the firm and include fixed effects for firm F and patent grant year Y. Standard errors clustered at the grant year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

et al. (2017) in Appendix Table E.4. We find that on average, a 10% increase in citations corresponds to a 0.1% to 1.7% increase in stock market valuation, which is substantially smaller than the direct effect of a 30% increase on stock market valuations. Thus, the quality of innovation as measured by future citations does not see a rise comparable to the increase in stock market valuation, in line with our model predictions.

Only value spillovers are dampened in bubbles. Our model predicts that value spillovers from the product market are dampened during bubbles while real spillovers not. We test this empirically using the empirical specification:

$$\begin{aligned} \log X_{i,t} = \; & \alpha + \beta \left(B_{i,t} \times \log spillsic_{i,t}\right) + \gamma_1 \log spillsic_{i,t} \\ & + \gamma_2 \log spilltech_{i,t} + \gamma_3 B_{i,t} + \delta Z_{i,t} + \varepsilon_{i,t}, \end{aligned} \tag{4.3}$$

where $X_{i,t}$ is either the market value (Tobins's q) or output of the firm (sales normalized by an industry price index) and $B_{i,t}$ is an indicator of whether firm i is in an industry facing a bubble in year t. The controls $Z_{i,t}$ are taken from Bloom, Schankerman, and Van Reenen (2013) and include firm and year fixed effects. Value spillovers, obtained from taking $X_{i,t}$ to be the market value of the firm, correspond to the wedge under disagreement, which falls with speculation as investors ignore the business-stealing effect. On the other hand, real spillovers, measured by taking $X_{i,t}$ to be output, correspond the wedge under agreement, regardless of the level of speculation.

Table 5 shows that the regression results are consistent with our model's predictions. The coefficient on the interaction term captures the change in spillovers accompanying an increase in speculation. The coefficient is significantly positive in the value spillover regressions, indicating a reduction in the business-stealing effect. Moreover, the point estimate is larger than the coefficient on $spillsic$, suggesting that the business-stealing effect vanishes during bubbles, consistent with our high speculation asymptotics in Proposition 2 with inelastic entry ($\gamma\theta > \eta$). In contrast, the presence of a bubble does not affect real spillovers, as predicted by our model.

Table 5
Social Value of Innovation in Times of Bubbles

	Value Spillovers		Real Spillovers	
	Jaffe (1)	Mahalanobis (2)	Jaffe (3)	Mahalanobis (4)
Bubble x Spill-SIC	0.152***	0.200***	0.004	−0.000
	(0.027)	(0.037)	(0.009)	(0.013)
Spill-SIC	−0.088***	−0.103***	−0.021***	−0.021**
	(0.016)	(0.033)	(0.006)	(0.010)
Spill-Tech	0.405***	0.844***	0.175***	0.159***
	(0.145)	(0.174)	(0.025)	(0.040)
Fixed Effects	Y, F	Y, F	Y, F	Y, F
Observations	8,896	8,946	8,775	8,825
R^2	0.74	0.74	0.99	0.99

Note: Table 5 presents panel regressions of firm value (Tobin's q or log of sales) on a measure of competition from Bloom, Schankerman, and Van Reenen (2013) interacted with a dummy from Greenwood, Shleifer, and You (2018) that captures whether the firm is an industry that is in a bubble or not. We control for the technological spillover measure that corresponds to public firms that issue patent in similar technological space. We follow the specification from Tables III and V of Bloom, Schankerman, and Van Reenen (2013) for the Tobin's q and sales regressions, respectively. We also include fixed effects for firm F and patent grant year Y. Standard errors clustered at the year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

These results emphasize the importance of distinguishing between value and real spillovers during speculative episodes. The general equilibrium version of our model in Section 5 makes further predictions about the effect of speculation on real and value spillovers, including their relationship with various industry characteristics. However, testing these predictions requires a more sophisticated econometric analysis, which is beyond the scope of this paper.

5 Speculation and Spillovers in General Equilibrium

We now build on our model from Section 2 to include a richer set of spillovers. With speculation, a subset of these spillovers are dampened under the market-based measures relative to their outcome-based counterparts. As a result, the effect of industry characteristics is reversed. Moreover, while these characteristics are sufficient statistics for real spillovers, the value spillovers with speculation depend on the microeconomic structure of the economy.

Specifically, instead of taking an exogenously determined profit function, we consider a neoclassical benchmark where firms have a production technology with decreasing returns to scale, using an input in fixed supply.[12] The new spillovers arise as entrants do not internalize their impact on the input market.

We discuss the robustness of our results to several extensions in Appendix D. In particular, we study the role of an elastic supply of labor input, a variable number of producing firms M, a setup in which firms compete to participate, and a setting where fixed costs determine the set of producing firms, as in Melitz (2003). For all these models, we obtain simple generalizations of the wedge formula and show that the comparative statics of the wedge remain valid.

5.1 Model where firms compete for a scarce input

5.1.1 Setup

We endogenize the profit function $\pi(a)$ at date T, by adding labor in fixed supply. We maintain the same preferences, beliefs and firm creation technology as the model of Section 2.

Households are endowed with a fixed quantity of labor L. Firms use labor to produce a homogenous good according to a decreasing returns to scale technology. The production function for a given productivity level a is:

$$y(a) = a \cdot \frac{\sigma}{\sigma - 1} \ell^{\frac{\sigma-1}{\sigma}}, \tag{5.1}$$

where y is firm output, ℓ is firm labor input, and the parameter $\sigma \in [1, \infty]$ controls the returns to scale in labor.

Labor trades at a competitive wage w. The market clearing condition for labor is:

$$\int \ell_i di = L. \tag{5.2}$$

5.1.2 Equilibrium properties

Before describing the entry wedge, it is useful to discuss several properties of the economy in date T. We formally define an equilibrium of the economy in Appendix B.

Profit function. In equilibrium, the firm profit function given productivity level a is:

$$\pi(a) = \frac{1}{\sigma - 1} \cdot w^{1-\sigma} a^{\sigma}. \tag{5.3}$$

Relative to Section 2, profits are still isoelastic with respect to productivity a. However, they now also depend on the equilibrium wage w, which is affected by equilibrium entry.

[12]This perfectly competitive approach, introduced in Hellwig and Irmen (2001), differs from the most commonly used models with imperfect substitution and monopolistic competition. We study these models and the role of the demand complementarities they induce in Section 5.2.

Labor and profit shares. The production technology implies that a fraction $(\sigma-1)/\sigma$ of the revenue for each firm is used to pay for labor and the remaining $1/\sigma$ accrues to profits. Because this is true for each firm, aggregate labor income is equal to a fraction $(\sigma-1)/\sigma$ of aggregate consumption $\mathcal{C}$, and aggregate profits are a fraction $1/\sigma$:

$$wL = \frac{\sigma-1}{\sigma} \cdot \mathcal{C}. \tag{5.4}$$

Elasticities with respect to firm creation. The aggregate production frontier is homogeneous of degree 1 in the distribution of productivities. Under the Pareto distribution, increasing M_e by one percent is equivalent to increasing all productivities by $1/\gamma$ percent. For the date 1 economy, the first welfare theorem holds and production is efficient. Therefore the elasticity of aggregate consumption with respect to firm creation $\mathcal{E}_{\mathcal{C}}$ is:

$$\mathcal{E}_{\mathcal{C}} = \frac{1}{\gamma}. \tag{5.5}$$

Since aggregate labor expenditure is a fixed fraction of consumption (equation (5.4)) and labor is in fixed supply, the elasticity of wages with respect to firm creation $\mathcal{E}_w$ is also:

$$\mathcal{E}_w = \frac{1}{\gamma}. \tag{5.6}$$

5.1.3 Social value

Entry wedge and spillovers. Taking the ratio of social and private value, as in Section 3.2, we obtain the entry wedge, which again captures the real and value spillovers introduced by the introduction of a fixed labor supply. As before, τ_1 captures both measures of spillovers without speculation. With speculation, τ_1 and τ_n capture real and value spillovers respectively.

Proposition 3 (Spillovers in general equilibrium)**.** *With speculation, the value spillover τ_n is:*

$$\tau_n = \underbrace{-\mathcal{E}_{\mathcal{I}_n}}_{\textit{bus. stealing}} + \underbrace{1 + \mathcal{E}_{\mathcal{I}_1} - \mathcal{E}_{\mathcal{C}}}_{\textit{general equilibrium}} - \underbrace{(\sigma-1)\mathcal{E}_{\mathcal{C}} \cdot \frac{\mathcal{I}_1}{\mathcal{I}_n}}_{\textit{appropriability effect}} \tag{5.7}$$

and the real spillover τ_1 is:

$$\tau_1 = 1 - \sigma\mathcal{E}_{\mathcal{C}} \tag{5.8}$$

There are three sources of the spillovers. The *business-stealing externality* is the only externality in the model of Section 2. As before, the contribution to the wedge from the business-stealing externality is $-\mathcal{E}_{\mathcal{I}_n}$, which arises because entrants displace marginal firms. As before, this is a negative externality that is weaker with disagreement than with agreement.

The *general equilibrium externality* corresponds to the change in profits due to the change in equilibrium wages, $\mathcal{E}_w \cdot d\log\pi/d\log w = -(1+\mathcal{E}_{\mathcal{I}_1} - \mathcal{E}_{\mathcal{C}})$. As new firms are created, the set of producing firms is more productive. Aggregate demand for labor increases, pushing wages up. The negative effect on firm value depends on the wage elasticity and on the impact of wages on profits. Since a change in the wage has the same relative effect on all firms regardless of their productivity, the general equilibrium externality does not depend on disagreement.

Thirdly, an externality arises from the *appropriability effect* because investors on financial markets do not take into account the surplus accruing to workers. When more firms enter, aggregate output increases, and the surplus for workers increases as well since the labor share is constant. Therefore this is a positive externality. Unlike on the market for firms, there is no speculation in labor markets, thus the surplus for workers is evaluated under the population distribution F. On the other hand, the private value of firms is evaluated under the subjective distribution F^n, accounting for the ratio $\mathcal{I}_1/\mathcal{I}_n$.

We now contrast the value and real spillovers in the high speculation limit. As in Section 2, we focus on situation of high disagreement with inelastic entry. We view this as the empirically relevant case because it implies that speculation has a large positive impact on firm entry and asset prices. We characterize the cases with elastic entry in Appendix B.1.

High speculation limit. The following proposition shows that in the high speculation limit, only the general equilibrium effects from the tax formula in equation (5.7) remain.

Proposition 4 (General model with high disagreement)**.** *Assume $\theta > 1/\gamma$. In the high disagreement limit ($n \to \infty$), the value spillover is:*

$$\tau_\infty := \lim_{n\to\infty} \tau_n = 1 + \mathcal{E}_{\mathcal{I}_1} - \mathcal{E}_{\mathcal{C}}$$

With inelastic entry, the business-stealing externality, $-\mathcal{E}_{\mathcal{I}_n}$, disappears with high speculation. This result is analogous to Theorem 2 in Section 3, but with a weaker parameter restriction for the externality to disappear. The fixed labor supply dampens the impact of disagreement on firm entry, thus increasing the difference between the average and marginal valuations.

The appropriability effect also disappears in the high speculation limit. As disagreement increases, households believe that they are investing in better firms while their labor income stagnates at the equilibrium wage determined by the physical distribution of productivity. Therefore, they expect an increasing share of their welfare to come from their capital investment profits rather than labor income. Formally, the ratio $\mathcal{I}_1/\mathcal{I}_n$ converges to zero.

Comparative statics. As a result of the vanishing business-stealing and appropriabilities effects, speculation can reverse the effect of industry characteristics on value spillovers. From equation (5.8), we have the formula for real spillovers:

$$\tau_1 = \frac{\gamma - \sigma}{\gamma}.$$

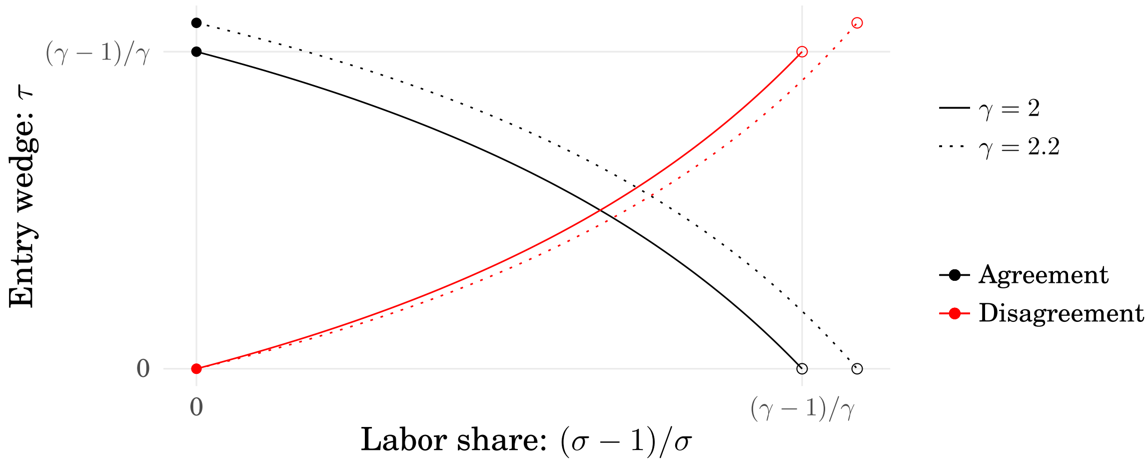

Figure 4
Comparisons of taxes with and without disagreement

On the other hand, the value spillover with a high level of disagreement corresponds to the general equilibrium effect:

$$\tau_\infty = \frac{\sigma - 1}{\gamma}.$$

The following proposition formalizes the result that disagreement reverses the effect of the labor share and Pareto tail on value spillovers.

Proposition 5 (Comparative statics of real and value spillovers)**.** *The real spillover is decreasing in both the labor share and the Pareto tail parameter, γ. With high disagreement ($n \to \infty$ and $\theta > 1/\gamma$), the value spillover is increasing in both the labor share and the Pareto tail parameter, γ.*

As the labor share increases, the appropriability effect becomes more important, which decreases the real spillovers. With speculation, the value spillovers depend only on the general equilibrium effect. As the labor share increases, firms rely more on the labor input. Hence, firm profits become more sensitive to the wage, which increases the general equilibrium effect. The two opposite forces have similar strength. When the labor share moves through its range from zero to $(\gamma - 1)/\gamma$, the wedge varies between zero and $(\gamma - 1)/\gamma$ in both cases.[13]

Similarly, the tail of the productivity distribution affects the spillovers in opposite directions. As the tail becomes thinner, γ is higher, and the elasticities of both aggregate consumption and the wage with respect to firm entry decrease. This is because additional entry increases the number of superstar firms less. Based on outcome measures, this implies that aggregate consumption exhibits more decreasing returns to scale. The growth rate of the economy in response to entry becomes even slower than implied by firms' private valuations. This leads to larger real spillovers. With disagreement, this

[13] The labor share is bounded from above by $(\gamma - 1)/\gamma$ and not one, due to the integrability condition $\sigma < \gamma$.

slowdown generates a smaller response of the wage, hence less negative externalities and smaller value spillovers .

5.2 Alternative general equilibrium externalities

We further investigate the implications of value spillovers depending only on general equilibrium effects with high speculation. In particular, we study two other sources of general equilibrium externalities—aggregate demand and knowledge spillovers.

Propositions 3 and 4 still hold. However, the general equilibrium effects, and therefore value spillovers with high speculation, depend on the nature of firms' interaction. In contrast, only the behavior of the economy's aggregates matters for real spillovers. We leave details of the models and derivations for the models of aggregate demand and knowledge spillovers to Appendix B.2 and Appendix B.3 respectively.

Aggregate demand. To capture the role of aggregate demand, we study an economy with differentiated goods where firms operate under monopolistic competition at date 1. Each firm produces a differentiated variety and household utility over the set of goods produced is:

$$\mathcal{C} = \left(\int_0^{M_e} \int_{F^{-1}\left(1 - \frac{M}{M_e}\right)}^{\infty} c\left(a, i\right)^{\frac{\sigma-1}{\sigma}} dF(a) di \right)^{\frac{\sigma}{\sigma-1}}.$$

Firms operate a linear technology in labor and output for a firm with productivity a is $y = a\ell$. Date 0 is unchanged from Section 5.1.

The economy is similar to the previous model at the macroeconomic level—the profit share is $1/\sigma$, the aggregate production function is homogeneous of degree one in the distribution of productivities, and the relative labor allocations are efficient.[14] The macroeconomic elasticities of aggregate consumption and wages to firm entry are therefore $\mathcal{E}_{\mathcal{C}} = \mathcal{E}_w = 1/\gamma$.

However the microeconomics of firms' interactions is different. Profits are:

$$\pi(a) = \frac{1}{\sigma} \cdot \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} w^{1-\sigma} \cdot \mathcal{C} \cdot a^{\sigma-1}.$$

The elasticity of profits to individual firm productivity is $\sigma - 1$ instead of σ. However, profits are now increasing in aggregate demand, $\mathcal{C}$, because of imperfect substitution across goods.

Knowledge spillovers. We capture the role of knowledge spillovers by assuming a firm's productivity combines its own type, a, and an aggregate of all the active firms'

[14]This result was first shown in Lerner (1934). It is the consequence of the homogeneous distortions at the firm level when markups are constant.

productivity, A. We assume the aggregator is homogenous of degree one in the productivity distribution.[15] The production function is:

$$y = \frac{\sigma}{\sigma - 1} a^{1-\alpha} A^{\alpha} \ell^{\frac{\sigma-1}{\sigma}},$$

where α is the intensity of the knowledge spillovers. This implies aggregate productivity has an elasticity $\mathcal{E}_A = 1/\gamma$ with respect to firm creation.

Again, the macroeconomic features of the economy with decreasing returns to scale are preserved: the labor share is $(\sigma - 1)/\sigma$ and $\mathcal{E}_{\mathcal{C}} = \mathcal{E}_w = 1/\gamma$. However, the microeconomics of firms' interactions differ from the model with decreasing returns to scale. Profits are:

$$\pi(a) = \frac{1}{\sigma - 1} \cdot w^{1-\sigma} \cdot A^{\alpha\sigma} \cdot a^{(1-\alpha)\sigma}.$$

As the intensity of knowledge spillovers increases (larger α), firm profits become less sensitive to individual types and more sensitive to the aggregate distribution.

5.2.1 Social value.

In both of these economies, the labor share is $(\sigma - 1)/\sigma$ and profits are isoelastic in firms' productivity. These two conditions are sufficient to obtain Propositions 3 and 4.

Sufficiency of macroeconomic aggregates. The following proposition stresses how the economics of the wedge with and without disagreement differ sharply.

Proposition 6 (Role of the microeconomic structure)**.** *The real spillover in our baseline economy (decreasing returns to scale), the economy with aggregate demand, and the economy with knowledge spillovers is identical:*

$$\tau_1 = \frac{\gamma - \sigma}{\gamma}.$$

With high disagreement ($n \to \infty$) and $\theta > 1/\gamma$, the value spillovers differ, taking the forms:

$$\tau_{\infty}^{DRS} = \frac{\sigma - 1}{\gamma}$$
$$\tau_{\infty}^{AD} = \frac{\sigma - 2}{\gamma}$$
$$\tau_{\infty}^{KS} = \frac{(1-\alpha)\sigma - 1}{\gamma}$$

for the decreasing returns to scale, aggregate demand, and knowledge spillovers models respectively.

[15] In Appendix B.3 we derive the case of Hölder mean of degree q, $A = \left(M_e/M \int_{\underline{a}}^{\infty} a^q dF(a)\right)^{1/q}$, where the parameter $q < \gamma$ controls how aggregate knowledge comes from the top firms.

Real spillovers only depend on the macroeconomic aggregates, and not on the microeconomic structure of the economy. The size of the appropriability effect relative to profits is determined by the labor share. The private value of profits assume a linear growth with firm creation M_e, whereas social value takes into account the elasticity of aggregate consumption $\mathcal{E}_{\mathcal{C}}$.

The irrelevance of microeconomic structure does not hold for value spillovers. With high levels of speculation, only the general equilibrium externality contributes to value spillovers. The microeconomic structure of the economy is therefore crucial to understanding the wedge. In the baseline model, the general equilibrium effect operates through the wage, giving rise to a tax of $-(1-\sigma)\mathcal{E}_w$. In the aggregate demand model, aggregate demand influences profits, leading to an additional positive externality that lowers the tax by $\mathcal{E}_{\mathcal{C}} \cdot d\log\pi/d\log\mathcal{C} = 1/\gamma$. Similarly, knowledge spillovers cause increases in aggregate productivity to induce a positive externality. The value spillover is lower than in the baseline model by $\mathcal{E}_A \cdot d\log\pi/d\log A = \alpha\sigma/\gamma$. The stronger the knowledge spillover, the lower the total value spillover.

The difference arises from how households value their portfolio relative to the firm population. With agreement, households hold representative portfolios of all firms in the economy. The expected performance of the portfolio can thus be summarized by the macroeconomic characteristics of the economy. Similarly, an outcome-based measure of spillovers depends on the aggregate distribution of firms in the economy. On the other hand, in the presence of disagreement, each household perceives the distribution of firm productivities in her portfolio to differ from the population. The microeconomic structure of the economy determines the profits from this non-representative portfolio of firms given the macroeconomic aggregates. Since stock prices are determined by households' perceptions of their own firms, market-based measures of spillovers are affected by the microeconomic structure.

Sign reversal. In both cases, the labor share continues to have opposite effects on the tax with and without disagreement. In addition, the additional positive externalities arising from the aggregate demand and knowledge spillover channels can lower the wedge below zero. These two features generate conditions under which the wedge changes sign in the presence of disagreement.

Proposition 7 (Sign reversal of spillovers)**.** *With demand externalities or knowledge spillovers, if the labor share is close to zero, the value spillover is positive with agreement and negative with large disagreement. The converse happens when the labor share is close to its upper-bound.*

Proposition 7 implies that value spillovers and real spillovers can have the opposite signs in the presence of speculation. The sign reversal is true throughout the range of the labor share whenever $\gamma = 2$ with demand externalities or $\alpha = 1 - 1/\gamma$ with knowledge spillovers. For instance, when the labor share is low, the labor surplus is relatively small, and the dominant force for the wedge is that firms do not internalize the aggregate decreasing returns to scale of the economy, leading to negative real spillovers. With disagreement however, since firms do not rely much on labor, the general equilibrium effect is small, hence the demand or knowledge externality dominates, leading to positive value spillovers.

6 Welfare Implications

The entry wedge is the optimal tax for a planner under the Pareto criterion, which respects each household's beliefs and evaluates the utility of each household under her own beliefs. Under this criterion, an allocation is more efficient than another one if it makes all households better off. The main alternative is a paternalistic approach, where the planner knows the true distribution and evaluates allocations under this distribution.[16]

6.1 Non-paternalistic criterion

The Pareto criterion is particularly well-suited for our object of study. We are interested in episodes when there is little information about new firms. Households thus rely on their priors to evaluate these firms, and there is no way to forecast who is correct. Our criterion respects this difficulty. The planner is no better judge of firms' futures than any investor. In contrast, a paternalistic approach might be more appropriate when heterogenous beliefs are viewed as inherently inefficient, due to a failure of communication or the irrationality of some agent.

A more positive reason to favor the Pareto criterion is that if households in our economy are offered a vote between two Pareto ranked allocations, they would choose the one that is more efficient under this criterion. Our characterization of efficiency therefore indicates policies that agents in our economy would support. In particular, our analysis highlights why policies to reduce firm entry may not receive support even in periods when investors agree that there is excess entry.

We are interested in optimal entry, and thus study allocations where the planner freely chooses the level of entry, leaving financial markets unaffected. The symmetry we assumed across agents guarantees a unambiguous ranking of allocations. Further, this simple policy approach reflects the policy debate of whether firm creation should be subsidized or taxed.

We could also consider a more general constrained efficient planner problem, where the planner allocates date-1 consumption using positive linear combinations of firm profits, reflecting the limits on trading in financial markets in the competitive equilibrium. In our simple model, because production is efficient, the allocation chosen by a planner who can only choose entry is Pareto efficient for the constrained planner. In fact, it is the most efficient allocation if we impose additionally that welfare weights are equal across agents.

6.2 Paternalistic criterion

Under the paternalistic approach, the planner maximizes aggregate consumption net of entry costs, $\mathcal{C} - W(M_e)$. This yields an entry wedge of:

$$\tau_n^{\text{pater}} = 1 - \sigma \mathcal{E}_{\mathcal{C}} \frac{\mathcal{I}_1}{\mathcal{I}_n}.$$

[16] Brunnermeier, Simsek, and Xiong (2014) propose a variation which avoid taking a stance on the true distribution by considering efficiency across any convex combination of agents' beliefs.

The new entry wedge is similar to the case of agreement, except that the relative social value is weighted by the ratio $\mathcal{I}_1/\mathcal{I}_n$. This reflects the distortion in the entry decision due to heterogeneous beliefs, which is a friction under this welfare criterion. As disagreement increases, more firms enter even though these firms have a low value when evaluated under the population distribution. Therefore, the wedge increases. In the high disagreement ($n \to \infty$) limit, the market values firms infinitely more than their social value, $\mathcal{I}_n/\mathcal{I}_1 \to \infty$. The planner can only rein in this exhuberance by taxing entry completely, $\tau_n^{\text{pater}} \to 1$.

This result contrasts with our analysis, where the additional entry generated by disagreement is not always undesirable. Instead, we showed that high disagreement often results in less over-entry than agreement, and even under-entry in some cases.

7 Conclusion

Given evidence that speculation and high levels of innovation tend to coincide, it is important to understand how speculation alters the private and social value of innovation. Failing to do so can distort our qualitative and quantitative understanding of the value of innovation, leading to erroneous answers to both positive and normative questions about innovation.

To that end, this paper introduces a model with sharp predictions about how market-based and outcome-based measures for the value of innovation are affected differently by speculation. Based on market-based measures, speculation increases the private value of innovation and decreases spillovers to other firms. However, neither of these effects occur in outcome-based measures of value. We show that these predictions are true for empirical measures of the value of innovation used in the literature.

In the general equilibrium version of our model, speculation reverses the effect of industry characteristics on market-based measures of spillovers. In addition, macroeconomic aggregates that determine these market-based spillovers in the absence of speculation are no longer sufficient with speculation, as the microeconomic structure of firms' interactions becomes crucial for investors' valuations of their firms. Our framework is flexible and tractable enough to extend to richer models, allowing us to understand the effect of speculation on the value of innovation in a wide range of settings.

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Appendix

A Derivations for Simple Model

A.1 General results

We first express the wedge in integral form, then prove Proposition 1 for general functions F and π.

A.1.1 General formulas

The wedge between the competitive equilibrium and the planner problem is

$$\tau_n(M_e) = -[M_e V^{(n)'}(M_e)]/[V^{(n)}(M_e)].$$

The numerator and denominator have interpretable expressions. First rewrite the denominator in the following integral form:

$$\begin{aligned} V^{(n)} &= \int_{F^{-1}}^{\infty} \pi(x) dF_n(x) \\ &= \int_{F^{-1}}^{\infty} \pi(x) \frac{F_n'}{F'}(x)\, dF(x), \end{aligned} \tag{A.1}$$

where we denote $F^{-1}(1 - M/M_e)$ by F^{-1} for convenience. Now the numerator can be written:

$$-M_e \frac{dV^{(n)}}{dM_e} = -\frac{M}{M_e} \cdot \pi\left[F^{-1}\right] \cdot \frac{F_n'}{F'}\left[F^{-1}\right] \tag{A.2}$$

$$= \int_{F^{-1}}^{\infty} \pi\left[F^{-1}\right] \frac{F_n'}{F'}\left[F^{-1}\right] dF(x). \tag{A.3}$$

This leads to the following formula for the wedge:

$$\tau_n = \frac{\int_{F^{-1}}^{\infty} \pi\left[F^{-1}\right] \frac{F_n'}{F'}\left[F^{-1}\right] dF(x)}{\int_{F^{-1}}^{\infty} \pi(x) \frac{F_n'}{F'}(x)\, dF(x)}. \tag{A.4}$$

A.1.2 Comparing the wedges

Lemma A.1. *Holding M_e constant, the wedge is larger with agreement than with disagreement.*

Proof. First recall the wedge $\tau_n(M_e) = \frac{-M_e V^{(n)'}(M_e)}{V^{(n)}(M_e)}$. We have the derivative:

$$V^{(n)'} = -\frac{1}{M_e}\frac{M}{M_e}\pi\left[F^{-1}\right] \cdot n\left(1 - \frac{M}{M_e}\right)^{n-1},$$

and we can bound $V^{(n)}$:

$$\begin{aligned} V^{(n)} &= \int_{F^{-1}}^{\infty} \pi(x) n F^{n-1}(x) dF(x) \\ &\geq \int_{F^{-1}}^{\infty} \pi(x) n F^{n-1}\left[F^{-1}\right] dF(x) \\ &\geq n\left(1 - \frac{M}{M_e}\right)^{n-1} \int_{F^{-1}}^{\infty} \pi(x) dF(x). \end{aligned}$$

Therefore we are able to bound the wedge for a given M_e and n:

$$\tau_n(M_e) \leq \frac{\int_{F^{-1}}^{\infty} \pi\left[F^{-1}\right] dF(x)}{\int_{F^{-1}}^{\infty} \pi(x) dF(x)} \leq \tau_1(M_e), \tag{A.5}$$

where the second inequality comes from the definition of $\tau_1(M_e)$. ∎

A.2 Power case derivations

We now outline the derivations for the case we focus on in the main text, with $F(a) = 1 - a^{-\gamma}$ and $\pi(a) = a^{\eta} \cdot \mathbf{1}\{a \geq \underline{a}\}$.

First, define

$$\underline{a} = F^{-1}(1 - \frac{M_e}{M}) = \left(\frac{M_e}{M}\right)^{1/\gamma}.$$

The ex-ante value of a firm, $V^{(n)}(M_e)$, is:

$$V^{(n)}(M_e) = \int_{\left(\frac{M_e}{M}\right)^{\frac{1}{\gamma}}}^{\infty} x^{\eta}\gamma n x^{-\gamma-1}\left(1 - x^{-\gamma}\right)^{n-1} dx \tag{A.6}$$

$$= \gamma n \underline{a}^{\eta-\gamma} \int_1^{\infty} t^{\eta-\gamma-1}\left(1 - \underline{a}^{-\gamma} t^{-\gamma}\right)^{n-1} dt, \tag{A.7}$$

The first derivative with respect to entrants is:

$$\frac{dV^{(n)}}{dM_e} = -\frac{1}{M_e} \cdot \frac{M}{M_e}\left(\frac{M_e}{M}\right)^{\frac{\eta}{\gamma}} \cdot n\left(1 - \frac{M}{M_e}\right)^{n-1}. \tag{A.8}$$

It is convenient to express $-M_e V^{(n)'}(M_e)$ as:

$$-M_e V^{(n)'}(M_e) = \underline{a}^{\eta-\gamma} \cdot n\left(1 - \underline{a}^{-\gamma}\right)^{n-1}. \tag{A.9}$$

A.2.1 Wedge and firm entry under agreement

Lemma A.2. *With agreement ($n = 1$), the wedge does not depend on the level of entry:*

$$\tau_1(M_e) = \frac{\gamma - \eta}{\gamma}. \tag{A.10}$$

The level of entry is:

$$\frac{M_e}{M} = \left(f_e \frac{\gamma - \eta}{\gamma}\right)^{-\frac{\gamma}{\gamma(\theta+1)-\eta}}. \tag{A.11}$$

Proof. Under agreement, $n = 1$, and we can derive an exact solution for the mass of firms entering in equilibrium, M_e. The value of a firm is:

$$V^{(1)}(M_e) = \gamma \underline{a}^{\eta-\gamma} \int_1^{\infty} t^{\eta-\gamma-1} dt$$

$$= \frac{\gamma}{\gamma - \eta}\underline{a}^{\eta-\gamma} = \frac{\gamma}{\gamma - \eta}\left(\frac{M_e}{M}\right)^{\frac{\eta-\gamma}{\gamma}}. \tag{A.12}$$

From equation (A.9) with $n = 1$, we have the numerator of the wedge:

$$-M_e \frac{dV^{(1)}}{dM_e} = \left(\frac{M_e}{M}\right)^{\frac{\eta-\gamma}{\gamma}}, \tag{A.13}$$

which leads directly to the desired formula (A.10) for the wedge. Finally, we can rewrite equation (2.6):

$$f_e\left(\frac{M_e}{M}\right)^{\theta} = \frac{\gamma}{\gamma - \eta}\left(\frac{M_e}{M}\right)^{\frac{\eta-\gamma}{\gamma}},$$

which reduces to (A.11) as desired. ∎

A.2.2 Disagreement asymptotics

Lemma A.3. *If $\theta \geq 0$, then as disagreement increases ($n \to \infty$) the mass of entrants also increases and goes to infinity:* $\lim_{n\to\infty} M_e = \infty$.

Proof. We define $\underline{a}_n = (M_e/M)^{1/\gamma}$, where M_e now depends on n, and show that $\underline{a}_n \to \infty$. Equation (2.6) implies an implicit definition of the sequence $\underline{a}_n$:

$$f_e \underline{a}_n^{\gamma(\theta+1)-\eta} = \gamma n \int_1^\infty t^{\eta-\gamma-1} \left(1 - \underline{a}_n^{-\gamma} t^{-\gamma}\right)^{n-1} dt.$$

Suppose $\underline{a}_n$ has a finite limit that is strictly larger than zero, i.e. $\underline{a}_\infty > 0$.[17] Then there exists N large enough such that $\forall n > N$, $\underline{a}_n > A = \underline{a}_\infty - \epsilon > 0$. We obtain a lower bound for the right-hand side of the implicit equation above:

$$\begin{aligned} I_n &= \gamma n \int_1^\infty t^{\eta-\gamma-1} \left(1 - \underline{a}^{-\gamma} t^{-\gamma}\right)^{n-1} dt \\ &> \gamma n \int_1^\infty t^{\eta-\gamma-1} \left(1 - A^{-\gamma} t^{-\gamma}\right)^{n-1} dt. \end{aligned}$$

Consider an arbitrary threshold T_n that depends on n and satisfies:

$$\begin{aligned} I_n &> \gamma n \int_{T_n}^\infty t^{\eta-\gamma-1} \left(1 - A^{-\gamma} t^{-\gamma}\right)^{n-1} dt \\ &> \gamma n \left(1 - A^{-\gamma} T_n^{-\gamma}\right)^{n-1} \int_{T_n}^\infty t^{\eta-\gamma-1} dt \\ &= \frac{\gamma}{\gamma-\eta} \cdot n \cdot T_n^{\eta-\gamma} \left(1 - A^{-\gamma} T_n^{-\gamma}\right)^{n-1}. \end{aligned}$$

Choose the threshold $T_n = n^{1/\gamma}$. The bound becomes:

$$\begin{aligned} I_n &> \frac{\gamma}{\gamma-\eta} \cdot n^{\frac{\eta}{\gamma}} \exp\left(-(n-1)\log(1 - A^{-\gamma} n^{-1})\right) \\ &> \frac{\gamma}{\gamma-\eta} \cdot n^{\frac{\eta}{\gamma}} \exp\left(-(n-1) A^{-\gamma} n^{-1} + \mathcal{O}(n^{-1})\right). \end{aligned}$$

Since $\gamma(\theta+1) - \eta \geq \gamma - \eta > 0$, this implies $I_n \to \infty$, contradicting $\underline{a}_\infty < \infty$. ∎

Lemma A.4 (Asymptotics for firm creation). *In the high disagreement limit ($n \to \infty$), we have the following asymptotics for the mass of firms created, M_e:*

- *If $\gamma\theta < \eta$, then $M_e/M = \left(\frac{1}{f_e}\frac{\gamma}{\gamma-\eta} \cdot n\right)^{\frac{\gamma}{\gamma(\theta+1)-\eta}}$.*
- *If $\gamma\theta = \eta$, then $\lim_{n\to\infty} M_e/M = \alpha_\infty n$, where α_∞ is a constant defined below.*

Proof. Substituting $\underline{a}$ into (2.6), we have:

$$\begin{aligned} f_e &= \gamma \underline{a}^{\eta-\gamma(\theta+1)} n \int_1^\infty t^{\eta-\gamma-1} \left(1 - \underline{a}^{-\gamma} t^{-\gamma}\right)^{n-1} dt \\ &\simeq \gamma \underline{a}^{\eta-\gamma(\theta+1)} n \int_1^\infty t^{\eta-\gamma-1} \exp\left(-(n-1)\underline{a}^{-\gamma} t^{-\gamma}\right) dt, \end{aligned}$$

where we have used the fact that $\underline{a} \to \infty$ from Lemma A.4, and $\log(1-x) = -x + \mathcal{O}(x^2)$. To find a solution, we guess the asymptotics of $\underline{a}(n)$. We rewrite $\underline{a} = \alpha(n) n^{1/(\gamma(1+\theta)-\eta)}$ and show that $\alpha(n)$ converges to a finite limit α. The above equation becomes:

$$f_e = \gamma \alpha(n)^{\eta-\gamma(\theta+1)} \int_1^\infty t^{\eta-\gamma-1} \exp\left(-\alpha(n)^{-\gamma} \frac{n-1}{n^{\frac{\gamma}{\gamma(1+\theta)-\eta}}} t^{-\gamma}\right) dt.$$

[17] Since the mass of firms producing cannot be higher than the mass of firms created, $\underline{a}_n \geq 1$.

Suppose $\gamma\theta < \eta$. Then the exponential term converges to zero and we have:

$$f_e = \gamma\alpha^{\eta-\gamma(\theta+1)} \int_1^\infty t^{\eta-\gamma-1} = \alpha^{\eta-\gamma(\theta+1)} \frac{\gamma}{\gamma-\eta},$$

such that we have the following asymptotics for firm entry:

$$\frac{M_e}{M} = \left(\frac{1}{f_e}\frac{\gamma}{\gamma-\eta} \cdot n\right)^{\frac{\gamma}{\gamma(\theta+1)-\eta}}. \tag{A.14}$$

Suppose $\gamma\theta = \eta$. Then $\underline{a}$ is defined by:

$$f_e = \gamma \underline{a}_n^{-\gamma} n \int_1^\infty t^{\eta-\gamma-1} \left(1 - \underline{a}^{-\gamma} t^{-\gamma}\right)^{n-1} dt.$$

Since $\underline{a} = (M_e/M)^{1/\gamma}$, it is sufficient to guess and verify that $\underline{a}_n = \alpha(n)^{-1/\gamma} n^{1/\gamma}$, and $\alpha(n)$ has a finite limit, α_∞ defined by:

$$\begin{aligned} f_e &= \gamma\alpha(n) \int_1^\infty t^{\eta-\gamma-1} \exp\left(-(n-1)(\alpha(n)n^{-1} + \mathcal{O}(\alpha(n)^2 n^{-2}))t^{-\gamma}\right) dt \\ &\xrightarrow[n\to\infty]{} \gamma\alpha_\infty \int_1^\infty t^{\eta-\gamma-1} e^{-\alpha_\infty t^{-\gamma}} dt, \end{aligned}$$

where we take the limit when $n \to \infty$. The wedge with agreement implies:

$$\begin{aligned} f_e &> \gamma\alpha_\infty e^{-\alpha_\infty} \int_1^\infty t^{\eta-\gamma-1} dt \\ &> \alpha_\infty e^{-\alpha_\infty} \frac{\gamma}{\gamma-\eta} \end{aligned}$$

and thus

$$\frac{\alpha_\infty e^{-\alpha_\infty}}{f_e} < \frac{\gamma-\eta}{\gamma}, \tag{A.15}$$

which implies a finite bound on α_∞. ∎

Using the asymptotics derived in Lemma A.4, we now prove Proposition 2.

Proof. **(Proposition 2)** Suppose $\gamma\theta < \eta$. Substitute the asymptotics derived in equation (A.14) into the formula for the wedge:

$$\tau_n(M_e) = \frac{\frac{M}{M_e}\left(\frac{M_e}{M}\right)^{\frac{\eta}{\gamma}} n \left(1 - \frac{M}{M_e}\right)^{n-1}}{f_e \left(\frac{M_e}{M}\right)^\theta} \tag{A.16}$$

$$\simeq \frac{1}{f_e} \cdot f_e \frac{\gamma-\eta}{\gamma}\frac{1}{n} \cdot n \left(1 - \frac{M}{M_e}\right)^{n-1} \to \frac{\gamma-\eta}{\gamma}, \tag{A.17}$$

where we have used the fact that $(1 - M/M_e)^{n-1} \to 1$.[18] The wedge therefore converges to the wedge with agreement in this case.

Now suppose $\gamma\theta > \eta$. We write the wedge directly:

$$\tau_n(M_e) = \frac{n\underline{a}^{\eta-\gamma}(1-\underline{a}^{-\gamma})^{n-1}}{f_e \underline{a}^{\gamma\theta}}.$$

[18]This follows from $(1 - M/M_e)^{n-1} = \exp\left[-(n-1)\log(M_e/M)\right]$ and using the asymptotics derived above for $\gamma\theta < \eta$: $(1 - M/M_e)^{n-1} = \exp\left[-(n-1)\left(f_e^{-1}\frac{\gamma}{\gamma-\eta}n\right)^{-\frac{\gamma}{\gamma(1+\theta)-\eta}}\right] \to 1$.

First suppose $\underline{a} \to \infty$. We rewrite the competitive equilibrium condition (2.6):

$$n\underline{a}^{-\gamma} = \frac{f_e \underline{a}^{\gamma\theta-\eta}}{\gamma \int_1^\infty t^{\eta-\gamma-1}(1-\underline{a}^{-\gamma}t^{-\gamma})^{n-1}dt}.$$

The denominator is bounded from above by $\gamma \int_1^\infty t^{\eta-\gamma-1}dt$, which implies $n\underline{a} \to \infty$. Using a first-order approximation, we have:

$$(1-\underline{a}^{-\gamma})^{n-1} \simeq \exp\left(-n\underline{a}^{-\gamma}\right).$$

Therefore, the wedge in the limit is:

$$\tau \simeq \frac{n\underline{a}^{-\gamma}\exp\left(-n\underline{a}^{-\gamma}\right)}{f_e \underline{a}^{\gamma\theta-\eta}} \to 0, \tag{A.18}$$

since the numerator goes to zero and the denominator goes to infinity. Suppose instead that $\underline{a}$ has a finite limit. We obtain the expression for τ:

$$\tau = \frac{n(1-\underline{a}^{-\gamma})^{n-1}}{f_e \underline{a}^{\gamma(1+\theta)-\eta}} = \frac{n\exp\left((n-1)\log(1-\underline{a}^{-\gamma})\right)}{f_e \underline{a}^{\gamma(1+\theta)-\eta}} \to 0, \tag{A.19}$$

since the denominator has a finite limit and the numerator goes to 0.

Lastly, consider the case where $\gamma\theta = \eta$. The tax expression simplifies to:

$$\tau = \frac{1}{f_e} \cdot n\underline{a}^{-\gamma}\left(1-\underline{a}^{-\gamma}\right)^{n-1}.$$

Using Lemma A.4 and the result that $\underline{a}^{-\gamma} = \alpha(n)/n$, and $\alpha(n) \to \alpha_\infty$ we have:

$$\tau \simeq \frac{1}{f_e}\alpha(n)\exp\left(-(n-1)\alpha(n)/n\right) \tag{A.20}$$

$$\simeq \frac{1}{f_e}\alpha(n)\exp\left(-\alpha(n)\right) \to \frac{1}{f_e}\alpha_\infty e^{-\alpha_\infty}. \tag{A.21}$$

Moreover using Lemma A.4, this also proves that in the limit τ is below the wedge with agreement $(\gamma-\eta)/\gamma$. ∎

B Derivations for General Equilibrium Model

Recall the definition of the average of a power function in productivity under measure $F^{(n)}$:

$$\mathcal{I}_n(M_e, \sigma) = \int_{\underline{a}}^{\infty} a^{\sigma} dF^n(a).$$

The integral with no disagreement is $\mathcal{I}_1$. We will use $\mathcal{I}_n$ when the dependence of the integral to M_e or σ is unambiguous. Under the Pareto distribution with parameter γ, we have the following result:

$$\mathcal{I}_1 = \frac{\gamma}{\gamma - \sigma} \cdot \left(\frac{M_e}{M}\right)^{\frac{\sigma}{\gamma}-1}. \tag{B.1}$$

B.1 Model with decreasing returns to scale

B.1.1 Equilibrium

The firm optimization problem given the production function and the competitive input price w is:

$$\max_{\ell(a)} \pi(a) = a \cdot \frac{\sigma}{\sigma - 1} \ell(a)^{\frac{\sigma-1}{\sigma}},$$

The first-order condition leads to demand for labor at the firm level:

$$\ell(a) = \left(\frac{w}{a}\right)^{-\sigma}.$$

Output and profit at the firm level are:

$$\begin{aligned} y(a) &= \frac{\sigma}{\sigma - 1} \cdot w^{1-\sigma} \cdot a^{\sigma} \\ \pi(a) &= \frac{1}{\sigma - 1} \cdot w^{1-\sigma} \cdot a^{\sigma}. \end{aligned}$$

Market clearing on the input market yields:

$$L = M_e \cdot w^{-\sigma} \int_{\underline{a}}^{\infty} a^{\sigma} dF(a) = M_e \cdot w^{-\sigma} \cdot \mathcal{I}_1, \tag{B.2}$$

which, given (B.1), leads to the following wage in equilibrium under a Pareto distribution for F:

$$w = \left(\frac{\gamma}{\gamma - \sigma}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{M_e}{L}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{M_e}{M}\right)^{\frac{1}{\gamma}-\frac{1}{\sigma}}.$$

Given the equilibrium quantities, we can decompose aggregate output into the profit and labor shares. First, observe that aggregate ouput is:

$$\mathcal{C} = M_e \cdot \int_{\underline{a}}^{\infty} y(a) dF(a) = M_e \cdot \frac{\sigma}{\sigma - 1} w^{1-\sigma} \mathcal{I}_1,$$

From this expression we immediately conclude that:

$$w^{1-\sigma} \cdot \mathcal{I}_1 = \frac{\sigma - 1}{\sigma} \cdot \frac{\mathcal{C}}{M_e},$$

and we are able to simplify the ex-ante valuation of firms:

$$\begin{aligned} V^{(n)}(M_e) &= \int_{\underline{a}}^{\infty} \pi(a) dF^n(a) = \frac{1}{\sigma - 1} \cdot w^{1-\sigma} \cdot \mathcal{I}_n(M_e) \\ &= \frac{1}{\sigma} \cdot \frac{\mathcal{C}}{M_e} \cdot \frac{\mathcal{I}_n(M_e)}{\mathcal{I}_1(M_e)}. \end{aligned}$$

Finally, we express the wage as function of the equilibrium mass of firms:

$$w = \left(\frac{\gamma - \sigma}{\gamma} L\right)^{-\frac{1}{\sigma}} \cdot M^{\frac{\gamma-\sigma}{\gamma\sigma}} \cdot M_e^{\frac{1}{\gamma}}.$$

The equilibrium condition that determines entry in equilibrium is:

$$W'(M_e) = \frac{1}{\sigma} \cdot \frac{\mathcal{C}}{M_e} \cdot \frac{\mathcal{I}_n(M_e)}{\mathcal{I}_1(M_e)}. \tag{B.3}$$

B.1.2 Entry wedge

The optimal entry tax is such that the level of entry in the competitive equilibrium with a tax τ is the same as in the planner problem. The planner maximizes expected consumption while respecting the individual household's belief. Consumption for household j is the product of labor income and profits from its investment:

$$\mathcal{C}_j = \underbrace{\frac{\sigma - 1}{\sigma} \cdot \mathcal{C}}_{\text{labor income: } wL} + \underbrace{\frac{1}{\sigma} \cdot \frac{\mathcal{I}_n}{\mathcal{I}_1} \cdot \mathcal{C}}_{\text{firm profits: } V^{(n)}} .$$

Hence the planner optimization sets the marginal cost of labor to equal its effect on perceived aggregate output $\mathcal{C}_j$:

$$W'(M_e) = \frac{1}{\sigma} \cdot \frac{d\left(\mathcal{I}_n/\mathcal{I}_1 \cdot \mathcal{C}\right)}{dM_e} + \frac{\sigma - 1}{\sigma} \cdot \frac{d\mathcal{C}}{dM_e}. \tag{B.4}$$

To find the tax formula we take the ratio of the expression for the planner entry to the competitive equilibrium entry and find:

$$1 - \tau_n = \frac{M_e}{\mathcal{C}} \frac{\mathcal{I}_1}{\mathcal{I}_n} \cdot \frac{d\left(\mathcal{I}_n/\mathcal{I}_1 \cdot \mathcal{C}\right)}{dM_e} + (\sigma - 1) \cdot \frac{M_e}{\mathcal{C}} \frac{\mathcal{I}_1}{\mathcal{I}_n} \cdot \frac{d\mathcal{C}}{dM_e}.$$

This leads us immediately to the general optimal entry tax formula:

$$\tau_n = -\mathcal{E}_{\mathcal{I}_n} + (1 + \mathcal{E}_{\mathcal{I}_1} - \mathcal{E}_{\mathcal{C}}) - (\sigma - 1)\mathcal{E}_C \cdot \frac{\mathcal{I}_1}{\mathcal{I}_n}. \tag{B.5}$$

The asymptotic environment is similar to that of Section 2. We start by studying the asymptotics of the business stealing effect.

Lemma B.1 (Asymptotics for business stealing distortion)**.** *In the high disagreement limit ($n \to \infty$), the business stealing distortion converges to a limit that depends on the marginal cost of firm creation θ:*

- *If* $\theta\gamma < 1$, *then* $\lim_{n\to\infty} \mathcal{E}_{\mathcal{I}_n} = \mathcal{E}_{\mathcal{I}_1} = \frac{\sigma}{\gamma} - 1$.
- *If* $\theta\gamma > 1$, *then* $\lim_{n\to\infty} \mathcal{E}_{\mathcal{I}_n} = 0$.
- *If* $\theta\gamma = 1$, *then* $\lim_{n\to\infty} \mathcal{E}_{\mathcal{I}_n} = \alpha_\infty e^{-\alpha_\infty}/f_e$.

Proof. The free entry condition equation (B.3) leads to:

$$(\sigma - 1)\left(\frac{\gamma - \sigma}{\gamma} L\right)^{\frac{1-\sigma}{\sigma}} \cdot f_e = M^{\theta + \frac{\sigma-\gamma}{\gamma}\frac{\sigma-1}{\sigma}} \cdot M_e^{\frac{1-\sigma}{\gamma} - \theta} \cdot \mathcal{I}_n$$

We recast the free entry condition using $\underline{a}$ to be able to use the asymptotic results from Lemma A.4

$$\text{constant} = \underline{a}^{1-\sigma-\gamma\theta} \int_{\underline{a}}^{\infty} x^\sigma dF_n(x).$$

Writing $\tilde{\theta} = \theta + (\sigma - 1)/\gamma$ and $\tilde{\eta} = \sigma$, we recognize the first-order condition from equation Lemma A.4 and use Proposition 2. ∎

For the labor surplus term, we study the behavior of $\mathcal{I}_1/\mathcal{I}_n$.

Lemma B.2 (Asymptotics for labor surplus distortion)**.** *In the high disagreement limit ($n \to \infty$), the labor surplus distortion disappears:*

$$\lim_{n\to\infty} (\sigma - 1)\mathcal{E}_{\mathcal{C}} \frac{\mathcal{I}_1}{\mathcal{I}_n} = 0$$

Proof. Since $\tilde{\theta} > 0$, Lemma A.3 gives $\lim_{n\to\infty} M_e = \infty$. The proof of Lemma A.3 implies $\lim_{n\to\infty} \mathcal{I}_n = \infty$. Finally, because $\sigma < \gamma$,

$$\mathcal{I}_1 = \frac{\gamma}{\gamma - \sigma} \left(\frac{M_e}{M}\right)^{\frac{\sigma-\gamma}{\gamma}} \to 0$$

as $n \to \infty$. Therefore $\mathcal{I}_1/\mathcal{I}_n$ converges to 0. ∎

B.2 Differentiated goods

B.2.1 Date 1 economy

The introduction of differentiated goods in Section 5.2 changes the production stage. We therefore focus on the equilibrium conditions in date 1.

Firms produce a mass M of differentiated goods, indexed by (a, i), where a is firm productivity and i indexes the firms. We drop the i index when unambiguous. Household utility aggregates consumption of these goods with constant elasticity of substitution σ across goods. At date 1, household j with total expenditure E_j solves:

$$\begin{aligned} \mathcal{C}(E_j) = \max_{\{c(a,i)\}} & \left(\int_0^{M_e} \int_{F^{-1}\left(1-\frac{M}{M_e}\right)}^{\infty} c\,(a,i)^{\frac{\sigma-1}{\sigma}} \, dF(a) di \right)^{\frac{\sigma}{\sigma-1}} \\ \text{s.t.} & \int_0^{M_e} \int_{F^{-1}\left(1-\frac{M}{M_e}\right)}^{\infty} p\,(a,i)\, c\,(a,i)\, dF(a) di \leq E_j. \end{aligned}$$

For reasons that will soon be clear, we denote by $1/\mathcal{P}$ the Lagrange multiplier on the budget constraint. Because the objective function is homogeneous of degree one in consumption and the budget constraint is linear, $\mathcal{C}(E_j)$ is linear in E_j. Thus we have $\mathcal{C}(E_j) = E_j/\mathcal{P}$. Therefore $\mathcal{P}$ is the price of one unit of the consumption basket. We use this consumption basket as the numeraire at date 1 by normalizing $\mathcal{P} = 1$. The linearity also implies that to aggregate individual demands, it is sufficient to know the aggregate expenditure in the economy, and not the whole distribution of individual expenditures. The first-order condition in the problem above implies the demand curve:

$$c(p) = \mathcal{C} p^{-\sigma}.$$

Output for a firm with productivity a is $y = a\ell$. Firms face monopolistic competition. They maximize profits by setting prices, taking as given the demand curve from each household:

$$\max_{p(a)} \; p(a) y\,(p\,(a)) - \frac{wy\,(p\,(a))}{a} = \mathcal{C} \left[p(a)^{1-\sigma} - \frac{w}{a} p(a)^{-\sigma} \right].$$

The optimal price is therefore

$$p(a) = \frac{\sigma}{\sigma - 1} \frac{w}{a}.$$

Firms charge a markup $\sigma/(\sigma - 1)$ over their marginal cost w/a.

We can then compute output y, revenue py, labor expenditure $w\ell$ and profits π as functions of productivity:

$$
\begin{aligned}
y &= \mathcal{C}w^{-\sigma}a^{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \\
py &= \mathcal{C}w^{1-\sigma}a^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \\
w\ell &= \frac{\sigma-1}{\sigma}\mathcal{C}w^{1-\sigma}a^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \\
\pi &= \frac{1}{\sigma}\mathcal{C}w^{1-\sigma}a^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}
\end{aligned}
$$

We see that labor expenditure is a fraction $(\sigma-1)/\sigma$ of revenues, and profits make up the remaining $1/\sigma$ share.

Labor market clearing gives $\mathcal{C}(\sigma-1)/\sigma = wL$. In equilibrium, aggregate expenditure is equal to aggregate consumption, so we have:

$$
\begin{aligned}
\mathcal{C} &= \mathcal{C}\left(\frac{\sigma}{\sigma-1}w\right)^{1-\sigma} M_e\mathcal{I}_1(M_e,\sigma-1) \\
&= M_e^{\frac{1}{\sigma-1}}\left(\frac{\gamma}{\gamma-(\sigma-1)}\right)^{\frac{1}{\sigma-1}}\left(\frac{M_e}{M}\right)^{\frac{(\sigma-1)-\gamma}{(\sigma-1)\gamma}} \cdot L \\
&= \left(\frac{\gamma}{\gamma-(\sigma-1)}\right)^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}-\frac{1}{\gamma}} M_e^{\frac{1}{\gamma}} \cdot L.
\end{aligned}
$$

Therefore we have $\mathcal{E}_{\mathcal{C}} = \mathcal{E}_w = 1/\gamma$. Alternatively, notice that the labor allocation is efficient with monopolistic competition and the aggregate production function is homogenous of degree 1 in the distribution of productivity. Because an increase in M_e increases all productivities with an elasticity $1/\gamma$, this results in an elasticity of aggregate consumption of $1/\gamma$.

B.2.2 Entry wedge

All arguments behind Proposition 3 apply, so the proposition is still valid, but with $\mathcal{I}_1$ and $\mathcal{I}_n$ now evaluated with parameter $\sigma-1$.

With agreement, because the aggregate consumption elasticity is unchanged, the entry wedge is unchanged: $\tau_1 = (\gamma-\sigma)/\gamma$.

With speculation, the free entry condition is:

$$
\left(\frac{M_e}{M}\right)^{\theta} = \frac{1}{\sigma}\mathcal{C}\left(\frac{\mathcal{C}}{L}\right)^{1-\sigma}\mathcal{I}_n,
$$

which we can rewrite as:

$$
KM_e^{\theta-(1-(\sigma-1))/\gamma} = \mathcal{I}_n,
$$

where K does not depend on M_e and n. This is again the same condition as the homogeneous goods model, with σ replaced by $\sigma-1$. The condition for the convergence of $\mathcal{E}_{\mathcal{I}_n}$ from Lemma B.1, still applies as well. In the high disagreement limit with $\theta > 1/\gamma$, the tax becomes:

$$
\tau_\infty = 1 + \mathcal{E}_{\mathcal{I}_1} - \mathcal{E}_{\mathcal{C}} = \frac{\sigma-2}{\gamma}. \tag{B.6}
$$

The upper panel of Figure B.1 shows the sign reversal of the entry wedge, as described in Proposition 7.

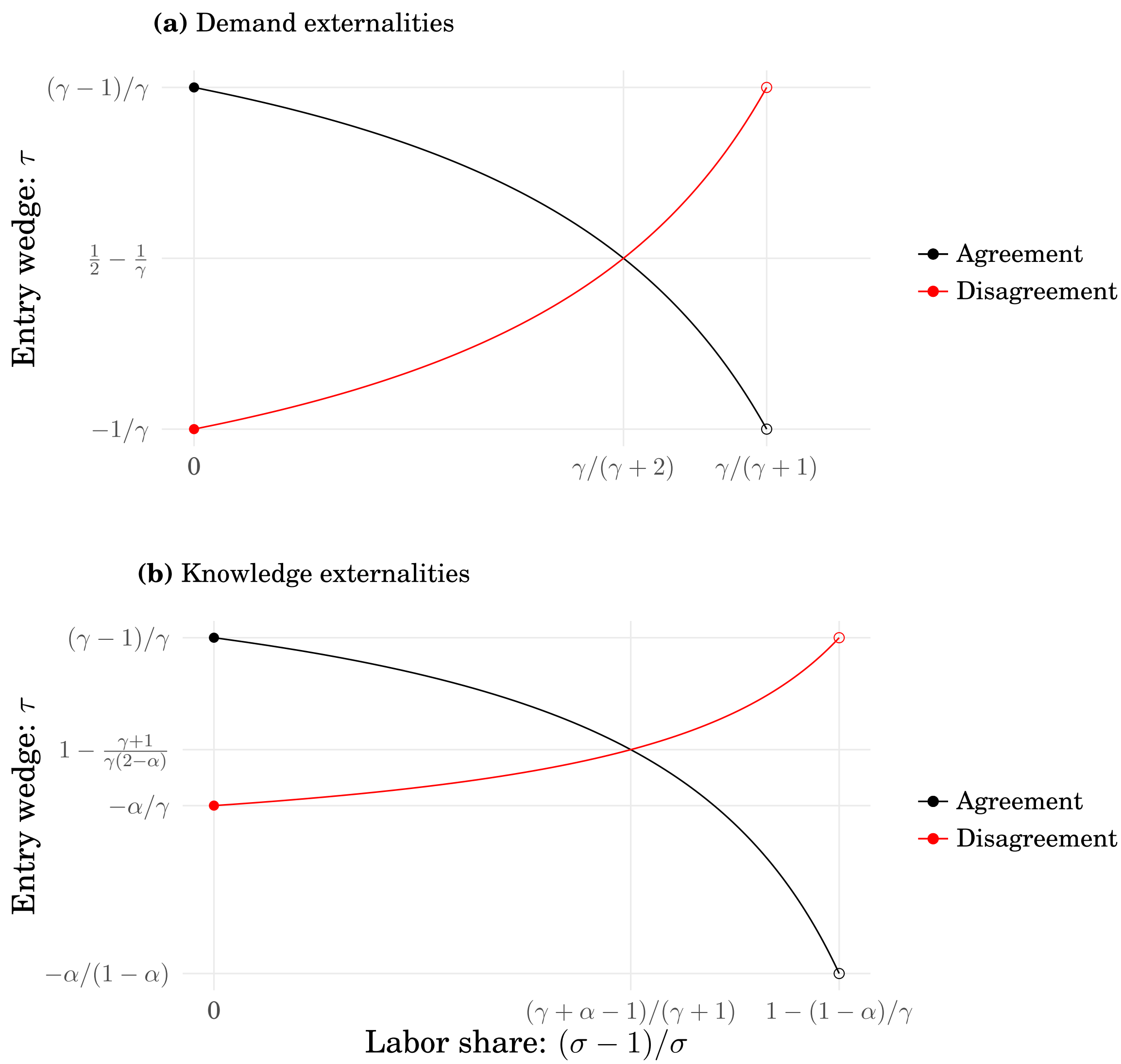

Figure B.1
Comparisons of taxes with and without disagreement

B.3 Knowledge externalities

B.3.1 Date 1 economy

We again focus on the date 1 economy and return to a setting with decreasing return to scale.

Now firm productivity depends on the productivity of other firms producing. In particular, consider the production function:

$$y = \frac{\sigma}{\sigma-1} a^{1-\alpha} A^{\alpha} \ell^{\frac{\sigma-1}{\sigma}},$$

where a is firm productivity and A is an aggregator of all producing firms' productivities. $\alpha > 0$ captures the intensity of knowledge spillovers. We use a Hölder mean of the productivity of all firms producing:

$$A=\left(\frac{M_e}{M}\int_{\left(\frac{M_e}{M}\right)^{\frac{1}{\gamma}}}a^q dF(a)\right)^{\frac{1}{q}}.$$

Imposing $q<\gamma$ so that the integral is well defined, we have:

$$A=\left(\frac{\gamma}{\gamma-q}\right)^{\frac{1}{q}}\left(\frac{M_e}{M}\right)^{\frac{1}{\gamma}}.$$

Our results generalize to any aggregator that is homogeneous of degree one in the productivity distribution of producing firms. Such aggregators similarly yield an elasticity $1/\gamma$ with respect to M_e.

Firms maximize their profits taking the wage as given:

$$\max_{\ell}\frac{\sigma}{\sigma-1}a^{1-\alpha}A^{\alpha}\ell^{\frac{\sigma-1}{\sigma}}-w\ell.$$

The demand for labor is therefore

$$\ell=\left(\frac{w}{a^{1-\alpha}A^{\alpha}}\right)^{-\sigma},$$

and we have:

$$\begin{aligned}
y(a)&=\frac{\sigma}{\sigma-1}\left(a^{1-\alpha}A^{\alpha}\right)^{\sigma}w^{1-\sigma}\\
w\ell(a)&=\left(a^{1-\alpha}A^{\alpha}\right)^{\sigma}w^{1-\sigma}=\frac{\sigma-1}{\sigma}y(a)\\
\pi(a)&=\frac{1}{\sigma-1}\left(a^{1-\alpha}A^{\alpha}\right)^{\sigma}w^{1-\sigma}=\frac{1}{\sigma}y(a)
\end{aligned}$$

The labor share is still $(\sigma-1)/\sigma$.

The market clearing condition for labor is:

$$\begin{aligned}
L&=w^{-\sigma}A^{\alpha\sigma}M_e\mathcal{I}_1\left(M_e,(1-\alpha)\sigma\right)\\
&=\left(\frac{\gamma}{\gamma-q}\right)^{\frac{\alpha\sigma}{q}}\left(\frac{M_e}{M}\right)^{\frac{\alpha\sigma}{\gamma}}M_e\frac{\gamma}{\gamma-(1-\alpha)\sigma}\left(\frac{M_e}{M}\right)^{\frac{(1-\alpha)\sigma}{\gamma}-1}w^{-\sigma}\\
&=\left(\frac{\gamma}{\gamma-q}\right)^{\frac{\alpha\sigma}{q}}\frac{\gamma}{\gamma-(1-\alpha)\sigma}M\left(\frac{M_e}{M}\right)^{\frac{\sigma}{\gamma}}w^{-\sigma}\\
w&=\left(\frac{M}{L}\right)^{\frac{1}{\sigma}}\left(\frac{\gamma}{\gamma-q}\right)^{\frac{\alpha}{q}}\left(\frac{\gamma}{\gamma-(1-\alpha)\sigma}\right)^{\frac{1}{\sigma}}\left(\frac{M_e}{M}\right)^{\frac{1}{\gamma}}.
\end{aligned}$$

We still have $(\sigma-1)/\sigma\mathcal{C}=wL$, and the same elasticities: $\mathcal{E}_w=\mathcal{E}_{\mathcal{C}}=\mathcal{E}_A=1/\gamma$.

B.3.2 Entry wedge

Proposition 3 and Lemma B.1 still apply, with $\mathcal{I}_1$ and $\mathcal{I}_n$ evaluated with parameter $(1-\alpha)\sigma$. The wedge with agreement is unchanged: $\tau_1=(\gamma-\sigma)/\gamma$. The wedge in the high disagreement limit with $\theta>1/\gamma$ becomes:

$$\tau_{\infty}=1+\mathcal{E}_{\mathcal{I}_1}-\mathcal{E}_{\mathcal{C}}=\frac{(1-\alpha)\,\sigma-1}{\gamma}. \tag{B.7}$$

The lower panel of Figure B.1 shows the sign reversal of the entry wedge, as described in Proposition 7.

C Extensions to Simple Model

C.1 Generalizing the business-stealing effect

We now consider more general functions for the business-stealing effect. In particular, suppose the expected profit of a firm with productivity a is:

$$\pi(a) = a^{\eta}\delta\left(r(a, M_e)\right)$$

where $r(a, M_e) \equiv (1 - F(a))\, M_e$ is the ranking of the firm, or the mass of firms with productivity greater than a. We can interpret δ as being the probability of producing conditional on a firm's ranking r. The main text focused on the special case of $\delta(r) = \mathbf{1}\{r \leq M\}$.

We continue to focus on the case with $F(a) = 1 - a^{-\gamma}$.

C.1.1 Wedge under agreement

Lemma C.1. *With agreement ($n = 1$) and $\delta(M_e) = 0$, the wedge does not depend on the level of entry:*

$$\tau_1(M_e) = \frac{\gamma - \eta}{\gamma}. \tag{C.1}$$

Proof. Under agreement, $n = 1$, and we can derive an exact solution for the mass of firms entering in equilibrium, M_e. Integrating by parts, the value of a firm is:

$$\begin{aligned} V^{(1)}(M_e) &= \int_1^{\infty} \gamma x^{\eta-\gamma-1}\delta(M_e x^{-\gamma})dx \\ &= \frac{\gamma}{\gamma - \eta}\left[\delta(M_e) - \gamma M_e \int_1^{\infty} x^{\eta-2\gamma-1}\delta'(M_e x^{-\gamma})dx\right]. \end{aligned} \tag{C.2}$$

In addition, we have:

$$-M_e\frac{dV^{(1)}}{dM_e} = -\gamma M_e \int_1^{\infty} x^{\eta-2\gamma-1}\delta'(M_e x^{-\gamma})dx \tag{C.3}$$

Recalling that $\tau_1 = -M_e\frac{dV^{(1)}}{dM_e}/V^{(1)}$, we have the desired formula (C.1) for the wedge. ∎

C.1.2 Disagreement asymptotics with multiple cutoffs

We now consider the generalization of δ to allow for multiple cutoffs. In particular, suppose we have cutoffs $\underline{a}_1 < ... < \underline{a}_K$, with $\underline{a}_k \equiv F^{-1}(1 - \frac{M_k}{M_e})$, and constants $\Delta_1, ..., \Delta_K$ so that

$$\delta(r) = \sum_{k=1}^{K}\Delta_k\mathbf{1}\left\{r \leq M_k\right\} \tag{C.4}$$

Notice that this imples that $V^{(n)} = \sum_{k=1}^{K}\Delta_k V_k^{(n)}$, where $V_k^{(n)} \equiv \int_{\underline{a}_k}^{\infty} x^{\eta}dF_n(x)$ and

$$-M_e\frac{dV^{(n)}}{dM_e} = -\sum_{k=1}^{K}\left(\Delta_k\frac{M_k}{M_e}\cdot a^{\eta}\cdot\frac{F_n'}{F'}(\underline{a}_k)\right)$$

For convenience, we normalize the cost of producing blueprints so that $W(b) = f_e b^{\theta+1}M_K^{-\theta}/(\theta+1)$.

Lemma C.2. *Holding M_e constant, the wedge is larger with agreement than with disagreement.*

Proof. Apply the proof for Proposition 1 for each k. ∎

Theorem C.3 (Asymptotics for the wedge, τ, with multiple cutoffs)**.** *With business-stealing of the form (C.4), in the high disagreement limit ($n \to \infty$), the wedge between the planner problem and the competitive equilibrium converges to a finite limit.*

- *If $\gamma\theta < \eta$, then $\tau \to (\gamma - \eta)/\gamma$.*
- *If $\gamma\theta > \eta$, then $\tau \to 0$.*
- *If $\gamma\theta = \eta$, then $\tau \to \frac{1}{f_e}\sum_{k=1}^{K}\left[\Delta_k\left(\frac{M_K}{M_k}\right)^{\frac{\eta-\gamma}{\gamma}}\alpha_\infty \exp\left(-\frac{M_k}{M_K}\alpha_\infty\right)\right]$.*

Proof. Suppose $\gamma\theta < \eta$. As before, conjecture that we can write $\underline{a}_K = \alpha(n)n^{1/(\gamma(1+\theta)-\eta)}$. This yields

$$f_e \simeq \sum_{k=1}^{K}\Delta_k\left(\frac{M_k}{M_K}\right)^{\frac{\gamma-\eta}{\gamma}}\alpha^{\eta-\gamma(\theta+1)}\frac{\gamma}{\gamma-\eta} \tag{C.5}$$

Therefore, we have the asymptotics for firm entry:

$$\frac{M_e}{M_K} = \left[\frac{1}{f_e}\sum_{k=1}^{K}\Delta_k\left(\frac{M_k}{M_K}\right)^{\frac{\gamma-\eta}{\gamma}}\frac{\gamma}{\gamma-\eta}\cdot n\right]^{\frac{\gamma}{\gamma(\theta+1)-\eta}}. \tag{C.6}$$

Substituting this into the formula for the wedge, we have:

$$\tau_n = \frac{\sum_{k=1}^{K}\Delta_k\left(\frac{M_k}{M_e}\right)^{\frac{\gamma-\eta}{\gamma}} n\left(1-\frac{M_k}{M_e}\right)^{n-1}}{f_e\left(\frac{M_e}{M_K}\right)^{\theta}} \to \frac{\gamma-\eta}{\gamma} \tag{C.7}$$

as desired.

Now suppose $\gamma\theta > \eta$. Then we have

$$\begin{aligned}\tau_n &= \frac{n\sum_{k=1}^{K}\underline{a}_k^{\eta-\gamma}\left(\underline{a}_k^{-\gamma}\right)^{n-1}}{f_e\underline{a}_K^{\gamma\theta}} \\ &= \frac{n\sum_{k=1}^{K}\left(\frac{M_K}{M_k}\right)^{\frac{\eta-\gamma}{\gamma}}\underline{a}_K^{-\gamma}\left(1-\frac{M_k}{M_K}\underline{a}_K^{-\gamma}\right)^{n-1}}{f_e\underline{a}_K^{\gamma\theta-\eta}}\end{aligned} \tag{C.8}$$

Suppose $\underline{a}_K \to \infty$. Then we can write the first-order condition for firm creation as:

$$\frac{f_e\underline{a}_K^{\gamma\theta-\eta}}{n\underline{a}_K^{-\gamma}} = \sum_{k=1}^{K}\Delta_k\left(\frac{M_K}{M_k}\right)^{\frac{\eta-\gamma}{\gamma}}\gamma\int_1^\infty t^{\eta-\gamma-1}\left(1-\underline{a}_k^{-\gamma}t^{-\gamma}\right)^{n-1}dt \tag{C.9}$$

Since the integral on the right-hand side is bounded from above by $\int_1^\infty t^{\eta-\gamma-1}dt$, we have that $n\underline{a}_K^{-\gamma} \to \infty$, which implies that we can use a similar approximation to the proof of Proposition 2 to show that

$$\tau_n \simeq \frac{n\sum_{k=1}^{K}\left(\frac{M_K}{M_k}\right)^{\frac{\eta-\gamma}{\gamma}}\underline{a}_K^{-\gamma}\exp\left(-n\frac{M_k}{M_K}\underline{a}_K^{-\gamma}\right)}{f_e\underline{a}_K^{\gamma\theta-\eta}} \to 0 \tag{C.10}$$

Finally, suppose $\gamma\theta = \eta$. We then have

$$f_e = \sum_{k=1}^{K}\Delta_k\gamma n\left(\frac{M_K}{M_k}\right)^{\frac{\eta-\gamma}{\gamma}}\underline{a}_K^{-\gamma}\int_1^\infty t^{\eta-\gamma-1}\left(1-\frac{M_k}{M_K}\underline{a}_K^{-\gamma}t^{-\gamma}\right) \tag{C.11}$$

As before, we conjecture that $\underline{a}_K = \alpha(n)^{-1/\gamma} n^{1/\gamma}$. Then

$$\begin{aligned} f_e &\simeq \sum_{k=1}^{K} \Delta_k \left(\frac{M_K}{M_k}\right)^{\frac{\eta-\gamma}{\gamma}} \gamma\alpha(n) \int_1^\infty t^{\eta-\gamma-1} \exp\left[-(n-1)\alpha(n) n^{-1} \frac{M_k}{M_K} t^{-\gamma}\right] dt \\ &\to \sum_{k=1}^{K} \Delta_k \left(\frac{M_K}{M_k}\right)^{\frac{\eta-\gamma}{\gamma}} \gamma\alpha_\infty \int_1^\infty t^{\eta-\gamma-1} \exp\left[-\frac{M_k}{M_K}\alpha_\infty t^{-\gamma}\right] dt \end{aligned} \tag{C.12}$$

By analogous reasoning to the proof in Proposition 2, we can obtain a finite bound on α_∞. We therefore have the wedge

$$\begin{aligned} \tau_n &= \frac{1}{f_e} \sum_{k=1}^{K} \left[\Delta_k n \left(\frac{M_K}{M_k}\right)^{\frac{\eta-\gamma}{\gamma}} \underline{a}_K^{-\gamma} \left(1 - \frac{M_k}{M_K} \underline{a}_K^{-\gamma}\right)\right] \\ &\to \frac{1}{f_e} \sum_{k=1}^{K} \left[\Delta_k \left(\frac{M_K}{M_k}\right)^{\frac{\eta-\gamma}{\gamma}} \alpha_\infty \left(1 - \frac{M_k}{M_K}\alpha_\infty\right)\right] \end{aligned} \tag{C.13}$$

as desired. ∎

C.1.3 Disagreement asymptotics with continuous business-stealing

We now consider a continuous function for the business-stealing effect

$$\delta(r) = \begin{cases} 1 & \text{if } r < 1 \\ r^{-\zeta} & \text{if } r \geq 1 \end{cases} \tag{C.14}$$

so that ζ parameterizes the business stealing effect for low productivity firms with $r \geq 1$. Larger ζ implies that low productivity firms have a lower probability of producing, with $\zeta = 0$ corresponding to the case with no business-stealing effect. With $\zeta \to \infty$, this converges to the benchmark step function business-stealing effect with $M = 1$.

We now normalize the cost of producing blueprints so that $W(b) = f_e b^{\theta+1}/(\theta+1)$, and define $\underline{a} \equiv M_e^{\frac{1}{\gamma}}$ to be the cutoff above which $\delta(a, M_e) = 1$, i.e. firms produce with probability one.

It will be convenient to consider the decomposition $V^{(n)} = V_L^{(n)} + V_U^{(n)}$, where

$$\begin{aligned} V_L^{(n)} &\equiv \gamma n M_e^{-\zeta} \int_1^{\underline{a}} x^{\eta-\gamma(1-\zeta)-1} (1 - x^{-\gamma})^{n-1} dx \\ V_U^{(n)} &\equiv \gamma n \int_{\underline{a}}^\infty x^{\eta-\gamma-1} (1 - x^{-\gamma})^{n-1} dx \end{aligned}$$

capture the expected profit conditional on having productivity below and above $\underline{a}$ respectively. We can write

$$V_L^{(n)} = \gamma n \underline{a}^{\eta-\gamma(\theta+1)} \int_{\underline{a}^{-1}}^1 t^{\eta-\gamma(1-\zeta)-1} (1 - \underline{a}^{-\gamma} t^{-\gamma})^{n-1} dt \tag{C.15}$$

$$V_U^{(n)} = \gamma n \underline{a}^{\eta-\gamma(\theta+1)} \int_1^{\underline{a}^{-1}} t^{\eta-\gamma-1} (1 - \underline{a}^{-\gamma} t^{-\gamma})^{n-1} dt \tag{C.16}$$

Moreover, since

$$\begin{aligned} \frac{dV_L^{(n)}}{dM_e} &= -\zeta M_e^{-1} V_L^{(n)} + \gamma n M_e^{-\zeta} M_e^{\frac{\eta-\gamma(1-\zeta)-1}{\gamma}} \left(1 - M_e^{-1}\right)^{n-1} \\ &= -\zeta M_e^{-1} V_L^{(n)} - \frac{dV_U^{(n)}}{dM_e} \end{aligned}$$

we have that

$$-M_e \frac{dV^{(n)}}{dM_e} = \zeta V_L^{(n)} \tag{C.17}$$

Theorem C.4 (Asymptotics for the wedge, τ, with continuous business-stealing)**.** *Suppose we have business-stealing of the form (C.14) and $\zeta > \frac{\gamma-\eta}{\gamma}$. In the high disagreement limit ($n \to \infty$), the wedge between the planner problem and the competitive equilibrium converges to a finite limit.*

- *If $\gamma\theta < \eta$, then $\tau \to (\gamma - \eta)/\gamma$.*
- *If $\gamma\theta \geq \eta$, then $\tau \to 0$.*

Proof. Suppose $\gamma\theta < \eta$. Conjecture that $\underline{a} = \alpha(n) n^{1/(\gamma(1+\theta)-\eta)}$. We have from the proof of Proposition 2 that $V_U^{(n)} \to \alpha^{\eta-\gamma}\frac{\gamma}{\gamma-\eta}$. In addition, we have from (C.15) that $V_U^{(n)} \to \alpha^{\eta-\gamma}\frac{\gamma}{\eta-\gamma(1-\zeta)}$. Therefore, we have

$$f_e = \left(\frac{\gamma}{\gamma-\eta} - \frac{\gamma}{\gamma(1-\zeta)-\eta}\right)\alpha^{\eta-\gamma(\theta+1)} \tag{C.18}$$

which verifies the conjecture. We thus have the asymptotic wedge

$$\tau_n = \zeta\frac{V_L^{(n)}}{V^{(n)}} \to \frac{\gamma-\eta}{\gamma} \tag{C.19}$$

as desired.

Suppose $\gamma\theta > \eta$. Suppose $\underline{a} \to \infty$. Then we can rewrite (2.6) as

$$n\underline{a}^{-\gamma} = \frac{f_e \underline{a}^{\gamma\theta-\eta}/\gamma}{\int_{\underline{a}^{-1}}^{1} t^{\eta-\gamma(1-\zeta)-1}(1-\underline{a}^{-\gamma}t^{-\gamma})^{n-1}dt + \int_1^\infty t^{\eta-\gamma-1}(1-\underline{a}^{-\gamma}t^{-\gamma})^{n-1}dt} \tag{C.20}$$

The two terms in the denominator of the right-hand side are bounded from above by $\int_0^1 t^{\eta-\gamma(1-\zeta)-1}dt$ and $\int_1^\infty t^{\eta-\gamma-1}dt$ respectively, which implies that $n\underline{a}^{-\gamma} \to \infty$. Using the approximation $(1-\underline{a}^{-\gamma})^{n-1} \simeq \exp(-n\underline{a}^{-\gamma})$, we have that

$$\tau_n = \frac{\zeta}{f_e \underline{a}^{\gamma\theta}} V_L^{(n)} \to 0. \tag{C.21}$$

If $\underline{a}$ has a finite limit, we can show that $V_L^{(n)} \to 0$ since $n(1-\underline{a}^{-\gamma}t^{-\gamma})^{n-1} \to 0$ for $t \in (\underline{a}^{-1}, 1)$, which implies that $\tau_n \to 0$ as well.

Suppose $\gamma\theta = \eta$. Using an analogous proof to Lemma A.4, we can show that $\underline{a}_n = \alpha(n)^{-1/\gamma}n^{1/\gamma}$ where $\alpha(n)$ has a finite limit α_∞. Since we can bound $V_L^{(n)}$ from above by:

$$\begin{aligned} V_L^{(n)} &= \gamma\alpha(n)\int_{\underline{a}^{-1}}^{1} t^{\eta-\gamma(1-\zeta)-1}(1-\underline{a}^{-\gamma}t^{-\gamma})^{n-1}dt \\ &\leq \gamma\alpha(n)\int_{\underline{a}^{-1}}^{1} t^{\eta-\gamma(1-\zeta)-1}dt \to \frac{\gamma\alpha_\infty}{\eta-\gamma(1-\zeta)} \end{aligned}$$

we have that $\tau_n = \frac{\zeta V_L^{(n)}}{f_e \underline{a}^{\gamma\theta}} \to 0$. ∎

C.2 Results with a zero cutoff profit condition

Our results are robust to an extension in which the marginal firm earns zero profits. Our baseline model specifies that the M most productive firms will be allowed to produce, which allows for tractability but results in the marginal firm earning positive profits, $\pi(\underline{a}) > 0$. We now augment the model with an intermediate stage where firms, after entering the market, compete to be among one of the M firms producing. The competition stage ensures that the business-stealing externality remains. We keep the belief and production structure of the model intact, and show that the main features of the entry wedge remain unchanged despite the introduction of a zero cutoff profit (ZCP) condition for the marginal firm.

In the new intermediate decision stage, firms can use some of their production as advertisement to reach consumers, a deadweight loss. Only the M firms that spend the most on advertising produce in equilibrium. Formally, each firm chooses how much of its production to use on advertisement, $h_i \leq \pi(a_i)$. In doing so, firms take as given the equilibrium level $\underline{h}$ of advertising necessary to attract consumers. Their profit function is therefore $\pi(a_i)\mathbf{1}\{h_i \geq \underline{h}\} - h_i$. The optimal advertisement choice is $h_i = \underline{h}$ if $\pi(a_i) \geq \underline{h}$ and 0 otherwise. The equilibrium value of $\underline{h}$ is such that exactly M firms choose to spend on advertisement. Keeping the definition of the production cutoff $\underline{a}$ from earlier, this implies

$$\underline{h} = \pi(\underline{a}). \tag{C.22}$$

Firms must spend the profits of the marginal firm to be able to produce, resulting in zero profits for the marginal firm.

C.2.1 General derivations

Firm value in this model is modified to account for the cost of advertisement:

$$\widetilde{V}^{(n)}(M_e) = \int_{\underline{a}}^{\infty} \left(\pi(a) - \pi(\underline{a})\right) dF^n(a), \tag{C.23}$$

We can define the corresponding integral $\widetilde{\mathcal{I}}_n$. With this new definition of firm value, the remainder of the competitive equilibrium and the planner problem are unchanged. In particular, the entry wedge is $\tau = -\mathcal{E}_{\widetilde{\mathcal{I}}_n}$.

Decompose firms' valuations into the revenue (from (C.23)) and advertising cost components:

$$V^{(n)}(M_e) = \int_{F^{-1}\left(1-\frac{M}{M_e}\right)}^{\infty} \pi(a) dF_n(a) - \left(\frac{M}{M_e}\right)^{1-\frac{\eta}{\gamma}} \cdot n \left(1 - \frac{M}{M_e}\right)^{n-1}. \tag{C.24}$$

The first derivative of $V^{(n)}$ is:

$$-M_e \cdot \frac{dV^{(n)}(M_e)}{dM_e} = \frac{\eta}{\gamma} \left(\frac{M_e}{M}\right)^{\frac{\eta}{\gamma}} \cdot \left[1 - \left(1 - \frac{M}{M_e}\right)^n\right].$$

Using the free-entry condition, $V^{(n)}(M_e) = W'(M_e)$, we have following formula for the wedge between planner problem and competitive equilibrium:

$$\tau_n(M_e) = \frac{\eta}{\gamma} \cdot \frac{1}{f_e} \cdot \left(\frac{M_e}{M}\right)^{\frac{\eta}{\gamma}-\theta} \cdot \left[1 - \left(1 - \frac{M}{M_e}\right)^n\right]. \tag{C.25}$$

C.2.2 Wedge and firm entry

Lemma C.5. *In the model with a ZCP condition and agreement ($n = 1$), the wedge between the competitive equilibrium and the planner problem is:*

$$\tau = \frac{\gamma - \eta}{\gamma}.$$

Proof. The free-entry condition with $n = 1$ gives us:

$$\left(\frac{M_e}{M}\right)^{\frac{\gamma(1+\theta)-\eta}{\gamma}} = \frac{1}{f_e} \cdot \frac{\eta}{\gamma - \eta}.$$

Given the derivation of the wedge in (C.25), we have:

$$\tau_1(M_e) = \frac{\eta}{\gamma} \cdot \frac{1}{f_e} \cdot \left(\frac{M_e}{M}\right)^{\frac{\eta}{\gamma}-\theta} \cdot \frac{M}{M_e} = \frac{\gamma-\eta}{\gamma}, \tag{C.26}$$

where we have used our equilibrium solution for M_e/M. ∎

Lemma C.6. *In the high disagreement limit ($n \to \infty$), the mass of entrants also increases and goes to infinity:* $\lim_{n\to\infty} M_e = \infty$.

Proof. We adapt the proof from Lemma A.3, again defining the sequence $\underline{a}_n = (M_e/M)^{1/\gamma}$ and showing that $\underline{a}_n \to \infty$. Equation (C.24) implies the implicit definition of the sequence $(\underline{a}_n)_n$ in this case:

$$f_e \underline{a}_n^{\gamma(\theta+1)-\eta} = \gamma n \int_1^\infty t^{\eta-\gamma-1} \left(1 - \underline{a}_n^{-\gamma} t^{-\gamma}\right)^{n-1} dt - n(1-\underline{a}_n^{-\gamma})^{n-1}.$$

Assume that $\underline{a}_n$ has a finite limit that is strictly larger than zero, $\underline{a}_\infty > 0$. Then there exists N large enough such that $\forall n > N, \underline{a}_n > A = \underline{a}_\infty - \epsilon > 0$. For any arbitrary threshold T_n we have

$$I_n > n\left[\frac{\gamma}{\gamma-\eta} \cdot T_n^{\eta-\gamma}\left(1 - A^{-\gamma}T_n^{-\gamma}\right)^{n-1} - 1\right].$$

As in Lemma A.3, we conclude by considering the threshold $T_n = n^{1/\gamma}$. ∎

Lemma C.7 (Asymptotics for firm creation)**.** *In the high disagreement limit ($n \to \infty$), we have the following asymptotics for the mass of firms created M_e:*

- *If $\gamma\theta < \eta$, then* $\lim_{n\to\infty} M_e/M = \alpha_\infty^\gamma n^{\frac{\gamma}{\gamma(1+\theta)-\eta}}$.
- *If $\gamma\theta = \eta$, then* $\lim_{n\to\infty} M_e/M = \alpha_\infty^{-1} n$.

In each case, α_∞ is a constant defined below.

Proof. We adapt the proof from Lemma A.4. Starting from (2.6), and using $\underline{a}$:

$$f_e \simeq \gamma \underline{a}^{\eta-\gamma(\theta+1)} n \int_1^\infty (t^\eta - 1)\, t^{-\gamma-1} \exp\left(-(n-1)\underline{a}^{-\gamma}t^{-\gamma}\right) dt,$$

where we have used the fact that $\underline{a} \to \infty$ and $\log(1-x) = -x + \mathcal{O}(x^2)$. To find a solution, we guess that asymptotically $\underline{a} \simeq \alpha(n) n^{\frac{1}{\gamma(1+\theta)-\eta}}$ and show that $\alpha(n)$ converges to a finite limit α. The first-order condition becomes:

$$f_e \simeq \gamma\alpha(n)^{\eta-\gamma(\theta+1)} \int_1^\infty (t^\eta - 1)\, t^{-\gamma-1} \exp\left(-\alpha(n)^{-\gamma} \frac{n-1}{n^{\frac{\gamma}{\gamma(1+\theta)-\eta}}} t^{-\gamma}\right) dt.$$

If $\gamma\theta < \eta$, then the exponential term converges to zero and we have:

$$\alpha_\infty = \left(\frac{1}{f_e} \cdot \frac{\eta}{\gamma-\eta}\right)^{\frac{1}{\gamma(1+\theta)-\eta}}. \tag{C.27}$$

If $\gamma\theta = \eta$, then $\underline{a}$ is defined by:

$$f_e = \gamma \underline{a}_n^{-\gamma} n \int_1^\infty t^{\eta-\gamma-1}\left(1 - \underline{a}^{-\gamma}t^{-\gamma}\right)^{n-1} dt.$$

We guess and verify that $\underline{a}_n = \alpha(n)^{-1/\gamma} n^{1/\gamma}$, and $\alpha(n)$ has a finite limit, α_∞:

$$f_e = \gamma\alpha_\infty \int_1^\infty (t^\eta - 1)\, t^{-\gamma-1} e^{-\alpha_\infty t^{-\gamma}} dt,$$

where we took the limit when $n \to \infty$. Moreover we are able to bound the wedge above the wedge with agreement using a bound on α_∞:

$$\frac{\alpha_\infty e^{-\alpha_\infty}}{f_e} < \frac{\gamma - \eta}{\eta}, \tag{C.28}$$

which verifies that α_∞ is finite. ∎

We now show that, despite the presence of the ZCP, we have the same result as in Proposition 2.

Theorem C.8. *In the high disagreement limit ($n \to \infty$), the wedge between the planner problem and the competitive equilibrium converges to a finite limit.*

- *If $\gamma\theta < \eta$, then $\tau \to (\gamma - \eta)/\gamma$.*
- *If $\gamma\theta > \eta$, then $\tau \to 0$.*
- *If $\gamma\theta = \eta$, then $\tau \to \frac{\eta}{\gamma}\frac{1}{f_e} e^{-\alpha_\infty}$.*

Proof. If $\gamma\theta > \eta$, then given equation (C.25), we use that $M_e \to \infty$ to conclude that $\lim_{n\to\infty} \tau = 0$.

If $\gamma\theta < \eta$, then we can use the asymptotics from C.7 and the formula for the wedge from (C.25):

$$\begin{aligned}
\tau_n(M_e) &\simeq \frac{\eta}{\gamma}\frac{1}{f_e}\alpha_\infty^{\eta-\theta\gamma} \cdot n^{\frac{\eta-\theta\gamma}{\gamma(1+\theta)-\eta}} \cdot \left[1 - \left(1 - \alpha_\infty^{-\gamma} n^{\frac{-\gamma}{\gamma(1+\theta)-\eta}}\right)^n\right] && \text{(C.29)}\\
&\simeq \frac{\eta}{\gamma}\frac{1}{f_e}\alpha_\infty^{\eta-\theta\gamma} \cdot n^{\frac{\eta-\theta\gamma}{\gamma(1+\theta)-\eta}} \cdot \left[1 - \exp\left(-\alpha_\infty^{-\gamma} n^{\frac{\gamma\theta-\eta}{\gamma(1+\theta)-\eta}}\right)\right] && \text{(C.30)}\\
&\simeq \frac{\eta}{\gamma}\frac{1}{f_e}\alpha_\infty^{\eta-\theta\gamma} \cdot n^{\frac{\eta-\theta\gamma}{\gamma(1+\theta)-\eta}} \cdot \alpha_\infty^{-\gamma} n^{\frac{\gamma\theta-\eta}{\gamma(1+\theta)-\eta}} \to \frac{\eta}{\gamma}\frac{1}{f_e}\alpha_\infty^{\eta-\gamma(1+\theta)}. && \text{(C.31)}
\end{aligned}$$

Using the definition of α_∞ from the proof above, we conclude $\lim_{n\to\infty} \tau_n(M_e) = (\gamma - \eta)/\gamma$.

In the knife-edge case with $\gamma\theta = \eta$, we have:

$$\tau_n(M_e) \simeq \frac{\eta}{\gamma}\frac{1}{f_e} \cdot \left[1 - \left(1 - \alpha_\infty n^{-1}\right)^n\right] \to \frac{\eta}{\gamma}\frac{1}{f_e} \cdot e^{-\alpha_\infty}. \tag{C.32}$$

We can bound the wedge in the limit: $\lim_{n\to\infty} \tau_n(M_e) < \alpha_\infty^{-1} \cdot (\gamma - \eta)/\gamma$. ∎

Our conclusions are therefore robust to including competition to enter. Intuitively, marginal firms drive the externality in both settings. In our baseline, the externality operates at the extensive margin: more entry displaces the profits of excluded marginal firms. In this model, the externality is on the intensive margin: firm entry increases the productivity of the marginal firm and thus advertisement costs for all producing firms.

D Extensions to General Equilibrium Model

D.1 Elastic labor supply

D.1.1 Setting and equilibrium

We now consider the case of variable labor supply. Households can provide labor L by exerting an effort cost $S(L)$. We assume

$$S'(L) = f_l \left(\frac{L}{L_0}\right)^{1/\kappa},$$

where κ is the Frisch elasticity of labor supply. As κ converges to 0, the model converges to a constant labor supply L_0. The remainder of the model is unchanged.

In equilibrium, we have $S'(L) = w$, which implies that $\mathcal{E}_L = \kappa \mathcal{E}_w$. Noting that we still have a constant labor share ($(\sigma-1)/\sigma \mathcal{C} = wL$), we also have $\mathcal{E}_C = \mathcal{E}_w + \mathcal{E}_L = (1+\kappa)\mathcal{E}_w$.

Market clearing for labor yields:

$$L = M_e \cdot w^{-\sigma} \int_{\underline{a}}^{\infty} a^{-\sigma} dF(a) = M_e \cdot w^{-\sigma} \cdot \mathcal{I}_1$$

Which given the expression for $\mathcal{I}_1$ in (B.1) under a Pareto leads to the following restriction:

$$wL^{1/\sigma} = \left(\frac{\gamma}{\gamma-\sigma}\right)^{\frac{1}{\sigma}} \cdot M_e^{\frac{1}{\sigma}} \cdot \left(\frac{M_e}{M}\right)^{\frac{1}{\gamma}-\frac{1}{\sigma}}$$

Using the labor supply equation, we obtain:

$$\mathcal{E}_w = \frac{1}{\gamma}\frac{\sigma}{\sigma+\kappa}$$
$$\mathcal{E}_{\mathcal{C}} = \frac{1}{\gamma}\frac{\sigma+\kappa\sigma}{\sigma+\kappa}$$

As we increase the elasticity of labor supply, the wage becomes less elastic to firm entry and consumption becomes more elastic to firm entry as it becomes less costly to expand labor.

D.1.2 Entry wedge

Asymptotics. To study the asymptotic behavior of the wedge, recall the free entry condition:

$$f_e \left(\frac{M_e}{M}\right)^{\theta} = \frac{1}{\sigma-1} w^{1-\sigma} \mathcal{I}_n.$$

We define

$$\tilde{\theta} = \theta + \frac{\sigma-1}{\gamma}\frac{\sigma}{\kappa+\sigma} \geq 0$$

and recognize the free entry condition from the pure business stealing model. This guarantees that $\lim_{n\to\infty} \mathcal{I}_1/\mathcal{I}_n = 0$. In addition, $\mathcal{E}_{\mathcal{I}_n}$ converges to 0 if:

$$\gamma\tilde{\theta} > \sigma \tag{D.1}$$
$$\Leftrightarrow \gamma\theta > 1 + (\sigma-1)\frac{\kappa}{\kappa+\sigma} \tag{D.2}$$

Planner problem and wedge. The planner objective is now:

$$\max_{M_e} \frac{1}{\sigma}\mathcal{C}\frac{\mathcal{I}_n}{\mathcal{I}_1} + \frac{\sigma-1}{\sigma}\mathcal{C} - S(L) - W(M_e)$$

The first-order condition is:

$$W'(M_e) = \frac{1}{\sigma}\mathcal{C}\frac{\mathcal{I}_n}{\mathcal{I}_1}\frac{\mathcal{I}_n'}{\mathcal{I}_n} + \frac{1}{\sigma}\mathcal{C}'\frac{\mathcal{I}_n}{\mathcal{I}_1} - \frac{1}{\sigma}\mathcal{C}\frac{\mathcal{I}_n}{\mathcal{I}_1}\frac{\mathcal{I}_1'}{\mathcal{I}_1} + \frac{\sigma-1}{\sigma}\mathcal{C}' - \underbrace{S'(L)L}_{\frac{\sigma-1}{\sigma}\mathcal{C}}\frac{1}{L}\frac{dL}{dM_e}.$$

The wedge is similar to Proposition 3. However, the appropriability term now accounts for the utility cost of expanding the labor supply:

$$\tau_n = -\mathcal{E}_{\mathcal{I}_n} + \mathcal{E}_{\mathcal{I}_1} + 1 - \mathcal{E}_{\mathcal{C}} - (\sigma-1)\underbrace{(\mathcal{E}_{\mathcal{C}} - \mathcal{E}_L)}_{\mathcal{E}_w}\frac{\mathcal{I}_1}{\mathcal{I}_n}.$$

With agreement, we have:

$$\tau_1 = 1 - \mathcal{E}_{\mathcal{C}} - (\sigma-1)\mathcal{E}_w = \frac{\gamma-\sigma}{\gamma}. \tag{D.3}$$

With high speculation and large θ, we have:

$$\tau_\infty = (\sigma-1)\mathcal{E}_w = \frac{\sigma-1}{\gamma}\frac{\sigma}{\kappa+\sigma}. \tag{D.4}$$

The wedge with agreement is unchanged from the baseline model. The wedge with high disagreement is increasing in σ and decreasing in γ, as is the case in Proposition 5. The elasticity of labor supply does not affect the wedge with agreement, but with disagreement a higher κ lowers the tax. As labor supply becomes more elastic, the wage becomes less responsive to entry, and firms have less influence on each other through general equlibrium effects. With perfectly elastic labor supply, there are no general equilibrium effects, and the economy with disagreement is efficient, i.e. $\tau_\infty = 0$.

D.2 Variable number of participating firms

D.2.1 Setting and equilibrium

We study a model where the number of participating firms, M, responds to firm creation M_e, which can be interpreted as households' consumption bundles becoming more or less concentrated as more firms enter the economy. We assume that M varies exogenously with the level of firm entry M_e:

$$M = \frac{1}{M_0^{\chi-1}} \cdot M_e^{\chi},$$

where χ is the elasticity of firms producing to firms created and M_0 a normalization constant. We assume that $\chi \leq 1$ such that we always have $M \leq M_e$.

The cost of creating a firm only depends on M_0 through:

$$W'(M_e) = f_e\left(\frac{M_e}{M_0}\right)^{\theta}.$$

The productivity threshold to produce is now:

$$\underline{a} := F^{-1}\left(1 - \frac{M}{M_e}\right) = \left(\frac{M_e}{M_0}\right)^{\frac{1-\chi}{\gamma}}.$$

The model still features a constant labor share and firm profits are still isoelastic in the productivity:

$$\pi(a) = \frac{1}{\sigma-1} \cdot w^{1-\sigma} \cdot a^{\sigma} = \frac{1}{\sigma}\frac{\mathcal{C}}{M_e} \cdot \frac{a^{\sigma}}{\mathcal{I}_1},$$

where we have redefined the integrals $\mathcal{I}_1$ and $\mathcal{I}_n$ to adjust for the new expressions for the productivity threshold $\underline{a}$:

$$\mathcal{I}_n(\chi) = \int_{\left(\frac{M_e}{M_0}\right)^{\frac{1-\chi}{\gamma}}}^{\infty} a^{\sigma} dF_n(a)$$

$$\mathcal{I}_1(\chi) = \frac{\gamma}{\gamma-\sigma} \cdot \left(\frac{M_e}{M_0}\right)^{(\chi-1)\frac{\gamma-\sigma}{\gamma}}.$$

The market-clearing condition $L = M_e w^{-\sigma} \mathcal{I}_1$ implies the equilibrium wage:

$$w = \left(\frac{\gamma}{\gamma-\sigma}\right)^{\frac{1}{\sigma}} \cdot L^{-\frac{1}{\sigma}} M_0^{(1-\chi)\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right)} \cdot M_e^{\frac{\chi}{\sigma}+\frac{1-\chi}{\gamma}},$$

so that the labor elasticity is :

$$\mathcal{E}_w = \frac{1}{\gamma} + \chi \cdot \left(\frac{1}{\sigma} - \frac{1}{\gamma}\right).$$

We obtain aggregate consumption by aggregating individual output $\mathcal{C} = M_e\sigma/(\sigma-1)w^{1-\sigma}\mathcal{I}_1$, which yields equilibrium aggregate consumption and elasticity:

$$\mathcal{C} = \frac{\sigma}{\sigma-1}\frac{\gamma}{\gamma-\sigma} \cdot L^{\frac{\sigma-1}{\sigma}} \cdot M_0^{(1-\chi)\left(\frac{1-\sigma}{\sigma}+\frac{\gamma-1}{\gamma}\right)} \cdot M_e^{\frac{1}{\gamma}+\chi\cdot\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right)}$$

$$\mathcal{E}_{\mathcal{C}} = \mathcal{E}_w = \frac{1}{\gamma} + \chi \cdot \left(\frac{1}{\sigma} - \frac{1}{\gamma}\right)$$

D.2.2 Entry wedge

Given the constant labor share and isoelastic profits, we can apply Proposition 3 and obtain the wedge:

$$\tau_n = -\mathcal{E}_{\mathcal{I}_n(\chi)} + 1 + \mathcal{E}_{\mathcal{I}_1(\chi)} - \mathcal{E}_{\mathcal{C}} - (\sigma-1)\mathcal{E}_{\mathcal{C}} \cdot \frac{\mathcal{I}_1(\chi)}{\mathcal{I}_n(\chi)}. \tag{D.5}$$

From the expression for $\mathcal{I}_n$ we have the following change in the elasticities:

$$\mathcal{E}_{\mathcal{I}_n(\chi)} = (1-\chi)\mathcal{E}_{\mathcal{I}_n(\chi=0)} = (1-\chi)\mathcal{E}_{\mathcal{I}_n} \tag{D.6}$$

Asymptotics. We now turn to the high disagreement limit. The first-order condition for firm creation is:

$$f_e \left(\frac{M_e}{M_0}\right)^{\theta} = \frac{1}{\sigma-1} w^{1-\sigma} \mathcal{I}_n(\chi)$$

$$\iff \quad \text{constant} = \underline{a}^{-\theta\frac{\gamma}{1-\chi}+(1-\sigma)\frac{\gamma}{1-\chi}\left(\frac{1}{\gamma}+\chi\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right)\right)} \int_{\underline{a}}^{\infty} a^{\sigma} dF_n(a).$$

We define:

$$\tilde{\theta} = \frac{1}{1-\chi}\left(\theta + \frac{\sigma-1}{\gamma} + \chi(\sigma-1)\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right)\right),$$

and recognize the entry condition of the baseline model. We apply our previous results, changing the condition for $\mathcal{E}_{\mathcal{I}_n(\chi)} \to 0$ to $\gamma\tilde{\theta} > \sigma$, which reduces to:

$$\gamma(\theta+\chi) > 1 + \chi\left(\frac{\gamma}{\sigma} - 1\right). \tag{D.7}$$

If this condition is satisfied, then $\lim_{n\to\infty} \mathcal{E}_{\mathcal{I}_n(\chi)} = 0$. When the inequality is reversed, $\lim_{n\to\infty} \mathcal{E}_{\mathcal{I}_n(\chi)} = \mathcal{E}_{\mathcal{I}_1(\chi)} = (1-\chi)(\sigma-\gamma)/\gamma$. With equality, the elasticity admits a finite limit between these two values.

Behavior of the wedge. The wedge with agreement is

$$\tau_1 = 1 - \sigma\mathcal{E}_{\mathcal{C}} = \frac{\gamma - \sigma}{\gamma} - \chi\sigma\left(\frac{1}{\sigma} - \frac{1}{\gamma}\right). \tag{D.8}$$

The wedge with high disagreement, when θ is large enough, is:

$$\tau_\infty = 1 + \mathcal{E}_{\mathcal{I}_1(\chi)} - \mathcal{E}_{\mathcal{C}} = \frac{\sigma - 1}{\gamma} + \chi\left(\sigma - 1\right)\left(\frac{1}{\sigma} - \frac{1}{\gamma}\right) \tag{D.9}$$

As long as $|\chi| < 1$, Proposition 5 still holds with the generalized version of $\mathcal{I}_n$. A higher elasticity of firm participation with respect to firm entry leads to opposite results with and without disagreement. A large elasticity χ dampens the wedge with agreement because it diminishes business stealing. However it leads to a higher wedge with disagreement: in response to firm entry, labor demand responds at the intensive margin with more productive firms, and at the extensive margin with more participating firms.

D.3 Advertising to participate

We augment the previous model with an intermediate stage after market entry when firms compete to be one of M firms producing, as in Appendix C.2.

D.3.1 Setting and equilibrium

In the new intermediate decision stage, firms choose to spend on advertisement to reach consumers. We assume that only the M firms that spend the most on advertising produce in equilibrium. Formally if a firm with productivity a_i spends h_i in advertising, its profit is: $\pi(a_i)\mathbf{1}\{h_i \geq \underline{h}\} - h_i$. Hence there is a threshold level of advertising, $\underline{h}$, below which firms cannot reach any consumers and above which firms do produce. Firms take the threshold as given and decide on their choice of advertising. Thus the advertising equilibrium is such that the threshold matches the profit of the marginal firm: $\underline{h} = \pi(\underline{a})$.

Profits are modified with respect to the standard model of Section B.1 to incorporate the advertisement payments:

$$\pi(a) = \frac{1}{\sigma} w^{1-\sigma}\left(a^\sigma - \underline{a}^\sigma\right),$$

The ex-ante firm valuation is therefore:

$$V^{(n)}(M_e) = \frac{1}{\sigma} \cdot \frac{\mathcal{C}}{M_e} \cdot \frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1},$$

where we define the adjusted integral $\widetilde{\mathcal{I}}_n$ as

$$\widetilde{\mathcal{I}}_n(M_e) = \int_{\left(\frac{M_e}{M}\right)^{\frac{1}{\gamma}}}^{\infty} \left(a^\sigma - \left(\frac{M_e}{M}\right)^{\frac{\sigma}{\gamma}}\right) dF_n(a).$$

The entry condition in the competitive equilibrium is now:

$$W'(M_e) = \frac{1}{\sigma} \cdot \frac{\mathcal{C}}{M_e} \cdot \frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1}.$$

D.3.2 Entry wedge

The planner maximizes expected utility:

$$\max_{M_e} \; -W(M_e) + \frac{1}{\sigma}\mathcal{C}\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1} + \frac{\sigma - 1}{\sigma}\mathcal{C}$$

The corresponding optimality condition is:

$$W'(M_e) = \frac{1}{\sigma}\mathcal{C}\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1}\frac{\widetilde{\mathcal{I}}_n'}{\widetilde{\mathcal{I}}_n} + \frac{1}{\sigma}\mathcal{C}'\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1} - \frac{1}{\sigma}\mathcal{C}\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1}\frac{\mathcal{I}_1'}{\mathcal{I}_1} + \frac{\sigma-1}{\sigma}\mathcal{C}'.$$

The wedge is therefore

$$\tau_n = -\mathcal{E}_{\widetilde{\mathcal{I}}_n} + \mathcal{E}_{\mathcal{I}_1} + 1 - \mathcal{E}_{\mathcal{C}} - (\sigma-1)\mathcal{E}_{\mathcal{C}}\frac{\mathcal{I}_1}{\widetilde{\mathcal{I}}_n}. \tag{D.10}$$

With agreement, the wedge is:

$$\tau_1 = 1 - \mathcal{E}_{\mathcal{C}} - (\sigma-1)\mathcal{E}_{\mathcal{C}}\frac{\gamma}{\sigma} = \frac{1}{\sigma} - \frac{1}{\gamma}, \tag{D.11}$$

where we have used that, as in the baseline model, $\mathcal{E}_{\mathcal{C}} = \mathcal{E}_w = 1/\gamma$. However, the profit share is now lower because of advertisement costs. A fraction $(\sigma-1)/\sigma$ of output accrues to labor but only a fraction $1/\gamma$ ($< 1/\sigma$) is collected as profits. The larger importance of labor relative to profits gives rise to a lower tax with agreement. As in the standard model, τ_1 is decreasing in σ and increasing in γ.

To derive the wedge with high disagreement, notice that $\mathcal{I}_1/\widetilde{\mathcal{I}}_n \to 0$ and therefore, if $\theta > 1/\gamma$, then τ_∞ is unchanged from the standard model:

$$\tau_\infty = \frac{\sigma-1}{\gamma}. \tag{D.12}$$

Therefore, Proposition 5 still holds.

D.3.3 Advertising with demand or knowledge spillovers.

Introducing advertisement costs to the aggregate demand and knowledge externalities models from Sections B.2 and B.3 yields the same formula for τ_n as in Equation (D.10), so that we have the limit with high disagreement $\tau_\infty = \sigma/\gamma - 1/\gamma$.

The main difference between wedge in the standard model and the models in Section B.2 and B.3 arises through the differences in the profit function, which affects the ratio $\mathcal{I}_1/\widetilde{\mathcal{I}}_n$. With aggregate demand externalities, the ratio is $\gamma/(\sigma-1)$ and the wedge with agreement is:

$$\tau_1^{\text{AD}} = -\frac{1}{\gamma}. \tag{D.13}$$

For the model with knowledge spillovers the ratio is $\gamma/(1-\alpha)\sigma$ and the wedge with agreement is:

$$\tau_1^{\text{KS}} = 1 - \frac{1}{\gamma} - \frac{\sigma-1}{(1-\alpha)\sigma}. \tag{D.14}$$

D.4 Participation costs in the baseline model

Another way to ensure the marginal firm makes zero profits is to assume firms invest in infrastructure to produce. In particular, suppose that upon entry all firms can participate on the goods market, but firms must buy one unit of infrastructure to reach all of their customers. Households produce infrastructure competitively at a cost of effort Φ. In an equilibrium with M producing firms, the price of infrastructure is:

$$\Phi'(M) = \varphi(M) = \varphi_0 \cdot M^\nu$$

with $\nu > 0$, so that the cost of infrastructure is increasig in the mass of producing firms M.

D.4.1 Participating firms

Given M_e and M, profits before the infrastructure costs are unchanged from the standard model with decreasing returns to scale:

$$\pi(a) = \frac{1}{\sigma - 1} \cdot w^{1-\sigma} \cdot a^{\sigma}.$$

The equilibrium wage is also unchanged:

$$w = \left(\frac{\gamma}{\gamma - \sigma}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{M_e}{L}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{M_e}{M}\right)^{\frac{1}{\gamma} - \frac{1}{\sigma}}.$$

The marginal firm has productivity $\underline{a}$ and spends all of its profit in infrastructure. Therefore, we have the zero cutoff profit condition $\Phi'(M) = \pi(\underline{a})$, which implies:

$$M^{\nu + \frac{1}{\gamma} + \frac{\sigma-1}{\sigma}} = \frac{1}{\varphi_0} \frac{1}{\sigma - 1} \left(\frac{\gamma - \sigma}{\gamma}\right)^{\frac{\sigma-1}{\sigma}} L^{\frac{\sigma-1}{\sigma}} \cdot M_e^{\frac{1}{\gamma}},$$

where we use the fact that $\underline{a} = (M_e/M)^{1/\gamma}$. In Section D.2, we specified an exogenous set of producing firms $M = M_e^{\chi}/M_0^{\chi-1}$. This arises endogenously through our cost of infrastructure with

$$\chi = \frac{1}{\gamma}\left(\nu + \frac{1}{\gamma} + \frac{\sigma - 1}{\sigma}\right)^{-1}$$

$$M_0^{1-\chi} = \left(\frac{1}{\varphi_0} \frac{1}{\sigma - 1} \left(\frac{\gamma - \sigma}{\gamma}\right)^{\frac{\sigma-1}{\sigma}} L^{\frac{\sigma-1}{\sigma}}\right)^{\left(\nu + \frac{1}{\gamma} + \frac{\sigma-1}{\sigma}\right)^{-1}},$$

where the exponent satisfies $\chi \leq 1$.

We can also compute the elasticity $\mathcal{E}_{\mathcal{C}}$:

$$\mathcal{E}_{\mathcal{C}} = \frac{1}{\gamma} + \chi\left(\frac{1}{\sigma} - \frac{1}{\gamma}\right) = \frac{1}{\gamma} \cdot \frac{1 + \nu}{1 + \nu + 1/\gamma - 1/\sigma} = \chi \cdot (1 + \nu).$$

D.4.2 Equilibrium

The equilibrium condition in the competitive equilibrium is:

$$W'(M_e) = \frac{1}{\sigma} \cdot \frac{\mathcal{C}}{M_e} \cdot \frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1},$$

where we define the modified $\widetilde{\mathcal{I}}_n$ integral to account for the infrastructure expenditures of the firm:

$$\widetilde{\mathcal{I}}_n(M_e, \chi) = \int_{\left(\frac{M_e}{M_0}\right)^{\frac{1-\chi}{\gamma}}}^{\infty} \left(a^{\sigma} - \left(\frac{M_e}{M_0}\right)^{\sigma\frac{1-\chi}{\gamma}}\right) dF_n(a).$$

With $n = 1$, we have:

$$\widetilde{\mathcal{I}}_1(M_e, \chi) = \frac{\sigma}{\gamma - \sigma} \cdot \left(\frac{M_0}{M_e}\right)^{(1-\chi)\frac{\gamma-\sigma}{\gamma}} = \frac{\sigma}{\gamma} \cdot \mathcal{I}_1(M_e, \chi).$$

Aggregate profits therefore represent a fraction σ/γ of aggregate revenue after labor costs, while aggregate infrastructure costs account for the other $(\gamma - \sigma)/\gamma$. Therefore aggregate profits represent a share $1/\gamma$ of consumption and aggregate infrastructure costs $1/\sigma - 1/\gamma$.

D.4.3 Entry wedge

The planner maximizes expected utility:

$$\max_{M_e} \; - W(M_e) + \frac{1}{\sigma}\mathcal{C}\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1} + \frac{\sigma-1}{\sigma}\mathcal{C} + \underbrace{\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right)\mathcal{C}}_{\text{consumption from infrastructure}} - \underbrace{\Phi(M)}_{\text{cost of infrastructure}} .$$

The corresponding optimality condition is:

$$\begin{aligned} W'(M_e) =& \frac{1}{\sigma}\mathcal{C}\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1}\frac{\widetilde{\mathcal{I}}'_n}{\widetilde{\mathcal{I}}_n} + \frac{1}{\sigma}\mathcal{C}'\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1} - \frac{1}{\sigma}\mathcal{C}\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1}\frac{\mathcal{I}'_1}{\mathcal{I}_1} \\ &+ \frac{\sigma-1}{\sigma}\mathcal{C}' + \left(\frac{1}{\sigma}-\frac{1}{\gamma}\right)\mathcal{C}' - \underbrace{\Phi'(M)M}_{(\frac{1}{\sigma}-\frac{1}{\gamma})\mathcal{C}} \cdot \frac{1}{M}\frac{dM}{dM_e}. \end{aligned}$$

The wedge is therefore:

$$\tau_n = -\mathcal{E}_{\widetilde{\mathcal{I}}_n} + \mathcal{E}_{\mathcal{I}_1} + 1 - \mathcal{E}_{\mathcal{C}} - (\sigma-1)\mathcal{E}_{\mathcal{C}} \cdot \frac{\mathcal{I}_1}{\widetilde{\mathcal{I}}_n} - \underbrace{\frac{\gamma-\sigma}{\gamma}(\mathcal{E}_{\mathcal{C}} - \mathcal{E}_M) \cdot \frac{\mathcal{I}_1}{\widetilde{\mathcal{I}}_n}}_{\text{surplus from participation costs}},$$

where the last term accounts for the surplus from infrastructure creation.

In particular, the wedge with agreement ($n = 1$) is:

$$\tau_1 = 1 - \mathcal{E}_{\mathcal{C}} - \left[(\sigma-1)\mathcal{E}_{\mathcal{C}} + \left(1-\frac{\sigma}{\gamma}\right)(\mathcal{E}_{\mathcal{C}} - \chi)\right]\frac{\gamma}{\sigma}.$$

Using the values of $\mathcal{E}_{\mathcal{C}}$ and χ, we obtain:

$$\tau_1 = 1 - \chi\gamma\left(1+\nu-\frac{1}{\sigma}+\frac{1}{\gamma}\right) = 0, \tag{D.15}$$

given the formula above for χ. Participation is now a good traded on a competitive market, hence the first welfare theorem applies and the competitive equilibrium is efficient with agreement.

Now we apply the reasoning from Section D.1 to find the condition for convergence when θ is large. The condition for convergence of $\mathcal{E}_{\widetilde{\mathcal{I}}_n}$ is the same as for $\mathcal{E}_{\mathcal{I}_n}$:

$$\gamma(\theta+\chi) > 1 + \chi\left(\frac{\gamma}{\sigma}-1\right) \tag{D.16}$$

$$\iff \quad \gamma\theta > 1 + \frac{1}{\nu+\frac{1}{\gamma}+\frac{\sigma-1}{\sigma}}\left(\frac{1}{\sigma}-\frac{1}{\gamma}-1\right) \tag{D.17}$$

As $n \to \infty$, we have that $\widetilde{\mathcal{I}}_n \to \infty$ and $\mathcal{I}_1 \to 0$, and therefore $\mathcal{I}_1/\widetilde{\mathcal{I}}_n \to 0$.

For the high disagreement wedge we have:

$$\tau_\infty = (\sigma-1)\cdot\mathcal{E}_w = \frac{\sigma-1}{\gamma} + (\sigma-1)\chi\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right) \tag{D.18}$$

$$= \frac{\sigma-1}{\gamma}\cdot\left(\frac{1+\nu}{1+\nu+1/\gamma-1/\sigma}\right). \tag{D.19}$$

Again, Proposition 5 holds. Moreover τ_∞ is decreasing in ν since $1/\gamma < 1/\sigma$. As the cost of producing infrastructure becomes steeper, the sensitivity of firm participation to firm creation is smaller, and the wedge is less responsive. In the limit with $\nu \to \infty$, a fixed number of firms produces, we are back to our baseline $\tau_\infty = (1-\sigma)/\gamma$. Finally, τ_∞ is increasing in σ.

D.5 Melitz (2003) model: participation costs and Dixit-Sitglitz

We now introduce Dixit-Stiglitz preferences to the above model, as in Melitz (2003).

D.5.1 Participating firms

Given M_e and M, profits before the infrastructure costs are unchanged from the standard model with decreasing returns to scale:

$$\pi(a) = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \cdot \mathcal{C} \cdot w^{1-\sigma} \cdot a^{\sigma-1}.$$

The equilibrium consumption is also unchanged:

$$\frac{\mathcal{C}}{L} = \left(\frac{\gamma}{\gamma-(\sigma-1)}\right)^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}-\frac{1}{\gamma}} M_e^{\frac{1}{\gamma}}.$$

The marginal firm has productivity $\underline{a}$ and spends all of its profit in infrastructure. Therefore, we have the zero cutoff profit condition $\Phi'(M) = \pi(\underline{a})$, which implies:

$$M^{\nu+\frac{1}{\gamma}+\frac{\sigma-2}{\sigma-1}} = \frac{1}{\varphi_0}\frac{1}{\sigma}\left(\frac{\gamma-(\sigma-1)}{\gamma}\right)^{\frac{\sigma-2}{\sigma-1}} L \cdot M_e^{\frac{1}{\gamma}},$$

where we use the fact that $\underline{a} = (M_e/M)^{1/\gamma}$. In Section D.2, we specified an exogenous set of producing firms $M = M_e^{\chi}/M_0^{\chi-1}$. This arises endogenously through our cost of infrastructure with

$$\chi = \frac{1}{\gamma}\left(\nu + \frac{1}{\gamma} + \frac{\sigma-2}{\sigma-1}\right)^{-1},$$

$$M_0^{1-\chi} = \left(\frac{1}{\varphi_0}\frac{1}{\sigma}\left(\frac{\gamma-(\sigma-1)}{\gamma}\right)^{\frac{\sigma-2}{\sigma-1}} L\right)^{\left(\nu+\frac{1}{\gamma}+\frac{\sigma-2}{\sigma-1}\right)^{-1}},$$

where the exponent satisfies $\chi \leq 1$ if and only if $\nu + \frac{\sigma-2}{\sigma-1} \in (-\infty, -1/\gamma) \cup [0, \infty)$. Otherwise, all firms participate as M_e grows to infinity.

Finally, we derive the elasticity $\mathcal{E}_{\mathcal{C}}$:

$$\mathcal{E}_{\mathcal{C}} = \frac{1}{\gamma} + \chi\left(\frac{1}{\sigma-1} - \frac{1}{\gamma}\right) = \frac{1}{\gamma} \cdot \frac{1+\nu}{1+\nu+1/\gamma-1/(\sigma-1)} = \chi \cdot (1+\nu).$$

D.5.2 Equilibrium

The equilibrium condition in the competitive equilibrium is:

$$W'(M_e) = \frac{1}{\sigma} \cdot \frac{\mathcal{C}}{M_e} \cdot \frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1}.$$

Aggregate profits represent a fraction $(\sigma-1)/\gamma$ of aggregate revenue after labor costs, and aggregate infrastructure costs account for the other $(\gamma-(\sigma-1))/\gamma$. Therefore aggregate profits represent a share $(\sigma-1)/(\sigma\gamma)$ of consumption and aggregate infrastructure costs $(\gamma-(\sigma-1))/(\sigma\gamma)$.

D.5.3 Entry wedge

The planner maximizes expected utility:

$$\max_{M_e} \; -W(M_e) + \frac{1}{\sigma}\mathcal{C}\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1} + \frac{\sigma-1}{\sigma}\mathcal{C} + \left(\frac{\gamma-(\sigma-1)}{\sigma\gamma}\right)\mathcal{C} - \Phi(M).$$

The corresponding optimality condition is

$$\begin{aligned}W'(M_e) &= \frac{1}{\sigma}\mathcal{C}\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1}\frac{\widetilde{\mathcal{I}}'_n}{\widetilde{\mathcal{I}}_n} + \frac{1}{\sigma}\mathcal{C}'\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1} - \frac{1}{\sigma}\mathcal{C}\frac{\widetilde{\mathcal{I}}_n}{\mathcal{I}_1}\frac{\mathcal{I}'_1}{\mathcal{I}_1} \\ &\quad + \frac{\sigma-1}{\sigma}\mathcal{C}' + \left(\frac{\gamma-(\sigma-1)}{\sigma\gamma}\right)\mathcal{C}' - \Phi'(M)M\frac{1}{M}\frac{dM}{dM_e}.\end{aligned}$$

The wedge is therefore

$$\tau_n = -\mathcal{E}_{\widetilde{\mathcal{I}}_n} + \mathcal{E}_{\mathcal{I}_1} + 1 - \mathcal{E}_{\mathcal{C}} - \left[(\sigma-1)\mathcal{E}_{\mathcal{C}} + \left(1-\frac{\sigma-1}{\gamma}\right)(\mathcal{E}_{\mathcal{C}}-\chi)\right]\frac{\mathcal{I}_1}{\widetilde{\mathcal{I}}_n}. \tag{D.20}$$

In particular, the wedge with agreement ($n=1$):

$$\begin{aligned}\tau_1 &= 1 - \mathcal{E}_{\mathcal{C}} - \left[(\sigma-1)\mathcal{E}_{\mathcal{C}} + \left(1-\frac{\sigma-1}{\gamma}\right)(\mathcal{E}_{\mathcal{C}}-\chi)\right]\frac{\gamma}{\sigma-1} && \text{(D.21)}\\ &= -\frac{1}{\sigma-1}\cdot\frac{1+\nu}{1+\nu+1/\gamma-1/(\sigma-1)}. && \text{(D.22)}\end{aligned}$$

Now we apply the reasoning from Section D.1 to find the condition for convergence when θ is large. The condition for convergence of $\mathcal{E}_{\widetilde{\mathcal{I}}_n}$ is the same as for $\mathcal{E}_{\mathcal{I}_n}$:

$$\begin{aligned}&\gamma(\theta+\chi) > 1 + \chi\left(\frac{\gamma}{\sigma-1}-1\right) && \text{(D.23)}\\ \iff\quad &\gamma\theta > 1 + \frac{1}{\nu+\frac{1}{\gamma}+\frac{\sigma-2}{\sigma-1}}\left(\frac{1}{\sigma-1}-\frac{1}{\gamma}-1\right) && \text{(D.24)}\end{aligned}$$

As $n\to\infty$, we have $\widetilde{\mathcal{I}}_n\to\infty$ and $\mathcal{I}_1\to 0$, and therefore $\mathcal{I}_1/\widetilde{\mathcal{I}}_n\to 0$.

For the high disagreement wedge we have:

$$\tau_\infty = (\sigma-1)\cdot\mathcal{E}_w - \mathcal{E}_{\mathcal{C}} = \frac{\sigma-2}{\gamma}\cdot\left(\frac{1+\nu}{1+\nu+1/\gamma-1/(\sigma-1)}\right). \tag{D.25}$$

When $\nu\to\infty$, there is a fixed supply of infrastructure and thus a fixed number of firms, which implies:

$$\begin{aligned}\tau_1 &= -\frac{1}{\sigma-1},\\ \tau_\infty &= \frac{\sigma-2}{\gamma}.\end{aligned}$$

E Data Appendix

E.1 Data construction details

Bubbles. Following Greenwood, Shleifer, and You (2018), we identify bubbles as episodes in which stock prices of an industry have increased over 100% in terms of both raw and net of market returns over the previous two years, followed by a decrease in absolute terms of 40% or more. Industries are classified according to the Fama French 49 industry scheme and the data begin in 1928.

Value of Innovation. We use the stock market value of patents at the patent and at the firm level directly from Kogan et al. (2017), as well as the number of citations that accrue to a patent.[19]

Compustat Segments. We merge the Compustat funda file with the Compustat segments file. We estimate the number of segments with different industry codes. The Compustat segment file provides both a six and a four digit industry code, which gives two measures of the number of different types of industries within a public firm.

Value of Spillovers. We obtain information on the quantity of competition spillovers (variable spillsic) as well as technological spillovers (variable spilltec) from the replication files in Bloom, Schankerman, and Van Reenen (2013). The exposure to spillovers from product market, *spillsic*, is defined as the correlation of the sales across two firms' Compustat segments. If we consider the vector of average sales share across each industry for a given firm i, $\mathbf{S}_i$. Product market proximity between firm i and j is defined by the uncentered correlation: $\text{SIC}_{ij} = \mathbf{S}_i\mathbf{S}_j'/\left(\sqrt{\mathbf{S}_i\mathbf{S}_i'}\sqrt{\mathbf{S}_j\mathbf{S}_j'}\right)$. The product market spillover is the average stock of R&D that are in the product market proximity of firm i:

$$spillsic_i = \sum_{j\neq i} \text{SIC}_{ij} G_j,$$

where G_j is the stock of R&D for firm j. The exposure to knowledge spillovers is constructed the same way, where we define for firm i a vector of share of patents across technology classes from the USPTO as $\mathbf{T}_i$. The uncentered correlation of technology between firm i and j is: $\text{TECH}_{ij} = \mathbf{T}_i\mathbf{T}_j'/\left(\sqrt{\mathbf{T}_i\mathbf{T}_i'}\sqrt{\mathbf{T}_j\mathbf{T}_j'}\right)$. The product market spillover is the average stock of R&D that are in the product market proximity of firm i:

$$spilltech_i = \sum_{j\neq i} \text{TECH}_{ij} G_j,$$

To look at the effect of spillovers we use sales item from Compustat funda file and Tobin's q (market-to-book ratio) from the CRSP-Compustat merged file.

[19]We thank Dimitris Papanikolaou for graciously sharing his data with us.

E.2 Supplementary Tables

Table E.1
Innovation in times of Bubbles

	Patents (#)	Log Patents (#)			
	(1)	(2)	(3)	(4)	(5)
Bubble	1.385**	0.148***	0.154**	0.169***	0.178**
	(0.578)	(0.056)	(0.066)	(0.063)	(0.075)
Lagged Citations	0.982***				
	(0.026)				
Lagged Log Citations		0.824***	0.795***	0.827***	0.799***
		(0.008)	(0.008)	(0.007)	(0.008)
Fixed Effects	C	–	C	Y	C, Y
Observations	106,176	106,278	106,176	106,278	106,176
R^2	0.91	0.67	0.68	0.68	0.68

Note: Table E.1 presents panel regressions of the quantity of innovation, measured by the number of patents issued at the USPTO three digit class level, on a dummy from Greenwood, Shleifer, and You (2018) that captures whether the firm is an industry that is in a bubble or not. We control for the lagged number of patents for column one and lagged logarithm for columns two to five. Depending on the specification, we include fixed effects for the patent class level C and patent grant year Y. Standard errors clustered at the grant year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table E.2
Private value of innovation in bubbles

	Patent Level			Firm Level		
	(1)	(2)	(3)	(4)	(5)	(6)
Bubble dummy	0.317***	0.289***	0.277***	0.514***	0.429***	0.431***
	(0.094)	(0.090)	(0.083)	(0.114)	(0.123)	(0.080)
Log Market Cap			0.543***			0.625***
(lagged)			(0.026)			(0.020)
Fixed Effects	Y, F	Y, F	Y, F	Y, F	Y, F	Y, F
Interaction	NA	Ind x Cite	Ind x Cite	NA	Ind x Cite	Ind x Cite
Observations	1,171,806	1,118,675	1,116,740	47,886	47,886	47,484
R^2	0.68	0.69	0.74	0.89	0.94	0.96

Note: Table E.2 presents panel regressions of the value of innovation, as measured in Kogan et al. (2017) at the patent firm levels, on a dummy from Greenwood, Shleifer, and You (2018) that captures whether the firm is an industry that is in a bubble or not. We control for the forward looking number of citations generated by a patent (or firm) from Kogan et al. (2017), and the lagged market capitalization of the firm. We include firm fixed effects F and patent grant year fixed effects Y. Depending on the specification, we also use industry fixed effects (from the Fama-French 49 industry classification) interacted with the log number of forward looking citations to allow for different slopes in the relation between private valuation and the patent quality across industries. Standard errors clustered at the grant year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table E.3
Diversity and private value of innovation in bubbles

	Patent Level			Firm Level		
	(1)	(2)	(3)	(4)	(5)	(6)
Bubble x Segments	-0.562^{***}	-0.550^{***}	-0.488^{***}	-0.388^{**}	-0.370^{***}	-0.295^{***}
(NAICS 6 digits)	(0.178)	(0.175)	(0.164)	(0.155)	(0.094)	(0.072)
Bubble	1.456^{***}	1.425^{***}	1.329^{***}	1.459^{***}	1.389^{***}	1.167^{***}
	(0.232)	(0.227)	(0.235)	(0.317)	(0.342)	(0.289)
Segments	0.122	0.122	0.112	0.010	-0.031	-0.026
(NAICS 6 digits)	(0.096)	(0.095)	(0.101)	(0.044)	(0.048)	(0.041)
Log Citations		0.049^{***}	0.047^{***}		0.047^{***}	0.047^{***}
(forward looking)		(0.010)	(0.009)		(0.009)	(0.009)
Log Market Cap			0.156^{***}			0.287^{***}
(lagged)			(0.042)			(0.046)
Fixed Effects	Y, F	Y, F	Y, F	Y, F	Y, F	Y, F
Observations	180,636	180,636	177,911	10,426	10,426	10,256
R^2	0.71	0.71	0.72	0.88	0.93	0.94

Note: Table E.3 presents panel regressions of the value of innovation, as measured in Kogan et al. (2017) at the patent and firm levels, on a bubble dummy interacted with the number of industries spanned by the different segments of the parent firm. The bubble dummy is from Greenwood, Shleifer, and You (2018) and captures whether the firm is an industry that is in a bubble or not. Compustat segments are measured at the six digit NAICS code level from the Compustat segment file. We control for the forward looking number of citations generated by a patent (or firm) from Kogan et al. (2017) and the lagged market capitalization of the firm. We also include fixed effects for firm F and patent grant year Y. Standard errors clustered at the grant year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table E.4
Forward Citations and Patent Market Values

	Table II from Kogan et al. (2017)					**Firm-Year Fixed Effects**		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log citations	0.174^{***}	0.099^{***}	0.054^{***}	0.013^{***}	0.004^{***}	0.023^{***}	0.019^{***}	0.012^{***}
	(0.017)	(0.010)	(0.005)	(0.001)	(0.001)	(0.004)	(0.003)	(0.002)
Controls								
Firm Size	—	✓	✓	✓	✓	—	✓	✓
Volatility	—	—	✓	✓	—	—	—	✓
Fixed Effects	CxY	CxY	CxY	CxY, F	CxY, FxY	Y, F	Y, F	Y, F
Observations	1,801,301	1,801,301	1,801,301	1,801,301	1,801,301	1,801,301	1,801,301	1,801,301
R^2	0.20	0.71	0.79	0.93	0.95	0.82	0.91	0.92

Note: Table E.4 presents panel regressions of the stock market value of innovation on the logarithm of the number of citations received by a patent until the end of the sample in 2010. Depending on the specification, we control for the logarithm of firm size and volatility. Depending on the specification, we include patent grant times year fixed effects, CxY, year fixed effects F, and firm fixed effects Y. Columns (1) to (5) reproduce Table II from Kogan et al. (2017), while columns (6) to (8) only include Year and Firm fixed effects to be comparable with the Tables above. Standard errors clustered at the grant year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.