

How Competitive is the Stock Market?

Theory, Evidence from Portfolios, and Implications for the Rise of Passive Investing*

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Abstract

The conventional wisdom in finance is that competition is fierce among investors: if a group changes its behavior, others adjust their strategies such that nothing happens to prices. We estimate a demand system with flexible strategic responses for institutional investors in the US stock market. When less aggressive traders surround an investor, she adjusts by trading more aggressively. However, this strategic reaction only counteracts half of the impact of the initial change in behavior. In light of these estimates, the rise in passive investing over the last 20 years has made the demand for individual stocks 15% more inelastic.

JEL Codes: G1, G2, D4, L1.

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1 Introduction

What happens to equilibrium prices when a subset of investors changes its behavior? For example, what are the implications of investors switching to passive strategies, which has occurred on a large scale over the last few decades?¹ Answering such questions relies crucially on how other investors react to changes. Under the common view that financial market participants compete fiercely with each other, the answer is simple: nothing happens, because other investors pick up any slack left by those changing their behavior.² Casually said: if you stop looking for \$20 bills on the floor, someone else will replace you. This paper proposes a framework to quantify these strategic responses, combining information from prices and portfolio positions. We implement the framework for the U.S. stock market and study its implications for the rise of passive investing.

We find that investors react to the behavior of others in the market: when an investor is surrounded by less aggressive traders—that is, with a lower price elasticity of demand—she trades more aggressively. While this reaction mitigates the equilibrium consequences of changes in individual behavior, it is not nearly as strong as in the common view of irrelevance of market organization. For instance, our estimates suggest that this strategic response reduces the direct impact of an increase in passive investing by half. An increase as large as the one observed over the last 20 years leads to substantially more inelastic aggregate demand curves for individual stocks, by 15%.

To get to these answers, we proceed in three steps. Intuitively and in line with many theories, we first formalize the degree of strategic response between investors: how much does my demand elasticity respond to the elasticity of others? When investors compete strongly for trading opportunities, their strategies respond more to how others are trading. Second, we provide a framework to quantify these strategic responses and their implications for prices. We write down a demand

¹For example, the ICI factbook (*Investment Company Fact Book – A Review of Trends and Activities in the Investment Company Industry*, 2020) reports that the total assets of passive mutual funds in the U.S. have increased from \$11b to \$2.8t between 1993 and 2020.

²In his discussion of Fama’s work on efficient markets, Cochrane (2013) emphasizes how intensely financial market participants look for investment opportunities: “other fields are not so ruthlessly competitive as financial markets.” Thaler (2015) also discusses the common view among economists that even if investors blunder, prices fix themselves in equilibrium, what he calls the “invisible handwave argument.” While rarely formalized, this view underlines most of the literature assuming away the structure of the financial sector and focusing on representative agent models.

system (à la Koijen and Yogo (2019)) where not only prices but also demand elasticities are the equilibrium result of investors' interactions. Third, we estimate the model using detailed portfolio positions of institutional investors in the U.S. stock market. We quantify the impact of a rise in passive investing and decompose the sources of evolution in demand for individual stocks.

Why is the degree of strategic response so central to financial markets? A more elastic demand curve implies more aggressive trading: the investor increases their position a lot when the asset is cheap. In standard price theory, only a consumer's preferences determine her demand elasticity; your demand for apples depends on how you trade off money and apples. In contrast, an investor's choice of elasticity in financial markets also depends on the behavior of other investors. If others are not trading aggressively, investment opportunities arise, and you have more incentives to trade aggressively. In an idealized view, there is always somebody on the lookout for good deals, and this response is so strong that it compensates for any initial change in investor behavior. In practice, many aspects limit the strength of this reaction. Changing your strategy might require new information to identify profitable trades (Grossman and Stiglitz, 1980), overcoming contractual frictions (e.g. investment mandates) that limit flexibility in setting trading strategies, having incentives to maximize risk-adjusted returns (Chevalier and Ellison, 1997), or having high cognitive sophistication (Eyster et al., 2019). More generally, investors face limits to arbitrage (Shleifer and Vishny, 1997). Finally, while the issue of how investors compete in setting their trading strategies is distinct from whether there is perfect competition for the asset (price-taking behavior), market power also weakens the degree of strategic response (Kyle, 1989).³ Despite their pervasiveness in the theoretical literature, strategic responses have been absent from the recent literature building empirical equilibrium models of investor demand.⁴

We entertain all of these mechanisms by taking a semi-structural approach: investors follow exogenous but flexible investment strategies, and the market must be in equilibrium. We assume that each investor's demand elasticity combines an investor-specific component and a reaction to the aggregate demand elasticity prevalent in the market. The degree of strategic response is the

³Going back to Kreps and Scheinkman (1983), it is understood that price-taking is not the only aspect shaping competition.

⁴As such our framework addresses a common Lucas critique of these models that the demand for assets is not a fixed characteristic of investors.

intensity of this reaction. An equilibrium combines two layers. First, the elasticities of all investors must be consistent with each other: the average of all investor elasticities must be equal to the aggregate elasticity. Second, the asset price is such that the sum of all demand curves evaluated at this price equals the supply of the asset. The simplicity of this framework does not impede its richness. We show that all of the aforementioned foundations for investor strategic responses map to the structure of our model.

What happens when a group of investors becomes passive? Their investment strategy turns irresponsive to the price of the asset; hence their demand elasticity goes to zero. This change pushes the aggregate elasticity down, prompting other investors to respond, potentially compensating for the direct effect. When the strategic response is strongest, this reaction completely offsets the direct effect. The equilibrium market elasticity remains unchanged, and so is the behavior of the asset price. This situation corresponds to the null hypothesis of “fiercely competitive financial markets.” On the other extreme, if investors do not react, the elasticity provided by the traders who became passive is just lost. More generally, we derive a simple formula for the pass-through of a rise in passive-investing to aggregate elasticities as a function of the degree of strategic response.

We parametrize the demand system in the style of Kojen and Yogo (2019) to take it to the data. In particular, the specification entertains rich heterogeneity across investors. However, unlike in Kojen and Yogo (2019), one cannot independently estimate the demand of each institution. Because of the strategic response, the demand elasticities of all investors are intertwined and must be solved simultaneously. This elasticity equilibrium creates three challenges that we overcome.

First, the interaction between investors through their elasticity decisions introduces a reflection problem (Manski, 1993): a market with high elasticity could result from either high individual elasticities or strong positive spillovers. The cross-section of stocks provides a solution to this issue: the same investor faces a different mix of competing investors for each stock, therefore a different aggregate demand elasticity. This variation allows us to isolate the spillover from the individual-specific component of elasticity. This argument faces a chicken-and-egg question. We need to know the elasticities of other investors to implement this comparison. But estimating these investors’ elasticities requires knowing the initial investor’s elasticity in the first place. We derive

and verify conditions on the graph of investor-stock connections under which these problems can be solved simultaneously.

Second, both the price and the aggregate elasticity are equilibrium quantities and therefore depend on portfolio decisions, leading to an endogeneity challenge in demand estimation. We construct an instrument for each of these variables using variations in investment universe across investors. Stocks that more investors can buy naturally have more money chasing them and a higher price, an instrument introduced in Kojien and Yogo (2019). For the aggregate elasticity, we introduce a new model-based instrument combining the variation in investment universe with the estimated individual component of elasticities.

Third, the inclusion of rich investor heterogeneity, the need to solve for an elasticity equilibrium, and the presence of a model-based instrument all concur to a seemingly intractable estimation. We develop a computationally efficient algorithm that estimates the model.

Our estimates suggest a substantial amount of strategic response. If the aggregate elasticity for a stock increases by 1, an individual investor decreases her elasticity of demand by 2.2. We confirm the robustness of this finding in a battery of specifications: alternative instrument construction, more weights on large investors, additional controls, etc. Across these specifications, the estimated strategic response remains between 1.9 and 2.5. This competition among investors stabilizes the levels of aggregate elasticity. Intuitively, when a very aggressive investor trades a specific stock, other investors in this stock adjust by becoming less aggressive. This force implies about 50% less cross-sectional variation in elasticity across stocks than estimates that ignore strategic interactions, highlighting the importance of these interactions.

We use these estimates to assess the impact of a rise in passive investing. To do so, we ask how equilibrium elasticities change when a fraction of investors exogenously becomes passive. We obtain a simple formula for the pass-through of a change in the fraction of active investors to the aggregate elasticity. This pass-through solely depends on the degree of strategic response and the initial fraction of active investors. It is decreasing in both quantities. Empirically, we find this pass-through to be about 0.4. A little less than half of a change in the fraction of active investors translates into a reduction in demand elasticity. Given the 30% decrease in active investing over

the last 20 years, this effect yields a reduction in elasticities of about 15%. This is a sizable change: in the context of many models, it would lead to less informative and more volatile prices, as well as more price impact — we confirm these connections empirically in the cross-section. This result is our main empirical conclusion: while the effects of competition in strategies are strong, the stock market is far from the common view.⁵

A potential concern is that the model ignores some forces to maintain tractability. For example, some theories predict that the strategic response depends on who is switching to passive investing beyond their initial elasticity. Or, these interactions could occur not only through existing investors changing their strategies but also through the entry or exit of new investors. To assess the presence of these other mechanisms, we regress changes in aggregate elasticity on changes in passive investing at the stock level, zooming in on several sources of variation. Confirming our model estimate, we find a pass-through of about 0.4 irrespective of whether we include stock or date fixed effects or even instrumenting for passive investing using index inclusions.

The model also provides an account of the actual evolution of the demand for stocks over the last 20 years. Our model finds two important sources of changes in elasticity. First, the fraction of passive investors has increased steadily over our sample. Second, the investor-specific component of the elasticity of active investors has also experienced significant changes: initially increasing until 2007, then trending downwards, and dropping overall. This second dimension is interesting because it suggests a role for market-wide shifts in individual strategies beyond the rise of passive investing, such as developments in computing power and access to big data.⁶ The presence of these other long-term changes in investor behavior also highlights the danger of assessing theories of the impact of passive investing purely based on aggregate trends, an issue that our structural approach steers clear of. We also find that strategic responses played an important role: active investors also increased their equilibrium elasticity in response to the broad decrease in aggregate elasticities. In a counterfactual exercise in which we shut down the strategic responses, we find that elasticities would have decreased twice as much. In contrast, they would have barely moved with strong strategic responses.

⁵In this case, the pass-through is 0: a rise in passive investing has no impact. On the other hand, without strategic effects, the pass-through is 1, leading to a 30% decrease in elasticity.

⁶Farboodi and Veldkamp (2020) develop a theory of the effect of growth in financial data technology.

Taken together, our results highlight the importance of a more nuanced approach to how investors compete in financial markets. No, it is not the case that “financial markets are fiercely competitive” and that all shocks are fully absorbed by other investors. But also, no, it does not mean that investors do not interact at all. This framework is a first step towards quantifying the degree of strategic response and its implications. Our estimates suggest that these interactions played an essential role in shaping the response to the rise of passive investment. This strategic response is likely important for many other questions about investor demand; we sketch the implications of our framework beyond the rise in passive investing. What happens when a large set of financial institutions must change their trading because of new regulations? What happens when some sophisticated specialized investors get in financial trouble?

Contribution to the existing literature. The idea that investors compete with each other when choosing their strategies has a long history in finance. Grossman and Stiglitz (1980) first formalize the notion of competition for information between investors and show it does not lead to informationally efficient markets.⁷ Kyle (1989) highlights how market power also creates interactions among investors. These seminal contributions have led to a large theoretical literature pointing out rich ways in which investors react to each other and choose their trading strategies. In the context of the rise of passive investing, Subrahmanyam (1991) is an early contribution highlighting liquidity concerns. More recent work includes Bond and García (2018), Malikov (2019), Lee (2020), Buss and Sundaresan (2020), and Kacperczyk et al. (2020). Kacperczyk et al. (2016) focus on cyclical changes in investor attention. Gârleanu and Pedersen (2018) and Gârleanu and Pedersen (2021) focus on the interaction between the market for asset managers and the market for assets. Farboodi and Veldkamp (2020) focus on the choice between information about fundamentals or about demand in the context of the rise in big data. However, these theories are rarely confronted to portfolio data. Our new approach, summarizing strategic responses through choices of demand elasticity, allows us to bring the theory to the data.

We also contribute to a recent literature on estimating demand systems accounting for the

⁷Coles et al. (2022) show that an increase in passive investing does not affect price informativeness in this baseline model.

large heterogeneity in portfolio holdings, started by Kojien and Yogo (2019). Kojien et al. (2021), Kojien and Yogo (2020), Kojien et al. (2020), and Jiang et al. (2020) also apply this approach. Balasubramaniam, Campbell, Ramadorai and Ranish (2021) estimate a factor model of portfolio holdings. Dou et al. (2020) study how mutual funds change their portfolios in response to common fund flows. Gabaix and Kojien (2020) estimate the aggregate demand for stocks. Our key innovation on that front is to incorporate strategic interactions between investors, a long-theorized feature we find to be quantitatively important.

More broadly our paper relates to a wider literature studying the relation between portfolio quantities and asset prices. De Long et al. (1990) argue that noise trader shocks can affect prices. These ideas have found applications across multiple asset classes: stocks (Shleifer (1986), Warther (1995)), government bonds (Vayanos and Vila (2021), Greenwood and Vayanos (2014), Haddad and Sraer (2020)), options (Gârleanu et al. (2009)), currency markets (Gabaix and Maggiori (2015), Greenwood et al. (2019), Gourinchas et al. (2019)), or corporate bonds (Haddad et al. (2021)). While our estimates concentrate on the stock market, we bring to the forefront the importance of strategic interactions between investors, which likely also matter in other markets.

Finally our results provide new insights in the debate on the consequences of the long-term rise in passive investing. French (2008) and Stambaugh (2014) provide empirical evidence of a shift towards passive investing. Zooming in on portfolios, we uncover how passive investing is altering how all investors trade and therefore its equilibrium implications. Other work focuses on quasi-natural experiments around index or ETF inclusion such as Chang et al. (2014), Ben-David et al. (2018), or Coles et al. (2022). Sammon (2021) studies the response of stock prices around earnings announcements. Bai et al. (2016), Dávila and Parlato (2018), and Farboodi et al. (2021) document long-term trends in price informativeness.

2 An Equilibrium Model of Financial Markets with Investor Competition

We present our framework of investor interactions in financial markets. The key idea is that there are two layers to an equilibrium in financial markets. First, the price is such that the sum of investor demands equals the supply of the assets. Second, investors compete with each other in setting their strategies: they choose how aggressively they trade as a function of how others trade. This aggressiveness is measured by their demand elasticity. First, we introduce the two layers, then we highlight the implications of our framework for the rise of passive investing. Table 1 summarizes the model.

2.1 First layer: the asset price clears the market given demand curves

For the sake of simplicity, we focus on the case of a single asset in fixed supply S and a continuum of investors indexed by i . We generalize to multiple assets when moving to the data in Section 4. In an equilibrium, each investor decides how much they buy as a function of the price P of the asset: a demand curve $D_i(P)$, which we can log linearize around a baseline value for the price \bar{P} :

$$d_i = \underline{d}_i - \mathcal{E}_i \times (p - \bar{p}), \quad (1)$$

where lowercase letters represent log values.⁸ The elasticity of this demand curve, \mathcal{E}_i , determines how aggressive the investor is.⁹ An investor with $\mathcal{E}_i = 0$ does not react to changes in prices, while an investor with large \mathcal{E}_i increases her position a lot when the asset is cheap. Beyond the price, other aspects can also affect the choice of positions. For example, an investor could care about the risk profile of the asset or have a preference for environmental, social, and governance (ESG) investing. We collect these other aspects inside the constant \underline{d}_i ; the empirical analysis will be more flexible about modeling \underline{d}_i .

Investors' elasticities play an important role in the determination of equilibrium prices. The

⁸The assumption of demand curves does not necessarily imply price-taking. For example, in the rational expectation equilibrium with imperfect competition of Kyle (1989), investors also post demand curves.

⁹Similarly Gabaix and Koijen (2020) consider log-linear demand curves around a reference price level.

Table 1. The 2-layer model of asset market.

	Individual Decision	Equilibrium Condition
Demand	$d_i = \underline{d}_i - \mathcal{E}_i \times (p - \bar{p})$	$\int D_i(p) = S$
Elasticity	$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \times \mathcal{E}_{agg}$	$\int \mathcal{E}_i D_i / S = \mathcal{E}_{agg}$

aggregate demand curve is $D_{agg}(P) = \int D_i(P)$, and the equilibrium price solves $D_{agg}(P^*) = S$. Aggregate demand has elasticity

$$\mathcal{E}_{agg} = \frac{\int \mathcal{E}_i D_i}{\int D_i}. \quad (2)$$

The (holdings-weighted) average of individual elasticities measures how strongly aggregate demand for the asset responds to the price. This aggregate elasticity shapes the behavior of the equilibrium price. If investors are very aggressive, aggregate demand is perfectly elastic, $\mathcal{E}_{agg} \rightarrow \infty$, and prices are pinned down at a fixed level. In such a situation, changes in individual investor characteristics \underline{d}_i or in supply S do not affect the price. This is what people sometimes describe as “efficient markets:” any deviation of the price from a fundamental value is immediately traded away by aggressive investors. On the other hand, when demand is more inelastic, small changes in the market structure can have a large effect on prices because investors are unwilling to change their positions.

For example, if elasticities are constant, a small uniform change $\Delta \underline{d}$ to the demand of all investors results in a price change of

$$\Delta p = \mathcal{E}_{agg}^{-1} \times \Delta \underline{d}. \quad (3)$$

If all investors want to increase the size of their position by one percent, the price increases by the multiplier $M_{agg} = \mathcal{E}_{agg}^{-1}$ percent. Consequently, more inelastic markets experience larger price

variation due to changing investor demands, and are therefore more volatile.¹⁰ A change in supply would have the opposite effect on the price with a multiplier $-M_{agg}$. More fleshed-out models such as the ones we present in Section 3 also relate the aggregate elasticity to other equilibrium properties such as price informativeness, liquidity, or limits to arbitrage. We confirm these relations empirically in Section 5.3.2.

2.2 Second layer: investors set their demand elasticity in response to others

In standard price theory, the elasticity of demand reflects only an individual's preference for a good. In particular, it does not depend on the decisions of other market participants. When choosing how many apples to put in your shopping cart, it does not matter what other shoppers are doing beyond their effect on the price level. However, in financial markets, it matters why the price is moving and consequently demand elasticities are not fixed.¹¹ Investors compete for trading opportunities. For example a common theoretical prediction is that, if many investors trade aggressively, fewer good deals are available; therefore, there are also fewer incentives to trade with a high elasticity.

This relation adds a second layer to the equilibrium, which captures how investors compete when choosing their strategies. At the individual level, the elasticity responds to the aggregate demand elasticity. But conversely, the aggregate demand elasticity is an average of individual elasticities. Formally, we represent this feedback by endogenizing individual demand elasticities as a function of the aggregate demand elasticity:

$$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \mathcal{E}_{agg}. \quad (4)$$

The parameter χ controls the strength of the response to the aggregate elasticity; it measures the

¹⁰See also Gabaix and Koijen (2020) for a discussion of the role of the elasticity of aggregate demand in financial markets.

¹¹A similar phenomenon arises in auction settings: a bidder's optimal bidding strategy often responds to the strategies of other participants in the auction.

extent of strategic substitution in demand elasticities.¹² $\underline{\mathcal{E}}_i$ is a baseline level of elasticity reflecting the investor’s own preferences for the asset, for example shaped by her risk aversion or her beliefs about the payoffs. Together, the individual decision equation (4) and the aggregation condition of equation (2) pin down the equilibrium of elasticities.

We refer to the parameter χ as the *degree of strategic response*. Large values of χ capture the narrative that “financial markets are fiercely competitive.” If a group of sophisticated investors goes away, other investors pick up the slack by trading more aggressively. In the extreme case where χ goes to infinity, the strategic response is so strong that the equilibrium aggregate elasticity \mathcal{E}_{agg} is pinned down at a fixed level.¹³ Changes in individual investor behavior or the composition of investors do not affect the aggregate elasticity.

On the other hand, when $\chi = 0$, individual investors do not respond to the aggregate elasticity. We are back to standard price theory: each investor follows a strategy that is independent of the actions of other investors. Under this view, if a group of sophisticated investors goes bankrupt, nobody else steps in to take advantage of the opportunities that are left untouched: the aggregate elasticity drops sharply.

The parameter χ offers a simple and flexible way to capture strategic interactions and their consequences. We do not take a stand on a specific microfoundation for the parameter χ . In many theories, demand elasticities are a key feature of investors’ strategies and exhibit substitutability or complementarity; we devote Section 3 to these theories.¹⁴ Rather than restricting ourselves to a specific foundation — many of these theories are operating side by side — we measure strategic responses directly from trading and portfolio data.

Next, we show how the degree of strategic response plays a crucial role in several applications. First, we study the effect of a rise in passive investing — our main empirical application. Second, we show that understanding how institutions react to each other in setting their strategies is crucial for intermediary asset pricing. Finally, Appendix Sections A.2 and A.3 consider implications for

¹²We consider strategic substitutes and complements in the sense of Bulow et al. (1985) and defined in chapter 4 of Veldkamp (2011).

¹³Equation 4 might suggest that elasticities are lower when χ is large, but it is not necessarily the case, as economies with a large χ will tend to also have a large $\underline{\mathcal{E}}_i$. See for example the calculation in the next section.

¹⁴Technically, other aspects of investor decisions may be the source of substitutability (e.g. information acquisition or social interactions). However, because elasticities are directly related to these other decisions, the substitutability manifests itself in the demand elasticity.

the asymmetry of mispricing and the dynamics of limits to arbitrage.

2.3 The effect of a rise in passive investing

Accounting for strategic responses is crucial in evaluating the effect of a rise in passive investing. Consider the following thought experiment. We start from an economy with homogeneous investors who, in this initial equilibrium, have elasticity $\mathcal{E}_i = \mathcal{E}_0$. The aggregate elasticity is therefore also \mathcal{E}_0 . What happens when a fraction $1 - \alpha$ of these investors becomes passive, that is keep the same holdings, but reduce their elasticity to zero?

The direct effect of this change is that now only a fraction α of investors contribute to the aggregate elasticity. If we only consider this effect, the aggregate elasticity decreases to $\mathcal{E}_{agg} = \alpha\mathcal{E}_i$ (from the aggregation equation (2)). But the story does not end here; the remaining active investors adjust their strategies. This is exactly what the degree of strategic response χ captures. Active investors change their own elasticity in response to the aggregate: $\Delta\mathcal{E}_i = -\chi\Delta\mathcal{E}_{agg}$ (from equation (4)). This response compensates the direct effect when $\chi > 0$. Each active investor responds again to the response of other active investors, until they reach a new equilibrium.¹⁵ Appendix Figure IA.2 illustrates this process. The new aggregate elasticity is:

$$\mathcal{E}_{NEW} = \underbrace{\alpha\mathcal{E}_0}_{\text{direct effect}} + \underbrace{(1 - \alpha)\mathcal{E}_0 \frac{\alpha\chi}{1 + \alpha\chi}}_{\text{strategic response}}. \quad (5)$$

With a large degree of strategic response, χ is large, and $\mathcal{E}_{NEW} = \mathcal{E}_0$, the aggregate elasticity is unchanged. The drop in elasticity due to the investors that became passive is exactly compensated by a greater elasticity of the remaining active investors. In contrast, when investors are insensitive to market conditions, χ close to zero, only the direct effect operates, and the elasticity declines by a factor α .

What does this imply quantitatively? In the estimation of Section 4, we find a degree of strategic response χ of 2. Over the last 20 years, the fraction of active investors has decreased by 30%, so we set $\alpha = 70\%$. This implies that the initial elasticity is multiplied by a factor of

¹⁵Formally, we do not model this tâtonnement, and instead focus directly on equilibria. We present details of the calculation in Appendix Section A.1.

$70\% + 30\%(70\% \times 2)/(1 + 70\% \times 2) = 0.875$. The rise of passive investing leads to a substantial drop in elasticity of 12.5%. This is about half of the direct effect that would have led to a decrease of 30%. However, it is still much more than the zero predicted by the common view.

In Section 4, we fully specify our framework to account for heterogeneity across investors and stocks, and estimate it using portfolio holdings data.¹⁶ This allows us to revisit the question of the rise in passive investing in the context of a realistic quantitative model in Section 5.1.

2.4 Intermediary asset pricing

How do markets change when some financial institutions get distressed or when they are more tightly regulated? As these institutions trade less aggressively they provide less elasticity to the market and we expect more unstable prices. Two aspects shape this response: how large the direct shock to the institutions is, but also how other competing investors respond. Consider how the aggregate elasticity responds to a combination of shocks to individual elasticities $\{\Delta \underline{\mathcal{E}}_i\}_i$; for example, only the affected institutions receive a negative shock to their elasticity. For simplicity, we assume that the price is at its baseline, $p = \bar{p}$, and leave the general case to Appendix A.¹⁷ We show that the change in the aggregate elasticity is

$$\Delta \mathcal{E}_{agg} = \frac{1}{1 + \chi} \mathbf{E}[\Delta \underline{\mathcal{E}}_i], \quad (6)$$

where $\mathbf{E}[\cdot]$ denote the demand-weighted population average.¹⁸ The change in aggregate elasticity combines the average direct elasticity shock $\mathbf{E}[\Delta \underline{\mathcal{E}}_i]$ and a mitigating factor due to the strategic response $1/(1 + \chi)$. With strong responses, $\chi \rightarrow \infty$, the shock to some investors has no effect on the aggregate elasticity. This is the view of those arguing that intermediaries cannot matter for asset prices. However, for lower values of χ , the direct effect is not mitigated. Theoretical models centered on intermediaries often assume $\chi = 0$ (e.g He and Krishnamurthy (2013) and

¹⁶Appendix Section A.4 shows that when χ differs across investors, what matters for the rise of passive investing is the demand-weighted average value among active investors.

¹⁷Unlike in the precedent calculation, we assume that there are no passive investors.

¹⁸Formally this corresponds to

$$\mathbf{E}[x_i] = \int x_i \frac{D_i}{S}.$$

Brunnermeier and Sannikov (2013)).

As such, when analyzing how the financial health of intermediaries matters for asset pricing and the economy, one must also take into account how other institutions compete with them. Consistent with this idea, Haddad and Muir (2021) show that in markets that are more sophisticated and hence with less intense competition, periods of distress in the financial sector are associated with stronger movements in risk premium. Eisfeldt et al. (2017) also emphasize this role of investor competition in markets for complex assets such as mortgage-backed securities. Siriwardane et al. (2021) document many situations in which shocks to one intermediary are imperfectly compensated by the reaction of other intermediaries.¹⁹

3 The Origins of Strategic Interactions in Financial Markets

In an idealized view of financial markets, investors are constantly on the lookout for good opportunities, and swiftly come in if another market participant steps down. This corresponds to $\chi \rightarrow +\infty$ in our framework. In practice, many forces limit this process of strategic response. We discuss the most prominent ones in this section: costly information acquisition, bounded rationality, liquidity, peer effects. Appendix Section C discusses the role of institutional frictions and endogenous risk. We show that our 2-layer equilibrium model captures the main insights of these theories in a parsimonious way.

3.1 Costly information acquisition

A basic idea of how investors interact with each other is that if some active investors exit the market, there are more investment opportunities to take advantage of, and other investors go after them by trading more aggressively. In practice, knowing that there are more investment opportunities is not enough, investors have to evaluate them. The costs of this process of learning (information gathering, hiring analysts, etc.) naturally limit the ability to compete.

¹⁹Other examples of large effects of intermediary health in specialized markets include Gabaix et al. (2007) and Siriwardane (2019).

We formalize this intuition in a model in the style of Grossman and Stiglitz (1980) with information acquisition as in Veldkamp (2011), and show it maps tightly to our two-layer equilibrium.²⁰ We focus here on the main results and leave details of the setting and derivations to Appendix B.

There is one period and one asset, and a continuum of agents indexed by i . Each agent has CARA preferences with risk aversion ρ_i . The gross risk-free rate is 1, and the (random) asset payoff is f . The asset is in noisy supply $\bar{x} + x$ with \bar{x} an exogenous fixed parameter and $x \sim \mathcal{N}(0, \sigma_x^2)$. Initially, each agent is endowed with an independent signal μ_i of the fundamental f , distributed $\mu_i \sim \mathcal{N}(f, \sigma_i^2)$.²¹ Obtaining more precise signals is more costly. Each agent can acquire an additional private signal $\eta_i \sim \mathcal{N}(f, \sigma_{i,\eta}^2)$ at monetary cost $c_i(\sigma_{i,\eta}^{-2} + \sigma_i^{-2})$, with $c_i(\cdot)$ a non-decreasing positive function.²² The signal being private implies in particular that signal realizations are uncorrelated across agents conditional on the fundamental f .

Optimal asset demand is linear in the price: $d_i = \underline{d}_i - \mathcal{E}_i p$.²³ The slope of the demand curve characterizes how aggressively an investor changes her portfolio when the price moves. We find (Appendix B.3):

$$\mathcal{E}_i = \frac{1}{\rho_i} (\sigma_i^{-2} + \sigma_{i\eta}^{-2}). \quad (7)$$

Two elements shape the investor's demand elasticity: her risk aversion and her private information. An investor with more precise information about the asset is more confident in her forecast of the asset returns, and therefore trades more aggressively. Looking ahead, we can already see that constraints to the ability to change information acquisition will limit the ability of the investor to change her elasticity.

Before that, we show that the aggregate elasticity, $\mathcal{E}_{agg} = \int_i \mathcal{E}_i di$, is the appropriate notion for

²⁰Bond and García (2018) and Malikov (2019) provide theoretical analyses of the rise of passive investing in this family of theories.

²¹Following Veldkamp (2011), we assume agents start with a flat prior on f , hence their initial belief is $f \sim \mathcal{N}(\mu_i, \sigma_i^2)$.

²²This parametrization is without loss of generality relative to a cost function that would only depend on the acquired signal $\sigma_{\eta,i}$.

²³For all of this subsection, we do a small abuse of notation: lowercase letters represent levels rather than logarithms and \mathcal{E}_i denotes the slope of the demand curve, rather than the elasticity *stricto sensu*. This approach lends itself to the linearity of the CARA-Normal framework, but is less appealing for empirical applications.

how the collective actions of all investors shape the price. In equilibrium, the price follows

$$p = A + f - \mathcal{E}_{agg}^{-1} x, \quad (8)$$

where A is a constant. The price responds one-to-one to the fundamental f , but is also affected by noise trading x . The aggregate elasticity controls the impact of noise: if everybody trades aggressively against abnormal price movements, noise traders cannot push the price far away from fundamentals. In line with this intuition, a market with higher aggregate elasticity also has less volatile returns ($\text{Var}(f - p) = \mathcal{E}_{agg}^{-2} \sigma_x^2$) and more informative prices ($\text{Var}(f|p)^{-1} = \mathcal{E}_{agg}^2 \sigma_x^{-2}$).

The strategic responses of investors to one another occur through information choices. The aggregate elasticity impacts price dynamics, which in turns affect the incentives to acquire information and trade in an elastic way. When choosing how much information to acquire, investors trade off the cost of a more precise signal with the benefit of a more informed trading strategy. The utility gain from precise information is proportional to knowledge of the fundamental, which combines private information (corresponding to \mathcal{E}_i) and information learned from prices (corresponding to \mathcal{E}_{agg}). Focusing on elasticities, this leads to the following optimization problem:

$$\max_{\mathcal{E}_i} \frac{1}{2} \log (\rho_i \mathcal{E}_i + \mathcal{E}_{agg}^2 \sigma_x^{-2}) - \rho_i c_i(\rho_i \mathcal{E}_i) \quad (9)$$

This problem is the counterpart to equation (4): the choice of individual elasticity \mathcal{E}_i depends on the aggregate elasticity \mathcal{E}_{agg} . To a first-order approximation, the degree of strategic response is the sensitivity of the optimal individual elasticity to the aggregate elasticity: $\chi = -\partial \mathcal{E}_i / \partial \mathcal{E}_{agg}$.²⁴ In this model, the degree of strategic response χ is always positive. If others acquire less information and become less aggressive, there are incentives to look for information and step in to replace them. However, these forces only partially offset the initial change, $\chi < \infty$. In particular, costs to adjust information limit the ability to react and result in lower χ . Formally, we show in Appendix B.5 that χ is decreasing in the curvature of the information cost function.²⁵

²⁴The relation between \mathcal{E}_i and \mathcal{E}_{agg} is not linear in general. In Appendix B.4 we find a two-parameter family of simple cost functions under which this relation is exactly linear as in equation (4). Each of the two parameters maps in closed-form to the degree of strategic response χ and individual elasticity $\mathcal{E}_{0,i}$.

²⁵Coles et al. (2022) show a full degree of strategic substitution in the baseline setup of Grossman and Stiglitz

3.2 Bounded rationality

Strategic interactions between investors rely on their understanding of market structure. For example, in the rational expectations equilibrium of Section 3.1, each investor knows the strategies followed by everyone else. Practically, how would investors figure out other people’s strategies? Both in our model and in the theories described above, investors only need to know the aggregate elasticity \mathcal{E}_{agg} , but not the actions of each of the other investors. In the real world, institutions can track changes in investment styles directly (e.g. industries, factors, arrival of activist investors) or through their impact on prices (e.g. price impact, volatility, price informativeness). While this information is useful, it is still a leap to assume investors can follow exactly the optimal policies in frictionless models.

First, the information available to investors about the aggregate elasticity might be imperfect.²⁶ In such a setting the response to other investors is dampened. For example, assume an investor wants to react to aggregate elasticity with a coefficient χ_0 , but she only observes a noisy signal about \mathcal{E}_{agg} . Then, her elasticity choice can be written as

$$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi_0 \theta \cdot \mathcal{E}_{agg} + \epsilon. \quad (10)$$

Because the investor cannot separate the noise from the information about \mathcal{E}_{agg} , she responds to her signal with a Bayesian shrinkage factor $0 < \theta < 1$. The residual ϵ is due to the noise in the signal. Appendix C.4 provides derivations and explicit expressions for these quantities. The effective degree of strategic response is $\chi_0 \theta$ and incorporates the baseline strategic response χ_0 with the dampening factor θ .

Second, investors have to be sophisticated enough to understand their strategy should react to what other investors are doing. A recent strand of research considers equilibria in which investors miss the actions of others (Eyster and Rabin (2005), Greenwood and Hanson (2014), Eyster et al. (2019), Bastianello and Fontanier (2021)). Neglecting equilibrium forces can either amplify or mitigate the degree of strategic response. On the one hand, investors could simply ignore how the

(1980) without adjustment costs.

²⁶Imperfect information about other investors’ strategies is different from imperfect information about fundamentals or noise traders.

elasticity choice of others affect their investment opportunities. In this case, we will not observe any strategic response. On the other hand, investors might understand the direct effect of changes in elasticity but fail to realize that others react to those as well, a form of partial equilibrium thinking as in Bastianello and Fontanier (2021). For example, all investors understand there is a rise in passive investing but fail to realize that others will react by trading more aggressively. We include partial equilibrium thinking into the calculation from Section 2.3 on the effect of a rise in passive investing. We show in Appendix C.5 that the new aggregate elasticity becomes

$$\mathcal{E}_{NEW}^{PET} = \alpha \mathcal{E}_0 + (1 - \alpha) \chi \alpha \mathcal{E}_0. \quad (11)$$

Because investors do not account for the response of others, they overreact to the initial change in elasticity. With partial equilibrium thinking, the strategic response is stronger than in the baseline (see equation (5)) by a factor $1 + \alpha \chi$. This leads to a relatively higher final level of aggregate elasticity, bringing the economy closer to the idealized view of financial markets.

3.3 Strategic complementarities

Finally, some forces generate strategic complementarity rather than substitutability, which yields negative values of the parameter χ . In these situations, when some investors become less aggressive, other investors also pull out of markets instead of replacing them.

One such case arises when investors worry about the price impact of their trades. In Appendix Section C.3, we show that a model of market power in the style of Kyle (1989) yields a negative value of χ .²⁷ Specifically, the standard CARA elasticity becomes

$$\mathcal{E}_i = \frac{1}{\rho_i \sigma^2 + \underbrace{(\mathcal{E}_{agg} - \mathcal{E}_i)^{-1}}_{\lambda_{-i}}}. \quad (12)$$

The investor responds to the price based on her risk aversion and the risk of the asset, $\rho_i \sigma^2$, and the slope of the residual demand curve for the asset, what Kyle (1989) calls λ_{-i} . When other

²⁷We also show that the measure of price impact Kyle's λ is closely related to the inverse of aggregate elasticity.

investors are more price elastic, it enhances liquidity in the market. In turn, this facilitates my ability to trade and I can be more responsive to prices. This type of complementarity holds in a broader family of theories of liquidity such as Vayanos and Wang (2007).

Strategic complementarities can also arise through social interactions. When investors follow their peers, as in Hong et al. (2004), changes in some investors are amplified by similar decisions from other investors.²⁸ If I see others around me trade a stock more aggressively, I also want to trade that stock more aggressively. This herding leads to negative values of χ .

4 Estimating the Degree of Strategic Response

In this section, we estimate the degree of strategic response χ and demand elasticities in the context of the U.S. stock market. First, we enrich our model to account for the heterogeneity of stocks and investors. Then, we design and implement a new identification strategy for demand estimation in the presence of strategic interactions.

4.1 Quantitative model

Individual decisions. In practice, agents invest in many assets. Therefore, an empirical model must make sure that portfolio positions add up to total assets for each investor. In addition, it should also account for the portfolio aspect of financial decisions, that is, substitution across assets. Kojien and Yogo (2019) show that a logit framework satisfies both of these requirements. We denote each security by the index k , the total assets of an investor by A_i , and the portfolio share of investor i in security k by w_{ik} . Therefore $d_{ik} = \log(w_{ik}A_i) - p_k$. The framework of Kojien and Yogo (2019) corresponds to specifying a log-linear model for relative portfolio shares w_{ik}/w_{i0} instead of the individual demand directly, with index 0 being the outside asset.²⁹ We follow this approach. For each investor, we take as given total assets under management, A_i , and the investment universe, \mathcal{K}_i , that is, the set of assets they can invest in.

Second, we need to specify the baseline levels of demand and elasticity \underline{d}_i and $\underline{\mathcal{E}}_i$. We assume

²⁸Hirshleifer (2020) more broadly emphasizes the importance of social interactions in finance.

²⁹Appendix D.4 details the empirical definition of the outside asset.

that each of those combines potentially distinct sets of asset characteristics using investor-specific coefficients. Going back to the setting of Section 3, an interpretation of this assumption is that investors form priors on different assets based on their characteristics; for example, characteristics could capture factor loadings. This corresponds to expressing the baseline demand as $\underline{d}_{ik} = \underline{d}_{0i} + \underline{d}'_{1i} X_k^{(d)} + \epsilon_{ik}$ and the baseline elasticity as $\underline{\mathcal{E}}_{ik} = \underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)}$, where the two vectors of characteristics are $X_k^{(d)}$ and $X_k^{(e)}$. We also account for asset-specific changes in demand by including a shock ϵ_{ik} in \underline{d}_{ik} . For example, this shock captures the private signal η and noise trading x of the model of Section 3.

Finally, we estimate the model separately each time period using information only from the cross-section. Thus, we allow all quantities and parameters of the model to depend on time. For ease of notation we drop the subscript t . Putting it all together, our model of portfolio demand is³⁰

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1i} X_k^{(d)} - \mathcal{E}_{ik} p_k + \epsilon_{ik}, \quad (13)$$

$$\mathcal{E}_{ik} = \underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi \mathcal{E}_{agg,k}. \quad (14)$$

Starting from the relative shares $\vartheta_{ik} = w_{ik}/w_{i0}$, the actual shares can be obtained by

$$w_{ik} = \frac{\vartheta_{ik}}{1 + \sum_{k \in \mathcal{K}_i} \vartheta_{ik}}, \quad (15)$$

$$w_{i0} = \frac{1}{1 + \sum_{k \in \mathcal{K}_i} \vartheta_{ik}}. \quad (16)$$

Interestingly, the demand system of Koijen and Yogo (2019) is a special case of this framework. In their model, demand elasticities are fixed structural parameters.³¹ This corresponds to setting $\underline{\mathcal{E}}_{1i} = 0$ and $\chi = 0$. Therefore, their model implicitly assumes no strategic response. Consequently, when some investors are removed from the markets, the other ones do not step in with larger

³⁰To match equation (13) with equation (1), recall that: $d_{ik} = \log \frac{A_i w_{ik}}{P_k}$. Then $\underline{d}_{ik} = \underline{d}_{0i} + \underline{d}'_{1i} X_k + \log(A_i) + \log(w_{0i}) + \epsilon_{ik}$.

³¹Technically in the logit model the demand elasticity is $1 - (1 - w_{ik})(1 - \mathcal{E}_{ik})$. For values of w_{ik} that are small relative to one, as in the data, this expression is close to \mathcal{E}_{ik} . Hence we refer to \mathcal{E}_{ik} as the demand elasticity throughout the paper.

elasticities. This is the polar opposite from the common view of “fiercely competitive financial markets,” which corresponds to $\chi \rightarrow \infty$. Our framework lets us quantify how close or far reality is from these two extremes.

Passive investors. We account separately for passive investors. By passive, we mean that these are investors whose demand does not respond to prices. Index funds are a specific example of such investors. Our notion is broader though, because it accommodates arbitrary fixed portfolios. To represent such behavior, we simply replace equation (14) by $\mathcal{E}_{ik} = 0$.³² Separating out these investors is important, not only because of their low level of elasticity, but also because they do not respond to aggregate trading conditions. We denote the set of active investors for asset k by $Active_k$ and the fraction of asset k held by this group of investors as $|Active_k|$.

Equilibrium prices and elasticities. Going from individual decisions to an equilibrium relies on market clearing. As in the model of Section 2, two equilibrium objects play a role in individual decisions: prices, p_k , and aggregate elasticities, $\mathcal{E}_{agg,k}$. The corresponding equilibrium conditions are

$$\sum_i w_{ik} A_i = P_k, \quad \forall k, \quad (17)$$

$$\sum_i \frac{w_{ik} A_i}{P_k} \mathcal{E}_{ik} = \mathcal{E}_{agg,k}, \quad \forall k. \quad (18)$$

We normalize the number of shares available to 1 to obtain the market-clearing condition for assets, equation (17). Said otherwise, p_k denotes the log market capitalization.

4.2 Data

We estimate the model for the U.S. stock market. We obtain stock-level data from CRSP: price, dividends, and shares outstanding. We merge the CRSP file with COMPUSTAT for balance

³²Importantly, the elasticity that we measure is for individual stocks, that is to the relative price of different stocks. It is possible that passive institutions contribute to the elasticity of the aggregate demand for stocks. Neither our model, nor any other empirical model to our knowledge, accomodates simultaneously these different levels of aggregation.

sheet information and compute additional stock-level characteristics: book equity, profitability, and investment.

We obtain portfolio holdings data from the 13F filings to the SEC from 2001 to 2020. We build the dataset from the SEC EDGAR website following the method of Backus et al. (2019, 2020). The SEC requires that every institution with more than \$100m of assets under management files a quarterly report of their stock positions. We find that collectively the holdings reported in the 13F filings account for 80% of the total stock market capitalization. We follow Koijen and Yogo (2019) to construct the final panel dataset. Appendix D provides additional details.

4.3 Identification

To estimate the model described above we have to overcome three difficulties: (i) a reflection problem induced by the interactions between investors; (ii) the classic problem of endogeneity in demand estimation; and (iii) how to implement the estimation given that one of the “regressors,” the aggregate elasticity, is unknown. We explain our identification strategy, starting from an ideal experiment, and progressively introducing the challenges brought on by the real world.

4.3.1 The reflection problem

In our setting, individual investor elasticities \mathcal{E}_{ik} depend on an investor-specific term, $\underline{\mathcal{E}}_{ik}$, and on the aggregate elasticity $\mathcal{E}_{agg,k}$:

$$\mathcal{E}_{ik} = \underline{\mathcal{E}}_{ik} - \chi \mathcal{E}_{agg,k}. \quad (19)$$

We need to disentangle whether investors are elastic because of their own characteristics or in response to other investors in the market. For example, if in a market we see that all investors behave in a very elastic manner, it could be that each of them is fundamentally very elastic, high $\underline{\mathcal{E}}_{ik}$. But it could also be the consequence of a strong positive feedback where $\chi < 0$. This identification problem is the reflection problem (Manski, 1993): are individual actions the fruit of individual traits or a reflection of the behavior of others?

The ideal experiment to answer this question would be to see two parallel worlds where the

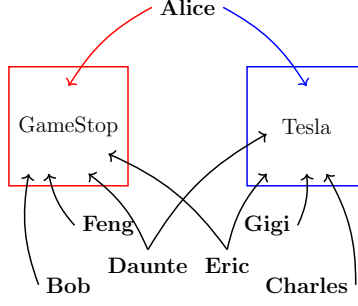


Figure 1. Illustration of identification strategy.

same investor faces different trading counterparts of known fixed elasticities for the same stock. Then, we would estimate the relation between the elasticity of this investor and the elasticity of the investors she is facing.

The real world provides us with a closely related situation: the same investor trades multiple stocks, each with a different mix of investors. Even assuming that investors are exogenously allocated and stocks are identical (so that $\underline{\mathcal{E}}_{ik} = \underline{\mathcal{E}}_i$), this is not quite the same as the ideal experiment. There is a chicken-and-egg problem: to estimate one investor's elasticity decisions, we need to already know how the other investors choose their elasticities. Figure 1 illustrates this idea: we need to compare how Alice trades differently when facing different groups of other investors, such as for GameStop and Tesla. To do so, we simultaneously need to figure out the elasticity for Bob, Charles, Daunte, etc. In particular, these investors' elasticities likely respond to others' elasticities, including Alice's.

The following theorem shows conditions under which we can back out the degree of strategic response from individual observations of demand elasticities. For simplicity, we focus on the case of constant individual-specific components $\underline{\mathcal{E}}_{ik} = \underline{\mathcal{E}}_i$.

Theorem 1. *A decomposition of demand elasticities $\{\mathcal{E}_{ik}\}_{i,k}$ into individual elasticities $\{\underline{\mathcal{E}}_i\}_i$ and the degree of strategic response χ is unique if:*

- (a) *The graph \mathcal{G} of investor-stock connections is connected.*
- (b) *Position-weighted averages of demand elasticities are not constant across stocks: there exists k and k' such that $\sum_{i \in I_k} w_{ik}/P_k A_i \underline{\mathcal{E}}_i \neq \sum_{i \in I_{k'}} w_{ik'}/P_{k'} A_i \underline{\mathcal{E}}_i$.*

Intuitively, this result states that if there is enough mixing in the allocation of investors to stocks, we can recover χ . For example, if investors are isolated (violating part (a)), or if in aggregate the composition of demand for each stock is the same (violating part (b)), it is not possible to escape the reflection problem. We derive and discuss this theorem in more depth in Appendix E.2. In particular, we explain that the two conditions for the result to apply are satisfied in our setting.

While this result explains how we can separate individual components of elasticities from strategic responses, it leaves aside some other challenges to identification. First, the same investor can behave differently for different stocks, that is $\underline{\mathcal{E}}_{ik}$ varies across stocks k within i . For example, the investor might trade differently small and large stocks, irrespective of who the other investors are in these stocks. Our model incorporates variation in individual elasticity across stocks conditional on observable characteristics of these stocks — the characteristics $X_k^{(e)}$ in equation (14). Theorem 1 still applies conditional on these characteristics.

Second, the demand of investors for various stocks also depends on the residual ϵ_{ik} (equation (13)), which are typically correlated across investors. Said otherwise, similar unobserved common determinants of demand affect different investors. Because elasticities are estimated from data on demand, these correlated residuals could contaminate the estimation of the degree of strategic response. Outside of our model, a related issue would arise if the individual elasticity varied due to unobserved sources of variations correlated across investors.³³ To overcome these challenges, one needs to compare situations where differences in aggregate elasticity \mathcal{E}_{agg} are the result of exogenous variation in the allocation of investors. We propose using instrumental variables that are plausibly exogenous to investor demand. This is the same issue as basic demand estimation, in which the price p_k is correlated with residual demand ϵ_{ik} . The next section presents this approach and discusses the specific instruments of our implementation.

³³This would correspond to changing equation (14) to $\mathcal{E}_{ik} = \underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi \mathcal{E}_{agg,k} + \nu_{ik}$, with ν_{ik} correlated across investors. While the instruments we use would overcome the identification problem from these residuals, we do not include such residuals in the model because the estimation becomes numerically intractable.

4.3.2 Instrumental variables

To understand better why instruments are crucial to the estimation, assume for a moment that the aggregate elasticity for each stock, $\mathcal{E}_{agg,k}$, is measured in our data.³⁴ Then, by combining equation (13) and (14), the model is a standard linear regression equation:

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1i} X_k^{(d)} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi \mathcal{E}_{agg,k} \right) p_k + \epsilon_{ik}. \quad (20)$$

The parameters are \underline{d}_{0i} , \underline{d}_{1i} , $\underline{\mathcal{E}}_{0i}$, $\underline{\mathcal{E}}_{1i}$, and χ . Challenges to identification come down to the correlation of the residual demand ϵ_{ik} and the regressors, in particular the equilibrium objects p_k and $\mathcal{E}_{agg,k}$.

The simplest possible identifying assumption takes residual demand as exogenous to all other variables to get the moment condition

$$\mathbf{E} \left[\epsilon_{ik} | X_k^{(d)}, X_k^{(e)}, p_k, \mathcal{E}_{agg,k} \right] = 0. \quad (21)$$

Then, we could estimate (20) using ordinary least squares. The independence of ϵ_{ik} from X_k is naturally motivated by taking the supply of assets as exogenous, as in endowment economies (Lucas, 1978). Furthermore, the independence from p_k and $\mathcal{E}_{agg,k}$ relies on the logic that residual demands do not matter for equilibrium outcomes because they “cancel out” in the aggregate. This rules out both the presence of non-atomistic investors and correlated demand shocks. Both of these last assumptions are not likely to hold for institutional investors. For example, there could be a fad where a large group of investors are enthusiastic about a specific stock, meaning a high ϵ_{ik} for this stock. This increase in demand would push the price p_k higher — see the equilibrium conditions in equations (17) and (18). This would generate a positive correlation between p_k and ϵ_{ik} which leads to a downward bias in estimates of elasticity.

Because the simple identifying assumption is not plausible, we relax it and propose an alternative identification strategy. We look for sources of variation in the aggregate variables that are

³⁴The discussion of the previous section explains how to deal with the elasticity equilibrium. We come back to this in our discussion of how to implement the estimation altogether.

orthogonal to the residual demand. Specifically, using instruments $(\hat{p}_k, \hat{\mathcal{E}}_{agg,k})$ for the aggregate variables $(p_k, \mathcal{E}_{agg,k})$ allows us to weaken the moment condition (21) to

$$\mathbf{E} \left[\epsilon_{ik} | X_k^{(d)}, X_k^{(e)}, \hat{p}_{i,k}, \hat{\mathcal{E}}_{agg,k} \right] = 0. \quad (22)$$

Provided that our instruments are relevant, we could estimate the model using two-stage least squares (again assuming that $\mathcal{E}_{agg,k}$ is known).

To construct these instruments, we use variation in total assets and the investment universe of institutions, an approach introduced by Koijen and Yogo (2019). The instrument for the price of asset k follows

$$\hat{p}_{k,i} = \log \left(\sum_{j \neq i} A_j \frac{\mathbf{1}_{k \in \mathcal{K}_j}}{|\mathcal{K}_j|} \right), \quad (23)$$

where $\mathbf{1}_{k \in \mathcal{K}_j}$ is an indicator variable of when stock k is in investor j investment universe. This instrument corresponds to how much money would flow to stock k if all investors other than i had an equal-weighted portfolio.³⁵ For example, a stock with large investors has more money flowing towards it. Given our assumption of downward-sloping demand for stocks, a larger exogenous demand generates higher prices that are uncorrelated with residual demand.

In addition to the price of each asset, our setting includes another equilibrium variable, the aggregate elasticity $\mathcal{E}_{agg,k}$, for which we develop a new instrument:

$$\hat{\mathcal{E}}_{agg,k} = \frac{1}{1 + \chi |\widehat{Active}_k|} \frac{\sum_{j \in Active_k} A_j / |\mathcal{K}_j| \cdot \mathbf{1}_{k \in \mathcal{K}_j} \cdot \mathcal{E}_{jk}}{\sum_j A_j / |\mathcal{K}_j| \cdot \mathbf{1}_{k \in \mathcal{K}_j}}. \quad (24)$$

This instrument is the solution to the elasticity equilibrium defined by equations (14) and (18), where we have replaced the endogenous weights w_{ik} with counterfactual weights under the assumption that each investor holds an equal-weighted portfolio.³⁶ The variation in this instrument also comes from variation across investors' investment universes. However, the asset flows are weighted

³⁵We consider an alternative with portfolio weights proportional to book equity.

³⁶Our instrument for aggregate elasticity is the solution to the following problem:

$$\hat{\mathcal{E}}_{ik} = \mathcal{E}_{ik} - \chi \hat{\mathcal{E}}_{agg,k}; \quad \sum_j \hat{w}_{jk} A_j / \exp(\hat{p}_k) \hat{\mathcal{E}}_{jk} = \hat{\mathcal{E}}_{agg,k},$$

by individual elasticity: a stock with more intrinsically inelastic investors (for example, passive mutual funds) will tend to have a lower aggregate elasticity. The degree of strategic response χ is the response of asset demand to the interaction of aggregate elasticity with the price (see equation (20)). To isolate this interaction from linear effects, we also include a linear control for aggregate elasticity in the model, similar to $X_k^{(d)}$.³⁷

The instrument $\hat{\mathcal{E}}_{agg,k}$ depends on the model parameters ($\underline{\mathcal{E}}_{0i}$ and $\underline{\mathcal{E}}_{1i}$). This is not an issue for identification as parameters are by definition not endogenous. However, this precludes us from using standard methods such as two-stage least squares to estimate the model; anyways, the elasticity equilibrium already prevents us from using these standard methods.

Why is it plausible that these instruments satisfy the identifying restriction in the moment condition equation (22)? For both instruments, it is sufficient to assume that the variation in total assets and the investment universe is exogenous to the residual demand, an assumption shared with Kojien and Yogo (2019). They argue that the investment universe is often determined by mandates, which are predetermined rules on which assets can be held. For example, to the extent that investment universes are very stable over time, fads do not affect them. A potential concern would be that persistent shocks to residual demands are correlated with these long-term choices of universe. Motivated by this concern Kojien et al. (2020) document the robustness of demand system estimates to many variations of this instrument: excluding hedge funds, measuring the investment universe for different time frames, and also expanding the investment universe with a placebo.

Appendix Section E.1 lists the unconditional moments derived from condition (22) that we use for estimation. In Section 4.3.3, we detail our numerical procedure for estimating the model.

Relevance condition. To evaluate the strength of our instruments, we run what would be a first-stage regression in a standard two-stage least square estimation. First, we regress the price

where the counterfactual weights \hat{w}_{jk} are defined as:

$$\hat{w}_{jk} = \frac{\mathbf{1}_{k \in \mathcal{K}_j}}{|\mathcal{K}_j|}.$$

³⁷To maintain tractability in estimation we assume that the coefficient on this linear control, like χ , is constant across investors. See Appendix Section E.3.

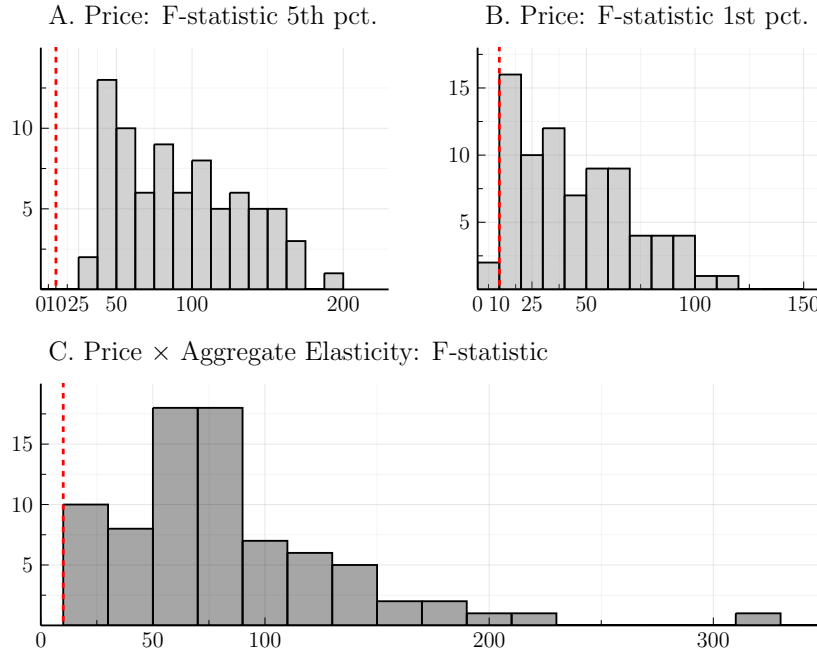


Figure 2. Relevance condition for the price and elasticity instruments.

Figure 2 shows the F-statistic of the first-stage regression for the price and aggregate elasticity variables. For the price, we estimate the F-statistic (Kleibergen-Paap) at the manager level for each year. We summarize these statistics at every date with the 5th percentile (Panel A) and 1st percentile (Panel B). The vertical red dashed line indicates the critical value of 10. In Panel C, we regress the elasticity interacted with the price onto their instrumented version and report the F-statistic for each date. The sample period is 2001–2020.

onto the instrument and the other characteristics for each manager. For each date, we compute the first and the fifth percentile of the Kleibergen and Paap (2006) F-statistics across managers. Figure 2 reports the histogram of these percentiles across all dates. At least 95% of the F-statistics in any given date are above 18 (panel A); panel B reports the first percentile. We also confirm the relevance of the elasticity instrument. In the panel, we regress the product of the price interacted with the aggregate elasticity onto their instrumented version and the other characteristics. We represent the histogram of the F-statistic of this regression for each date in panel C; the F-statistic is always above 10.³⁸

4.3.3 Implementation

Last, we need to implement the estimation free of the identification issues discussed above. We cannot estimate (20) using off-the-shelf methods. This is because the degree of strategic response

³⁸Appendix Figure IA.6 reports similar results for the book-equity weighted instrument.

χ and the aggregate elasticities $\mathcal{E}_{agg,k}$ must not only satisfy moment conditions but also respect the two-layer equilibrium relations. A naïve approach to solve all these conditions simultaneously is computationally untractable due to the large dimension of the parameter space.

However, we develop an algorithm that leads to rapid computation: estimating the model for a given quarter takes about two minutes on a personal computer. The basic idea of our method is to focus on two nested equilibrium questions. On the one hand, if one knows the coefficient on aggregate elasticity, solving the values of aggregate elasticities can be done using an iteration process: run standard instrumental regressions at the investor level to estimate their demand, then update aggregate elasticities using equation (18); repeat until obtaining convergence. On the other hand, if one knows the equilibrium quantities, finding the coefficient on the interaction of prices and aggregate elasticities involves a single large panel regression. Put together, finding a set of elasticity estimates and the parameter for strategic interactions χ that are internally consistent with one another corresponds to a low-dimensional fixed-point problem that we solve using the standard Newton method. Appendix Section E.3 details this estimation procedure.

4.4 Estimates

We estimate the model for each quarter from 2001Q1 to 2020Q4. Recall that our identification comes from the cross-section, such that the model is estimated independently for each time period.

4.4.1 Degree of strategic response χ

The average value of the degree of strategic response is $\chi = 2.2$. We show a summary of the estimates of the parameter χ across quarters in row 1 of Table 2; Appendix Figure IA.3 represents the whole distribution. The estimates show little variation around their median. There are no detectable trends in the time series of the estimates as seen in Appendix Figure IA.4.

A degree of strategic response of 2.2 implies substantial reactions at the individual level. If all other investors become more aggressive and increase their elasticity by 1, an atomistic investor would respond by decreasing her elasticity by 2.2. However, this estimate of χ points to an equilibrium behavior far from both the common view of $\chi \rightarrow +\infty$ and the no-strategic-response

benchmark of $\chi = 0$. For example, our simple calculation in equation (5) shows that we need large values of χ for strong equilibrium effects. Making 50% of investors passive, a value of χ of at least 18 is necessary to compensate 90% of the drop in aggregate elasticity. This is an order of magnitude larger than our main estimate of 2.2, and actually than all of our estimates. Going back to the theories of Section 3, a positive value for χ pushes more towards theories based on strategic substitution (e.g. models of information acquisition as in Grossman and Stiglitz (1980)) rather than complementarity (e.g. models of market power as in Kyle (1989)). Furthermore, the low value of χ suggests the presence of significant frictions impeding investor reaction. We investigate the quantitative implications of our value of χ for the impact of the rise of passive investing in Section 5.

Robustness. We assess the robustness of the estimates along several dimensions. Table 2 reports the results of the estimation for these alternative specifications in rows 2 to 7. Overall, the estimates of competition χ do not vary substantially across specifications — the median value of χ across quarters is always between 1.91 and 2.51.

First, in row 2, we consider an alternative construction of the instrument where the counterfactual portfolio positions are weighted by book equity instead of being equally weighted. While these weights are potentially more realistic and can strengthen the relevance condition, their ad-hoc nature might weaken the plausibility of the exogeneity condition. This leads to a median χ of 1.91, close to our baseline. Row 3 includes additional controls for stock characteristics to the regression (X_k): profitability, investment, and dividend yield. The estimates do not change much. However, we find that including many additional parameters sometimes hinders the convergence of the estimation algorithm (Appendix Table IA.1 restricts to a sample where all the methods converge, 2003Q3 to 2020Q4, and finds very similar results). Rows 4 and 5 consider an alternative weighting scheme. In our baseline, all investors contribute equally to the estimate of χ , while row 4 weighs them by their assets under management. Therefore, if our model was misspecified and the competitive response varied by investor size, this change would lead to different estimates. This is not the case here, with extremely close estimates, suggesting that we capture the empirically relevant moment for the rise in passive investing. Row 5 weighs by book assets under management

Table 2.**Estimates of the degree of strategic response χ under alternative specifications**

	Estimates for χ		
	Median	25th pct.	75th pct.
(1) Baseline Specification	2.15	1.81	2.76
(2) BE-weighted Instrument for \mathcal{E}_{agg}	1.91	1.52	2.31
(3) Additional Controls	2.51	2.09	3.5
(4) AUM-weighted Regression	2.3	1.81	2.8
(5) Book-weighted Regression	2.27	1.76	2.78
(6) Investor-Type Grouping	2.42	1.93	2.94
(7) Constant χ	1.95		
(8) No Instrument for \mathcal{E}_{agg}	1.21	0.77	1.56
(9) No Instruments	0.96	0.67	1.38

Table 2 presents statistics of estimates of χ across dates (2001Q1–2020Q4) under various specifications. Our baseline specification (1) estimates χ given aggregate elasticities $\mathcal{E}_{agg,k}$ each period via the regression:

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1i} X_k^{(d)} + \xi \mathcal{E}_{agg,k} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi \mathcal{E}_{agg,k} \right) p_k + \epsilon_{ik},$$

where $X_k^{(d)}$ contains log book equity and log book equity squared. $X_k^{(e)}$ is log book equity. Active investors with fewer than 1,000 stock holdings are pooled together based on their assets under management, such that each group on average contains 2,000 stock holdings. The regression is weighted such that each group's weights sum to the same constant. Specification (2) shows estimates of χ based on the book-equity weighted instrument. Specification (3) adds additional characteristics to $X_k^{(d)}$; profitability, investment and dividends relative to book equity. Specification (4) value-weights the regression by weighting investors by their AUM. Specification (5) similarly value-weights the regression by weighting investors by their book assets. Specification (6) groups investors both by investor type and AUM. Institutional investors whose type we cannot determine are bundled together in a separate group. Specification (7) imposes for χ to be constant across time in the estimation, with each year receiving equal weight. Specification (8) reports results without instrumenting for the aggregate elasticity \mathcal{E}_{agg} . Specification (9) additionally removes the instrument for prices.

to avoid contamination by prices, also leading to virtually identical estimates. Row 6 addresses the details of how we deal with investors with few positions. In the baseline, active investors with fewer than 1,000 stock holdings are grouped together based on their assets under management such that each group on average contains 2,000 stock holdings.³⁹ A finer way to construct these groups is to make them based on investor types, but data coverage is incomplete. The estimates in row 6 updates our estimation based on these data, which results in little change. Finally, row 7 constrains the degree of strategic response χ to be constant over time. This pooled estimation is more computationally demanding. It yields a value of χ of 1.95, close to the baseline.

³⁹This grouping ensures enough observations for each group to avoid incidental parameter issues.

We also estimate the model without using instruments. Row 8 removes the instrument for aggregate elasticity. In this case we find an average value of χ of 1.21. This estimate, far below that from any other specification, suggests that it is important to account for the endogeneity of elasticities — because they depend on actual portfolio weights, which themselves depend on residual demand. Also removing the instruments for prices — row 9 — leads to even lower estimates, suggesting a deeper bias.

4.4.2 Stock-level elasticities

The model delivers estimates of aggregate elasticity, $\mathcal{E}_{agg,k}$, for each stock. Figure 3 represents these elasticities as a function of stock market capitalization for 2011Q3. Each green dot corresponds to an elasticity estimate of one stock in our model for that date. We compare our estimates to a model where individual-level elasticities are fixed, that is, where $\underline{\mathcal{E}}_{1,i} = 0$ and $\chi = 0$. These estimates are represented by red squares.

There is substantial cross-sectional variation in elasticities, lending credence to our ability to identify the degree of strategic response χ . In both sets of estimates, the demand curve for individual stocks is inelastic with average values around 0.45. This magnitude is far from the asset-pricing benchmark of perfectly horizontal demand curves with infinite elasticity.⁴⁰ However, it is consistent with other empirical estimates, in particular based on portfolio data; see for example the discussion in Chang et al. (2014) and Koijen and Yogo (2019).

Figure 3 demonstrates a few ways in which accounting for the endogeneity of demand elasticities is important. First, the full model estimates exhibit less variation than the model with constant elasticities. With constant individual elasticities, variation in investor composition directly translates into variation in aggregate elasticities. However, with a positive degree of strategic response χ , investors react to each other and soften such variation. For example, if an active investor with high elasticity takes position in a stock, other investors respond by trading less aggressively. Thus, stocks become more similar to each other.

Second, the full model exhibits a stronger negative relation between the size of a stock and its elasticity. Koijen and Yogo (2019) point out that large stocks tend to have more inelastic

⁴⁰Petajisto (2009) shows that standard models with risk aversion and many assets also imply very large elasticities.

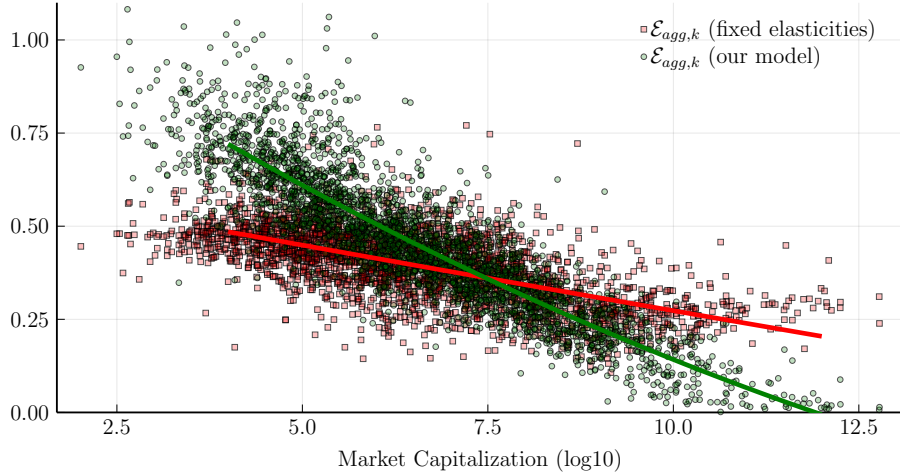


Figure 3. Aggregate elasticity at the stock level: $\mathcal{E}_{agg,k}$.

Figure 3 represents estimates of the aggregate elasticity $\mathcal{E}_{agg,k}$ as a function of their market capitalization (in logarithm) for the date 2011Q3. Each point represents a stock; green circles are our estimates, while red squares correspond to a model where elasticities are fixed.

investors overall. Once we allow individual elasticities to respond to stock characteristics and the aggregate elasticity, the data reveals an additional source for this relation: the same investor behaves more inelastically for large stocks than small stocks. This additional source of variation within investor rather than across investors leads to a steeper relation between size and elasticity. For computational tractability, we estimate a linear relation between size and elasticity at the investor-level; this linearity yields the tiny values of elasticity for the very largest stocks.

Table 3 shows that these conclusions hold not only for this specific date, but across our sample. We report the distribution across dates of various statistics of the cross-section of \mathcal{E}_{agg} . In particular, we confirm that our estimates have a steeper relation between elasticity and stock size (Panel B), and less residual variation in elasticity across stocks (Panel C), by about 50%.

The negative relation between size and elasticity might appear surprising given existing evidence suggesting that large stocks are more informationally efficient.⁴¹ However, there are good reasons to think that institutions are more reluctant to change their positions for large stocks than for small stocks. Mechanically, the largest stocks occupy a larger share of portfolios. As of July 2021, the five largest corporations in the U.S. stock market account for about 18% of total market capitalization.⁴² As a consequence, a large change in portfolio weight would have a large effect on

⁴¹See Lo and MacKinlay (1990), Jegadeesh and Titman (1993), Lakonishok et al. (1994), and Hong et al. (2000).

⁴²The total market capitalization of Apple, Microsoft, Amazon, Alphabet (Google), and Facebook amount to

Table 3.
Properties of aggregate elasticity \mathcal{E}_{agg}

Panel A: Statistics of average elasticity across stocks				
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	0.489	0.411	0.46	0.54
Fixed elasticity	0.389	0.357	0.389	0.442
Panel B: Regression coefficient (by dates) of elasticity on size				
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	−0.109	−0.117	−0.1	−0.0855
Fixed elasticity	−0.0286	−0.0309	−0.0272	−0.0249
Panel C: Residual cross-sectional standard deviation of elasticity				
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	0.0498	0.0389	0.0441	0.0521
Fixed elasticity	0.0842	0.0739	0.0826	0.0917

Table 3 presents statistics of the aggregate elasticity $\mathcal{E}_{agg,k,t}$. We estimate the elasticities in our baseline model and in a specification with fixed elasticities ($\chi = 0$ as in Koijen and Yogo (2019)). Panel A has summary statistics of the average elasticity by date. Panel B shows summary statistics of the coefficient β_t from the regression $\mathcal{E}_{agg,k,t} = \alpha_t + \beta_t p_{k,t} + \varepsilon_{k,t}$ by date. Panel C reports summary statistics of the cross-sectional standard deviation of the residual from the regression described in Panel B. The sample period is 2001–2020.

an institution’s portfolio return. Many institutions are either benchmarked to the index or have hard dollar limits on how much they can trade a given stock, and hence they would be unwilling to take on such large changes. As an illustration, Figure 4 decomposes trading activity—the sum of squared relative change in portfolio position—across percentiles of portfolio weights; Appendix Section F details this calculation. There is much less trading activity for the larger portfolio positions: the top 50% of portfolio positions only account for 9% of trading activity. As such, the interpretation of our results is not so much that large stocks experience more mispricing but rather that high investor elasticity cannot be the explanation for the evidence on their returns.⁴³

\$8.8tn for total U.S. market capitalization of \$49tn.

⁴³In the model of Section 3, both elasticity and the quantity of noise trading determine price informativeness. Farboodi et al. (2021) use a richer structural model to decompose informativeness into data, growth, and volatility.

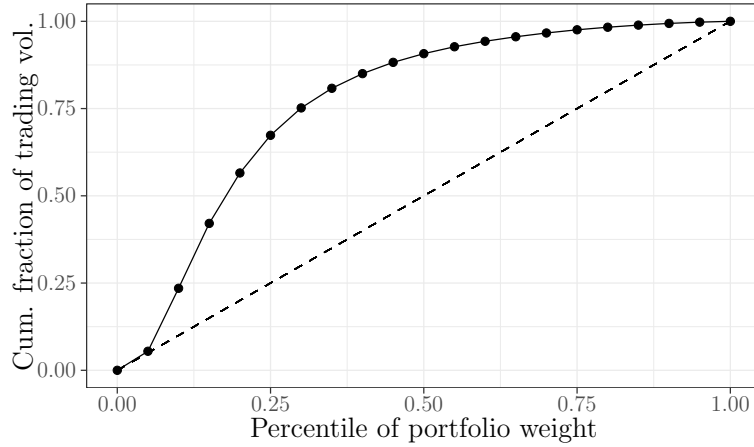


Figure 4. Trading activity across portfolio positions. Figure 4 presents the cumulative share of trading activity (defined in equation (IA.155)) by quantiles of investor portfolio weights. The dashed line is the 45 degree line.

5 Implications

5.1 The rise of passive investing

The last 20 years have seen a large increase in passive investing, a fact documented in French (2008). More recently, Stambaugh (2014) shows that both the fraction of mutual funds that are actively managed and the active share of the portfolio of active equity mutual funds have declined. We update and confirm these trends in Figure 5. The share of passive funds of the U.S. stock market has grown from nearly zero at the beginning of the 1990s to more than 15% in 2019. Concurrently, the share of active funds topped out at the end of the 1990s and has declined from 20% to 15% from 2000 to 2019.⁴⁴ Our model takes a more comprehensive view of who are the passive investors, not restricting ourselves to mutual funds.⁴⁵ With this approach we find that the share of passive strategies has grown by 22 percentage points from 19% to 41% over the last 20 years (see Appendix Figure IA.8). These larger numbers are consistent with the view that some institutions beyond mutual funds follow passive strategies, or that some funds follow “closet indexing” strategies (Cremers and Petajisto, 2009). Chinc0 and Sammon (2022) find a large passive investing share in line with our estimates in a completely different empirical strategy using excess

⁴⁴We report the dollar numbers in Figure IA.7. Net assets of passive funds has grown from virtually zero to \$5.4t in 2019, whereas the net assets of active funds only increased from \$600b in 1993 to \$5.5t in 2019.

⁴⁵Our methodology for measuring passive investing as inelastic demand is described further in Appendix D.3.

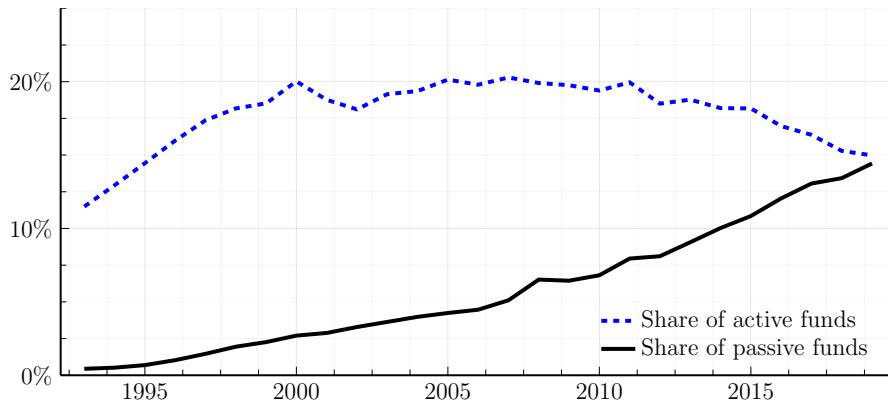


Figure 5. Share of passive and active funds. Figure 5 shows the share of domestic mutual funds and ETFs as a fraction of the US stock market capitalization for passive funds (black solid line) and active funds (blue dashed line).

volume around index reconstitutions.

Has the shift to passive portfolios impacted the behavior of prices? Understanding how investors react to changes in the behavior of other investors is crucial to answer this question. In the common view of “fiercely competitive markets,” when some investors stop looking for profitable trading opportunities, some other investors step in to replace them; prices do not change. In contrast, if investors do not respond to others, the demand for stocks becomes more inelastic, which strongly affects the behavior of prices. For example in the theory of Section 3.1, more inelastic demand leads to prices that are more volatile and less informative. Our model, and in particular the parameter χ , accounts for the strength of this reaction. We use the estimated parameters to quantify the impact of the rise in passive investing on aggregate demand elasticities.

Starting with the demand system from Section 4, we consider the following counterfactual: we impose an exogenous change in the fraction of active investors and compute the new equilibrium elasticities. Of course the rise of passive investing is not a purely exogenous phenomenon. However, most plausible explanations of this phenomenon are independent from the rest of the demand system. For example, the development of financial technology made it cheaper to pursue passive strategies: fees on passive funds have dropped dramatically and ETFs have become available. Or, one subset of investors, maybe after listening to finance professors, realized they were making mistakes when pursuing active strategies.⁴⁶ Such shocks are equivalent to an exogenous change in

⁴⁶Bhamra and Uppal (2019) estimate sizable welfare costs from lack of diversification.

the fraction of passive investors as long as they do not directly affect the demand of the remaining investors.

Computing the effect of the rise of passive investing corresponds to the calculation of equation (5), accounting for heterogeneous investors. Combining the individual demand elasticity \mathcal{E}_{ik} in equation (14) with the equilibrium condition of (18), we have

$$\mathcal{E}_{agg,k} = |Active_k| \times \left(\sum_{i \in Active_k} \frac{w_{ik} A_i}{\sum_{j \in Active_k} w_{jk} A_j} \cdot \underline{\mathcal{E}}_{ik} - \chi \mathcal{E}_{agg,k} \right) \quad (25)$$

The aggregate elasticity combines three terms: (i) the fraction of the asset held by active investors, $|Active_k|$; (ii) the average baseline elasticity among active investors, weighted by their respective positions; and (iii) an adjustment for the strategic response of active investors to the aggregate elasticity, which depends on χ .⁴⁷

From this expression we obtain the effect of a change in the fraction of active investing. Changing $|Active_k|$ while holding everything else constant corresponds to the assumption that the set of active investors that become passive is a representative sample of the active population. This leads to a simple formula:

$$\frac{d \log \mathcal{E}_{agg,k}}{d \log |Active_k|} = \frac{1}{1 + \chi |Active_k|}. \quad (26)$$

The pass-through from a rise in active investment to aggregate elasticity is determined by two numbers: the degree of strategic response χ and the fraction of active investors.⁴⁸ When χ is large, the aggregate elasticity does not respond to a shift in passive investing, and the pass-through is zero. At the opposite end, when $\chi = 0$ such that investors do not respond to market conditions, the pass-through is 100%; an increase in the fraction of passive investors translates into a one-to-one decrease in aggregate demand elasticity. Furthermore, because only active investors change their

⁴⁷Using equation (25), we can solve for the equilibrium value of aggregate elasticity

$$\mathcal{E}_{agg,k} = \sum_{i \in Active_k} \frac{w_{ik} A_i}{\sum_{j \in Active_k} w_{jk} A_j} \cdot \underline{\mathcal{E}}_{ik} \times |Active_k| \times \frac{1}{1 + \chi |Active_k|}.$$

⁴⁸Appendix Section A.4 shows that with investor-specific χ_i , this expression remains unchanged, other than what matters now is the position-weighted average χ_i among active investors.

elasticities in response to others (passive investors always have an elasticity of zero), starting with a larger fraction of active investors leads to a smaller pass-through.

We can readily compute the pass-through: it solely depends on two observable quantities, χ and $|Active_k|$. In Section 4, we estimated the competition parameter and found that $\chi = 2.15$. Recall we measure the total quantity of passive investors as investors with an elasticity of zero in a Koijen-Yogo demand system. Not surprisingly, we find a trend down from 81% in 2001 to 59% in 2020. Taking the average across dates for the share of active investors, 68%, and for the degree of strategic response, $\chi = 2.15$, we find a value of the pass-through of⁴⁹

$$\frac{1}{1 + \chi |Active_k|} = \frac{1}{1 + 2.15 \times 0.68} = 40.6\%. \quad (27)$$

This implies that the strategic response is strong enough to compensate about 60% of the direct effect of a rise in passive investing. While substantial, this effect is far from the full cancellation of the idealized view of financial markets.

We multiply this pass-through by the rise in the proportion of passive investing to obtain the total effect on elasticity. We consider different takes for the size of the exogenous change. First we use our comprehensive measure of passive investing. The decline from 81% to 59% corresponds to a 32% drop, leading to elasticities lowered by $40.6\% \times 32\% = 13\%$. Translating the elasticities into price multipliers, this implies that the rise in passive investing causes an increase in the price impact of buying \$1 of a stock roughly from \$2.5 to \$2.9. Second, we look at a narrower measure of the rise in passive investing centered around the assets under management of passive mutual funds and ETFs. Their fraction of total market capitalization has increased by 15 percentage points in the last 30 years. Starting from a baseline of 81% of active investors, this change represents a 19% drop in the total fraction of active investors. With our pass-through of 0.4, this increase in passive investing by mutual funds reduces elasticities by 8%.

⁴⁹When the share of active investors is at 81% as in 2001 the pass-through is 36.5%, while when this share is at its lowest value of 59% at the end of the sample it is 44%.

5.2 Decomposing the evolution of the demand for stocks

In the previous exercise, we isolated the causal effect of a change in passive investing on equilibrium demand elasticities. Next, we propose a positive account of the data: we decompose the actual changes in elasticity over the last twenty years in light of our model.

5.2.1 The evolution of aggregate elasticity

Figure 6 presents the time series of the distribution of equilibrium elasticities across stocks. For each date, we compute quantiles of the cross-section of aggregate elasticities, $\mathcal{E}_{agg,k}$. Elasticities increase until before the financial crisis, and have been decreasing since. The average elasticity (bold solid line) goes from 0.46 to 0.31. This pattern holds throughout the distribution of elasticities. The episode of declining elasticities in the second part of the sample might be surprising because of the intuition that markets are continuously becoming more liquid due to the evolution of trading technologies. However, such a trend is not specific to our model estimates: Kojen and Yogo (2019) find similar behavior towards the end of their sample, and Kojen et al. (2020) attribute a large part of the decline in active trading to a decrease in elasticity between 2007 and 2019. Furthermore, microstructure measures of liquidity also do not particularly improve over this part of the sample (see Frazzini et al. (2018), Rösch et al. (2016)). In the next section, we further our understanding of what is behind the evolution of demand elasticities through a simple decomposition.

5.2.2 Sources of change in elasticity

In Section 4, we estimated the demand elasticities for each investor-stock in each quarter from 2001 to 2020. While our identification strategy is purely cross-sectional, we can use the time-series dimension of our estimates as a description of the evolution of the demand for stocks over time. To make parameters such as the investor-specific demand elasticity $\underline{\mathcal{E}}_i$ comparable across periods, we use the model estimates under the assumption that the degree of strategic response is constant over time at its sample median of 2.15.

We decompose changes in elasticity from year to year into three components by differentiating equation (25). We denote by $\langle \underline{\mathcal{E}}_{ik} \rangle$ the position-weighted average of the individual-specific compo-

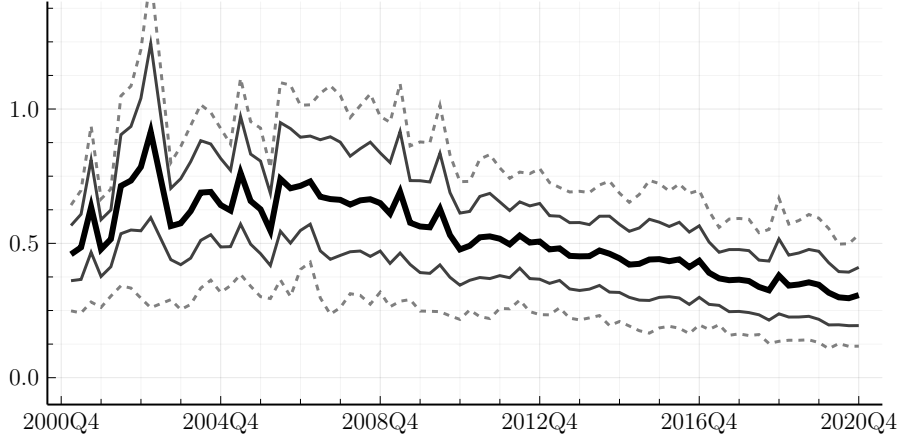


Figure 6. Distribution of aggregate elasticity across stocks. Figure 6 traces out the distribution of aggregate elasticity $\mathcal{E}_{agg,k}$ over time. The bold line represents the average elasticity across stocks for each year. The solid lines represents the 25th and 75th percentile and the dashed lines the 10th and 90th percentile.

ment of the elasticity of active investors, \mathcal{E}_{ik} ; this corresponds to the second term in equation (25).

We derive the effect of a change in investor composition,

$$\begin{aligned}
 \underbrace{\frac{d\mathcal{E}_{agg,k}}{\mathcal{E}_{agg,k}}}_{\text{Change in aggregate elasticity}} &= \underbrace{\frac{d|Active_k|}{|Active_k|}}_{\text{Share of active investors}} + \underbrace{|Active_k| \cdot \frac{d\langle \mathcal{E}_{ik} \rangle}{\mathcal{E}_{agg}}}_{\text{Individual elasticity of active investors}} \\
 &\quad - \underbrace{\chi |Active_k| \frac{d\mathcal{E}_{agg}}{\mathcal{E}_{agg}}}_{\text{Strategic response}}. \tag{28}
 \end{aligned}$$

The first component accounts for changes in the share of active investors over time and their ultimate effect on the elasticities. The second component corresponds to changes in the average individual-level elasticity component of active investors; how their own characteristics contribute to the elasticity. These forces correspond respectively to the extensive and intensive margin of individual elasticities. The last component corresponds to the strategic response to these two changes. If $\chi = 0$ there is no strategic response and this term disappears. Otherwise, the strategic response compensates the direct effects of both the share of active investors and their composition.

We accumulate the three terms of this decomposition over time in Figure 7.⁵⁰ We smooth the series to make the secular trends easier to identify. Consistent with the importance of the rise

⁵⁰Because we cannot continuously integrate equation (28), we use the natural discrete approximation of the first and third terms and compute the second one as a residual.

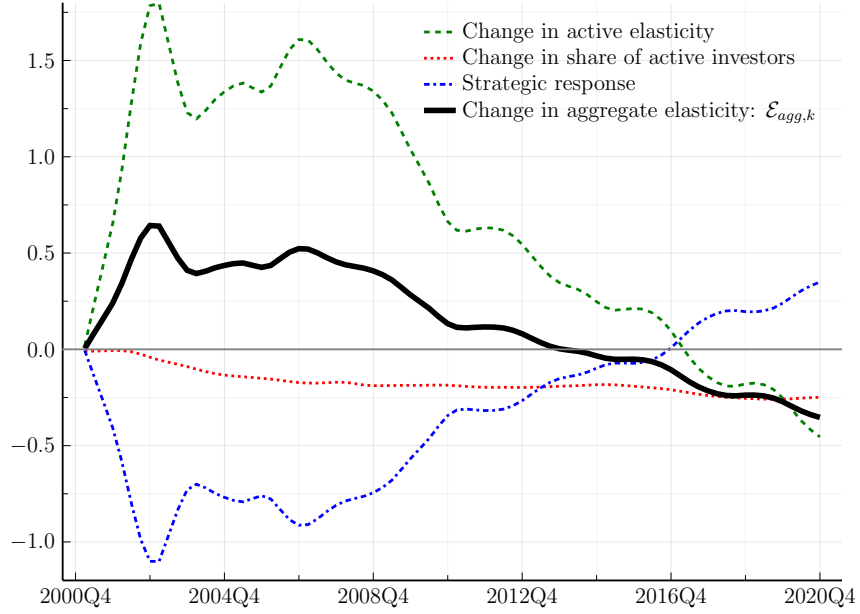


Figure 7. Decomposition of the change in aggregate elasticity. Figure 7 shows the decomposition derived in equation (28) over time. We compute each term of the decomposition for each date and accumulate the changes over time, scaled by the initial aggregate elasticity.

in passive investing discussed in Section 5.1, we find that the direct effect of the decrease in the fraction of active investors has similar magnitude to the total change in aggregate elasticities. Interestingly, investors also change their own elasticities at the intensive margin. While individual elasticities increase until before the financial crisis, they experience a sharp drop afterwards.⁵¹ Appendix Figure IA.9 confirms this pattern holds in the entire cross-section of investors. In net, this second direct force also contributes to a drop in aggregate elasticities. However, the strategic response strongly mitigates these individual changes in equilibrium. The strategic response reverses around half of the decline.

5.2.3 Evolution under counterfactual degrees of strategic response

Finally, we ask how the changes in the individual components of investor demand would have affected the aggregate elasticities under different strategic regimes. We start from the equilibrium levels of demand elasticity at the beginning of our sample (2001Q1). We feed into the model the two direct components highlighted above: how individual elasticities, \mathcal{E}_{ik} , change over time

⁵¹Relatedly, Pavlova and Sikorskaya (Forthcoming) documents a trend down in the tracking error of active mutual funds.

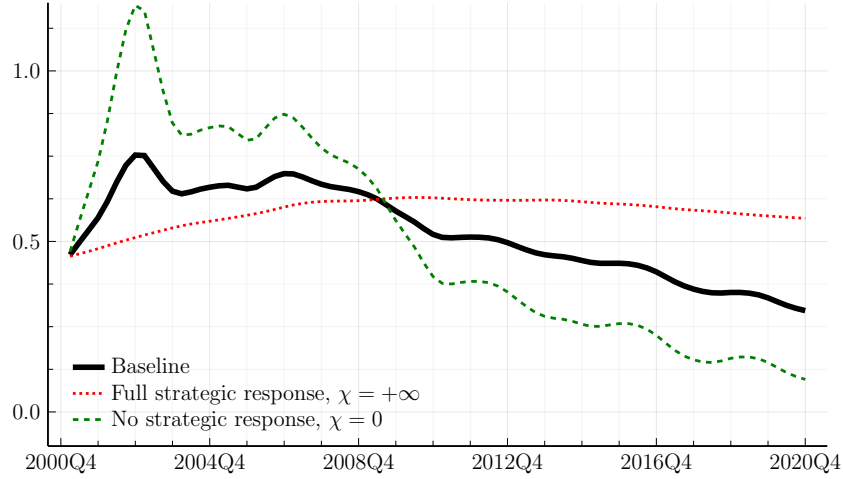


Figure 8. The evolution of aggregate elasticity under alternative competition regimes. Figure 8 shows the evolution of aggregate elasticity $\mathcal{E}_{agg,k}$ under alternative strategic regimes. The bold black line presents our baseline estimate. The dotted red line shows the elasticity with strong strategic response ($\chi \rightarrow +\infty$). The dashed green line shows the elasticity with no strategic response ($\chi = 0$).

and who becomes passive. We make different assumptions on how investors react to changes in the behavior of others. We show the time series of the results in Figure 8. The solid black line represents the actual evolution of the average aggregate elasticity across stocks; the colored dashed and dotted lines show the counterfactual results.

We first consider the case where interactions are strongest, corresponding to $\chi \rightarrow +\infty$. In this situation any change in individual behavior is completely counteracted by other investors. The aggregate elasticities for each stock are pinned down at their initial level. The only source of variation in the average elasticity over time are changes in the composition of the universe of stocks. This is the dotted red line in Figure 8, which experiences very little change over our sample. This result also confirms that the patterns in aggregate elasticities we have documented is not the consequence of changes in which stocks are traded.

The other extreme is the situation where investors do not react to others at all and $\chi = 0$. Then, all the changes in individual investor behavior directly feed into aggregate elasticities. This leads to a more dramatic drop in elasticities over time than our baseline estimates. This is the dashed green line in Figure 8. We observe a strong decrease, about twice as large as the baseline.

Overall these results confirm that changes in the behavior of investors have profoundly changed

the aggregate demand curves for individual stocks. Competition among investors in setting their strategies played an important role in mitigating the total impact of those changes. However, the strategic response was not strong enough to fully negate the course of a downward trend in aggregate elasticities.

5.3 Implications in the cross-section of stocks

5.3.1 The strategic response in the cross-section

In our model, the response to a change in the share of passive investors occurs through the strategic response: the other active investors change their elasticity. However, other types of adjustments could happen. For example, the composition of active investors could change. Also, the identity of who becomes passive might shape the response beyond their demand elasticity, as is the case in some more sophisticated theories.

While these possibilities are not explicitly part of our empirical model, they would manifest themselves through the changes in aggregate elasticity in response to changes in passive investing. We investigate their presence by zooming in on sources of variation in passive investing different from that driving our baseline estimates (the ones caused by our instrument).

We regress annual log changes in stock-level elasticity on changes in the fraction of active investors:

$$\log(\mathcal{E}_{agg,k,t}) - \log(\mathcal{E}_{agg,k,t-1}) = \beta (\log(|Active_{k,t}|) - \log(|Active_{k,t-1}|)) + \alpha_k + \gamma_t + e_{k,t}. \quad (29)$$

The inclusion of time and stock fixed effects allows to focus on variation independent of the average variation. A benchmark value for the coefficient β is the pass-through from equation (26), about 0.4. However, if changes in individual-level elasticities, or other types of changes in investor composition, are correlated with the active share, this would push β away from the theoretical pass-through. So effectively, we are assessing whether changes in investor behavior beyond the strategic response are correlated with changes in passive investing.

Table 4 presents the result, using the unconstrained cross-sectional model estimates. Column

1 is a univariate regression; columns 2 and 3 add date then stock fixed effects. Throughout, we find a coefficient of about 0.4, close to the theoretical pass-through.⁵² This result supports the interpretation that our measured degree of strategic response is the main driver of the response of aggregate elasticity to changes in passive investing. Furthermore, because our model estimates are only based on cross-sectional evidence, this result from including the time series dimension provides additional support for our theory. Going in this direction, in Appendix Table IA.3, we confirm that the regression results are mostly unchanged when using the estimates that impose a constant value of χ through time.

We also consider what happens around index inclusions and exclusions. For these events, the source of the variation in passive investing is known because index funds are forced to change their portfolio after reclassification. Following Chang et al. (2014), Ben-David et al. (2018), and Chinco and Sammon (2022), we exploit the mechanical rule that allocates stocks between the Russell 1000 and 2000 indexes.⁵³ We use the index-switching event as an instrument for the share of passive investors; column 5 of Table 4 reports the result. The first stage is significant with reclassification changing active ownership by about 5% (see Appendix Table IA.4).⁵⁴ The coefficient is 0.39, again very close to the theoretical pass-through.

5.3.2 Behavior of asset prices

Our empirical model focuses on the estimation of demand elasticities for two reasons. First, elasticities are the quantity through which investor strategic interactions manifest themselves across many theories. Second, these elasticities are a key determinant of the behavior of asset prices. For example, all else equal, aggressive investors limit the influence of excess fluctuations in prices, which often results in less volatility or more price informativeness.⁵⁵ Similarly, an asset with highly elastic investors will tend to be more liquid, because these investors are willing to provide liquidity. In this section, we document the relation between aggregate elasticity and some of these

⁵²Statistical significance is not completely meaningful in this setting, because the left-hand-side of the regression is model-generated.

⁵³We are grateful to Alex Chinco for sharing his data with us.

⁵⁴Glossner (Forthcoming) documents no change in institutional ownership around reclassifications. This suggests our variation is coming from a change in the composition of institutions.

⁵⁵In the theories we considered, this corresponds to holding the quantity of noise trading constant.

Table 4. Change in aggregate stock-level elasticity $\mathcal{E}_{agg,k}$ on the active share

	Change in Elasticity				
	(1)	(2)	(3)	(4)	(5)
Change in Active share	0.446*** (0.044)	0.475*** (0.036)	0.457*** (0.036)	0.426*** (0.034)	0.389*** (0.066)
Date Fixed Effects		Yes	Yes	Yes	Yes
Stock Fixed Effects			Yes		
Controls				Yes	Yes
Estimator	OLS	OLS	OLS	OLS	IV
N	50,292	50,292	49,661	50,292	10,619
R^2	0.076	0.461	0.497	0.569	0.748
First-stage F statistic					9.444
First-stage p value					0.000

Table 4 reports a panel regression of annual log change in stock level elasticity $\mathcal{E}_{agg,k}$ on the annual log change in the active share $|Active_k|$. Column 2 adds date fixed effects. Column 3 adds stock fixed effects. Column 4 uses date fixed effects and controls for lagged book equity and annual log changes of log book equity. Column 5 instruments the log change in the active share $|Active_k|$ between Q1 and Q2 in any given year by two indicator variables corresponding to stocks switching between Russell 1000 and 2000 in either direction. In this column, the sample is restricted to stocks with CRSP market capitalization ranked between 500 to 1500 as of the end of Q1. The sample period is 2001–2020 for columns 1-4, and 2007–2020 for column 5. Standard errors are 2-way clustered by date and stock for columns 1-4, and clustered by date for column 5.

aspects of asset prices in the cross-section.

In the spirit of our structural model, we run the following regressions:

$$Y_{k,t} = \beta \mathcal{E}_{agg,k,t} + \gamma'_t X_{k,t} + \alpha_t + e_{k,t}, \quad (30)$$

where $Y_{k,t}$ is a stock-level outcome, $X_{k,t}$ controls for stock characteristics, and α_t are time fixed effects. This OLS specification is likely biased because $\mathcal{E}_{agg,k,t}$ correlates with unobserved aspects of the stocks. Therefore, our preferred specification is 2SLS in which we instrument for $\mathcal{E}_{agg,k,t}$ using $\hat{\mathcal{E}}_{agg,k,t}$. We have already shown in Section 4.3.2 that the first stage of this estimation is strongly significant.

Table 5 reports the results. In columns 1 to 4, we measure the effect of aggregate elasticity on daily stock volatility. The first two columns use total volatility and the latter two use idiosyncratic volatility (from the three-factor model of Fama and French (1993)). While the relation is weak

without instrumenting, the IV specifications reveal a strongly negative relation. Consistent with most theories, stocks with more elastic investors have less volatile returns. This result also ties together our mechanism with the results of Ben-David et al. (2018) on index inclusions. When a stock has more passive investors following an index switch, its aggregate elasticity declines due to a low value of χ (Table 4), which results in more volatility, as documented in their paper.

Columns 5 and 6 consider the measure of price informativeness of Dávila and Parlato (2018). We find no significant relationship. However, the large standard errors reveal that the relation is difficult to estimate precisely rather than a tight zero. A number of papers attempt to measure directly the effect of passive investing on price informativeness, with conflicting findings. Sammon (2021) shows that an increase in passive ownership leads to a decrease in price informativeness while Coles et al. (2022) estimate that there is no effect. Columns 7 and 8 use the illiquidity measure of Amihud (2002). The IV specification is consistent with the theory: illiquidity is lower for stocks with more elastic investors. An interesting aspect of this connection is that our elasticity estimates focus on low-frequency aspects of portfolios while the Amihud (2002) measure highlights high-frequency properties of returns.

Overall, these results support the view that estimating the demand for stocks is useful to get to a better understanding of the behavior of financial markets. Specifically, demand elasticities appear to shape many aspects of this behavior.

6 Conclusion

The idea that investors interact with each other is fundamental in financial markets. A classic hypothesis, motivated by the view of “fiercely competitive markets,” states that changes in a group of investors’ behavior have no impact on prices because others step in to compensate. Many theories of financial decisions work through strategic responses: how others trade affects how you trade. While strategic responses permeate all of finance, an empirical understanding of their importance remains elusive. We put forward a framework that enables measurement of the degree of strategic response and the analysis of its impact on equilibrium outcomes.

In the U.S. stock market we find evidence that investors do react to each other: when an investor

is surrounded by less aggressive traders, she trades more aggressively. However, this response is much weaker than anticipated by the classic hypothesis. Strategic responses compensate only 60% of the effect of changes in investor behavior on the aggregate demand for a stock. This implies that the rise in passive investing leads to substantially more inelastic markets.

The ability to measure strategic responses opens a new path to address many other important issues in finance. To assess the impact of financial regulation on some market participants, for example the Basel III leverage constraint on banks, one cannot ignore how other institutions will respond. Likewise, to understand how the distress of some financial institutions creates fire-sale spillovers, one must realize that other investors will step up. Our framework measures how many actually will. Recent work in international finance emphasizes the importance of cross-border flows and global imbalances. What happens if a large sovereign institution stops investing in one market, like China with US treasuries? Again, strategic interactions among investors will be a crucial input in determining the final impact of such a momentous shift. Moreover, the rise and availability of big data promises to change the landscape of how financial institutions compete with each other.

Table 5. Stock-level elasticities $\mathcal{E}_{agg,k}$ and the behavior of asset prices

	Total Volatility		Idiosyncratic Volatility		Price informativeness		Illiquidity	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Elasticity	0.081 (0.068)	−0.757*** (0.164)	0.008 (0.049)	−0.740*** (0.134)	−0.078 (0.306)	−0.270 (0.731)	1.250*** (0.066)	−0.475** (0.228)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	IV	OLS	IV	OLS	IV	OLS	IV
N	219,531	219,531	206,140	206,140	66,677	66,677	216,983	216,983
R^2	0.218	0.178	0.247	0.209	0.020	0.019	0.744	0.610

Table 5 reports panel regressions of measures of volatility, price informativeness and illiquidity on stock level elasticity $\mathcal{E}_{agg,k}$. All variables are demeaned and standardized for each date. Odd columns show results from OLS regressions. Even columns show results from instrumental variables regressions that use our instrument for stock elasticity defined in equation (24). For columns (1) and (2), we compute the total daily volatility of stocks. For columns (3) and (4) we compute daily idiosyncratic volatility with respect to the Fama and French (1993) three-factor model based on daily CRSP data within a quarter. Columns (5) and (6) take the measure of price informativeness provided by Dávila and Parlato (2018). Columns (7) and (8) use the Amihud (2002) measure for illiquidity as the dependent variable, calculated again based on daily CRSP data within a quarter. All specifications are weighted by lagged market equity. We follow our main specification for the estimation of elasticity and control non-linearly for book equity. The sample period starts in 2001 for all columns, and ends in 2020 for specifications 3–4 and 7–8, 2019 for specifications 1–2, and 2017 for specifications 5–6, based on respective data availability. Standard errors are 2-way clustered by date and stock.

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Internet Appendix for
“How Competitive is the Stock Market?
Theory, Evidence from Portfolios, and Implications for the Rise of
Passive Investing”^{*}

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A Equilibrium Model of Financial Markets with Investor Competition

We derive the implications of the model of Section 2. First, we provide formal derivations for the effect of the rise of passive investing presented in Section 2.3. Then, we show how the degree of strategic response shapes the relative prevalence of overpricing and underpricing. Next, we illustrate the role of investor interactions for dynamic aspects of limits to arbitrage. Finally, we re-evaluate the effect of a rise of passive investing under heterogeneity.

A.1 The effect of a rise in passive investing

In Section 2.3, we ask how the aggregate elasticity changes when a fraction of investors becomes passive. We start with an economy with homogeneous investors and assume an initial equilibrium $p = \bar{p}$. Because all investors are identical, we have $\mathcal{E}_i = \mathcal{E}_{agg}$. Combined with elasticity decision equation (4), this gives $\underline{\mathcal{E}}_i = (1 + \chi)\mathcal{E}_i$. We denote the initial equilibrium elasticity of investors by $\mathcal{E}_i = \mathcal{E}_{agg} = \mathcal{E}_0$.

Assume that a fraction $1 - \alpha$ of investors becomes passive such that their elasticity becomes zero and their level of demand is unchanged. We have to determine the new elasticity of active investors \mathcal{E}_i . The individual decisions and equilibrium conditions are:

$$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi\mathcal{E}_{agg}, \quad (\text{IA.1})$$

$$\mathcal{E}_{agg} = \alpha\mathcal{E}_i + (1 - \alpha) \cdot 0. \quad (\text{IA.2})$$

Solving this system gives aggregate elasticity:

$$\mathcal{E}_{agg} = \frac{\alpha}{1 + \alpha\chi}\mathcal{E}_i. \quad (\text{IA.3})$$

$$\mathcal{E}_{agg} = \frac{\alpha}{1 + \alpha\chi}(1 + \chi)\mathcal{E}_0 = \alpha\mathcal{E}_0 + \frac{\alpha\chi}{1 + \alpha\chi}(1 - \alpha)\mathcal{E}_0. \quad (\text{IA.4})$$

The first part of the equation corresponds to the direct effect of the rise in passive investing, and the second part represents the compensation from the strategic response of the remaining active investors.

A.2 Asymmetry of mispricing

In this section, we show that in markets with strong strategic responses, prices are less responsive to demand when their levels are high than when they are low. The opposite happens for low degrees of strategic response. This distinction has important practical implications: when strategic responses are mild, as we find in the data, we expect to observe more situations of overpricing than underpricing, and more sensitivity to demand shocks for overpriced assets. This prediction lines up with the evidence in Stambaugh et al. (2012) and Stambaugh et al. (2015) across a large set of anomaly strategies.¹ In particular, they document that trading the short leg of the portfolio (overpriced stocks) is more profitable

¹The anomaly strategies are portfolios sorted on characteristics that predict unconditional returns, in the style of Fama and French (1993).

than the long leg (underpriced stocks) in episodes of high investment sentiment (demand shocks in our theory).²

Multiplier. First, we revisit the calculation of equation (3) in the presence of strategic responses. We show below (Appendix A.2.1) that the price multiplier to an aggregate demand shock becomes:

$$M_{agg} = \frac{1}{\mathcal{E}_{agg}} \cdot \frac{1}{1 + \frac{\chi}{1+\chi} \frac{\text{Var}[\mathcal{E}_i]}{\mathcal{E}_{agg}} (p - \bar{p})}, \quad (\text{IA.5})$$

where $\text{Var}[\mathcal{E}_i]$ is the demand-weighted cross-sectional variance of elasticity.³ For small deviations of the price from its baseline (small values of $p - \bar{p}$), the response of prices to a change in demand is still determined by the aggregate elasticity \mathcal{E}_{agg} . For example, at first order, the effect of a rise in passive investing on aggregate elasticity is reflected one-to-one into the multiplier. In “fiercely competitive markets,” the rise in passive investing does not affect the sensitivity of prices to demand, whereas it increases it when there are no strategic responses. We now turn to how the multiplier changes when the price deviates from its baseline \bar{p} .

No strategic response. Without strategic responses ($\chi = 0$), the aggregate multiplier is $M_{agg} = \mathcal{E}_{agg}^{-1}$. While each of the investors’ elasticities is fixed at \mathcal{E}_i , their contribution to aggregate elasticity depends on their relative demand. When the price is below its baseline, the more elastic investors have a stronger response than the less elastic investors: they buy relatively more of the asset. Thus more elastic investors represent a larger fraction of the market; aggregate elasticity increases. For example, in response to a supply shock, the aggregate elasticity moves in the opposite direction from the price: $\Delta \mathcal{E}_{agg} = -\text{Var}[\mathcal{E}_i] \Delta p$. How much investors differ from each other, the variance of individual elasticities, controls the strength of the composition effect. The market has a higher capacity to absorb demand shocks on the downside than on the upside. Without strategic responses, overpricing is more likely to happen than underpricing.

Strong strategic responses. Consider now the other extreme of a high degree of strategic response, $\chi \rightarrow \infty$. In this case the aggregate elasticity \mathcal{E}_{agg} is constant; if more elastic investors decrease their relative share, everybody becomes more elastic and exactly compensates the initial decline. However, the multiplier changes because the second term of the product in equation (IA.5) is not constant: it is smaller than one for $p > \bar{p}$ and larger than one otherwise. What happens then? If the price is above its baseline, the increase in individual elasticities due to the strategic response implies a decrease in demand. Because all investors become more aggressive, they demand less of the asset, which prevents the price from increasing much, leading to a smaller multiplier. The demand effect goes in the

²As such, imperfect competition is an alternative explanation to short-sell constraints (for example, Miller (1977) or Haddad et al. (2021)) for the pervasiveness of overpricing.

³Formally this corresponds to

$$\text{Var}[\mathcal{E}_i] = \int \left(\mathcal{E}_i^2 \frac{D_i}{S} \right) - \mathcal{E}_{agg}^2.$$

opposite direction when the price is below its baseline. Overall this leads to an opposite behavior of the multiplier M_{agg} relative to no strategic response. The multiplier is larger with underpricing than overpricing: underpricing is more likely to happen than overpricing.

A.2.1 Derivations

How do prices in the two-layer equilibrium decrease in response to an exogenous shift in the supply of a stock, ΔS ? We start by defining aggregation operators which weight investors' outcomes by the fraction of the stock they own:

$$\check{\mathbf{E}}[x_i] = \int x_i \frac{D_i}{S}, \quad (\text{IA.6})$$

$$\check{\text{Var}}[x_i] = \int x_i^2 \frac{D_i}{S} - \left(\int x_i \frac{D_i}{S} \right)^2 \quad (\text{IA.7})$$

$$\check{\text{Cov}}[x_i, y_i] = \int (x_i - \check{\mathbf{E}}[x_i])(y_i - \check{\mathbf{E}}[y_i]) \frac{D_i}{S}. \quad (\text{IA.8})$$

To see the impact of the change in supply on the equilibrium we start by deriving the change in individual demand:

$$\Delta D_i = \Delta e^{\underline{d}_i - \mathcal{E}_i(p - \bar{p})} = -\mathcal{E}_i D_i \Delta p - (p - \bar{p}) D_i \Delta \mathcal{E}_i, \quad (\text{IA.9})$$

such that change in aggregate demand, which corresponds to the change in aggregate supply, reads:

$$\Delta S = \Delta D = -\Delta p S \int \mathcal{E}_i D_i / S - (p - \bar{p}) S \int \Delta \mathcal{E}_i D_i / S \quad (\text{IA.10})$$

$$= -\Delta p S \mathcal{E}_{agg} + \chi(p - \bar{p}) S \Delta \mathcal{E}_{agg}, \quad (\text{IA.11})$$

where in the last equation we used the individual elasticity equation $\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \mathcal{E}_{agg}$ leading to the uniform change in individual investors' elasticity

$$\Delta \mathcal{E}_i = -\chi \Delta \mathcal{E}_{agg}. \quad (\text{IA.12})$$

To estimate the change in aggregate elasticity we write:

$$\Delta \mathcal{E}_{agg} = \int \Delta (\mathcal{E}_i D_i / S) = \int \Delta \mathcal{E}_i D_i / S - \frac{\Delta S}{S} \mathcal{E}_{agg} + \int \mathcal{E}_i \Delta D_i / S \quad (\text{IA.13})$$

$$= -\chi \Delta \mathcal{E}_{agg} - \frac{\Delta S}{S} \mathcal{E}_{agg} - \Delta p \check{\mathbf{E}}[\mathcal{E}_i^2] + \chi(p - \bar{p}) \mathcal{E}_{agg} \Delta \mathcal{E}_{agg}. \quad (\text{IA.14})$$

$$\Delta \mathcal{E}_{agg} = -\frac{1}{1 + \chi - \chi(p - \bar{p}) \mathcal{E}_{agg}} \left(\mathcal{E}_{agg} \frac{\Delta S}{S} + \check{\mathbf{E}}[\mathcal{E}_i^2] \Delta p \right), \quad (\text{IA.15})$$

Then we can plug the change in aggregate elasticity into the main equation expressing the change in aggregate demand and solve for Δp as a function of the change in supply ΔS .

$$\frac{\Delta S}{S} = -\mathcal{E}_{agg} \Delta p - \frac{\chi(p - \bar{p})}{1 + \chi - \chi(p - \bar{p}) \mathcal{E}_{agg}} \left(\mathcal{E}_{agg} \frac{\Delta S}{S} + \check{\mathbf{E}}[\mathcal{E}_i^2] \Delta p \right) \quad (\text{IA.16})$$

After some algebra, we find the change in prices in response to an exogenous change in supply:

$$\Delta p = \frac{1}{\mathcal{E}_{agg}} \cdot \frac{1}{1 + \frac{\chi}{1+\chi} \frac{p-\bar{p}}{\mathcal{E}_{agg}} \check{\text{Var}}[\mathcal{E}_i]} \frac{\Delta S}{S} = M_{agg} \cdot \frac{\Delta S}{S}, \quad (\text{IA.17})$$

where we define the aggregate multiplier M_{agg} as:

$$M_{agg} = \frac{1}{\mathcal{E}_{agg}} \cdot \frac{1}{1 + \frac{\chi}{1+\chi} \frac{p-\bar{p}}{\mathcal{E}_{agg}} \check{\text{Var}}[\mathcal{E}_i]}. \quad (\text{IA.18})$$

A.3 Limits to arbitrage

An important insight is that engaging in arbitrage trades (or more broadly exploiting mispricing) is a risky activity and this risk limits the effectiveness of arbitrageurs (De Long et al. (1990), Shleifer and Summers (1990), Shleifer and Vishny (1997), Brunnermeier and Pedersen (2008)).

If an asset is underpriced ($p < \bar{p}$), we expect arbitrageurs (high elasticity investors) to take on large positions. When the mispricing worsens, the arbitrageurs suffer large losses due to their large exposure. If they are unable to raise additional capital, they have to liquidate some of their positions which pushes the price down even further. This feedback creates a natural instability of arbitrage activity: shocks that worsen the mispricing hurt the arbitrageurs and deepen the mispricing. Mitchell et al. (2002) and Mitchell and Pulvino (2012) document this instability in action.

The degree of strategic response plays an important role in this process. Without strategic responses, the arbitrage capacity destroyed by arbitrageurs' losses is gone altogether. With strategic responses, other investors become more elastic in response to the decline of the arbitrageurs. They purchase more of the asset, thereby partially compensating the lower positions of the arbitrageurs.

We can illustrate this mechanism in our model by considering how the price responds to a demand shock that affects disproportionately investors with high elasticity. Below (Appendix A.3.1), we assume that each investor's baseline demand changes by an amount $\{\Delta \underline{d}_i\}_i$ and compute the equilibrium price response:

$$\Delta p = M_{agg} \times \left[\mathbf{E}(\Delta \underline{d}_i) + \frac{\chi}{1+\chi} (p - \bar{p}) \text{Cov}(\mathcal{E}_i, \Delta \underline{d}_i) \right], \quad (\text{IA.19})$$

where expectation and covariance represent demand-weighted moments. Without strategic responses, $\chi = 0$, the price response is simply the product of the aggregate multiplier M_{agg} with the average demand shock $\mathbf{E}(\Delta \underline{d}_i)$. A shock that hurts disproportionately investors with large positions (for example the asset is underpriced and these investors have a high demand elasticity) pushes the price down. The second term captures the role of strategic responses. When the asset is underpriced ($p < \bar{p}$), and demand decreases more for high elasticity investors ($\text{Cov}(\mathcal{E}_i, \Delta \underline{d}_i) < 0$), we obtain a positive response which compensates the direct effect. A symmetric compensation occurs when the asset is overpriced (see the derivations below). Interestingly, the stabilizing role of strategic responses is stronger the further away prices are from their baseline.⁴

⁴Duffie (2010) and Duffie and Strulovici (2012) study how the competitive response unfolds over time.

A.3.1 Derivations

We consider an experiment where demand across investors changes by $\{\Delta \underline{d}_i\}_i$. Solving for the equilibrium response of the price and elasticity is similar to the simple case of a change in aggregate supply. First, we consider the effect of a change in investors' demands on aggregate demand:

$$\Delta D = \Delta \int e^{\underline{d}_i - \mathcal{E}_i(p - \bar{p})} = \int \Delta \underline{d}_i D_i - \int \mathcal{E}_i \Delta p D_i - \int (p - \bar{p}) \Delta \mathcal{E}_i D_i \quad (\text{IA.20})$$

$$\frac{\Delta D}{S} = \check{\mathbf{E}}[\Delta \underline{d}_i] - \mathcal{E}_{agg} \Delta p + \chi(p - \bar{p}) \Delta \mathcal{E}_{agg}, \quad (\text{IA.21})$$

where we have used $\check{\mathbf{E}}[\mathcal{E}_i] = \mathcal{E}_{agg}$ and $\Delta \mathcal{E}_i = -\chi \Delta \mathcal{E}_{agg}$. To solve for the change in price as a function of the change in investors' demands we use the second market clearing condition of aggregate elasticities to find $\Delta \mathcal{E}_{agg}$:

$$\Delta \mathcal{E}_{agg} = \int \Delta \mathcal{E}_i D_i / S + \int \mathcal{E}_i \Delta D_i / S \quad (\text{IA.22})$$

$$= -\chi \Delta \mathcal{E}_{agg} + \int \mathcal{E}_i \Delta \underline{d}_i D_i / S - \Delta p \int \mathcal{E}_i^2 D_i / S - (p - \bar{p}) \int \mathcal{E}_i \Delta \mathcal{E}_i D_i / S \quad (\text{IA.23})$$

$$= (-\chi + \chi(p - \bar{p}) \mathcal{E}_{agg}) \Delta \mathcal{E}_{agg} + \int \mathcal{E}_i \Delta \underline{d}_i D_i / S - \Delta p \int \mathcal{E}_i^2 D_i / S \quad (\text{IA.24})$$

$$= \frac{1}{1 + \chi - \chi(p - \bar{p}) \mathcal{E}_{agg}} \cdot (\check{\mathbf{E}}[\mathcal{E}_i \Delta \underline{d}_i] - \check{\mathbf{E}}[\mathcal{E}_i^2] \Delta p). \quad (\text{IA.25})$$

We plug this expression back into the expression for the change in aggregate demand above:

$$\frac{\Delta D}{S} = \check{\mathbf{E}}[\Delta \underline{d}_i] - \mathcal{E}_{agg} \Delta p + \frac{\chi(p - \bar{p})}{1 + \chi - \chi p \mathcal{E}_{agg}} \cdot (\check{\mathbf{E}}[\mathcal{E}_i \Delta \underline{d}_i] - \check{\mathbf{E}}[\mathcal{E}_i^2] \Delta p) \quad (\text{IA.26})$$

The supply is fixed such that $\Delta D/S = 0$, which after rearranging terms gives the final expression for the change in price:

$$\Delta p = \frac{1}{\mathcal{E}_{agg}} \frac{1}{1 + \frac{\chi}{1 + \chi} \frac{p - \bar{p}}{\mathcal{E}_{agg}} \check{\mathbf{Var}}[\mathcal{E}_i]} \left(\check{\mathbf{E}}[\Delta \underline{d}_i] + \frac{\chi}{1 + \chi} (p - \bar{p}) \check{\mathbf{Cov}}(\mathcal{E}_i, \Delta \underline{d}_i) \right) \quad (\text{IA.27})$$

We recognize the first term as the standard aggregate multiplier (obtained when we derived the response to change in supply) and write the price response as:

$$\Delta p = M_{agg} \cdot \left(\underbrace{\check{\mathbf{E}}[\Delta \underline{d}_j]}_{\text{average demand shock}} + \frac{\chi}{1 + \chi} (p - \bar{p}) \underbrace{\check{\mathbf{Cov}}(\mathcal{E}_i, \Delta \underline{d}_i)}_{\text{average elasticity composition}} \right). \quad (\text{IA.28})$$

A.4 The effect of a rise in passive investing under heterogeneity

In Section A.1, we show how the elasticity changes when a fraction of investors become passive. Here, we revisit this calculation under heterogeneity in baseline elasticity \mathcal{E}_i (see

also Section 5.1) and degree of strategic response χ_i . Again, we start with an economy with an initial equilibrium $p = \bar{p}$.

Individual decisions and equilibrium conditions are:

$$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi_i \mathcal{E}_{agg}, \quad (\text{IA.29})$$

$$\mathcal{E}_{agg} = \int \mathcal{E}_i \frac{D_i}{S}. \quad (\text{IA.30})$$

Combining the two yields

$$\mathcal{E}_{agg} = \int \mathcal{E}_i \frac{D_i}{S} \quad (\text{IA.31})$$

$$= \int \underline{\mathcal{E}}_i \frac{D_i}{S} - \mathcal{E}_{agg} \int \chi_i \frac{D_i}{S} \quad (\text{IA.32})$$

$$= \check{\mathbf{E}}[\underline{\mathcal{E}}_i] - \check{\mathbf{E}}[\chi_i] \mathcal{E}_{agg} \quad (\text{IA.33})$$

$$= \check{\mathbf{E}}[\underline{\mathcal{E}}_i] \times \frac{1}{1 + \check{\mathbf{E}}[\chi_i]}. \quad (\text{IA.34})$$

Denote the set of active investors as *Active*, and the fraction of assets held by this group by $|Active|$. Passive investors have $\underline{\mathcal{E}}_i = \chi_i = 0$, such that

$$\mathcal{E}_{agg} = |Active| \times \check{\mathbf{E}}[\underline{\mathcal{E}}_i | i \in Active] \times \frac{1}{1 + |Active| \times \check{\mathbf{E}}[\chi_i | i \in Active]}. \quad (\text{IA.35})$$

The aggregate elasticity is the product of the active share, the average baseline elasticity among active investors, and an equilibrium term capturing the strategic responses of active investors. Note that this expression looks exactly like in the case with homogeneous χ (e.g. equation (25)), other than now the strength of strategic response is controlled by the average χ_i among active investors.

Now consider the effect of a change in the active share $|Active|$. We assume that random investors are switching, meaning that the set of active investors that become passive is a representative sample of the active population, such that $\check{\mathbf{E}}[\underline{\mathcal{E}}_i | i \in Active]$ and $\check{\mathbf{E}}[\chi_i | i \in Active]$ remain unaffected by the change. Then:

$$\frac{d \log \mathcal{E}_{agg}}{d \log |Active|} = \frac{|Active|}{\mathcal{E}_{agg}} \times \left[\frac{\check{\mathbf{E}}[\underline{\mathcal{E}}_i | i \in Active] - \frac{|Active| \times \check{\mathbf{E}}[\underline{\mathcal{E}}_i | i \in Active] \times \check{\mathbf{E}}[\chi_i | i \in Active]}{1 + |Active| \times \check{\mathbf{E}}[\chi_i | i \in Active]}}{1 + |Active| \times \check{\mathbf{E}}[\chi_i | i \in Active]} \right] \quad (\text{IA.36})$$

$$= 1 - \frac{|Active| \times \check{\mathbf{E}}[\chi_i | i \in Active]}{1 + |Active| \times \check{\mathbf{E}}[\chi_i | i \in Active]} \quad (\text{IA.37})$$

$$= \frac{1}{1 + |Active| \times \check{\mathbf{E}}[\chi_i | i \in Active]}. \quad (\text{IA.38})$$

The pass-through from a change in the active share mirrors equation (26). The only difference is that the average degree of strategic response χ_i among active investors is what determines the pass-through instead of the single parameter χ of the homogenous model.

A.5 Exogenous change in the baseline elasticity

Last, we consider a change in investors' baseline elasticity $\Delta \underline{\mathcal{E}}_i$. We start with the relative change in aggregate demand:

$$\frac{\Delta D}{S} = -(p - \bar{p}) \int \Delta \mathcal{E}_i D_i / S - \mathcal{E}_{agg} \Delta p \quad (\text{IA.39})$$

$$= -(p - \bar{p}) \check{\mathbf{E}} [\Delta \underline{\mathcal{E}}_i] + \chi(p - \bar{p}) \Delta \mathcal{E}_{agg} - \mathcal{E}_{agg} \Delta p \quad (\text{IA.40})$$

We use market clearing to get $\Delta D/S = 0$:

$$\Delta p = -\frac{1}{\mathcal{E}_{agg}} ((p - \bar{p}) \check{\mathbf{E}} [\Delta \underline{\mathcal{E}}_i] - \chi(p - \bar{p}) \Delta \mathcal{E}_{agg}) . \quad (\text{IA.41})$$

The change in aggregate elasticity in the case of changes in baseline elasticities is different than in the previous cases:

$$\Delta \mathcal{E}_{agg} = \int \Delta (\mathcal{E}_i D_i / S) = \int \Delta \underline{\mathcal{E}}_i D_i / S - \chi \Delta \mathcal{E}_{agg} + \int \mathcal{E}_i \Delta D_i / S \quad (\text{IA.42})$$

$$= \int \Delta \underline{\mathcal{E}}_i D_i / S - \chi \Delta \mathcal{E}_{agg} - (p - \bar{p}) \int \mathcal{E}_i \Delta \mathcal{E}_i D_i / S + \chi(p - \bar{p}) \mathcal{E}_{agg} \Delta \mathcal{E}_{agg} - \Delta p \int \mathcal{E}_i^2 D_i / S \quad (\text{IA.43})$$

$$= \check{\mathbf{E}} [\Delta \underline{\mathcal{E}}_i] - \chi \Delta \mathcal{E}_{agg} - (p - \bar{p}) \check{\mathbf{E}} [\mathcal{E}_i \Delta \underline{\mathcal{E}}_i] + \chi(p - \bar{p}) \mathcal{E}_{agg} \Delta \mathcal{E}_{agg} - \Delta p \check{\mathbf{E}} [\mathcal{E}_i^2] \quad (\text{IA.44})$$

After rearranging the terms and plugging in the expression for the change in the price, we find:

$$\left(1 + \chi + \chi(p - \bar{p}) \frac{\check{\text{Var}}[\mathcal{E}_i]}{\mathcal{E}_{agg}} \right) \Delta \mathcal{E}_{agg} = \left(1 + (p - \bar{p}) \frac{\check{\text{Var}}[\mathcal{E}_i]}{\mathcal{E}_{agg}} \right) \check{\mathbf{E}} [\Delta \underline{\mathcal{E}}_i] - (p - \bar{p}) \check{\text{Cov}} (\mathcal{E}_i, \Delta \underline{\mathcal{E}}_i) \quad (\text{IA.45})$$

Plugging back into the change in the price we have:

$$\Delta p = -M_{agg} \frac{p - \bar{p}}{1 + \chi} (\check{\mathbf{E}} [\Delta \underline{\mathcal{E}}_i] + (p - \bar{p}) \check{\text{Cov}} (\mathcal{E}_i, \Delta \underline{\mathcal{E}}_i)) . \quad (\text{IA.46})$$

B Model of Information Acquisition

B.1 Setup

There is one period and one asset, and a continuum of agents indexed by $i \in [0, 1]$. Each agent has CARA preferences with risk aversion ρ_i :

$$U_i = \mathbf{E}_i[-e^{-\rho_i W_i}], \quad (\text{IA.47})$$

and initial wealth W_i . The gross risk-free rate is 1, and the random asset payoff is f . The asset is in noisy supply $\bar{x} + x$ with \bar{x} an exogenous fixed parameter and $x \sim \mathcal{N}(0, \sigma_x^2)$.

Each agent has a prior that $f \sim \mathcal{N}(\mu_i, \sigma_i^2)$. Following Veldkamp (2011), agents start with a flat prior on f and receive signal μ_i such that the signal is distributed $\mu_i \sim \mathcal{N}(f, \sigma_i^2)$. Each agent can acquire a private signal $\eta_i \sim \mathcal{N}(f, \sigma_{i,\eta}^2)$ at cost $c_i(\sigma_i^{-2} + \sigma_{i,\eta}^{-2})$, with $c_i(\cdot)$ a non-decreasing positive function. That is, obtaining more precise signals is more costly. The signal being private implies in particular that signal realizations are uncorrelated across agents conditional on the fundamental f .

We focus on rational expectations equilibria, and among those linear equilibria specifically. These are equilibria in which the price takes the form:

$$p = A + Bf + Cx. \quad (\text{IA.48})$$

An equilibrium is a set of coefficient (A, B, C) , information choices $\sigma_{i,\eta}^2$, demand curves $D_i(p|\eta_i)$ such that:

- (a) Each demand function and information choice maximizes expected utility, taking as given the price function.
- (b) The market for the asset clears: $\bar{x} + x = \int D_i(p|\eta_i) di$.

To solve the model, we proceed in two steps: first we solve for the price given information decisions; second, we derive equilibrium information decisions.

B.2 Solving prices given information

We are going to solve for the price function $p = A + Bf + Cx$. First, we solve for allocations given the information choice and finally we use market clearing to pin down the price.

Agents form posterior beliefs on the fundamental f based on their prior μ_i , signal η_i , and based on prices. The signal agents can extract from prices about f is:

$$s(p) = \frac{p - A}{B} = f + \frac{C}{B}x. \quad (\text{IA.49})$$

Given the three signals, we are able to derive the posterior belief about f , which will be distributed as $\mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$ as follows:

$$\hat{\sigma}_i^{-2} = \sigma_i^{-2} + \sigma_{i\eta}^{-2} + \frac{B^2}{C^2} \sigma_x^{-2} \quad (\text{IA.50})$$

$$\hat{\mu}_i = \hat{\sigma}_i^2 \left(\sigma_i^{-2} \mu_i + \sigma_{i\eta}^{-2} \eta_i + \frac{B^2}{C^2} \sigma_x^{-2} s(p) \right) \quad (\text{IA.51})$$

Asset Demand. Abstracting from the cost of acquiring information, the expected utility function for a given asset holding q_i is:

$$U_i(q_i) = -\mathbf{E}[\exp(-\rho_i(fq_i - pq_i))] \quad (\text{IA.52})$$

$$= -\exp\left(-\rho_i q_i (\mathbf{E}[f] - p) + \frac{\rho_i^2}{2} q_i^2 \text{Var}[f]\right). \quad (\text{IA.53})$$

The first order condition with respect to q_i gives us immediately:

$$\begin{aligned} & -\rho_i (\mathbf{E}[f] - p) + \rho_i^2 q_i \text{Var}[f] = 0 \\ \iff & q_i = \frac{1}{\rho_i \text{Var}[f]} (\mathbf{E}[f] - p) \\ \iff & q_i = \frac{1}{\rho_i} \hat{\sigma}_i^{-2} (\hat{\mu}_i - p) \end{aligned} \quad (\text{IA.54})$$

Market Clearing. The market clearing condition reads:

$$\int q_i di = \bar{x} + x. \quad (\text{IA.55})$$

Given asset demand this translates into:

$$\int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} (\hat{\mu}_i - p) di = \bar{x} + x \quad (\text{IA.56})$$

The goal now is to find (A, B, C) , which we identify directly from the market clearing condition. First we replace the expressions for the price function and the posteriors mean and variances in the market clearing equation:

$$\int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} \left[f + \frac{B}{C} \frac{\hat{\sigma}_i^2}{\sigma_x^2} x \right] di - \int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} [A + Bf + Cx] di = \bar{x} + x \quad (\text{IA.57})$$

We identify all the terms that are linear in f and find that $B = 1$. Next, we group the terms that are linear in x , which yields

$$\begin{aligned} & \int \frac{1}{\rho_i} \frac{1}{C} \sigma_x^{-2} di - \int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} C di = 1 \\ \iff & \int \frac{1}{\rho_i} \left[\frac{1}{C} \sigma_x^{-2} - C \sigma_i^{-2} - C \sigma_{i\eta}^{-2} - \frac{1}{C} \sigma_x^{-2} \right] di = 1 \\ \iff & C = - \left[\int \frac{1}{\rho_i} (\sigma_i^{-2} + \sigma_{i\eta}^{-2}) di \right]^{-1}, \end{aligned} \quad (\text{IA.58})$$

where we used the expression of the posterior found above to substitute into the second equation. Last, we gather the constant terms to find A :

$$A = -\bar{x} \left[\int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} di \right]^{-1}. \quad (\text{IA.59})$$

B.3 Demand elasticity

We recall the demand schedule for agent i :

$$\begin{aligned}
q_i &= \frac{1}{\rho_i} \hat{\sigma}_i^{-2} (\hat{\mu}_i - p) \\
&= \frac{1}{\rho_i} \hat{\sigma}_i^{-2} (\hat{\sigma}_i^2 [\sigma_i^{-2} \mu_i + \sigma_{i\eta}^{-2} \eta_i + C^{-2} \sigma_x^{-2} s(p)] - p) \\
&= \frac{1}{\rho_i} (\sigma_i^{-2} \mu_i + \sigma_{i\eta}^{-2} \eta_i + C^{-2} \sigma_x^{-2} (p - A) - \hat{\sigma}_i^{-2} p) \\
&= \frac{1}{\rho_i} (\sigma_i^{-2} \mu_i + \sigma_{i\eta}^{-2} \eta_i + (C^{-2} \sigma_x^{-2} - \hat{\sigma}_i^{-2}) p - C^{-2} \sigma_x^{-2} A). \tag{IA.60}
\end{aligned}$$

We can read the demand elasticity off the terms proportional to the price p as:

$$\mathcal{E}_i = -\frac{dq_i}{dp} = -\frac{1}{\rho_i} (C^{-2} \sigma_x^{-2} - \hat{\sigma}_i^{-2}) = \frac{1}{\rho_i} (\sigma_i^{-2} + \sigma_{i\eta}^{-2}). \tag{IA.61}$$

In the model, a regression of q_i on p would not give us the proper elasticity. There is a bias in the regression because p is correlated with μ_i and η_i . It is still possible to recover the elasticity using an instrument; for example the supply shock x covaries with p but is uncorrelated with μ_i and η_i .

We define the aggregate demand elasticity as

$$\mathcal{E}_{agg} = \int \mathcal{E}_j dj. \tag{IA.62}$$

We can express the equilibrium in terms of demand elasticities. Taking (IA.61) and (IA.62) together, we express the solution for the equilibrium

$$C = - \left[\int \frac{1}{\rho_j} (\sigma_j^{-2} + \sigma_{j\eta}^{-2}) dj \right]^{-1} = \mathcal{E}_{agg}^{-1}. \tag{IA.63}$$

B.4 Optimal information

Computing expected utility. Conditional on the signal and the price, expected utility is:

$$U_i(q_i) = -\mathbf{E} [\exp(-\rho_i (f q_i - p q_i)) | p, \eta] \tag{IA.64}$$

$$= -\exp \left(-\rho_i q_i (\mathbf{E}[f | p, \eta] - p) + \frac{\rho_i^2}{2} q_i^2 \text{Var}[f | p, \eta] \right) \tag{IA.65}$$

$$= -\exp \left(-\frac{1}{2} \frac{(\mathbf{E}[f | p, \eta] - p)^2}{\text{Var}[f | p, \eta]} \right), \tag{IA.66}$$

where the last line is derived using standard properties of quadratic functions.⁵

⁵For a function $f(x) = ax^2 + bx$, the maximum is reached for $x^* = -b/(2a)$ and its value is $f(x^*) = -b^2/(4a)$.

We can write:

$$\mathbf{E}[f|p, \eta] - p = \underbrace{(\mathbf{E}[f|p, \eta] - \mathbf{E}[f|p])}_z + (\mathbf{E}[f|p] - p). \quad (\text{IA.67})$$

Conditional on p , z has mean 0 and its variance σ_z^2 can be obtained from:

$$\underbrace{f - \mathbf{E}[f|p]}_{\text{variance: } (\sigma_i^{-2} + \sigma_x^{-2}/C^2)^{-1}} = \underbrace{(f - \mathbf{E}[f|p, \eta])}_{\text{variance: } \hat{\sigma}_i^2} + \underbrace{z}_{\text{variance: } \sigma_z^2} \quad (\text{IA.68})$$

Using equation (7.32) in Veldkamp (2011), this maps to:⁶

$$F = -\frac{1}{2} \frac{1}{\hat{\sigma}_i^2} \quad (\text{IA.69})$$

$$G = -(\mathbf{E}[f|p] - p) \frac{1}{\hat{\sigma}_i^2} \quad (\text{IA.70})$$

$$H = -\frac{1}{2} (\mathbf{E}[f|p] - p)^2 \frac{1}{\hat{\sigma}_i^2} \quad (\text{IA.71})$$

So expected utility conditional on the price is:

$$\begin{aligned} U_0|_p &= -(1 - 2\sigma_z^2 F)^{-1/2} \exp \left(\frac{1}{2} G^2 (1 - 2\sigma_z^2 F)^{-1} \sigma_z^2 + H \right) \\ &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp \left(\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma}_i^2} \left[(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1} \frac{1}{\hat{\sigma}_i^2} \sigma_z^2 - 1 \right] \right) \\ &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp \left(\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma}_i^2} \left[(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1} (\frac{1}{\hat{\sigma}_i^2} \sigma_z^2 - 1 - \frac{\sigma_z^2}{\hat{\sigma}_i^2}) \right] \right) \\ &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp \left(-\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma}_i^2} \left[(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1} \right] \right) \\ U_0|_p &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp \left(-\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma}_i^2 + \sigma_z^2} \right). \end{aligned} \quad (\text{IA.72})$$

Expected utility is:

$$\begin{aligned} \mathbf{E}[U_0|_p] &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \mathbf{E} \exp \left(-\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma}_i^2 + \sigma_z^2} \right) \\ &= - \left[\frac{\sigma_i^{-2} + \sigma_x^{-2}/C^2}{\sigma_i^{-2} + \sigma_{i\eta}^{-2} + \sigma_x^{-2}/C^2} \right]^{1/2} \cdot \mathbf{E} \left[\exp \left(-\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{(\sigma_i^{-2} + \sigma_x^{-2}/C^2)^{-1}} \right) \right] \end{aligned} \quad (\text{IA.73})$$

where we use (IA.68) in the second equality.

⁶There is a general formula for the mean of the exponential of the quadratic of a normal variable. If we take the multivariate normal $z \sim \mathcal{N}(0, \Sigma)$:

$$\mathbf{E}[\exp(z' F z + G' z + H)] = |I - 2\Sigma F|^{-1/2} \exp \left(\frac{1}{2} G' (I - 2\Sigma F)^{-1} \Sigma G + H \right)$$

Optimal information. To derive the optimal information choice, investors trade off utility with the cost of acquiring information, which translates in utility terms to:

$$U_0^{(c)} = U_0 \cdot \exp(\rho_i c_i(\sigma_i^{-2} + \sigma_{i\eta}^{-2})). \quad (\text{IA.74})$$

The cost function $c_i(\cdot)$ is increasing in the signal precision and can be specific to investor i . We obtain the first-order condition that determines the information choice:

$$\begin{aligned} & \max_{\sigma_{i\eta}^{-2}} -\log(-U_0) - \rho_i c_i(\sigma_i^{-2} + \sigma_{i\eta}^{-2}) \\ \iff & \max_{\sigma_{i\eta}^{-2}} -\log(-U_0) - \rho_i c_i(\rho_i \mathcal{E}_i) \\ \iff & \frac{1}{2} \frac{1}{\sigma_i^{-2} + \sigma_{i\eta}^{-2} + \sigma_x^{-2}/C^2} = \rho_i c'_i(\sigma_i^{-2} + \sigma_{i\eta}^{-2}). \\ \iff & \frac{1}{2} \frac{1}{\rho_i \mathcal{E}_i + \mathcal{E}_{agg}^2 \sigma_x^{-2}} = \rho_i c'_i(\rho_i \mathcal{E}_i). \end{aligned} \quad (\text{IA.75})$$

Example 1: linear cost function. We consider the case of a constant marginal cost for any information acquired past the initial endowment: $c_i(x) = c_{1,i} \max(x - \sigma_i^{-2}, 0)$. Note that in this case not all agents acquire information since $\sigma_{i\eta}^{-2} > 0$, so the actual precision is

$$\sigma_{i\eta}^{-2} = \max\left(\frac{1}{2\rho_i c_{1,i}} - \sigma_i^{-2} - \mathcal{E}_{agg}^2 \sigma_x^{-2}, 0\right). \quad (\text{IA.76})$$

We can rewrite the choice of information as a choice of elasticity:

$$\mathcal{E}_i = \underbrace{\frac{1}{2\rho_i^2 c_{1,i}}}_{\text{Investor characteristics}} - \frac{\sigma_x^{-2}}{\rho_i} \underbrace{\mathcal{E}_{agg}^2}_{\text{market elasticity}} \quad (\text{IA.77})$$

Example 2: linear response to aggregate elasticity. To relate to the model of Section 2, we ask if there is a reasonable family of cost functions that exactly gives rise to equation (4). We are looking for a cost function such that $\mathcal{E}_i = \alpha - \beta \mathcal{E}_{agg}$. Equivalently, this corresponds to $\mathcal{E}_{agg} = \frac{1}{\beta}(\alpha - \mathcal{E}_i)$. Plugging in the first order condition, this gives:

$$2\rho_i^2 c'_i(\rho_i \mathcal{E}_i) = \frac{1}{\mathcal{E}_i + \frac{\sigma_x^{-2}}{\rho_i \beta^2} (\alpha - \mathcal{E}_i)^2} \quad (\text{IA.78})$$

$$\stackrel{\text{def}}{=} \tilde{c}'_i(\mathcal{E}_i) = \frac{1}{\frac{\sigma_x^{-2}}{\rho_i \beta^2} \mathcal{E}_i^2 + \left(1 - 2\frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2}\right) \mathcal{E}_i + \frac{\alpha^2 \sigma_x^{-2}}{\rho_i \beta^2}} \quad (\text{IA.79})$$

The denominator of the right-hand-side is a second degree polynomial, we solve for its roots. The discriminant is:

$$\Delta = \left(1 - 2\frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2}\right)^2 - 4\frac{\sigma_x^{-2}}{\rho_i \beta^2} \frac{\alpha^2 \sigma_x^{-2}}{\rho_i \beta^2} \quad (\text{IA.80})$$

$$= 1 - 4\frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2} \quad (\text{IA.81})$$

Let us assume $\Delta < 0$. This is equivalent to $\rho_i \beta^2 < 4\alpha \sigma_x^{-2}$. In this case, we have, using standard results on the primitive of the inverse of a polynomial:

$$\tilde{c}_i(\mathcal{E}_i) = \frac{2 \arctan \left(\frac{2 \frac{\sigma_x^{-2}}{\rho_i \beta^2} \mathcal{E}_i + \left(1 - 2 \frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2}\right)}{\sqrt{4 \frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2} - 1}} \right)}{\sqrt{4 \frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2} - 1}} + K \quad (\text{IA.82})$$

The cost function is convex as long as the argument of the arctangent is negative, so:

$$\mathcal{E}_i \leq \alpha - \frac{\rho_i \beta^2}{2\sigma_x^{-2}}. \quad (\text{IA.83})$$

We can see that if the right-hand-side of this condition is positive, the condition of $\Delta < 0$ is automatically satisfied.

After rescaling, $2\rho_i c_i(\rho_i \mathcal{E}_i) = \tilde{c}_i(\mathcal{E}_i)$, or equivalently $c_i(x) = \frac{1}{2\rho_i} \tilde{c}_i(x/\rho_i)$ we have:

$$c_i(x) = \frac{1}{\rho_i} \frac{1}{\sqrt{2\alpha\tilde{\beta} - 1}} \arctan \left(\frac{\tilde{\beta} \frac{x}{\rho_i} + (1 - \alpha\tilde{\beta})}{\sqrt{2\alpha\tilde{\beta} - 1}} \right) + \tilde{K} \quad (\text{IA.84})$$

with $\tilde{\beta} = 2\sigma_x^{-2}/(\rho_i \beta^2)$, and the condition $\alpha - \rho_i \beta^2/(2\sigma_x^{-2}) \geq 0$ becomes $\alpha\tilde{\beta} \geq 1$. We can collect these results in a proposition.

Proposition 1. *For any $a > 0$ and $b > 0$ so that $ab > 1$, assume the information cost follows the function:*

$$\begin{aligned} c_i(x) &= 0, \text{ if } x < 0, \\ c_i(x) &= \frac{1}{\rho_i} \frac{1}{\sqrt{2ab - 1}} \arctan \left(\frac{b \frac{x}{\rho_i} + (1 - ab)}{\sqrt{2ab - 1}} \right) + K, \text{ if } 0 \leq x/\rho_i \leq a - b^{-1} \\ c_i(x) &= +\infty, \text{ if } x/\rho_i \geq a - b^{-1}, \end{aligned} \quad (\text{IA.85})$$

where K is such that $c_i(0) = 0$. This cost function is increasing and convex. Then the optimal elasticity is:

$$\mathcal{E}_i = \mathcal{E}_{0,i} - \chi \mathcal{E}_{agg}, \quad (\text{IA.86})$$

with $\mathcal{E}_{0,i} = a$ and $\chi = \sqrt{(2\sigma_x^{-2})/(\rho_i b)}$.

B.5 Flexibility in information acquisition

We turn to the role of the flexibility in the acquisition of information for the degree of strategic response. We first take the inverse of (IA.75)

$$\rho_i \mathcal{E}_i + \mathcal{E}_{agg}^2 \sigma_x^{-2} = \frac{1}{2\rho_i} \cdot \frac{1}{c'_i(\rho_i \mathcal{E}_i)}. \quad (\text{IA.87})$$

Then we use the implicit function theorem:

$$\rho_i \frac{d\mathcal{E}_i}{d\mathcal{E}_{agg}} + 2\mathcal{E}_{agg}\sigma_x^{-2} = -\frac{1}{2\rho_i} \cdot \frac{\rho_i c_i''}{c_i'^2} \frac{d\mathcal{E}_i}{d\mathcal{E}_{agg}}. \quad (\text{IA.88})$$

This yields

$$\chi = -\frac{d\mathcal{E}_i}{d\mathcal{E}_{agg}} = 2\mathcal{E}_{agg}\sigma_x^{-2} \cdot \frac{1}{\rho_i + \frac{1}{2} \frac{c_i''}{c_i'^2}} \quad (\text{IA.89})$$

The response depends on the curvature of the information acquisition cost function. If the curvature is zero (as is the case in our linear cost example), then the response is highest. A larger curvature would elicit a weaker response.

B.6 Price informativeness

We define price informativeness for investor i as the ratio of the precision of their belief about the fundamental when they condition on their private information and on the price and the precision of their belief using private information only:

$$\begin{aligned} \mathcal{I}_i &= \frac{\text{Var}(f|\mu_i, \eta_i, p)^{-1}}{\text{Var}(f|\mu_i, \eta_i)^{-1}} = \frac{\sigma_i^{-2} + \sigma_{i\eta}^{-2} + \mathcal{E}_{agg}^2 \sigma_x^{-2}}{\sigma_i^{-2} + \sigma_{i\eta}^{-2}} \\ &= 1 + \mathcal{E}_{agg} \frac{\mathcal{E}_{agg}}{\rho_i \mathcal{E}_i} \sigma_x^{-2} \end{aligned} \quad (\text{IA.90})$$

We also define the absolute price informativeness of the price as

$$\mathcal{I}_{abs} = \text{Var}(f|p)^{-1} = \mathcal{E}_{agg}^2 \sigma_x^{-2}. \quad (\text{IA.91})$$

C Other Foundations for the Degree of Strategic Response χ

C.1 Alternative theories

We discuss other theories of what determines the strategic response.

One such aspect is risk. Following an aggressive high-elasticity trading strategy entails taking more extreme positions, and hence more risk. Risk itself is endogenous to the aggressiveness of other traders: in more efficient markets, prices are tightly related to fundamentals, while without any active traders, prices are more sensitive to the whims of noise traders.⁷ Thus, it is unappealing to follow aggressive strategies exactly when they are most needed, which limits the process of investor competition. In Appendix Section C.2, we present a setting where investors do not make information choices but learn from prices. We show that endogenous risk shapes strategic responses; for example when all risk is endogenous investors do not interact, $\chi = 0$, while otherwise there is a positive response.⁸

Another aspect is institutional. Many financial institutions face strong mandates from their ultimate investors in terms of what strategies they are allowed to follow. While these restrictions can be viewed as an optimal contract solving information asymmetry between final investors and the asset manager, they inhibit strategic responses. Investment mandates limit the ability of institutions to react to changes in the behavior of other investors, pushing χ down relative to an unconstrained setting. Beber et al. (2018) show how explicit mandates and constraints in active mutual funds prospectuses strongly limit their investment opportunity set. Investment strategies of banks and insurance companies are also restricted, this time by their regulatory framework (for example, Basel III capital regulation).

Similarly, asset managers might have different incentives than that of their investors which pushes their decisions away from maximizing risk-adjusted returns. For example, Chevalier and Ellison (1997) show that flow-performance sensitivity distorts mutual funds' investment choices.

C.2 Learning from prices

We consider a model in which agents can learn from prices, which highlights a distinct mechanism from that of the previous section. Two main assumptions differ: agents cannot acquire information, and there is residual uncertainty about the asset payoff that cannot be learned. This setting leads to a new determinant of demand elasticity, beyond risk aversion and prior information. When many traders are aggressive, prices are more informative. How should one react? On the one hand, the extra information implies that price variation are less indicative of future return, and that pushes the investor to trade less aggressively. On the other hand, the extra information implies that returns appear less risky, and that pushes the investor to trade more aggressively. Increased price informativeness reveals relatively more about the fundamental than the payoff risk, exactly because of the presence of resid-

⁷De Long et al. (1990) first highlighted the importance of endogenous risk for dynamic arbitrage.

⁸In the first case, the model coincides with that of the previous section when the information cost is infinitely steep, $c_i''/c_i'^2 \rightarrow \infty$.

ual uncertainty.⁹ Therefore the first effect dominates: the investor responds by being less aggressive, $\chi > 0$. This response is stronger when residual uncertainty is higher.

C.2.1 Setup

The asset trades at endogenous price p and pays off $f + \epsilon$, with $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. There is a continuum of mass 1 of agents indexed by i . Each agent has CARA preferences with risk aversion ρ_i . Each agent has a flat prior on f and receives an independent signal μ_i , such that $\mu_i \sim \mathcal{N}(f, \sigma_i^2)$. The asset is in noisy supply $\bar{x} + x$ with \bar{x} a constant and $x \sim \mathcal{N}(0, \sigma_x^2)$.

We look for a rational expectations equilibrium, with:

$$p = A + Bf + Cx. \quad (\text{IA.92})$$

C.2.2 Equilibrium

Learning from the price. After observing the price, agent i 's posterior belief about the fundamental f is $\mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$, with:

$$\hat{\sigma}_i^{-2} = \sigma_i^{-2} + \frac{B^2}{C^2} \sigma_x^{-2}, \quad (\text{IA.93})$$

$$\hat{\mu}_i = \hat{\sigma}_i^2 \left(\sigma_i^{-2} \mu_i + \frac{B^2}{C^2} \sigma_x^{-2} s(p) \right), \quad (\text{IA.94})$$

where the signal from the price is:

$$s(p) = \frac{p - A}{B} = f + \frac{C}{B}x. \quad (\text{IA.95})$$

Because priors are independent from preferences, we can compute the average belief for agents of type i (that is for a given σ_i and ρ_i):

$$\mathbb{E}_i[\hat{\mu}_i] = \hat{\sigma}_i^2 \left(\sigma_i^{-2} f + \frac{B^2}{C^2} \sigma_x^{-2} \left[f + \frac{C}{B}x \right] \right) \quad (\text{IA.96})$$

$$= f + \frac{B}{C} \frac{\hat{\sigma}_i^2}{\sigma_x^2} x. \quad (\text{IA.97})$$

Asset demand. Asset demand q_i is given by the standard optimum portfolio choice:

$$q_i = \frac{1}{\rho_i} \frac{\mathbb{E}[f + \epsilon | \mu_i, p] - p}{\text{Var}[f + \epsilon | \mu_i, p]} \quad (\text{IA.98})$$

$$= \frac{1}{\rho_i} \frac{\hat{\mu}_i - p}{\hat{\sigma}_i^2 + \sigma_\epsilon^2}. \quad (\text{IA.99})$$

⁹In the model of the previous section, the two effects exactly cancelled out. The response was coming from changes in information acquired, which is shut down here.

Market clearing. The total demand for the asset must equal its supply:

$$\int q_i di = \bar{x} + x. \quad (\text{IA.100})$$

Plugging in the demand, we obtain

$$\int \frac{1}{\rho_i} \frac{1}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left[\frac{\hat{\sigma}_i^2}{\sigma_i^2} \mu_i + \frac{B^2}{C^2} \frac{\hat{\sigma}_i^2}{\sigma_x^2} f + \frac{B}{C} \frac{\hat{\sigma}_i^2}{\sigma_x^2} x - A - Bf - Cx \right] di = \bar{x} + x. \quad (\text{IA.101})$$

We use the law of large numbers as in equation (IA.96) to obtain

$$\int \frac{1}{\rho_i} \frac{1}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left[f + \frac{B}{C} \frac{\hat{\sigma}_i^2}{\sigma_x^2} x - A - Bf - Cx \right] di = \bar{x} + x \quad (\text{IA.102})$$

$$(\text{IA.103})$$

This gives:

$$B = 1, \quad (\text{terms in } f) \quad (\text{IA.104})$$

$$\int \frac{1}{\rho_i} \frac{1}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left[\frac{B}{C} \frac{\hat{\sigma}_i^2}{\sigma_x^2} - C \right] di = 1. \quad (\text{terms in } x) \quad (\text{IA.105})$$

Plugging in the definition of $\hat{\sigma}_i^2$, we obtain

$$\int \frac{1}{\rho_i} \frac{1}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left[\frac{1}{C^2} \frac{\hat{\sigma}_i^2}{\sigma_x^2} - 1 \right] di = C^{-1}, \quad (\text{IA.106})$$

$$\int \frac{1}{\rho_i} \frac{1}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left[\frac{1}{C^2} \sigma_x^{-2} \frac{1}{\sigma_i^{-2} + \frac{1}{C^2} \sigma_x^{-2}} - 1 \right] di = C^{-1}, \quad (\text{IA.107})$$

$$\int \frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left[\frac{1}{C^2} \sigma_x^{-2} - \sigma_i^{-2} - \frac{1}{C^2} \sigma_x^{-2} \right] di = C^{-1}. \quad (\text{IA.108})$$

Therefore we have:

$$C^{-1} = - \int \frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \frac{1}{\sigma_i^2} di, \quad (\text{IA.109})$$

$$-C^{-1} = \int \frac{1}{\rho_i} \frac{1}{1 + \sigma_\epsilon^2 \left(\sigma_i^{-2} + \frac{1}{C^2} \sigma_x^{-2} \right)} \frac{1}{\sigma_i^2} di. \quad (\text{IA.110})$$

Define $\tilde{C} = -C$, which is positive. We can rewrite:

$$\tilde{C}^{-1} = \int \frac{1}{\rho_i} \frac{1}{1 + \sigma_\epsilon^2 \left(\sigma_i^{-2} + \frac{1}{\tilde{C}^2} \sigma_x^{-2} \right)} \frac{1}{\sigma_i^2} di. \quad (\text{IA.111})$$

The left-hand-side of this equation is decreasing in \tilde{C} . The right-hand-side is increasing in \tilde{C} . If $\tilde{C} \rightarrow 0$, the left-hand-side goes to ∞ and the right-hand-side goes to 0. If $\tilde{C} \rightarrow \infty$, the left-hand-side goes to 0 and the right-hand-side has a finite positive limit. Therefore, there is a unique solution to the equation, and a unique linear equilibrium.

C.2.3 Equilibrium elasticities

We now derive demand elasticities. We show how individual demand elasticities respond to the aggregate elasticity. Demand is given by:

$$q_i = \frac{1}{\rho_i} \frac{\hat{\mu}_i - p}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \quad (\text{IA.112})$$

$$= \frac{1}{\rho_i} \frac{\hat{\sigma}_i^2 \left(\sigma_i^{-2} \mu_i + \frac{B^2}{C^2} \sigma_x^{-2} s(p) \right) - p}{\hat{\sigma}_i^2 + \sigma_\epsilon^2}. \quad (\text{IA.113})$$

Therefore the slope of the demand curve is:

$$\mathcal{E}_i = -\frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left(\frac{1}{C^2} \sigma_x^{-2} - \hat{\sigma}_i^{-2} \right) \quad (\text{IA.114})$$

$$= -\frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left(\frac{1}{C^2} \sigma_x^{-2} - \sigma_i^{-2} - \frac{1}{C^2} \sigma_x^{-2} \right) \quad (\text{IA.115})$$

$$= \frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \frac{1}{\sigma_i^2}. \quad (\text{IA.116})$$

Here, we observe clearly the intuition for the role of price informativeness. When prices are more informative, low $\hat{\sigma}_i^2$, expected returns respond less to the price, the numerator of the first fraction. However, the perceived risk of the asset also decreases, the denominator of the first fraction. Because of residual uncertainty σ_ϵ^2 , the effect on the asset risk is weaker than the effect on expected returns: the ratio decreases and the trader becomes less aggressive.

More aggregate elasticity leads to more informative prices, so this mechanism will lead to a negative response of individual elasticity to aggregate elasticity. Formally, note that $\int_i \mathcal{E}_i = \mathcal{E}_{agg} = \tilde{C}^{-1}$. Plugging in, we obtain:

$$\mathcal{E}_i = \frac{1}{\rho_i} \frac{1}{1 + \sigma_\epsilon^2 (\sigma_i^{-2} + \mathcal{E}_{agg}^2 \sigma_x^{-2})} \frac{1}{\sigma_i^2} \quad (\text{IA.117})$$

$$= \frac{1}{\rho_i} \frac{1}{\sigma_i^2 + \sigma_\epsilon^2 + \sigma_i^2 \sigma_\epsilon^2 \sigma_x^{-2} \mathcal{E}_{agg}^2} \quad (\text{IA.118})$$

Clearly, the individual elasticity \mathcal{E}_i is decreasing in the aggregate elasticity \mathcal{E}_{agg} . Linearizing this expression, we obtain the counterpart of the degree of strategic response $\chi > 0$:

$$\chi = -\frac{d\mathcal{E}_i}{d\mathcal{E}_{agg}} \quad (\text{IA.119})$$

$$= \frac{1}{\rho_i} \frac{2\sigma_i^2 \sigma_\epsilon^2 \sigma_x^{-2} \mathcal{E}_{agg}}{(\sigma_i^2 + \sigma_\epsilon^2 + \sigma_i^2 \sigma_\epsilon^2 \sigma_x^{-2} \mathcal{E}_{agg}^2)^2}. \quad (\text{IA.120})$$

C.3 Price impact

We now consider a model in the style of Kyle (1989), in which investors have non-negligible price impact and take it into account when making trading decisions. This leads to a foundation for a negative degree of strategic response χ . Intuitively, when other traders are aggressive, I face a very elastic residual supply curve when sending orders to the market. This implies that my trades will not have a large price impact, hence I can also trade aggressively.

C.3.1 Price impact

There are I investors indexed by i . Each agent has CARA preferences with risk aversion ρ_i :

$$U_i = \mathbf{E}_i[-e^{-\rho_i W_i}], \quad (\text{IA.121})$$

and initial wealth W_i . The gross risk-free rate is 1, and the random asset payoff f is distributed $\mathcal{N}(\mu, \sigma^2)$. The asset is in noisy supply $\bar{x} + x$ with \bar{x} an exogenous fixed parameter and $x \sim \mathcal{N}(0, \sigma_x^2)$.

As in Kyle (1989) we are interested in rational expectation equilibria with imperfect competition. We look for a linear pricing rule $p = A + Cx$. We solve for the individual demand strategies and look for linear strategies of the form:

$$d_i = \underline{d}_i - \mathcal{E}_i p \quad (\text{IA.122})$$

C.3.2 Solving for optimal demand strategies

Investor i maximizes their profit taking as given the residual demand from other investors' demand schedule. We use market clearing to find the residual supply curve:

$$\begin{aligned} \sum_i d_i &= \bar{x} + x \\ d_i &= \bar{x} + x - \sum_{k \neq i} \underline{d}_k + \left(\sum_{k \neq i} \mathcal{E}_k \right) p \\ p(d_i) &= \underbrace{\left(\sum_{k \neq i} \mathcal{E}_k \right)^{-1}}_{\lambda_{-i}} d_i + \underbrace{\left(\sum_{k \neq i} \mathcal{E}_k \right)^{-1} \cdot \left(\sum_{k \neq i} \underline{d}_k - \bar{x} - x \right)}_{p_{-i}}. \end{aligned} \quad (\text{IA.123})$$

To find the optimal demand of investor i for the asset, we write their program¹⁰

$$\begin{aligned} \max_d \mathbf{E}\{f - p(d)|p_{-i}\}d - \frac{\rho_i}{2} \text{Var}\{f - p(d)|p_{-i}\}d^2, \\ \max_d (\mu - p_{-i})d - \lambda_{-i}d^2 - \frac{\rho_i}{2}d^2\sigma^2. \end{aligned} \quad (\text{IA.124})$$

The first order condition and replacing $p_{-i} = p - \lambda_{-i}d_i$ gives us:

$$d_i = \frac{\mu - p}{\rho_i \sigma^2 + \lambda_{-i}}. \quad (\text{IA.125})$$

We can already see that stronger λ_{-i} leads to less aggressive trading because of a larger price impact. Remember that λ_{-i} is the aggregate of demand elasticities of other investors, a quantity closely related to aggregate elasticity. We now close the equilibrium to show this relation more clearly.

¹⁰Note that expectation and variances are conditional on the residual demand curve p_{-i} , which is equivalent to conditioning on p .

C.3.3 Solving for aggregate demand elasticity

Given our original demand $d_i = \underline{d}_i - \mathcal{E}_i p$, we are able to identify the linear terms as:

$$\underline{d}_i = \frac{\mu}{\rho_i \sigma^2 + \lambda_{-i}}; \quad \mathcal{E}_i = \frac{1}{\rho_i \sigma^2 + \lambda_{-i}} = \frac{1}{\rho_i \sigma^2 + (\mathcal{E}_{agg} - \mathcal{E}_i)^{-1}}, \quad (\text{IA.126})$$

where we define the aggregate elasticity:

$$\mathcal{E}_{agg} = \sum_i \mathcal{E}_i. \quad (\text{IA.127})$$

Next we show that there is a unique solution for the aggregate elasticity. From the expression in equation (IA.126), we remark that \mathcal{E}_i solves a quadratic equation. We rule out the larger of the two roots and the solution is¹¹

$$\mathcal{E}_i = \frac{1}{2} \left(\frac{2}{\rho_i \sigma^2} + \mathcal{E}_{agg} - \sqrt{\left(\frac{2}{\rho_i \sigma^2} \right)^2 + \mathcal{E}_{agg}^2} \right) \quad (\text{IA.128})$$

To show that there is a unique stable equilibrium we consider the fixed point problem $F(x) = x$, with F defined by:

$$f_i(x) = \frac{1}{2} \left(\frac{2}{\rho_i \sigma^2} + x - \sqrt{\left(\frac{2}{\rho_i \sigma^2} \right)^2 + x^2} \right), \quad (\text{IA.129})$$

$$F(x) = \sum_i f_i(x). \quad (\text{IA.130})$$

The function F is positive, increasing, and concave. Moreover $F(0) = 0$, $F'(0) = I/2$, and $\lim_{x \rightarrow +\infty} F'(x) = 0$, we conclude that there is a unique non-zero solution for \mathcal{E}_{agg} as long as $I \geq 3$.

The relation derived in (IA.128) between \mathcal{E}_i and \mathcal{E}_{agg} is not linear. We can approximate this equation linearly by $\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \mathcal{E}_{agg}$ with

$$\chi = -\frac{1}{2} \left(1 - \frac{\mathcal{E}_{agg}}{\sqrt{\mathcal{E}_{agg}^2 + \left(\frac{2}{\gamma_i \sigma^2} \right)^2}} \right) < 0 \quad (\text{IA.131})$$

This expression gives bounds on the value of χ : $-1/2 \leq \chi < 0$.

C.4 Imperfect information

Assume your optimal elasticity is $\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \mathbf{E}[\mathcal{E}_{agg} | \mathcal{F}_i]$. With perfect knowledge, you obtain: $\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \mathcal{E}_{agg}$. Assume the agent observes a signal $\hat{\mathcal{E}}_{agg} = \mathcal{E}_{agg} + \epsilon$ (with variance σ_ϵ^2) and has a prior $\mathcal{E}_{agg} \sim \mathcal{N}(\bar{\mathcal{E}}, \sigma^2)$. Then we have:

$$\mathbf{E}[\mathcal{E}_{agg} | \hat{\mathcal{E}}_{agg}] = \frac{1}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma^2}} \left(\frac{1}{\sigma_\epsilon^2} \hat{\mathcal{E}}_{agg} + \frac{1}{\sigma^2} \bar{\mathcal{E}} \right). \quad (\text{IA.132})$$

¹¹The larger root is such that $\mathcal{E}_i > \mathcal{E}_{agg}$ which violates $\sum_i \mathcal{E}_i = \mathcal{E}_{agg}$.

Therefore:

$$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \frac{1}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma^2}} \frac{1}{\sigma^2} \bar{\mathcal{E}} - \chi \underbrace{\frac{\frac{1}{\sigma_\epsilon^2}}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma^2}}}_{<1} \hat{\mathcal{E}}_{agg}. \quad (\text{IA.133})$$

C.5 Partial equilibrium thinking

We repeat the calculation of Section 2.3 on the rise in passive investing in a situation with partial equilibrium thinking.

We assume that all investors are homogenous and their initial elasticity is $\mathcal{E}_i = \mathcal{E}_0$. What happens to the economy if a fraction $1 - \alpha$ of these investors become passive? Their elasticity reduces to zero. To model partial equilibrium thinking, we assume that active investors only react to the effect of the switch to passive on aggregate elasticity and do not take into account the collective response of other active investors.

Because investors are infinitesimal, this corresponds to forecasting a change in elasticity of $\Delta \mathcal{E}_{agg}^{\text{forecast}} = -(1 - \alpha)\mathcal{E}_0$. This implies that each active investor changes her elasticity by

$$\Delta \mathcal{E}_i = -\chi \Delta \mathcal{E}_{agg}^{\text{forecast}} = \chi(1 - \alpha)\mathcal{E}_0. \quad (\text{IA.134})$$

Aggregating across all investors, the new aggregate elasticity is:

$$\mathcal{E}_{NEW}^{PET} = \alpha \mathcal{E}_0 (1 + \chi(1 - \alpha)) = \alpha \mathcal{E}_0 + (1 - \alpha)\chi \alpha \mathcal{E}_0. \quad (\text{IA.135})$$

The new elasticity with partial equilibrium thinking is in contrast to our baseline model in Section 2.3, \mathcal{E}_{NEW} :

$$\mathcal{E}_{NEW}^{PET} - \mathcal{E}_{NEW} = (1 - \alpha)\mathcal{E}_0 \alpha \chi \frac{\alpha \chi}{1 + \alpha \chi}. \quad (\text{IA.136})$$

The difference is positive when $\chi > 0$. Because investors do not account for the response of others, they overreact to the initial change in elasticity. This leads to a relatively higher final level of aggregate elasticity and is therefore akin to a larger degree of strategic response. For some parameter values, the response is so strong that it flips the sign of the change in aggregate elasticity.

D Data

D.1 Institutional holdings data

Institutional investment managers with \$100 million or more in assets under their investment discretion are required to disclose their ownership of Section 13(f) securities as of the end of each calendar quarter to the SEC within 45 days after the end of the calendar quarter. The filing requirement applies to both U.S. domestic investment managers, and, under certain conditions regarding the course of their business, foreign investment managers. The official list of 13(f) securities is made available by the SEC shortly after each quarter end. It primarily includes U.S. exchange-traded stocks, shares of closed-end investment companies, and shares of exchange-traded funds.

We obtain data on 13F filings from 2001Q1 until 2017Q4 from Backus et al. (2020). Mirroring their approach, we extend their sample until 2020Q4. To do so we start with a SEC linking table, which provides a list with links to all 13F-HR and 13F-HR/A filings of a given quarter. Based on those links, we scrape all filings, and subsequently parse the filings based on a Perl script generously provided by Backus et al. (2019). The script corrects common filing issues in 13F filings. Finally, we merge the scraped 13F data with CRSP and Compustat.

D.2 CRSP and Compustat

We obtain market capitalization data for stocks from CRSP and apply standard filters: we keep stocks traded on NYSE, NASDAQ and AMEX, and filter to ordinary common shares with share codes 10, 11, 12 and 18 in CRSP. These stocks make up the universe of all assets in our model.

Additional stock-level information comes from quarterly and annual Compustat files. In particular, we closely follow Kojen and Yogo (2019) and their data definitions for building a set of stock characteristics: book equity, investment (defined as growth in total assets), operating profitability (as defined in Fama and French (2015)), and dividend yield as a fraction of book equity. Characteristics are winsorized at the 2.5% and the 97.5% level each quarter. Characteristics that are denominated in dollar values, such as market equity and book equity, are denominated in million dollars.

We match CRSP and Compustat based on the standard linking table on WRDS. Finally, we use CUSIP identifier information from CRSP to merge the CRSP-Compustat merged stock-level data to 13F holdings. We exclude stocks for which institutional ownership is greater than 100% based on the 13F data.

D.3 Measuring passive investing

Passive investors are insensitive to prices. At each date, we identify passive investors as investors with elasticity close to zero in a Kojen and Yogo (2019) type demand system:

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1it} X_k^{(d)} - \mathcal{E}_i^{fixed} p_k + \epsilon_{ik}, \quad (\text{IA.137})$$

where $X^{(d)}$ contains log book equity. An investor is defined as passive if their fixed elasticity is close to zero, i.e. $\mathcal{E}_i^{fixed} < \kappa$, for small κ . We choose $\kappa = 0.06$ to calibrate the level of passive investing, and define a stock's passive share as the ownership-weighted average of an indicator that is 1 if the investor is passive, and zero otherwise. That is,

$$|Active_k| \equiv 1 - \sum_i \underbrace{\frac{w_{ik}A_i}{\exp(p_k)}}_{\text{ownership share}} 1_{\{\mathcal{E}_i^{fixed} < \kappa\}}. \quad (\text{IA.138})$$

Appendix Figure IA.8 shows the the cross-sectional median of $|Active_k|$ over time. We validate our measure in the setting of Russell index switching. Specification (4) of Appendix Table IA.4 shows that our measure of passive investing in the cross-section strongly responds to stocks switching between the Russell 1000 and 2000 indices, by about 4% of total ownership.

D.4 Additional data definitions

There are a number of additional data steps that define the final estimation sample.

Defining the outside asset. For the logit demand system we define any stock with missing stock characteristics or CRSP share code 12 or 18 in a given quarter as part of the outside asset for that particular quarter. Of the remaining stocks, any stock with fewer than 20 investment managers invested in it is also part of the outside asset.

Defining the household sector. Investment managers with fewer than 100 stocks in their portfolio are filtered out, such that their assets are part of the residual household sector. The residual household sector contains direct household holdings, but also an amalgamation of holdings from small investment managers with AUM below the reporting threshold, certain foreign investors, and investment managers with fewer than 100 stocks in their portfolio. As in Kojen and Yogo (2019), this residual household sector is modeled as one investor in the demand system, to ensure that the number of shares held adds up to the number of shares outstanding.

Measuring the investment universe. We define any stock that an investment manager has held over the past three years as part of her investment universe. This follows Kojen and Yogo (2019), who show that the measured investment universe using this approach is very stable over time. The investment universe is primarily used during the estimation procedure to construct our instruments for a stock's market equity as in equation (23), and a stock's aggregate demand elasticity as in equation (24).

Pooling investors during estimation. For our baseline estimation, we pool together investors that hold fewer than 1,000 stocks in a quarter and are classified as active. Investors are grouped based on their assets under management, with the number of groups chosen such that on average, each group holds 2,000 stocks. Specifically, we assume that all

investors within the same group have the same demand parameters, except for the constant of equation (IA.153), which we leave as investor-specific (for example, this absorbs variation in quantity of outside asset w_{i0}). That is, we estimate equation (IA.153) at the group level with an investor fixed effect.

Weighting investors during estimation. For the pooled regression in equation (IA.154), we weight each observation such that each investor-group contributes equally to the regression. That is, each observation receives a weight $1/|\mathcal{K}_i|$. For example, consider a simplified example with two investors and two assets. Investor A holds both assets, while investor B holds only one of the assets. We would assign weight of 0.5 to each position of investor A , and a weight of 1 to the observation for investor B . The estimate of χ is robust to different weighting schemes, as shown in Table 2.

E Identification Strategy

E.1 Moment conditions

We estimate the model using the method of moments. All of the moment conditions derive from the identifying assumption of equation (22). We list these moments here:

$$\mathbf{E} [\epsilon_{jk} \mathbf{1}_{\{j=i\}}] = 0, \forall i \quad (\text{IA.139})$$

$$\mathbf{E} [\epsilon_{jk} X_k^{(d)} \mathbf{1}_{\{j=i\}}] = 0, \forall i \quad (\text{IA.140})$$

$$\mathbf{E} [\epsilon_{jk} \hat{p}_{k,i} \mathbf{1}_{\{j=i\}}] = 0, \forall i \quad (\text{IA.141})$$

$$\mathbf{E} [\epsilon_{jk} X_k^{(e)} \hat{p}_{k,j} \mathbf{1}_{\{j=i\}}] = 0, \forall i \quad (\text{IA.142})$$

$$\mathbf{E} [\epsilon_{jk} \hat{\mathcal{E}}_{agg,k}] = 0 \quad (\text{IA.143})$$

$$\mathbf{E} [\epsilon_{jk} \hat{\mathcal{E}}_{agg,k} \hat{p}_{k,j}] = 0 \quad (\text{IA.144})$$

There are exactly as many moment conditions as model parameters.

E.2 Solving the reflection problem

One challenge for identification is the reflection problem. How can we separate the individual component of demand elasticity from the strategic response to other investors? We show that the presence of variation in investor population across stocks allows to solve this problem. To isolate this argument from other identification concerns, we assume that we observe individual elasticities, \mathcal{E}_{ik} . For exposition purposes, we focus on a simplified version of the model in which $\underline{\mathcal{E}}_i$ does not depend on asset characteristics.

We provide sufficient conditions for the uniqueness of a decomposition of the individual elasticities into investor-specific elasticities $\underline{\mathcal{E}}_i$ and the strategic response controlled by χ . After proving this result, we come back to the economic content and the empirical relevance of these conditions.

Before stating the theorem, we introduce a few notations. We define the undirected graph \mathcal{G} of investor-stock connections. The vertices and the nodes are the investors i and the stocks k . There is an edge between i and k if and only if $i \in I_k$. There are no edges between two investors or two stocks.

Theorem 2. *A decomposition of demand elasticities $\{\mathcal{E}_{ik}\}_{i,k}$ into individual elasticities $\{\mathcal{E}_i\}_i$ and the degree of strategic response χ is unique if:*

- (a) *The graph \mathcal{G} of investor-stock connections is connected.*
- (b) *Position-weighted averages of demand elasticities are not constant across stocks: there exists k and k' such that $\sum_{i \in I_k} w_{ik}/p_k A_i \underline{\mathcal{E}}_i \neq \sum_{i \in I_{k'}} w_{ik'}/p_{k'} A_i \underline{\mathcal{E}}_i$.*

Proof. Let us assume that there exist two distinct decompositions $(\{\underline{\mathcal{E}}_i^{(1)}\}_i, \chi^{(1)}) \neq (\{\underline{\mathcal{E}}_i^{(2)}\}_i, \chi^{(2)})$ and the two conditions (a) and (b) hold. Each decomposition for $l \in \{1, 2\}$

satisfies the two conditions of the elasticity layer

$$\mathcal{E}_{agg,k} = \sum_{i \in I_k} w_{ik} A_i / p_k \mathcal{E}_{ik}, \quad \text{for all } k \in \mathcal{K} \quad (\text{IA.145})$$

$$\mathcal{E}_{ik} = \underline{\mathcal{E}}_i^{(l)} - \chi^{(l)} \mathcal{E}_{agg,k}, \quad \text{for all } k \in \mathcal{K} \text{ and } i \in I_k. \quad (\text{IA.146})$$

We subtract the decomposition of \mathcal{E}_{ik} for $l = 1$ from the decomposition for $l = 2$ and obtain:

$$(\chi^{(2)} - \chi^{(1)}) \mathcal{E}_{agg,k} = \underline{\mathcal{E}}_i^{(2)} - \underline{\mathcal{E}}_i^{(1)}, \quad \text{for all } k \in \mathcal{K} \text{ and } i \in I_k. \quad (\text{IA.147})$$

Here we see immediately that if $\chi^{(1)} = \chi^{(2)}$, then for all i , $\underline{\mathcal{E}}_i^{(1)} = \underline{\mathcal{E}}_i^{(2)}$, thus violating the initial assumption of distinct decompositions. Hence, we focus on the case of $\chi^{(1)} \neq \chi^{(2)}$.

We define the function:

$$f(x) = \begin{cases} (\chi^{(2)} - \chi^{(1)}) \mathcal{E}_{agg,x} & \text{for } x \in \mathcal{K} \\ \underline{\mathcal{E}}_x^{(2)} - \underline{\mathcal{E}}_x^{(1)} & \text{for } x \in I. \end{cases} \quad (\text{IA.148})$$

We restate the equality of equation (IA.147) as:

$$f(x) = f(x'), \quad \text{if and only if there is an edge between } x \text{ and } x' \text{ on } \mathcal{G}. \quad (\text{IA.149})$$

Therefore, since the graph \mathcal{G} is connected: $\forall x, x', f(x) = f(x')$, and f is a constant. We write the constant $f = a$, and plug in the constant in the aggregation of individual elasticities:

$$\mathcal{E}_{agg,k} = \sum_{i \in I_k} w_{ik} A_i / p_k \mathcal{E}_{ik} = \sum_{i \in I_k} w_{ik} A_i / p_k \underline{\mathcal{E}}_i^{(1)} - \chi^{(1)} \sum_{i \in I_k} w_{ik} A_i / p_k \mathcal{E}_{agg,k} \quad (\text{IA.150})$$

$$\iff (1 + \chi^{(1)}) \mathcal{E}_{agg,k} = \sum_{i \in I_k} w_{ik} A_i / p_k \underline{\mathcal{E}}_i^{(1)} \quad (\text{IA.151})$$

$$\iff (1 + \chi^{(1)}) \frac{a}{\chi^{(2)} - \chi^{(1)}} = \sum_{i \in I_k} w_{ik} A_i / p_k \underline{\mathcal{E}}_i^{(1)} \quad \text{for all } k, \quad (\text{IA.152})$$

where we use $\mathcal{E}_{agg,k} = a / (\chi^{(2)} - \chi^{(1)})$. Equation (IA.152) violates assumption (b), which concludes the proof. \blacksquare

The intuition behind theorem 2 is that identification relies on comparing the behavior of one investor for two different stocks with different populations of investors. If this investor trades less aggressively when surrounded by more aggressive investors, we conclude that the degree of strategic response χ is positive. A challenge to implement this comparison is that we already need to know the elasticity of these other investors. This is a chicken-and-egg question. The ability to find a unique solution to this problem relies on being able to cycle through investors with enough variation in composition: this is the essence of conditions (a) and (b).

To better understand why these conditions are important, we show examples of how the model is not identified when either (a) or (b) is violated. Starting with (a), let us consider the case where each stock has its own non-overlapping population of investors. In this case, there is no identification. Because a given investor only invests in one stock, it

is not possible to tell if this investor is aggressive because of her own characteristics or in response to the other investors. As an example that violates condition (b), consider the case in which all investors have the same size and relative portfolio positions such that: $\forall k, k', w_{ik}A_i/p_k = w_{ik'}A_i/p_{k'}$. Investor composition is the same for all stocks and therefore there is no information in comparing different stocks. Relatedly, we could also consider a violation of (b) where all individual elasticities are identical across investors: $\underline{\mathcal{E}}_i = \underline{\mathcal{E}}$. Then, for all k we have $\sum_{i \in I_k} w_{ik}/p_k A_i \underline{\mathcal{E}}_i = \underline{\mathcal{E}}$: the aggregate elasticity for all stocks is identical. Intuitively, even though there is variation in investor composition across stocks, all investors behave the same way in terms of elasticity. This is equivalent to having a single investor, and we cannot separate individual elasticities from the response to other investors.

How can we assess these conditions empirically? The graph \mathcal{G} of investors-stocks connections can be observed directly in our data and we can assess immediately that condition (a) is satisfied using known algorithms such as depth-first-search. Condition (b) is potentially more challenging because it relies on parameter estimates $\underline{\mathcal{E}}_i$. However, inspecting the condition shows it holds generically. Condition (b) stipulates the equality of K linear forms applied to the vector $(\underline{\mathcal{E}}_i)_i$. It is violated if and only if $(\underline{\mathcal{E}}_i)_i \in \bigcap_{k \geq 1} (w_k - w_1)^\perp$, a set of measure 0 for almost all combinations of w_k . In addition, there is still the possibility of verifying whether the condition is satisfied empirically, once the econometrician has found a set of parameter estimates.

E.3 Numerical procedure

We describe our estimation procedure, which solves a series of nested problems.

Step 1. Given a guess for (χ, ξ) and $\{\mathcal{E}_{agg,k}\}_k$, we can estimate all remaining model parameters by two-stage least squares regression investor by investor. This corresponds to estimating the following regression for each investor i :

$$\log \frac{w_{ik}}{w_{i0}} - p_k - \chi \mathcal{E}_{agg,k} p_k - \xi \mathcal{E}_{agg,k} = \underline{d}_{0i} + \underline{d}'_{1it} X_k^{(d)} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} \right) p_k + \epsilon_{ik}, \quad (\text{IA.153})$$

where p_k and $X_k^{(e)} p_k$ are instrumented by $\hat{p}_{k,i}$ and $X_k^{(e)} \hat{p}_{k,i}$. Estimating these regressions is equivalent to solving the moment conditions (IA.139) to (IA.142).

Step 2. Given a guess for (χ, ξ) , we look for equilibrium values of $\{\mathcal{E}_{agg,k}\}_k$. We start from the aggregate elasticities implied by the model of Kojen and Yogo (2019). We run step 1 above. With the newly estimated $\underline{\mathcal{E}}_i$ and the parameter χ , we solve explicitly for the equilibrium elasticity they imply by solving the linear system of equations (14) and (18). We update our guessed aggregate elasticity by taking a weighted average of the previous iteration and these new implied values with weights of 75% and 25%. We repeat this updating process until the values of $\{\mathcal{E}_{agg,k}\}_k$ converge. This step ensures that our estimated model satisfies the 2-layer equilibrium.

Step 3. We estimate χ and ξ . We start from a guess for (χ, ξ) and run step 2 to find the aggregate elasticities it implies. With these values, we estimate the pooled regression of

equation (20):

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1it} X_k^{(d)} + \xi \mathcal{E}_{agg,k} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi \mathcal{E}_{agg,k} \right) p_k + \epsilon_{ik}, \quad (\text{IA.154})$$

using two-stage least squares with all the instruments of the investor-level regression, $\hat{\mathcal{E}}_{agg,k}$ and $\hat{\mathcal{E}}_{agg,k} \hat{p}_{k,i}$. This is a very large scale regression with many fixed effects and investor-specific coefficients. We speed up the estimation of this large-scale regression tremendously by taking advantage of the Frisch-Waugh-Lovell theorem. We absorb all individual-level variables using investor-specific regressions, and are left with only the coefficients χ and ξ to estimate in the pooled data.

Define as $f(\cdot)$ the function that maps the guess (χ, ξ) to estimates for χ and ξ in the pooled regression, and define the fixed point function $F(\chi, \xi) \equiv f(\chi, \xi) - (\chi, \xi)$ as the difference between the estimates from the pooled regression and the guess.

Step 4. The pooled regression gives $F(\chi, \xi)$. We use a bivariate quasi-Newton method, in which we approximate the Jacobian of F numerically via finite differences by varying χ and ξ by a small ϵ , to find a root of $F(\chi, \xi)$, which constitutes a fixed point for (χ, ξ) . With such a fixed point, we are sure that our estimates satisfy simultaneously all the moment conditions of Appendix Section E.1 and the 2-layer equilibrium.

Algorithm E.1 summarizes the numerical procedure to obtain a fixed point for (χ, ξ) in pseudo-code.

Algorithm E.1: Numerical procedure solving for a fixed point of (χ, ξ) .

```

1  begin
2    Initialize starting values  $(\chi^{(0)}, \xi^{(0)})$ 
3     $h \leftarrow 0$ 
4    while  $(\|F(\chi^{(h-1)}, \xi^{(h-1)})\| > \text{tol})$  or  $(h = 0)$ 
5      Initialize  $\{\mathcal{E}_{agg,k}^{(0)}\}_k$  at  $\{\mathcal{E}_{fixed,k}\}_k$ 
6      for  $n$  in  $1:N$ 
7        Update investor-specific parameters conditional on  $\{\mathcal{E}_{agg,k}^{(n-1)}\}_k$  and  $(\chi^{(h)}, \xi^{(h)})$  (Step 1).
8        Aggregate to determine  $\{\mathcal{E}_{agg,k}^{(n)}\}_k$  conditional on  $(\chi^{(h)}, \xi^{(h)})$  (Step 2).
9      end
10     Determine  $f(\chi^{(h)}, \xi^{(h)})$ , i.e. estimate  $(\chi, \xi)$  conditional on  $\{\mathcal{E}_{agg,k}^{(N)}\}_k$  (Step 3).
11      $F(\chi^{(h)}, \xi^{(h)}) \leftarrow f(\chi^{(h)}, \xi^{(h)}) - (\chi^{(h)}, \xi^{(h)})$ 
12      $\hat{J}(\chi^{(h)}, \xi^{(h)}) \leftarrow \frac{1}{\epsilon} (F(\chi^{(h)} + \epsilon, \xi^{(h)}) - F(\chi^{(h)}, \xi^{(h)}), F(\chi^{(h)}, \xi^{(h)} + \epsilon) - F(\chi^{(h)}, \xi^{(h)}))$ 
13      $(\chi^{(h+1)}, \xi^{(h+1)}) \leftarrow (\chi^{(h)}, \xi^{(h)}) - \hat{J}^{-1}(\chi^{(h)}, \xi^{(h)}) F(\chi^{(h)}, \xi^{(h)})$  (Step 4)
14      $h \leftarrow h + 1$ 
15   end
16   return  $(\chi^{(h)}, \xi^{(h)})$ 
17 end

```

Lines 2 and 3 initialize the numerical procedure. Starting values $(\chi^{(0)}, \xi^{(0)})$ are based on past experience with the algorithm as the Newton method may fail for starting values too far removed from a root of F . Line 4 starts a while loop that ends when a solution is found, i.e. when the norm of F is below some small tolerance level. Lines 5 to 9 solve for an elasticity equilibrium conditional on the current iteration $(\chi^{(h)}, \xi^{(h)})$: First, we initialize aggregate elasticities based on a fixed elasticity model (IA.137). Then, we iterate back and

forth between estimating investor-specific parameters conditional on aggregate elasticities and aggregating individual elasticities until an equilibrium is found. In line 10 we estimate the pooled regression of equation (IA.154) conditional on $\{\mathcal{E}_{agg,k}^{(N)}\}_k$. Line 11 updates the fixed point function. Line 12 approximates the Jacobian of F at $(\chi^{(h)}, \xi^{(h)})$ via a finite difference approach. Line 13 updates χ and ξ via a Newton step, and line 14 increases the iterator.

F Trading Big and Small Stocks

We investigate whether firms trade big and small stocks differently. Our estimates of elasticities by stocks suggest that the demand for large stocks is more inelastic (see Figure 3). To explain this result, one hypothesis is that large stocks mechanically tend to receive a high portfolio weight and that investors are unwilling to adjust their largest positions. For example, a 10% relative increase in portfolio weight would create much larger tracking error to the index for large positions than for small positions. Also, the granular nature of large stocks imply that they have fewer substitutes.

To complement our structural results and investigate this hypothesis, we compare the trading activity of investors across the distribution of their portfolio. For a given investor-quarter, we compute for each stock the squared relative change in the number of shares:

$$\text{Trading Activity}_{i,k,t} = \left[\left(\frac{A_{i,t}w_{ik,t}}{p_{k,t}} - \frac{A_{i,t-1}w_{ik,t-1}}{p_{k,t-1}} \right) / \frac{A_{i,t}w_{ik,t}}{p_{k,t}} \right]^2 \quad (\text{IA.155})$$

We sort positions by portfolio weights, and compute the ratio of the cumulative sum of trading activity to the total sum. This gives us a relation between the percentile of portfolio weight and the cumulative share of total trading activity. We average this relation within size groups of investors and present our results in Figure IA.1 for various dates.

If trading activity is as intense for all portfolio weights, this curve should coincide with the 45-degree line. Instead, we see that the curve is always above the 45-degree line and particularly flat along the largest investor positions. This implies that there is relatively less trading activity for the largest stocks. In addition, we observe that this pattern is more pronounced for the largest investors (panel D) than for small investors (panel A). Because larger investors are more important for the biggest stocks, this will amplify the lack of trading activity for the biggest stocks.

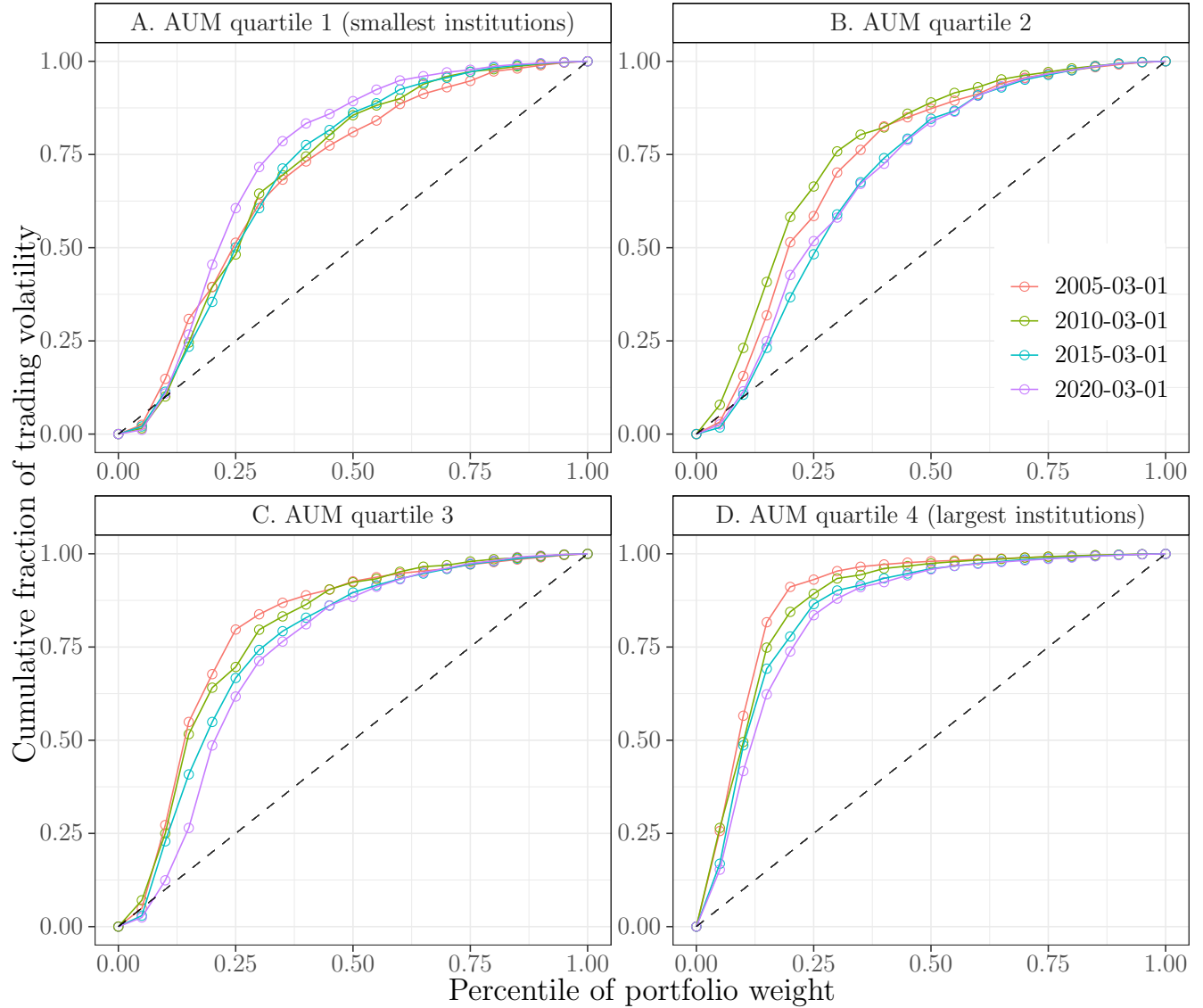


Figure IA.1. Trading activity across portfolio positions. Figure IA.1 presents the cumulative share of trading activity (defined in equation (IA.155)) by quantiles of investor portfolio weights. We aggregate the statistics by date and quartiles of assets under management.

G Appendix Tables

Table IA.1. Estimates of the degree of strategic response χ under alternative specifications starting 2003Q3

	Estimates for χ		
	Median	25th pct.	75th pct.
(1) Baseline Specification	2.21	1.85	2.75
(2) BE-weighted Instrument for \mathcal{E}_{agg}	1.91	1.54	2.29
(3) Additional Controls	2.49	2.09	3.28
(4) AUM-weighted Regression	2.3	1.86	2.77
(5) Book-weighted Regression	2.24	1.77	2.68
(6) Investor-Type Grouping	2.46	1.96	2.93
(7) Constant χ	2.07		
(8) No Instrument for \mathcal{E}_{agg}	1.27	0.88	1.59
(9) No Instruments	1.01	0.73	1.41

Table IA.1 presents statistics of estimates of χ across dates (2003Q3–2020Q4) under various specifications. Our baseline specification (1) estimates χ given aggregate elasticities $\mathcal{E}_{agg,k}$ each period via the regression:

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1i} X_k^{(d)} + \xi \mathcal{E}_{agg,k} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi \mathcal{E}_{agg,k} \right) p_k + \epsilon_{ik},$$

where $X_k^{(d)}$ contains log book equity and log book equity squared. $X_k^{(e)}$ is log book equity. Active investors with fewer than 1,000 stock holdings are grouped together based on their assets under management such that each group on average contains 2,000 stock holdings. The regression is weighted such that each group's weights sum to the same constant. Specification (2) shows estimates of χ based on the book-equity weighted instrument. Specification (3) adds additional characteristics to $X_k^{(d)}$; profitability, investment and dividends relative to book equity. Specification (4) value-weights the regression by weighting investors by their AUM. Specification (5) similarly value-weights the regression by weighting investors by their book assets. Specification (6) groups investors both by investor type and AUM. Institutional investors whose type we cannot determine are bundled together in a separate group. Specification (7) imposes for χ to be constant across time in the estimation, with each year receiving equal weight. Specification (8) reports results without instrumenting for the aggregate elasticity \mathcal{E}_{agg} . Specification (9) additionally removes the instrument for prices.

Table IA.2. Summary statistics of aggregate elasticity \mathcal{E}_{agg} with book-equity weighted instrument

	Panel A: Statistics of average elasticity across stocks			
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	0.504	0.428	0.482	0.569
Fixed elasticity	0.389	0.357	0.389	0.442
	Panel B: Regression coefficient (by dates) of elasticity on size			
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	−0.111	−0.12	−0.104	−0.091
Fixed elasticity	−0.0286	−0.0309	−0.0272	−0.0249
	Panel C: Residual cross-sectional standard deviation of elasticity			
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	0.0548	0.0438	0.05	0.0591
Fixed elasticity	0.0842	0.0739	0.0826	0.0917

Table IA.2 presents statistics of the aggregate elasticity $\mathcal{E}_{agg,k,t}$ using the book-equity weighted instrument. We estimate the elasticities in our baseline model and in a specification with fixed elasticities ($\chi = 0$ as in Kojen and Yogo (2019)). Panel A has summary statistics of the average elasticity by date. Panel B shows summary statistics of the coefficient β_t from the regression $\mathcal{E}_{agg,k,t} = \alpha_t + \beta_t p_{k,t} + \varepsilon_{k,t}$ by date. Panel C reports summary statistics of the cross-sectional standard deviation of the residual from the regression described in Panel B. The sample period is 2001–2020.

Table IA.3. Change in aggregate stock-level elasticity $\mathcal{E}_{agg,k}$ on the active share using estimates from the model with a constant χ

	Change in Elasticity				
	(1)	(2)	(3)	(4)	(5)
Change in Active share	0.321*** (0.058)	0.462*** (0.028)	0.441*** (0.026)	0.412*** (0.025)	0.428*** (0.049)
Date Fixed Effects		Yes	Yes	Yes	Yes
Stock Fixed Effects			Yes		
Controls				Yes	Yes
Estimator	OLS	OLS	OLS	OLS	IV
N	50,292	50,292	49,661	50,292	10,619
R^2	0.017	0.269	0.282	0.318	0.627
First-stage F statistic					9.444
First-stage p value					0.000

Table IA.3 reports a panel regression of annual log change in stock level elasticity $\mathcal{E}_{agg,k}$ on the annual log change in the active share $|Active_k|$. We use the estimates from the model with a constant value of χ over time. Column 2 adds date fixed effects. Column 3 adds stock fixed effects. Column 4 uses date fixed effects and controls for lagged book equity and annual log changes of log book equity. Column 5 instruments the log change in the active share $|Active_k|$ between Q1 and Q2 in any given year by two indicator variables corresponding to stocks switching between Russell 1000 and 2000 in either direction. In this column, the sample is restricted to stocks with CRSP market capitalization ranked between 500 to 1500 as of the end of Q1. The sample period is 2001–2020 for columns 1-4, and 2007–2020 for column 5. Standard errors are 2-way clustered by date and stock for columns 1-4, and clustered by date for column 5.

Table IA.4. Change in aggregate stock-level elasticity $\mathcal{E}_{agg,k}$ on the active share around Russell index reconstitution

	Log Change in Elasticity			Log Change in Active share
	(1)	(2)	(3)	(4)
Log Change in Active share	0.389*** (0.066)	0.431*** (0.021)		
Log Change in Book Equity	-1.158*** (0.049)	-1.159*** (0.049)	-1.151*** (0.050)	0.018 (0.018)
Lagged Book Equity	0.005 (0.007)	0.005 (0.007)	0.007 (0.007)	0.003 (0.002)
Switch from Russell 2000 to 1000			0.017*** (0.004)	0.035*** (0.008)
Switch from Russell 1000 to 2000			-0.011 (0.006)	-0.043*** (0.012)
Date Fixed Effects	Yes	Yes	Yes	Yes
Estimator	IV	OLS	OLS	OLS
N	10,619	10,619	10,619	10,619
R^2	0.748	0.748	0.670	0.090
First-stage F statistic	9.444			
First-stage p value	0.000			

Table IA.4 reports a panel regression of log change in stock level elasticity $\mathcal{E}_{agg,k}$ on the log change in the active share $|Active_k|$ around Russell index reconstitution, in particular between end of Q2 and Q1, from 2007 to 2020. The sample is restricted to stocks with CRSP market capitalization ranked between 500 to 1500 as of the end of Q1. The active share $|Active_k|$ is instrumented by two indicator variables corresponding to stocks switching between Russell 1000 and 2000 in either direction. We use the estimates for $\mathcal{E}_{agg,k}$ from the model with a variable value of χ over time. Columns 2 and 3 show the corresponding OLS and reduced-form regressions, respectively. Column 4 is the first-stage regression. Standard errors are clustered by date.

H Appendix Figures

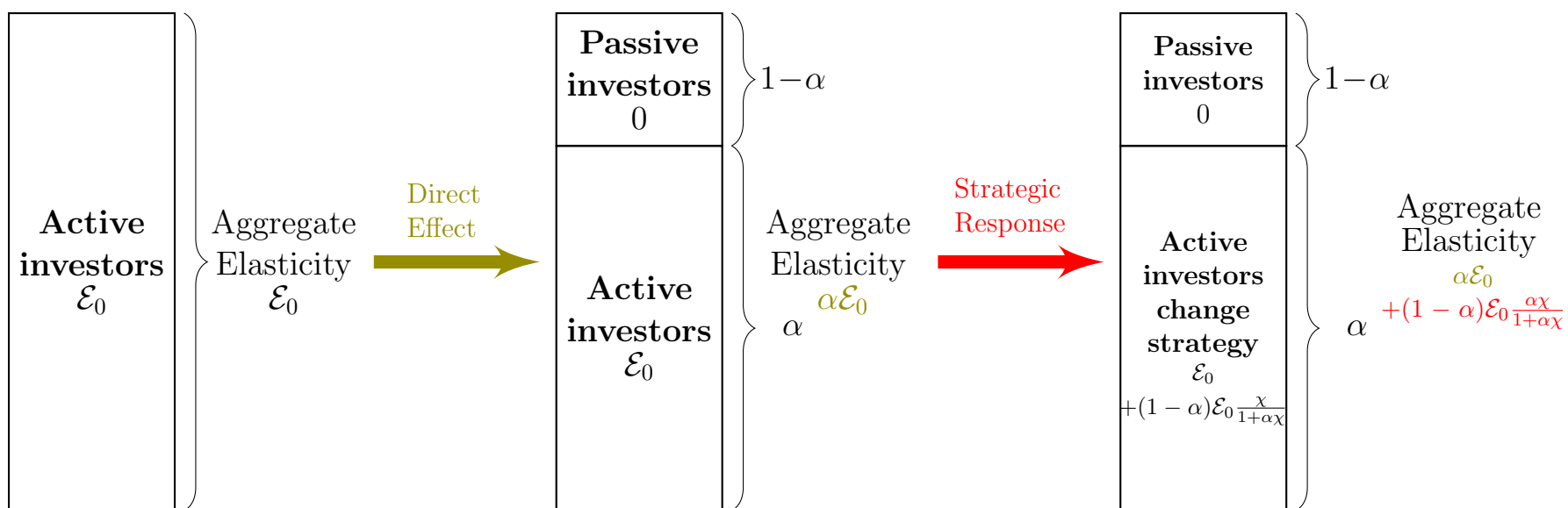


Figure IA.2. Effect of an increase in passive investing.

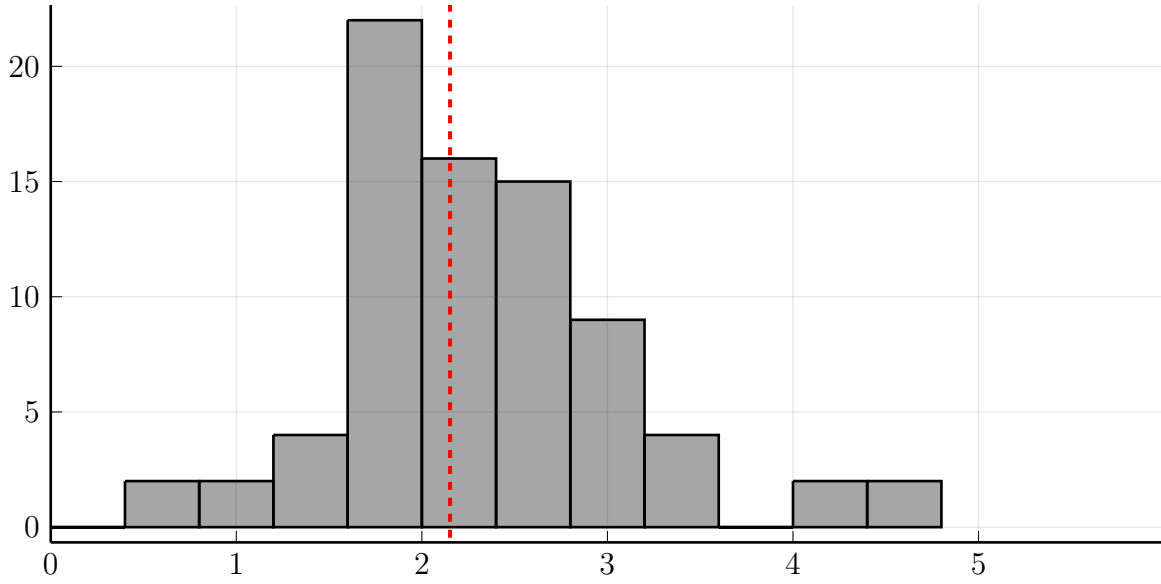


Figure IA.3. Distribution of the degree of strategic response χ across dates. Figure IA.3 presents an histogram of our estimates of the degree of strategic response χ for each date between 2001 and 2020. The median estimate over the time-period is $\chi = 2.15$ (dashed red line).

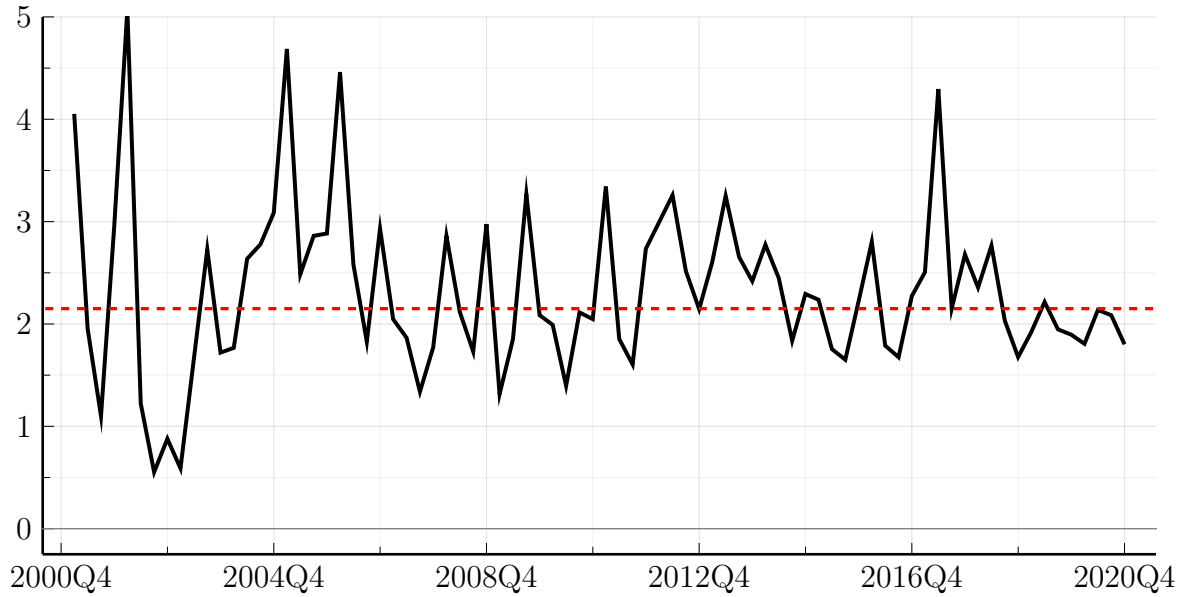


Figure IA.4. Time-series of the degree of strategic response χ . Figure IA.4 shows the time-series of the estimates for the degree of strategic response χ . The median estimate over the time-period is $\chi = 2.15$ (dashed red line).

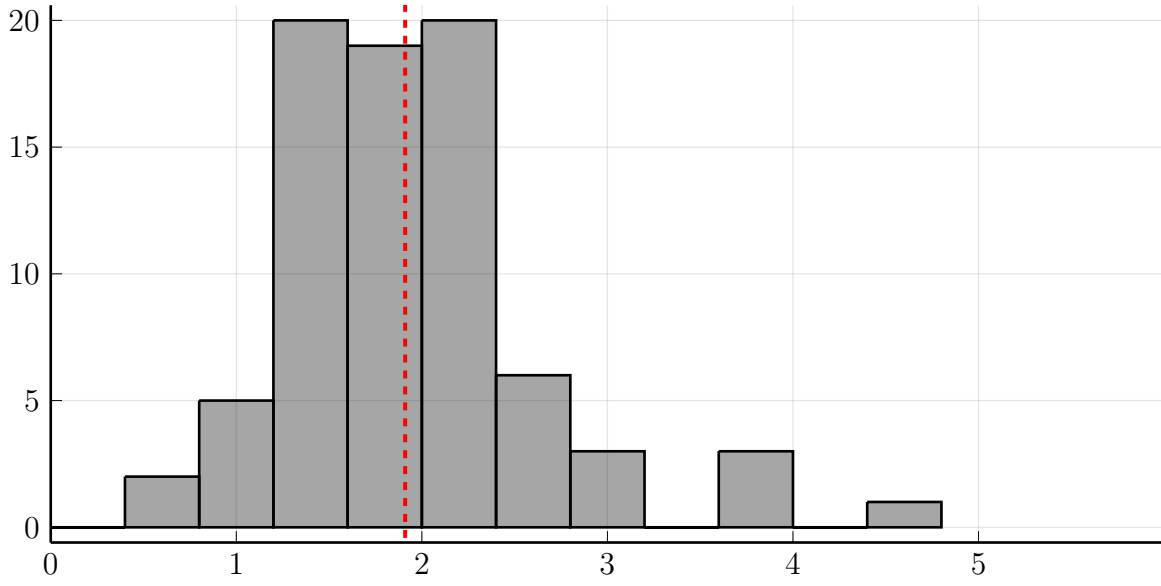


Figure IA.5. Estimates of χ using the book-equity weighted instrument. Figure IA.5 presents an histogram of our estimates of the degree of strategic response χ where the instrument for aggregate elasticity weights portfolios by book equity, for each date between 2001 and 2020. The median estimate over the time-period is $\chi = 1.91$ (dashed red line).

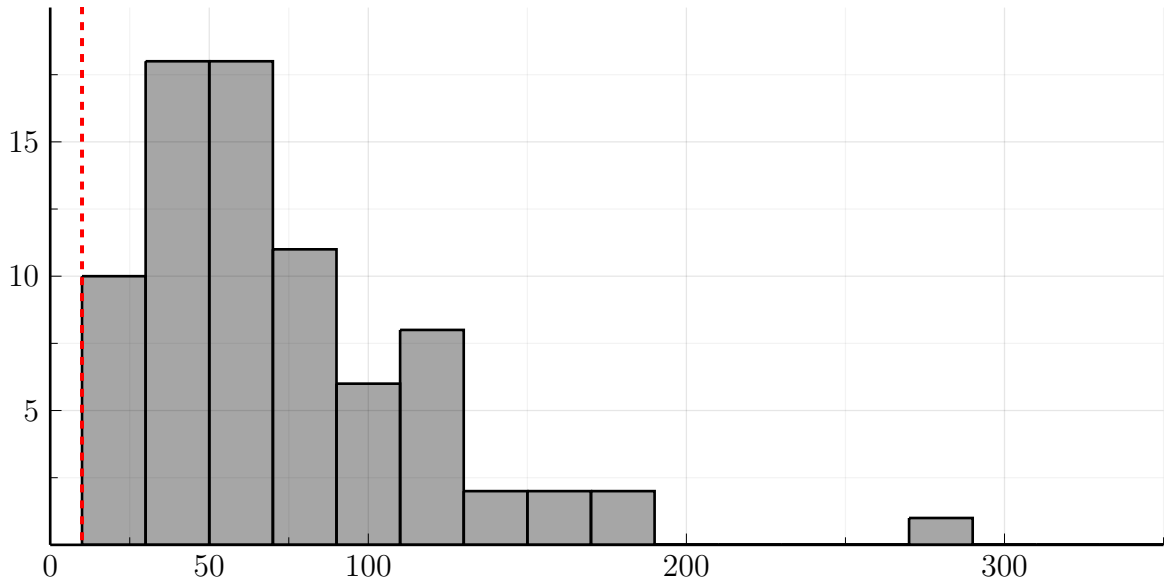


Figure IA.6. Relevance condition for the the book-equity weighted elasticity instrument. Figure IA.6 shows the first-stage F-statistics (Kleibergen-Paap) for the instrument of price times the aggregate elasticity when the instrument of the aggregate elasticity is weighted by book equity. The dashed red line is the critical value of 10.

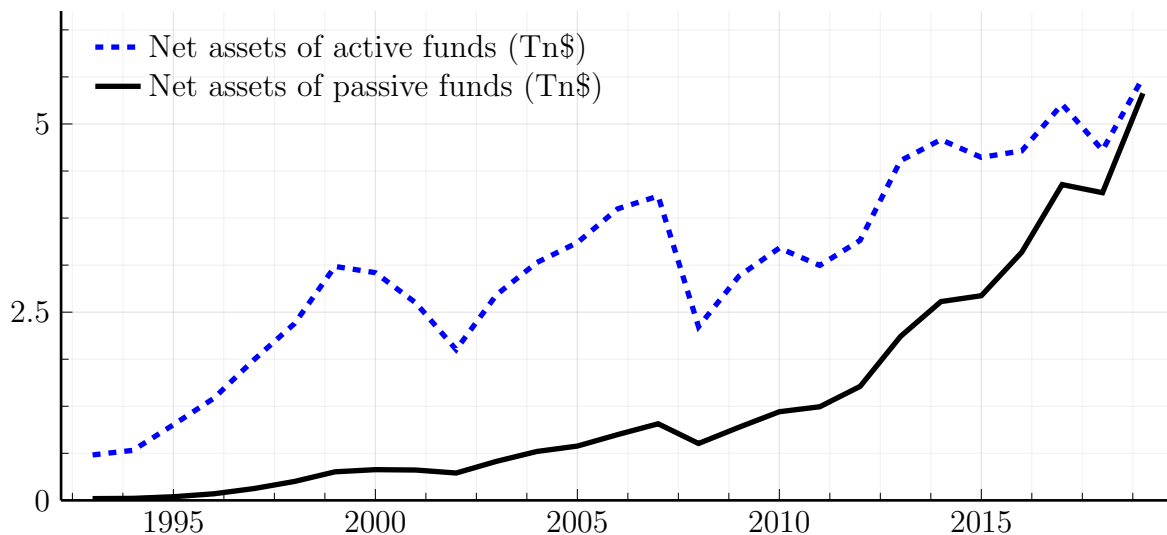


Figure IA.7. Net assets of passive and active funds. Figure IA.7 shows the net assets of domestic mutual funds and ETFs in trillions of dollars (year-end) for passive funds (black solid line) and active funds (blue dashed line). Source: *Investment Company Fact Book – A Review of Trends and Activities in the Investment Company Industry* (2020).

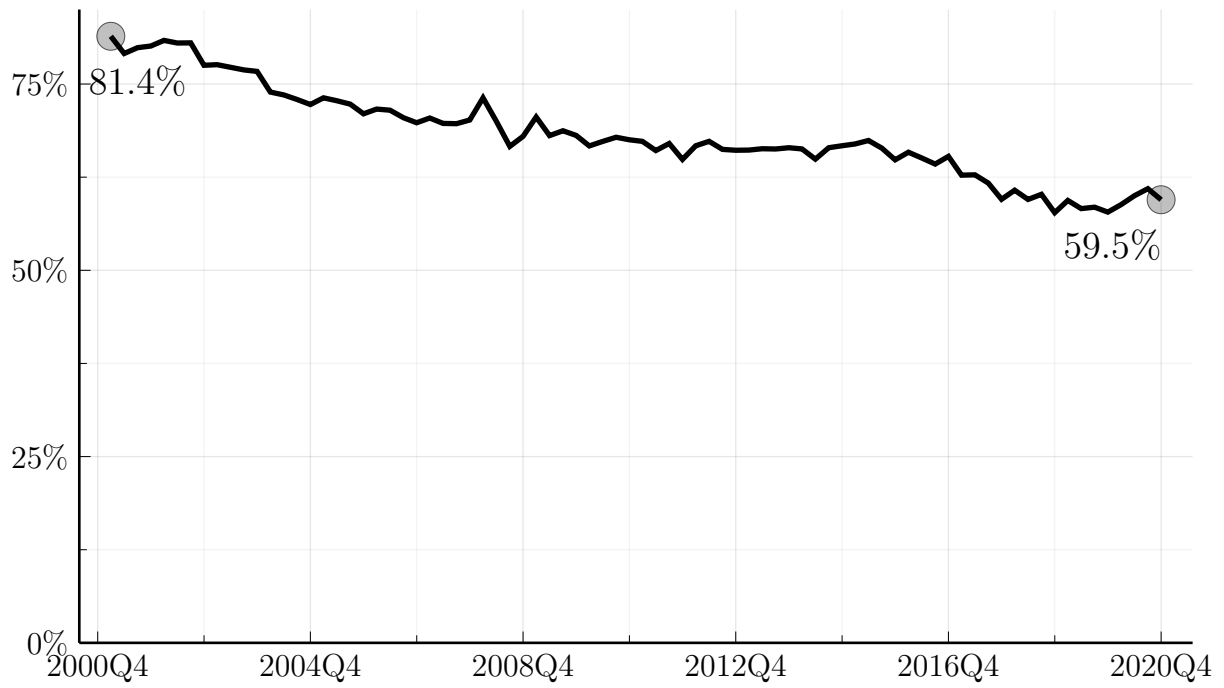


Figure IA.8. Fraction of active investors. Figure IA.8 reports the fraction of active investors according to our model. For each stock, we compute the ratio of total position of active investors and the market capitalization. We report the median across stocks.

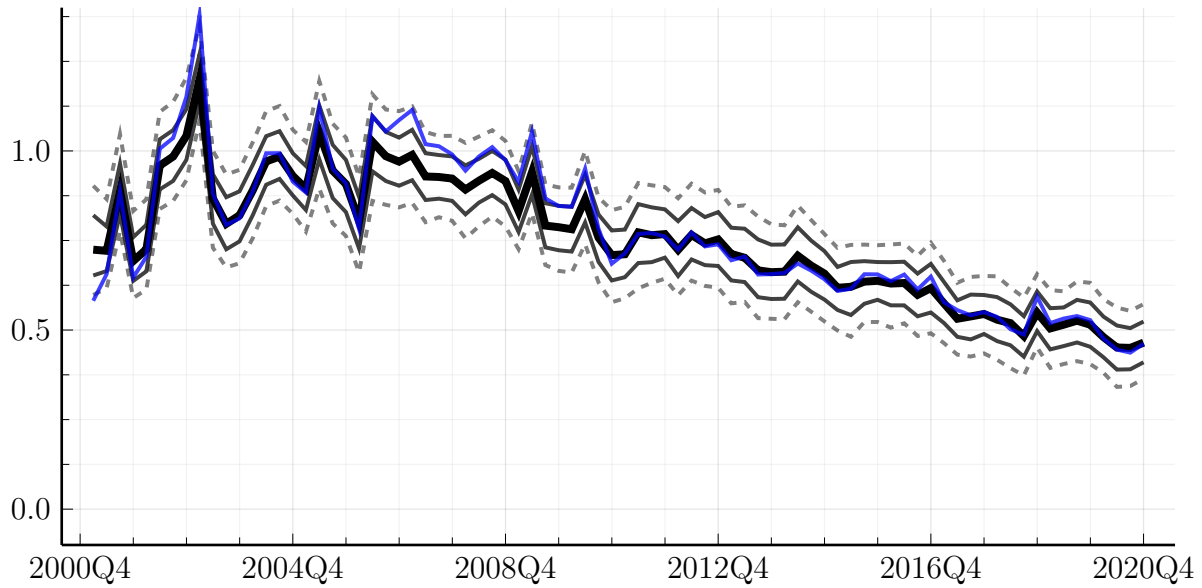


Figure IA.9. Distribution of individual-specific elasticities $\underline{\mathcal{E}}_{ik}$. Figure IA.9 shows the quantiles of the distribution of individual elasticities $\underline{\mathcal{E}}_{ik}$ across investors for each stock and each date. We average the quantiles for each date to plot their time series. The black bold line is the average across investors. The two thin grey lines represent the 25th and 75th percentiles. The two dashed grey lines represent the 10th and 90th percentiles. And the solid blue line represents the average individual elasticities of the household investor.

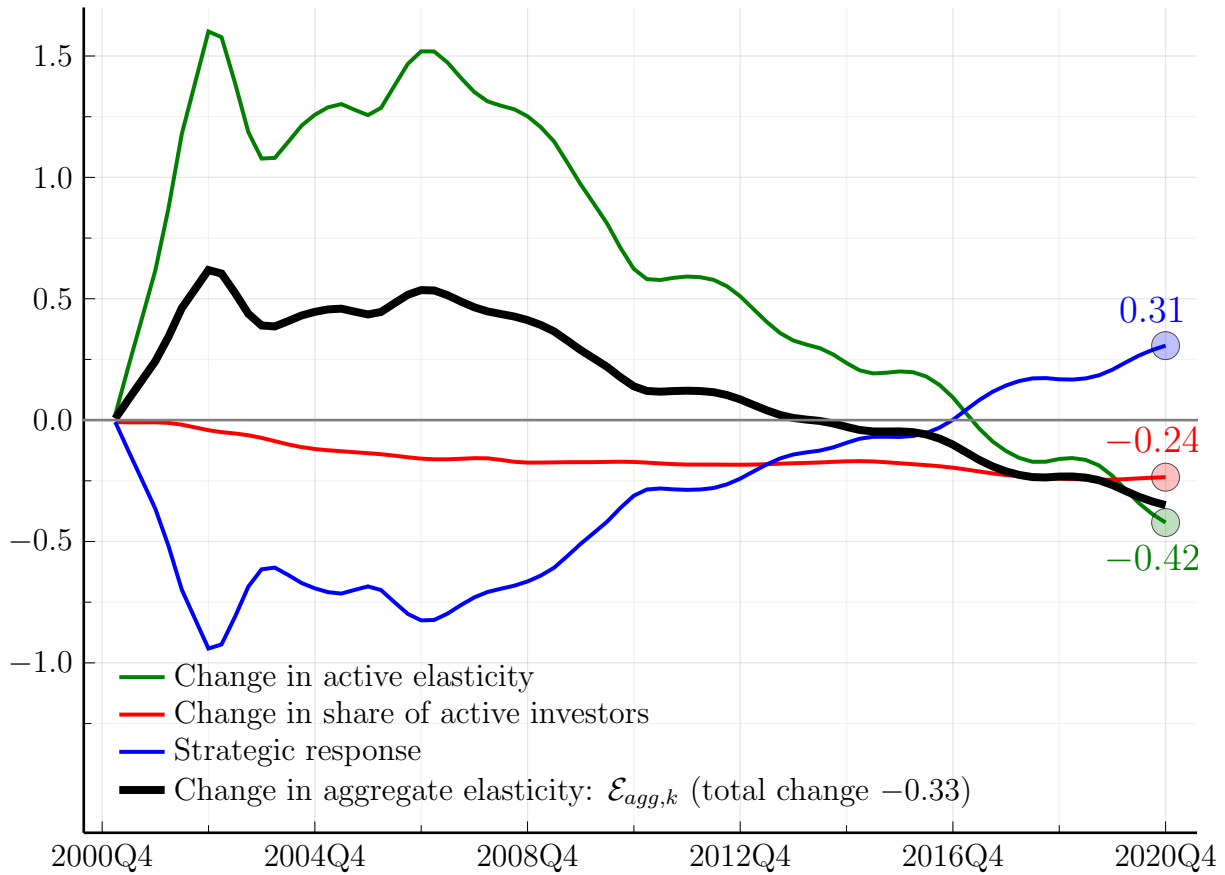


Figure IA.10. Decomposition of the change in aggregate elasticity for the the book-equity weighted elasticity instrument. Figure IA.10 shows the decomposition derived in equation (28) over time, based on elasticities estimated from the book-equity weighted instrument for $\mathcal{E}_{agg,k}$. We compute each term of the decomposition for each date and accumulate the changes over time.

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