Inelastic Capital

Matthieu Gomez   Émilien Gouin-Bonenfant

October, 2025

Q. How does the economy absorb a shock in the demand for savings?

(a) Increase in the quantity of capital

(b) Increase in the price of capital

Q. How does the economy absorb a shock in the demand for savings?

(a) Increase in the quantity of capital

(b) Increase in the price of capital

- In neoclassical growth model, output can be seamlessly transformed into capital
  - $\rightarrow$ saving shocks are entirely absorbed through higher capital quantity

Q. How does the economy absorb a shock in the demand for savings?

(a) Increase in the quantity of capital

(b) Increase in the price of capital

- In neoclassical growth model, output can be seamlessly transformed into capital
  - $\rightarrow$ saving shocks are entirely absorbed through higher capital quantity
- Yet, in reality, secular decline in r has led to large increases in asset valuations
  - $\rightarrow$ flipside of "missing investment" puzzle

Q: What is the effect of decline in interest rate (or capital tax τ) on price vs. quantity?

Q: What is the effect of decline in interest rate (or capital tax τ) on price vs. quantity?

	$\frac{d \log \text{Capital price}}{d \log(1-\tau)}$	$\frac{d \log \text{Capital Quantity}}{d \log(1-\tau)}$
Baseline macro model (NGM)	0	1/Labor share
Baseline finance model (Lucas tree)	1	0
Reality	?	?

→ Response depends on horizon and type of capital (machines vs. ideas vs. land)

→ Key statistic for the redistributive effect of saving shocks, capital taxes...

Roadmap

Theoretical framework:

- $\rightarrow$ Analytical formulas for response of capital prices to permanent discount rate shocks
- $\rightarrow$ Formulas depend on moments on production network + structural parameters

Measurement: Measuring expenditure shares across sectors

- $\rightarrow$ Investment sector is more labor intensive
- $\rightarrow$ Production of intangible capital particularly reliant on high-skilled workers

Quantitative model: Calibrated using input-output + labor supply elasticities

- $\rightarrow$ Capital prices absorb 85% of permanent tax (or discount rate) decline on impact
- $\rightarrow$ Number drops to 50% in the long-run
- $\rightarrow$ Heterogeneous effects for tangible vs. intangible vs. land
- $\rightarrow$ Use model to quantify welfare effect of saving shocks & capital taxes

Analytical results

- Whether discount rate shocks affect capital qty vs. prices depends on technology
- To see why, compare two economies:

  Economy 1: output can be seamlessly converted to capital

  Economy 2: capital is fixed (land)

- Goal: express price elasticity of capital to parameters of the input-output network

Two-sector model

- Two factors of production (K and L), two sectors (C and I):

$$
\begin{aligned}
C &= F_C(K_C, L_C) && \text{(consumption good sector)} \\
I &= F_I(K_I, L_I) && \text{(investment good sector)} \\
K &= K_C + K_I && \text{(capital allocation)} \\
L &= L_C + L_I && \text{(labor allocation)} \\
\dot{K} &= I - \delta K && \text{(capital accumulation)}
\end{aligned}
$$

where F_C and F_I are homogeneous functions of degree one.

Stylized model: Demand side

- Denote r interest rate and τ capital tax (taken as exogenous)
- SS capital valuation eq. implies capital price p_K increasing fction of rental rate r_K

$$(r+\delta)p_K = (1-\tau)r_K$$
$$\implies p_K = \frac{(1-\tau)r_K}{r+\delta}$$

Stylized model: Supply side

- Cost minimization in the consumption and investment sector:

$$0 = \alpha_C \,\mathrm{d}\log r_K + (1 - \alpha_C)\,\mathrm{d}\log w$$
$$\mathrm{d}\log p_K = \alpha_I \,\mathrm{d}\log r_K + (1 - \alpha_I)\,\mathrm{d}\log w$$

  where α_C, α_I are cost shares of each sector on capital services

- Combining gives capital price as fction of rental rate r_K

$$\mathrm{d}\log p_K = -\frac{\alpha_C - \alpha_I}{1 - \alpha_C}\,\mathrm{d}\log r_K$$

  - In the special case $\alpha_C = \alpha_I$, this implies $\mathrm{d}\log p_K = 0$.
  - In the relevant case $\alpha_C > \alpha_I$, this implies $\mathrm{d}\log p_K$ decreases in $\mathrm{d}\log r_K$

Combining long-run supply and demand sides implies

$$\mathrm{d}\log p_K = \frac{\alpha_C - \alpha_I}{1 - \alpha_I}\,\mathrm{d}\log(1-\tau)$$

Combining long-run supply and demand sides implies

$$d\log p_K = \frac{\alpha_C - \alpha_I}{1-\alpha_I}\, d\log(1-\tau)$$

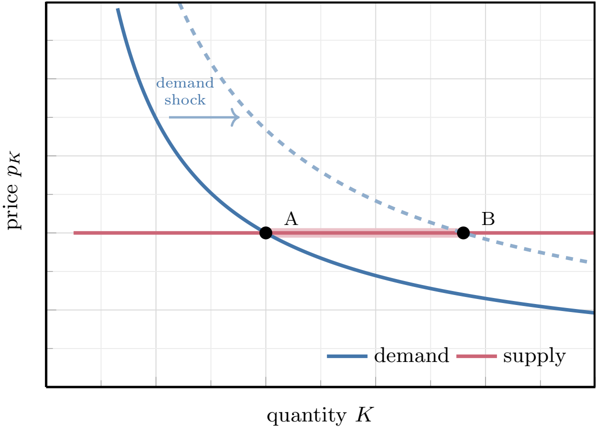


Figure 1: Special case $\alpha_C = \alpha_I$

Combining long-run supply and demand sides implies

$$\mathrm{d}\log p_K = -\frac{\alpha_C - \alpha_I}{1-\alpha_I}\,\mathrm{d}\log(1-\tau)$$

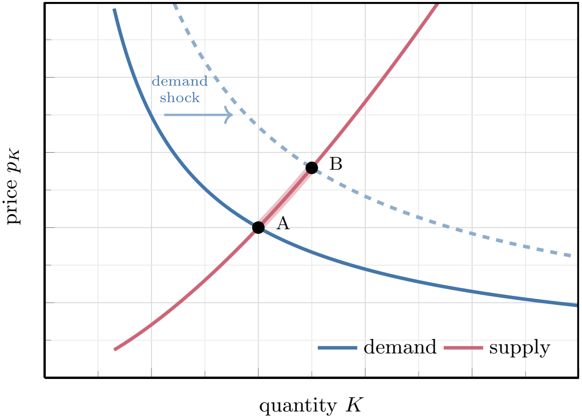


Figure 2: Empirically relevant case $\alpha_C > \alpha_I$

General model

- We now have arbitrary number of fixed factors $\{L_f\}_1^F$ (labor types, land, DRS...)

$$
\begin{aligned}
C &= F_C\left(K_C, L_C\right) && \text{(consumption good sector)}\\
I &= F_I\left(K_I, L_F\right) && \text{(investment good sector)}\\
K &= K_C + K_I && \text{(capital allocation)}\\
1 &= L_{C,f} + L_{I,f} \quad \forall f && \text{(labor allocation)}\\
K' &= (1-\delta)K + I && \text{(capital accumulation)}
\end{aligned}
$$

- Imperfect sectoral labor allocation

$$\frac{L_{I,f}}{L_{C,f}} = \frac{\mu}{1-\mu}\left(\frac{w_{If}}{w_{Cf}}\right)^{\phi}$$

  - Case $\phi = \infty$ corresponds to model above
  - Case $\phi = 0$ corresponds to sector-specific labor (Jones 1971)

Multiple factors

- Cost minimization in each sector

$$
\begin{aligned}
0 &= \alpha_C \, \mathrm{d} \log r_K + (1 - \alpha_C) \, \mathrm{d} \log w_C \\
\mathrm{d} \log p_K &= \alpha_I \, \mathrm{d} \log r_K + (1 - \alpha_I) \, \mathrm{d} \log w_I
\end{aligned}
$$

Multiple factors

- Cost minimization in each sector

$$
\begin{aligned}
0 &= \alpha_C \,\mathrm{d}\log r_K + (1-\alpha_C)\,\mathrm{d}\log w_C \\
\mathrm{d}\log p_K &= \alpha_I \,\mathrm{d}\log r_K + (1-\alpha_I)\,\mathrm{d}\log w_I
\end{aligned}
$$

- This implies, in the long-run

$$
\mathrm{d}\log p_K = \frac{\alpha_C - \alpha_I}{1-\alpha_I} \cdot \mathrm{d}\log(1-\tau) + (1-\alpha_C) \cdot \mathrm{d}\log\left(\frac{w_I}{w_C}\right)
$$

Multiple factors

- Cost minimization in each sector

$$
\begin{aligned}
0 &= \alpha_C \, \mathrm{d}\log r_K + (1-\alpha_C)\, \mathrm{d}\log w_C \\
\mathrm{d}\log p_K &= \alpha_I \, \mathrm{d}\log r_K + (1-\alpha_I)\, \mathrm{d}\log w_I
\end{aligned}
$$

- This implies, in the long-run

$$
\begin{aligned}
\mathrm{d}\log p_K =& \frac{\alpha_C - \alpha_I}{1-\alpha_I}\, \mathrm{d}\log(1-\tau) \\
&+ (1-\alpha_C) \cdot \mathrm{Cov}_f\left(\omega_{I,f} - \omega_{C,f}, \mathrm{d}\log w_f\right) \\
&+ (1-\alpha_C) \cdot \mathrm{E}^{\frac{\omega_{C,f}\omega_{I,f}}{\lambda_f}}\left[\mathrm{d}\log\left(\frac{w_{I,f}}{w_{C,f}}\right)\right]
\end{aligned}
$$

Multiple factors

- Cost minimization in each sector

$$\begin{aligned} 0 &= \alpha_C \, \mathrm{d} \log r_K + (1 - \alpha_C) \, \mathrm{d} \log w_C \\ \mathrm{d} \log p_K &= \alpha_I \, \mathrm{d} \log r_K + (1 - \alpha_I) \, \mathrm{d} \log w_I \end{aligned}$$

- In the long-run and assuming Cobb-Douglas production functions

$$\begin{aligned} \frac{\mathrm{d} \log p_K}{\mathrm{d} \log (1-\tau)} = & \frac{\alpha_C - \alpha_I}{1 - \alpha_C} \\ & + (1 - \alpha_C) \frac{\lambda_I \lambda_K}{\alpha_C} \cdot \mathrm{Cov}_f \left(\omega_{I,f} - \omega_{C,f}, \frac{(1-\alpha_I)\omega_{I,f} - (1-\alpha_C)\omega_{C,f}}{\lambda_f} \right) \\ & + (1 - \alpha_C) \frac{\lambda_K}{\alpha_C \lambda_C} \cdot \mathrm{E}^{\frac{\omega_{C,f}\omega_{I,f}}{\lambda_f}} \left[\frac{1}{1 + \phi_f} \right] \end{aligned}$$

Further extensions (see paper)

- Internal vs. external investment
- Input-output production network
- Markups
- Capital adjustment cost
- Open economy

Measurement

Methodology

- We document the input shares of the consumption and investment sectors
- We rely on BEA input-output matrices + KLEMS
  - We classify intermediary expenditures on professional services as investment $\rightarrow$ "organizational services" (Corrado et al., Eisfeldt and Papanikolaou)
  - Each commodity can be used in C or I but there is specialization We assume input shares to produce C is the same as I within commodity
  - We estimate "consolidated" labor shares of industry j through input-output matrix

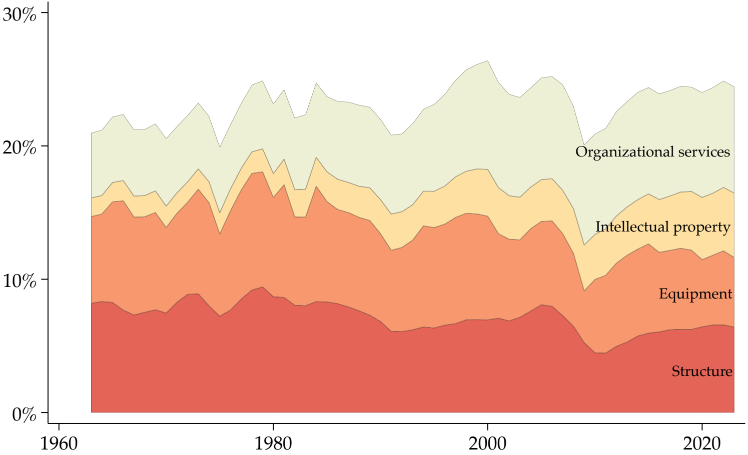


Figure 3: Investment expenditures as a fraction of GDP

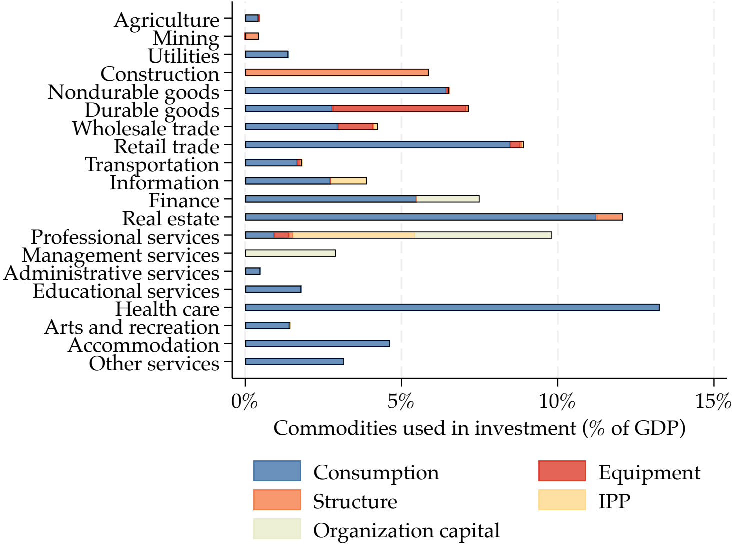


Figure 4: Composition of final consumption and investment (summary)

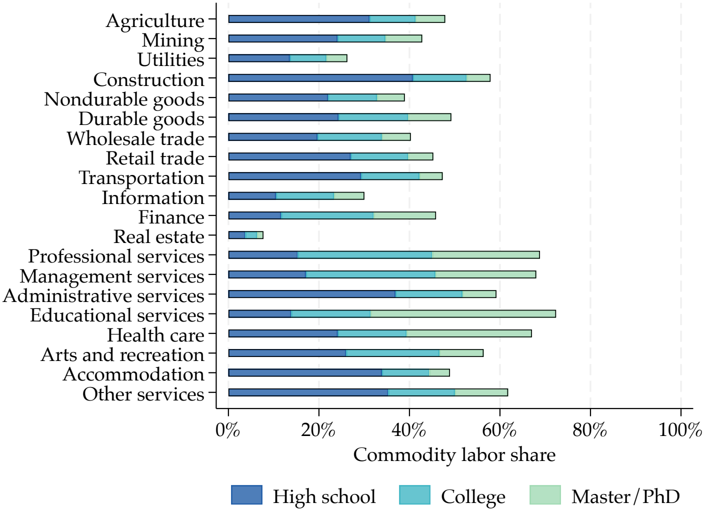
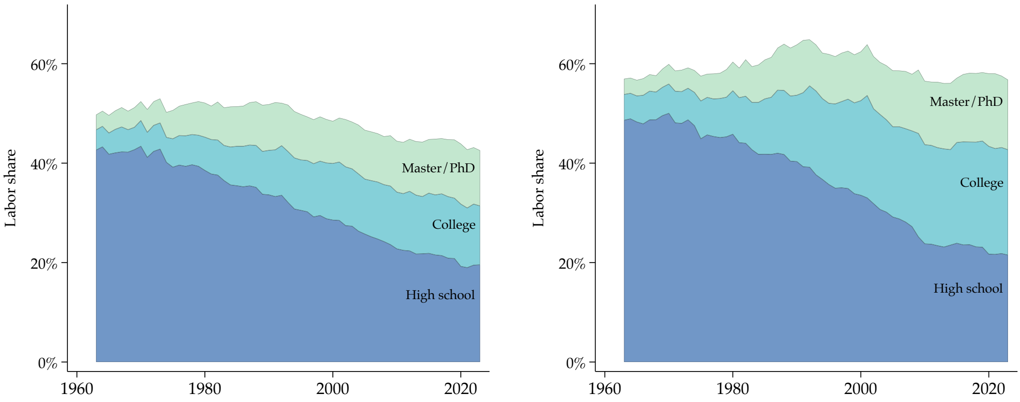


Figure 5: "Consolidated" labor share by commodity (summary)

Table 1: Labor share by sector (%)

	Sector		
	C	I_{tan}	I_{intan}
Total	43.8	51.5	63.9
Highschool	20.1	29.9	15.1
College	12.2	13.9	28.3
Master/Phd	11.6	7.8	20.5



(a) Consumption sector

(b) Investment sector

Figure 6: Evolution of labor share in consumption and investment sectors

Table 2: Alternative construction of total labor share (%)

	Sectors		
	C	I	$I-C$
Baseline	43.8	57.8	+14.0
Without capitalizing fin. services	45.1	59.1	+14.0
Without capitalizing fin. and prof. services	50.6	60.4	+9.8
Cost-shares instead of income-shares	60.1	80.5	+20.4
Using common industry-technology assumption	44.9	56.1	+11.2
Domestic labor share	40.4	51.6	+11.3
BEA labor share	39.8	57.7	+17.9
Using after-redefinition input-output tables	44.9	55.4	+10.5
Including gov consumption and investment	47.4	58.1	+10.8

Quantitative model

Quantitative model

1. Capital K (1−tangible, 2−intangible, 3−land)
2. Labor L (1−high school 2− College 3− Master/Phd)
3. Capital installment costs θ for tan. and intan. capital
4. Sectoral labor supply elasticity ϕ_n for each labor type $n \in \{1, 2, 3\}$

Environment

- Technology is summarized by

$$C = K_{1,C}^{\alpha_{K_1,C}} K_{2,C}^{\alpha_{K_2,C}} K_{3,C}^{\alpha_{K_3,C}} L_{1,C}^{\alpha_{L_1,C}} L_{2,C}^{\alpha_{L_2,C}} L_{3,C}^{\alpha_{L_3,C}} \qquad \text{(consumption good)}$$

$$I_j = K_{1,I_j}^{\alpha_{K_1,I_j}} K_{2,I_j}^{\alpha_{K_2,I_j}} K_{3,I_j}^{\alpha_{K_3,I_j}} L_{1,I_j}^{\alpha_{L_1,I_j}} L_{2,I_j}^{\alpha_{L_2,I_j}} L_{3,I_j}^{\alpha_{L_3,I_j}} \qquad \text{(inv. good for } j \in \{1,2\}\text{)}$$

$$\dot{K}_j = K_j^{\theta} I_j^{1-\theta} - \delta_j K_j \qquad \text{(capital accumulation for } j \in \{1,2\}\text{)}$$

$$K_j = K_{j,C} + K_{j,I_1} + K_{j,I_2} \qquad \text{(capital allocation for } j \in \{1,2,3\}\text{)}$$

$$L_n = L_{n,C} + L_{n,I_1} + L_{n,I_2} \qquad \text{(labor allocation for } n \in \{1,2,3\}\text{)}$$

- Imperfect labor reallocation across sectors

$$\frac{L_{n,I_j}}{L_{n,C}} = \frac{\mu_n}{1-\mu_n} \left(\frac{w_{n,I_j}}{w_{n,C}} \right)^{\phi_n}.$$

Identification

- Labor Cobb-Douglas exponents identified from sectoral cost shares
- Capital exponents + depreciation identified from sectoral investment rates

	Capital			Labor		
	Tangible	Intangible	Land	High school	College	PhD/Master
C sector	0.29	0.21	0.03	0.21	0.14	0.13
I_{tan} sector	0.22	0.26	0.01	0.31	0.14	0.08
I_{intan} sector	0.12	0.24	0.01	0.15	0.28	0.21

with depreciation rates $\delta_1 = 0.063$ and $\delta_2 = 0.09$

Identification

1. Internal adjustment costs θ inferred from PE elasticity of investment to taxes

    Chodorow Reich-Smith-Zidar-Zwick (2025) estimates imply $\theta \approx 0.25$

2. Sectoral labor supply elasticities ϕ_1 and ϕ_2 inferred from labor market literature

    Literature finds low elasticity for scientist, higher for low-skilled workers

    We take $\phi_1 = 2$, $\phi_2 = 1$, $\phi_3 = 0.3$

Response of capital price to capital tax shocks

	$\frac{\mathrm{d}\log p_{K,0}}{\mathrm{d}\log(1-\tau)}$	$\frac{\mathrm{d}\log p_{K,\infty}}{\mathrm{d}\log(1-\tau)}$
Neoclassical growth model	0	0
Endowment economy model	1	1
Calibrated model		
Baseline	0.96	0.54
No adjustment cost ($\theta = 0$)	0.84	0.54
Low sectoral labor frictions ($2 \times \phi$)	0.89	0.45
No sectoral labor frictions ($\phi = \infty$)	0.72	0.26

Welfare incidence

Welfare incidence of capital tax

- MIT shock. Consider a perturbation in capital tax $\mathrm{d}\tau$
- Define a factor "welfare gain" as cash-equivalent variation in factor payment

$$\text{Worker Welfare Gain}_{n,C} = \int_0^\infty e^{-rt} L_{n,C,t} \cdot \mathrm{d}w_{n,C,t} dt$$

$$\text{Worker Welfare Gain}_{n,I_j} = \int_0^\infty e^{-rt} L_{n,I_j,t} \cdot \mathrm{d}w_{n,I_j,t} dt$$

$$\text{Capitalist Welfare Gain}_{K_j} = K_{j,0}\,\mathrm{d}p_{K_j,0} + \int_0^\infty p_{K_j,t} K_{j,t}\,\mathrm{d}r_t dt$$

Welfare gains aggregate to mechanical decrease in government revenue

Welfare incidence of capital tax

Table 3: Welfare gains of decrease in capital tax rate (% of decrease in gov revenue)

	Capitalists	Workers
Neoclassical growth model (fully elastic capital)	0%	100%
Endowment economy model (fully inelastic capital)	100%	0%
Calibrated model		
Baseline	48%	52%
No adjustment cost ($\theta = 0$)	43%	57%
Low sectoral labor frictions ($2 \times \phi$)	45%	55%
No sectoral labor frictions ($\phi = \infty$)	37%	63%

Welfare heterogeneity within workers

Table 4: Welfare gains of decrease in capital tax rate (% of decrease in gov revenue)

	C-workers	I-workers
Neoclassical growth model (fully elastic capital)	65%	35%
Endowment economy model (fully inelastic capital)	0%	0%
Calibrated model		
Baseline	19%	33%
No adjustment cost ($\theta = 0$)	21%	36%
Low sectoral labor frictions ($2 \times \phi$)	25%	29%
No sectoral labor frictions ($\phi = \infty$)	42%	21%

Welfare heterogeneity within workers

Table 5: Welfare gains of decrease in capital tax rate (% of decrease in gov revenue)

	Highschool	College+
Neoclassical growth model (fully elastic capital)	42%	58%
Endowment economy model (fully inelastic capital)	0%	0%
Calibrated model		
Baseline	20%	32%
No adjustment cost ($\theta = 0$)	22%	35%
Low sectoral labor frictions ($2 \times \phi$)	21%	33%
No sectoral labor frictions ($\phi = \infty$)	24%	38%

Welfare heterogeneity within capitalists

Table 6: Welfare gains of decrease in capital tax rate (% of decrease in gov revenue)

	Tangible	Intangible	Land
Neoclassical growth model (fully elastic capital)	0%	0%	0%
Endowment economy model (fully inelastic capital)	52%	44%	4%
Calibrated model			
Baseline	21%	23%	4%
No adjustment cost ($\theta = 0$)	18%	21%	4%
Low sectoral labor frictions ($2 \times \phi$)	20%	21%	4%
No sectoral labor frictions ($\phi = \infty$)	16%	17%	4%

- We focus here on heterogeneity w.r.t. asset *holdings*
- In the real world, there is additional heterogeneity based on asset *sales*
  - Entrepreneurs who use external financing (Gomez and Gouin Bonenfant 2023)
  - Older households who sell assets (Fagereng et al. 2025)

Conclusion

- Theory. Analytical formulas for response of capital prices to discount rates
- Empirics. Large and growing heterogeneity in input shares in C vs. I sectors
- Quantitative model. Quantitative model calibrated on input-output matrix
  - → Capital is inelastic
  - → Tax cuts disproportionately benefit K-owners and L_I-workers
  - → Heterogeneous incidence within capitalists and within workers