Causal Inference for Asset Pricing

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November 2024

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Growing use of causal inference methods

- e.g. use IV/diff-in-diff to estimate the demand for financial assets
  - If the stock price of Tesla drops by 1%, how do you change your position?
  - If a group of investors starts buying GameStop, how does its price change?

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"This is not how we do asset pricing"

Causal Inference vs Asset Pricing

Key difference: substitution and spillovers between assets

- Natural substitution: assets are alternative ways to transfer wealth across time and states
- Equilibrium: all asset prices are jointly determined (CAPM, SDF, ...)

- Distinct from canonical causal inference
  - independent treatment, control, and excluded assets (SUTVA)

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$\rightarrow$ *our answer:* a large family of substitution patterns that make inference possible

This Paper

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$\rightarrow$ Design sample and experiments to satisfy these conditions
$\rightarrow$ Natural interpretation in standard asset pricing

  - Markowitz finance: covariance between assets determine substitutability

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- **Cross-section** only identifies **relative elasticity**:
  - If the price of the treatment changes relative to the control, how does my demand for the treatment changes relative to that for the control?
  - Difference between own-price and cross-price elasticity

$\rightarrow$ direct answer to micro-level counterfactuals (e.g. QE in one bond vs another)

This Paper

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  - "Missing intercepts" in the cross-section
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  - Aggregate + all observables driving substitution
  - In practice: which substitution patterns matter for your research question? Incorporate those!

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  - In practice: which substitution patterns matter for your research question? Incorporate those!
  - Alternative: using models for aggregation (CARA preferences, Logit, ...)

Takeaway

A guide for causal inference in asset pricing

- Precise and flexible formal conditions for identification with asset prices and quantities
- A lot (but not all) of what's already been done is reasonable

Related literature

- **Diff-in-diff**
  - Shleifer (1986); Coval, Stafford (2007), Lou (2012); Chang, Hong, Liskovich (2014); Da, Larrain, Sialm, Tessada (2018); Pavlova, Sikorskaya (2023); Ben-David, Li, Rossi, Song (2023); Lu, Wu (2023); Selgrad (2024); ...

- **Demand systems**
  - Koijen, Yogo (2019, 2024); Koijen, Richmond, Yogo (2024); Gabaix, Koijen (2024); Bretscher, Schmid, Sen, Sharma (2024); Davis (2024); Oh, Noh, Song (2023); Chaudhry (2023), van der Beck (2024); Li, Lin (2024); Jansen, Li, Schmid (2024); ...

- **What we don't do:**
  - Strategic responses (Haddad, Huebner, Loualiche, 2024)
  - Dynamics (Huebner, 2024; Haddad, Moreira, Muir, 2024)

Difference between Causal Inference and Asset Pricing

Causal Inference for Demand Estimation

All else equal, if the stock price of Tesla drops by 1%, how much do you increase your position?

$$\Delta D_i = \widehat{\mathcal{E}} \Delta P_i + \theta' X_i + \epsilon_i$$

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- Basic identification concern: changes in prices are correlated with shifts in your demand curve $\mathrm{cov}(\Delta P_i, \epsilon_i) \neq 0$
  $\rightarrow$ use an instrument Z_i for prices
  - e.g. shocks to the demand of others
  - *exclusion restriction*: instrument orthogonal to your demand shocks, $Z_i \perp \epsilon_i | X_i$

Examples

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- Koijen and Yogo (2019)
  - Estimate the demand curve of each institution (e.g. AQR)
  - $Z_i \approx$ how many institutions hold stock i
  - X_i: stock characteristics (book value, profitability, investment, beta)
  - Cross-section estimation in levels not differences

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- Selgrad (2024)
  - Estimate bond mutual fund response to shifts in price of treasuries
  - Z_i: unexpected Fed purchase of specific treasury in QE auction

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Markowitz and Samuelson: assets are just alternative means of transferring money across time and states of the world $\rightarrow$ close substitutes

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- Mean-variance demand:

$$\begin{pmatrix} D_1 \\ \vdots \\ D_n \end{pmatrix} = \frac{1}{\gamma}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - \begin{pmatrix} P_1 \\ \vdots \\ P_n \end{pmatrix})$$

$\rightarrow$ *All prices matter for all demands*

Asset Pricing vs Causal Inference

General asset pricing demand: matrix of elasticity $\mathcal{E}$

$$\Delta D_i = \mathcal{E}_{ii}\Delta P_i + \sum_{k\neq j}\mathcal{E}_{ik}\Delta P_k + \epsilon_i$$

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- $\mathcal{E}_{ik}$: capture substitution across assets

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$$\Delta D_i = \widehat{\mathcal{E}}\Delta P_i + \theta' X_i + \epsilon_i$$

Misspecified estimation: violation of SUTVA

- Because all prices are connected in equilibrium, shocking one price naturally shocks the other prices
- Even if you could only treat one asset, its price will affect demand for the control

Simple Misspecified regression example

- Setup:
  - 2 assets in estimation sample $\mathcal{S}$: Tesla, GM
  - 1 omitted asset: Nvidia
  - No shifts in demand curves ϵ_i or observables X_i
  - Exogenous supply shock $Z_{Tesla} = 1$ affects prices ($Z_{GM} = Z_{Nvidia} = 0$)

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- For large N: many asymmetric substitutes generally do not cancel out
  $\rightarrow$ may add up to have a large effect on $\hat{\mathcal{E}}$ (Chaudhary, Fu, Li, 2023)

Conditions for valid causal inference

Making Causal Inference Work

- Data-generating process: matrix of elasticities $\mathcal{E}$
- Empirical estimation with IV/diff-in-diff for some sample of assets $\mathcal{S}$

$$\Delta D_i = \widehat{\mathcal{E}} \Delta P_i + \theta' X_i + \epsilon_i$$
$$\Delta P_i = \lambda Z_i + \eta' X_i + u_i$$

Conditions on the elasticity matrix $\mathcal{E}$ such that $\widehat{\mathcal{E}}$ is a meaningful estimate?

Elasticity Identification Theorem

A1. **Homogenous substitution between assets**

→ *Two assets with same observables substitute the same with any third asset*

$$\forall i, j \in \mathcal{S}, l \neq i, j, \quad X_i = X_j \Rightarrow \mathcal{E}_{il} = \mathcal{E}_{jl} = \mathcal{E}_{\text{cross}}(X_i, X_l) = X_i' \mathcal{E}_S X_l$$

- X_i is a $K \times 1$ vector of observables
- $\mathcal{E}_S$ is a $K \times K$ matrix

A2. **Constant relative elasticity**

→ *Assets in the estimation sample with the same observables have the same relative elasticities*

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Mechanics of identification

- Take 2 assets with same characteristics, $X_1 = X_2$

First difference

$$\Delta D_1 = \mathcal{E}_{11}\Delta P_1 + \mathcal{E}_{12}\Delta P_2 + \sum_{k\geq 3}\mathcal{E}_{1k}\Delta P_k$$

$$\Delta D_2 = \mathcal{E}_{22}\Delta P_2 + \mathcal{E}_{21}\Delta P_1 + \sum_{k\geq 3}\mathcal{E}_{2k}\Delta P_k$$

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$$\Delta D_1 - \Delta D_2 = \underbrace{\left(\mathcal{E}_{11} - \mathcal{E}_{\text{cross}}(X_1, X_1)\right)}_{\widehat{\mathcal{E}}}\Delta P_1 - \underbrace{\left(\mathcal{E}_{22} - \mathcal{E}_{\text{cross}}(X_2, X_2)\right)}_{\widehat{\mathcal{E}}}\Delta P_2$$

$$= \widehat{\mathcal{E}}\left(\Delta P_1 - \Delta P_2\right)$$

Identification-Friendly Substitution under Symmetry

$$\mathcal{E} = \begin{array}{c} \begin{array}{cc} \overbrace{\qquad\qquad\qquad}^{\text{Estimation Sample}} & \overbrace{\qquad\qquad\qquad}^{\text{Excluded Assets}} \end{array} \\ \left[\begin{array}{c|c} \qquad\qquad\qquad & \qquad\qquad\qquad \\ \hline \qquad\qquad\qquad & \qquad\qquad\qquad \end{array}\right] \end{array}$$

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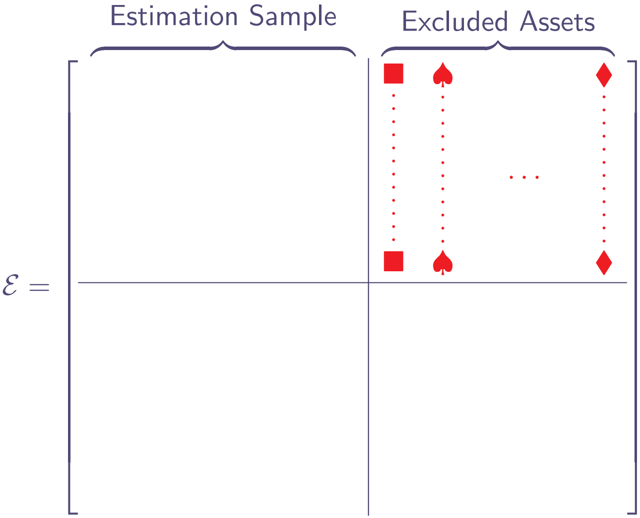

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  - Ex. Corporate bonds with same rating and duration
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- Logit (Koijen, Yogo, 2019): $\mathcal{E} = \alpha(I - \mathbf{1}w')$, $\hat{\mathcal{E}} = \alpha$

Broadly Applicable Identification Conditions

General A1: Homogeneous substitution conditional on observables: $\mathcal{E}_{cross}(X_i, X_j) = X_i' \mathcal{E}_S X_j$

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- Example: **Factor models** $X_i = \beta_i \Rightarrow \mathcal{E}_{cross}(X_i, X_j) = \beta_i' \mathcal{E}_\beta \beta_j$
  - Factor model with β standard exposure (Koijen, Yogo, 2019)

- Example: **Characteristics**
  - Substitution based on balancing a characteristic (e.g. average ESG score)

Broadly Applicable Identification Conditions

General A1: Homogeneous substitution conditional on observables: $\mathcal{E}_{cross}(X_i, X_j) = X_i' \mathcal{E}_S X_j$

- To estimate $\widehat{\mathcal{E}}$, **control for observables** X_i in a cross-sectional regression
  - No need to "know" $\mathcal{E}_S$ to estimate relative elasticities
  - For repeated cross-sections: control for observables X_i interacted with time fixed effects
- Example: **Homogeneity within groups** $\rightarrow$ observables X_i are group dummies
  - Pool all bonds but add narrowly defined credit & duration FEs (Chaudhary, Fu, Li, 2023)
- Example: **Factor models** $X_i = \beta_i \Rightarrow \mathcal{E}_{cross}(X_i, X_j) = \beta_i' \mathcal{E}_\beta \beta_j$
  - Factor model with β standard exposure (Koijen, Yogo, 2019)
- Example: **Characteristics**
  - Substitution based on balancing a characteristic (e.g. average ESG score)
- **Warning:** fixed effects or coefficient on X_i do not identify anything about $\mathcal{E}$
  - Cannot disentangle substitution from demand for characteristics

Average Treatment Effect

- **Heterogeneity independent of the instrument** $\rightarrow$ local average of relative elasticity

$$\widehat{\mathcal{E}} = \frac{\mathbf{E}_i \left\{\lambda_i(\mathcal{E}_{ii} - \mathbf{E}_j(\mathcal{E}_{ji}))\right\}}{\mathbf{E}_i(\lambda_i)}$$

  - Overweigh assets where instrument has large price impact

Example: Fuchs, Fukuda, Neuhann (2024)

- Setup: Representative agent $+$ log utility
- Payoffs of 3 assets (rows) in 3 states (cols)

	green state $p_g = \frac{1}{4}$	red state $p_r = \frac{1}{4}$	state 2 $p_2 = \frac{1}{2}$
green	$1 + \varepsilon$	$1 - \varepsilon$	0
red	$1 - \varepsilon$	$1 + \varepsilon$	0
2	0	0	1

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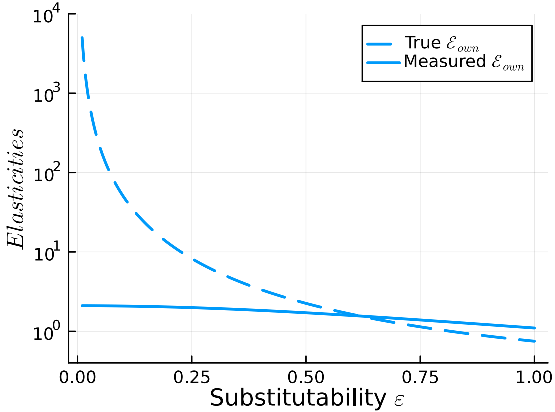

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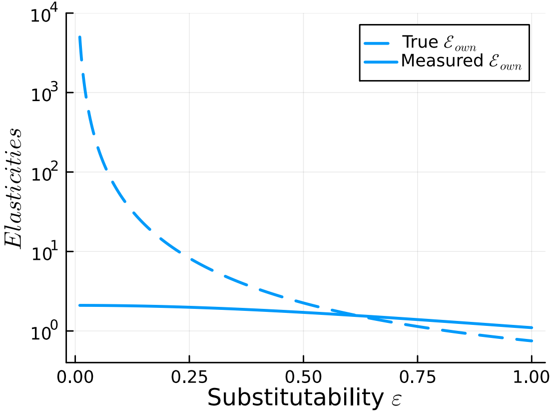
- Does regression of ΔD_g on ΔP_g identify $\mathcal{E}_{own}$? No.
- Does relative regression of $\Delta(D_g - D_r)$ on $\Delta(P_g - P_r)$ identify $\mathcal{E}_{own} - \mathcal{E}_{cross}$?



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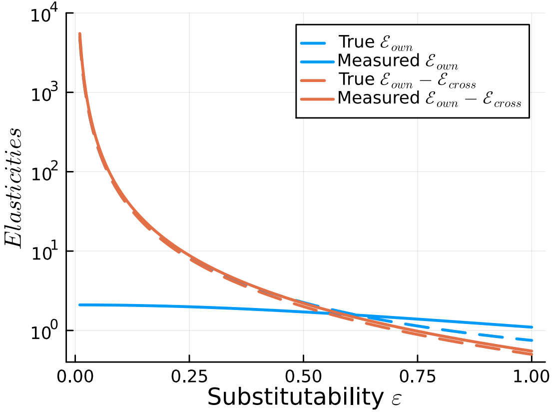

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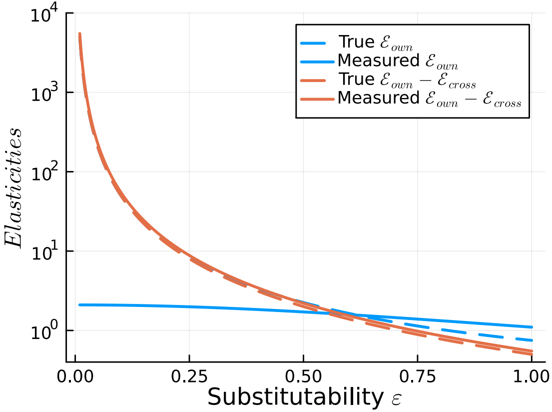
- Empirical literature (mostly) measures relative elasticities ✓



What can we do with these estimates?

Useful for micro counterfactuals

- Only recover relative elasticities $\mathcal{E}_{own} - \mathcal{E}_{cross}$
- Change the supply of one asset: how much does its price changes relative to another asset
- E.g.: price dispersion, effect of passive investing on individual stocks (Haddad, Huebner, Loualiche, 2024), ...
- For aggregate questions: relative elasticities not enough, need to separate $\mathcal{E}_{own}$ and $\mathcal{E}_{cross}$

Estimating Multipliers: Price Impact Regressions

Multiplier vs Elasticity

If demand for various assets shift, how do prices respond?

- Market clearing condition with aggregate demand: $D(P) = S$
  - ΔD: shift in the demand curve
  - Prices adjust so that aggregate demand does not move

$$0 = \underbrace{\Delta D}_{\text{shifts in demand}} + \underbrace{\mathcal{E}\Delta P}_{\text{move along demand curve}}$$

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- Notion of exogenous demand shock Z_i:

$$\Delta D = Z + u$$
$$Z \perp u$$

Price Impact Regressions

Univariate price impact regression:

$$\Delta P_i = \widehat{\mathcal{M}} Z_i + \epsilon_i$$

- Basic identification concern: changes in realized demand are correlated with shifts in the aggregate demand curve
  - *identification restriction*: $Z_i \perp$ all other demand shifts, no first stage
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Proposition. If $\mathcal{M}$ satisfies A1, A2, and the demand shock is exogenous, estimator of $\widehat{\mathcal{M}}$ identifies the **relative multiplier**:

$$\widehat{\mathcal{M}} = \mathcal{M}_{\text{own}} - \mathcal{M}_{\text{cross}}$$

Example: Corporate Bond Relative Multiplier

- Group investment-grade corporate bonds in 5 buckets based on duration
- Z_{it}: flow-induced demand: fund flow in mutual funds $\times$ portfolio composition (Coval Stafford 2007, Lou 2012)
- X_i: average duration of each bucket of corporate bonds

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- Z_{it}^{idio}: residual of instrument regressed on a date fixed effect and $X_i \times$ date fixed effect

Example: Corporate Bonds

Relative multiplier $\widehat{\mathcal{M}} = 1.232$

	Price ΔP_{it}				
	(1)	(2)	(3)	(4)	(5)
Z_{it}	4.051*	0.995	**1.232**		
	(1.990)	(1.399)	(1.048)		
Z_{it}^{idio}				**1.232**	**1.232**
				(1.048)	(1.046)
Date Fixed Effects		Yes	Yes	Yes	Yes
$X_i \times$ Date Fixed Effects			Yes	Yes	
N	230	230	230	230	230
R^2	0.064	0.724	0.981	0.981	0.724

An inversion result

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$$\mathcal{M}_{\text{own}} \neq -\mathcal{E}_{\text{own}}^{-1}$$

$$\mathcal{M}_{\text{cross}} \neq -\mathcal{E}_{\text{cross}}^{-1}$$

Estimating Substitution Patterns

Aggregate substitution

Moving to aggregate effects

- E.g.: the Fed decides to purchase all corporate bonds
- E.g.: the Fed decides to purchase all corporate bonds *with long duration*
- Change the supply of all assets or a large group of assets
  $\rightarrow$ need to figure out individually $\mathcal{E}_{own}$ and $\mathcal{E}_{cross}$ and other substitutions

Aggregation under Homogenous Substitution

- **Fully symmetric case** with constant $\mathcal{E}_{\text{own}}$ and $\mathcal{E}_{\text{cross}}$:

$$\text{Relative elasticity } \hat{\mathcal{E}} = \mathcal{E}_{\text{own}} - \mathcal{E}_{\text{cross}}$$
$$\text{Aggregate elasticity } \bar{\mathcal{E}} = \mathcal{E}_{\text{own}} + (N-1)\mathcal{E}_{\text{cross}}$$

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- **Separation** of relative and aggregate demand well-defined:

$$\Delta P_{agg} = \frac{1}{N}\sum_j \Delta P_j; \quad \Delta D_{agg} = \frac{1}{N}\sum_j \Delta D_j$$

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  - Symmetric is a special case leading to equal-weighted

Estimating the Aggregate Elasticity

How can we estimate $\bar{\mathcal{E}}$?

- *Time series instrument* for ΔP_{agg}, and run a single time series regression
  - Identify shocks exogenous to my demand that make the price of all assets higher or lower
  - Example: Granular IV $=$ idiosyncratic shocks to large institutions (Gabaix and Koijen, 2024)
  - Alternatively use large event at high frequency: Fed introduces QE
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- *Use models*:
  - Mean variance: ratio $\bar{\mathcal{E}}/\hat{\mathcal{E}}$ increases with correlation, and linearly in number of assets, decreases with segmentation
  - Logit: $\bar{\mathcal{E}} = 1 - \alpha\omega_0$, if no outside assets, $\bar{\mathcal{E}} = 1$ (not driven by the data at all)

Example: Corporate Bond Aggregate Multiplier

Relative multiplier: $\widehat{\mathcal{M}} = 1.232$

Aggregate multiplier: $\bar{\mathcal{M}} = 5.314$

	Price ΔP_{it}		Aggregate Price ΔP_t^{agg}
	(1)	(2)	(3)
$Z_{it}^{idio} \equiv Z_{it} - Z_t^{agg} - X_i Z_t^X$	1.232	1.232	
	(1.046)	(1.048)	
$Z_t^{agg} \equiv \frac{1}{N}\sum_{i=1}^{N} Z_{it}$		**5.314**	**5.314**
		(2.901)	(2.921)
N	230	230	46
R^2	0.001	0.078	0.107

Aggregation with Richer Substitution

- **Observable-based substitution**: additional "aggregate" components:
  - Does duration affect substitution?
  - Would an "operation twist" shock shift aggregate bond prices?

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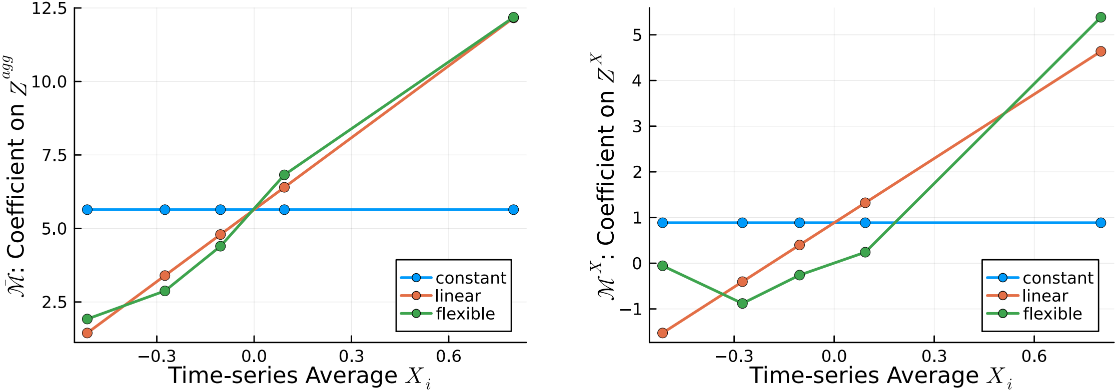
- Additional missing intercepts: c-s again does not help with observable-based substitution
- Unlike for $\widehat{\mathcal{E}}$, the aggregate $\bar{\mathcal{E}}$ requires assumptions on which X drive substitution
  - Warning: omitting drivers of substitution may lead to omitted variable bias
- In practice:
  - For ΔP_{agg}: check at the disaggregated level that ΔP_i are "parallel", don't line up with X_i
  - Incorporate substitution among observables plausibly relevant for your research question

Example: Corporate Bond Multipliers

	Price ΔP_{it}		Aggregate Price ΔP_t^{agg}	Factor Price ΔP_t^X
	(1)	(2)	(3)	(4)
Z_{it}^{idio}	1.232	1.232		
	(1.053)	(1.058)		
X_i	0.005	0.007		
	(0.005)	(0.005)		
Z_t^{agg}	5.640*	5.640*	5.640	8.154*
	(2.768)	(2.780)	(2.807)	(3.135)
Z_t^X	0.886	0.886	0.886	4.690
	(3.666)	(3.682)	(3.717)	(3.751)
$Z_t^{agg} \times X_i$		8.154*		
		(3.105)		
$Z_t^X \times X_i$		4.690		
		(3.716)		
N	230	230	46	46
R^2	0.085	0.116	0.109	0.120

Nonparametric Version

- Allow each bond to respond to each aggregate shock



- Can we go fully nonparametric: each bond on each demand shock? **No!**
  - Need to assume all instruments are orthogonal to all prices
  - Instruments are correlated $\Rightarrow$ multicolinearity problem...

Conclusion

- Key challenge for causal methods in asset pricing: substitution across assets

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- Key challenge for causal methods in asset pricing: substitution across assets
- **Simple conditions on substitution** for valid inference, conditional on observables
  - A1. homogenous substitution between assets (within and outside the estimation sample)
  - A2. constant relative elasticity for assets (within the estimation sample)
- Standard cross-sectional causal inference method identifies **relative elasticity** or its inverse, **relative multiplier**
  - Guidance on designing settings such that assumptions are plausible
  - Compatible with usual covariance matrix assumptions
- Aggregation well-defined but
  - Creates "missing intercepts": use time-series variation
  - Need to consider all dimensions of substitution jointly

    $\rightarrow$ Decide which type of substitution patterns are relevant for your question, and assess them