

A Tale of Two Networks: Common Ownership and Product Market Rivalry

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SFS Cavalcade 2024

Market Power, Markups and Asset Prices

Common Ownership

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- **Rising Common Ownership** ([Gilje, Gormley & Levit 2020](#); [Backus, Conlon & Sinkinson, 2021](#)) → Huge policy/research interest:
 - ▶ Consolidation in asset management industry is putting stock ownership in the hands of a few large institutional investors.

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 2. Common Ownership: 13(f) institutional shareholding data

Literature

- **Macro-Finance and Industrial Organization:** Pellegrino (2020).
- **Declining Competition:** Covarrubias, Gutierrez & Philippon (2019); Gutierrez & Philippon (2019); DeLoecker, Eeckhout & Unger (2020).
- **Rise of Common Ownership (CO):** Backus, Conlon & Sinkinson (2019a, 2019b); Gilje, Gormley and Levit (2020).
- **Empirical evidence of Common Ownership conduct:** Azar, Schmalz & Tecu (2018), Dennis, Gerardi & Schenone (2019), Lewellen & Lowry (2019); Anton, Ederer, Giné & Schmalz (2019); Boller & Scott Morton (2020); Xie & Gerakos (2020); Shekita (2021); Park & Seo (2019); Backus, Conlon & Sinkinson (2022).
- **This Paper:** first structural GE model to quantify the effects of common ownership.
 - ▶ What we don't do: provide evidence or test for conduct, identify mechanism.
 - ▶ What we do: macro-style counterfactual analyses. Start from product demand system → vary firms' conduct (objective function) → study changes in welfare.

Theory: product markets

Generalized Hedonic-Linear Demand (Pellegrino '19)

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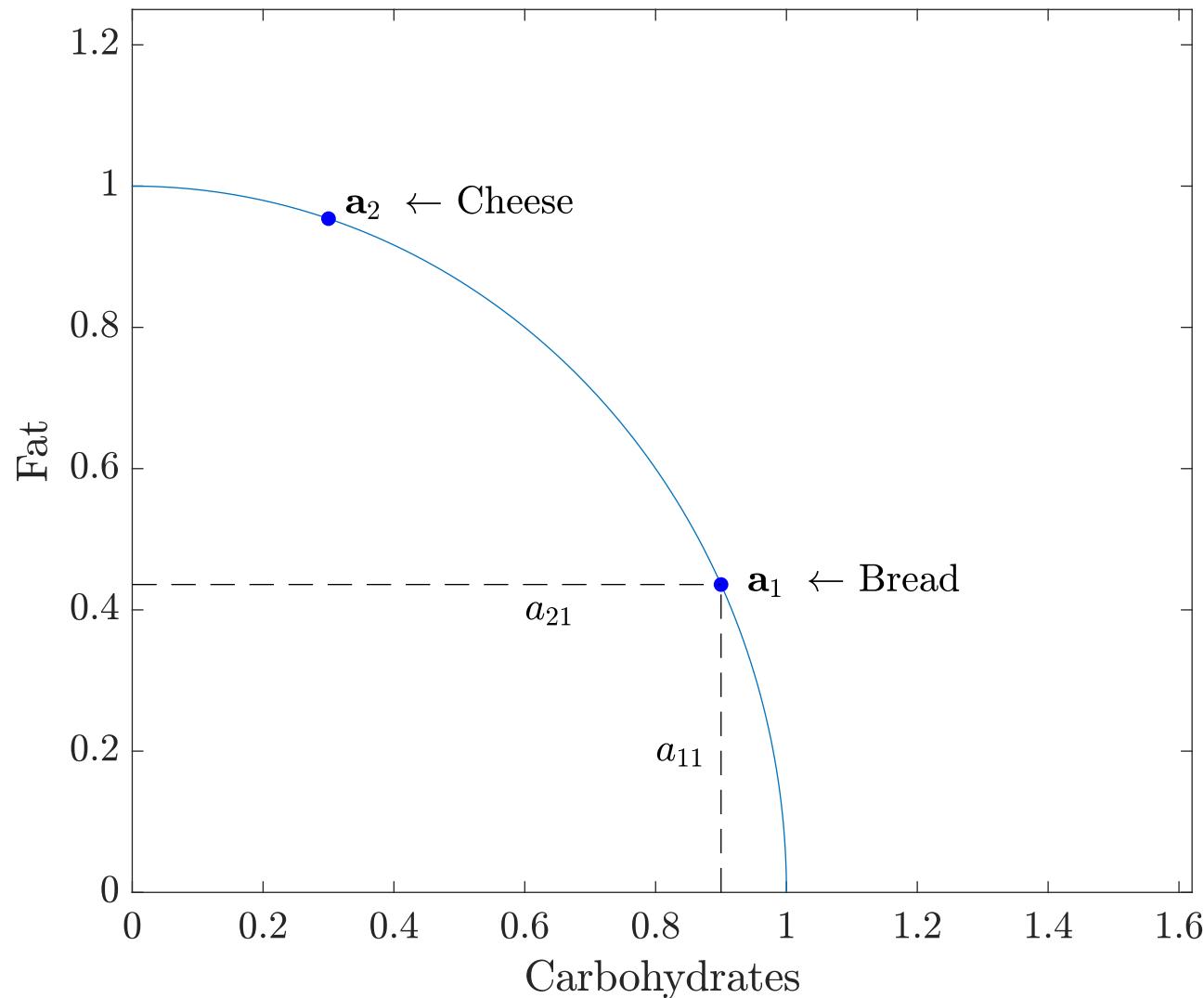
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- Each product is described by a vector of characteristics \mathbf{a}_i (of length normalized to 1)

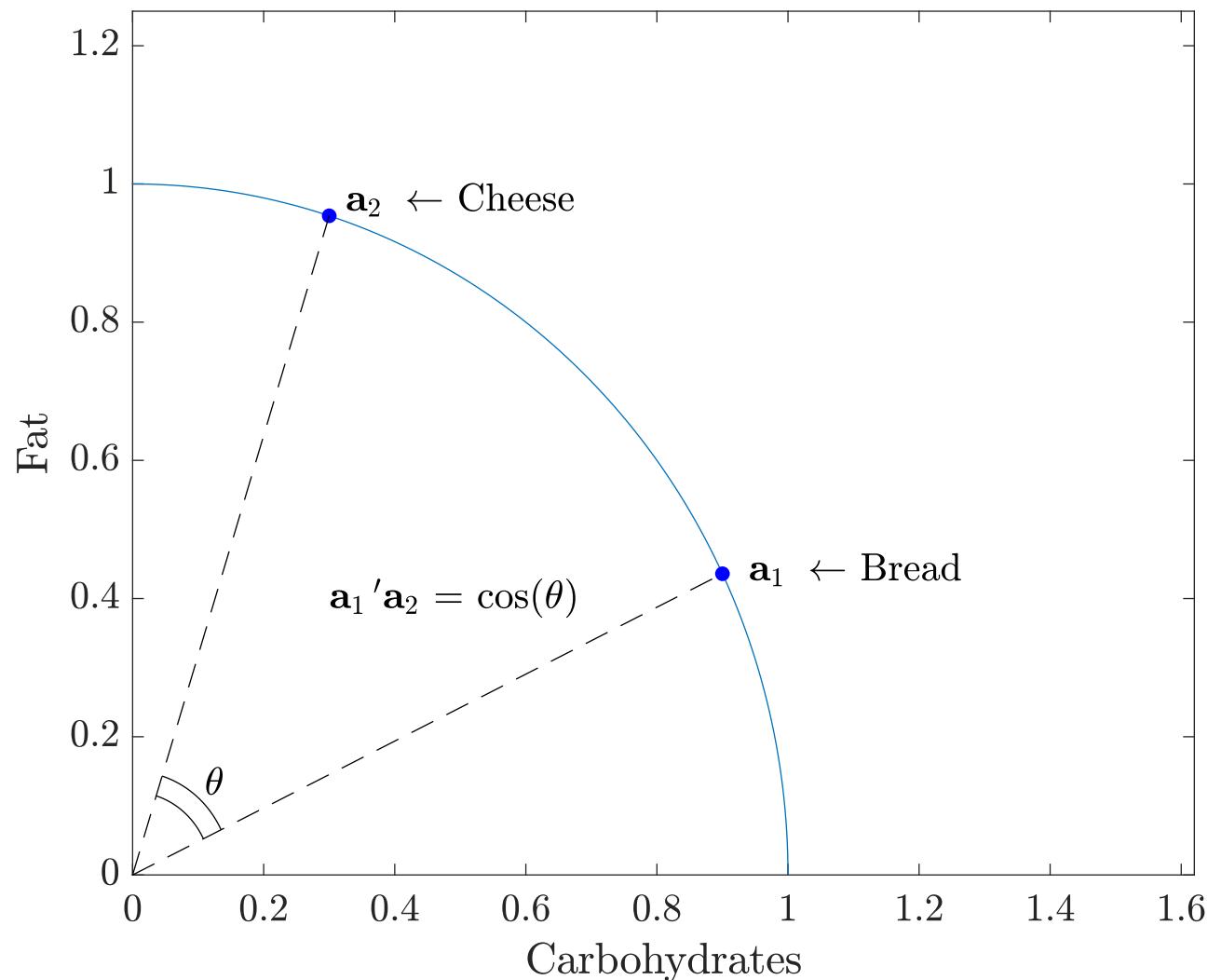
A basic example: 2 firms, 2 characteristics



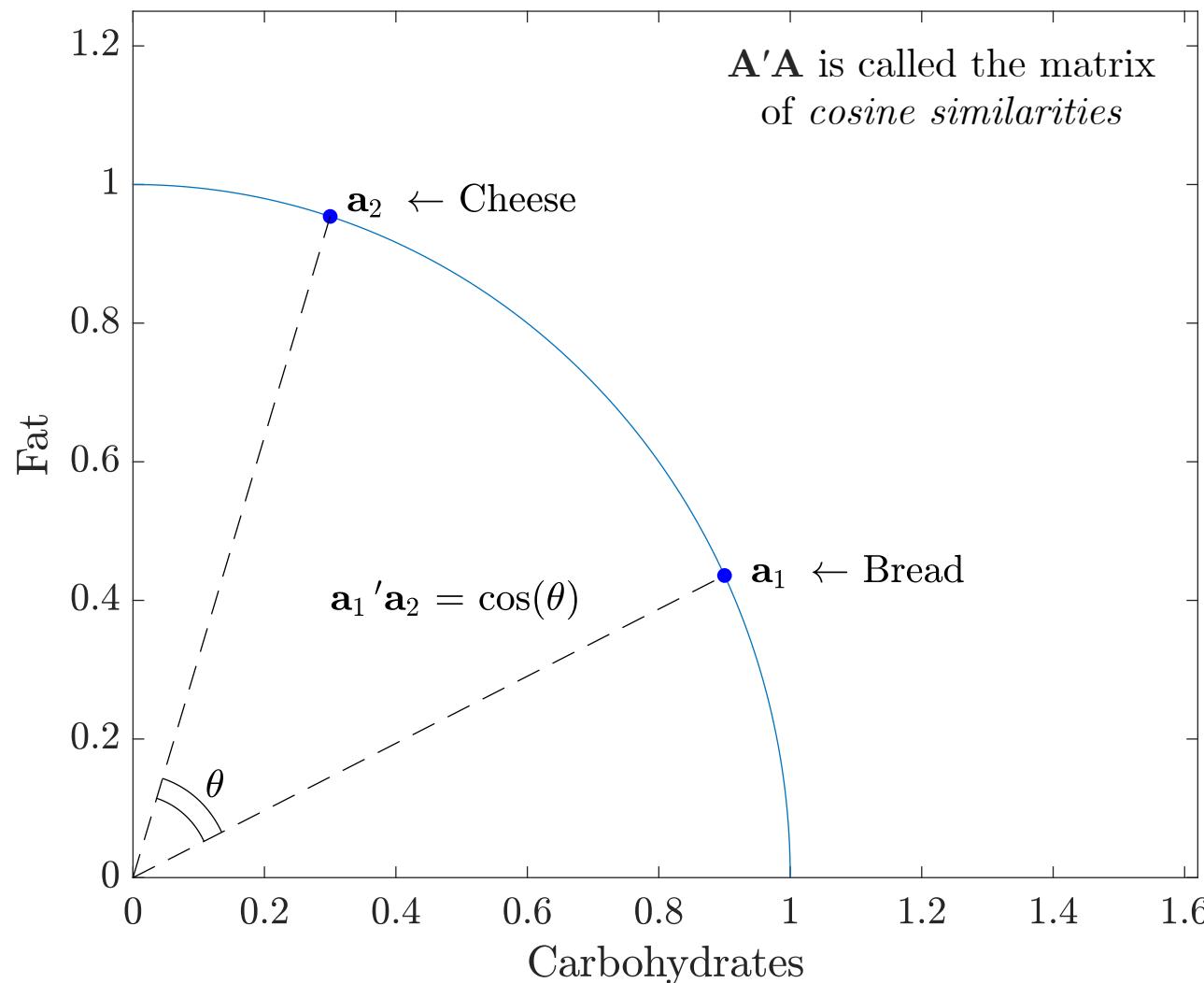
Aggregating common characteristics

Characteristics (Dietary Intake)	Matrix of Coordinates (Nutrition Facts)	Product Bundle
$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix}$	$=$	$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{M1} & a_{M2} & \cdots & a_{Mn} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}$
\mathbf{x}	$=$	\mathbf{Aq}

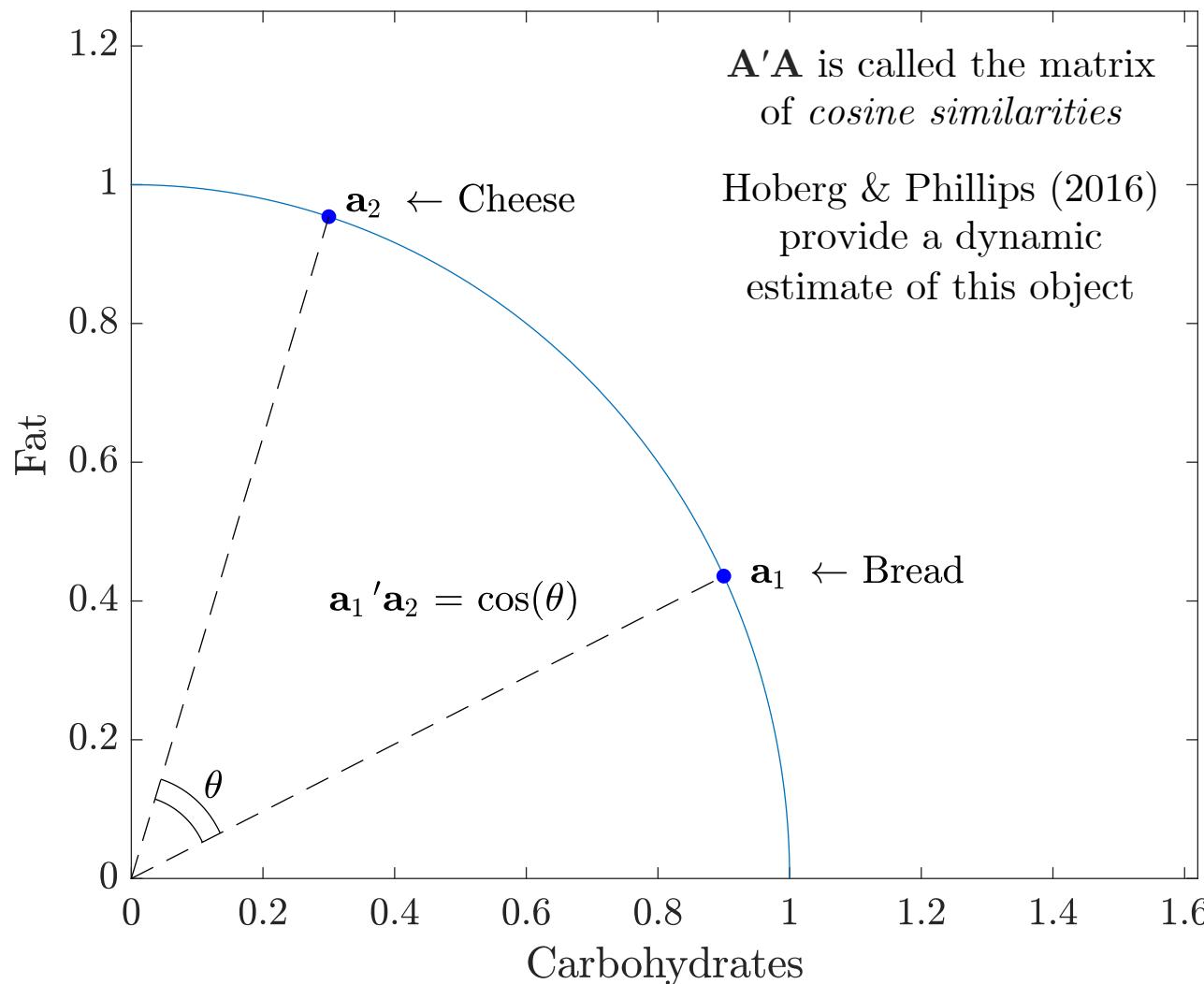
Defining Cosine Similarity



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Generalized Hedonic-Linear Demand

- Utility quadratic in characteristics \rightarrow Inverse demand

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where

$$\boldsymbol{\Sigma} \propto \mathbf{A}'\mathbf{A} - \mathbf{I}$$

Theory

ownership and firms' objectives

Common Ownership and the Firm's Objective

- There are Z funds indexed by $z = 1, 2, \dots, Z$. Fund z own shares s_{iz} in company i . Then fund z 's total income is:

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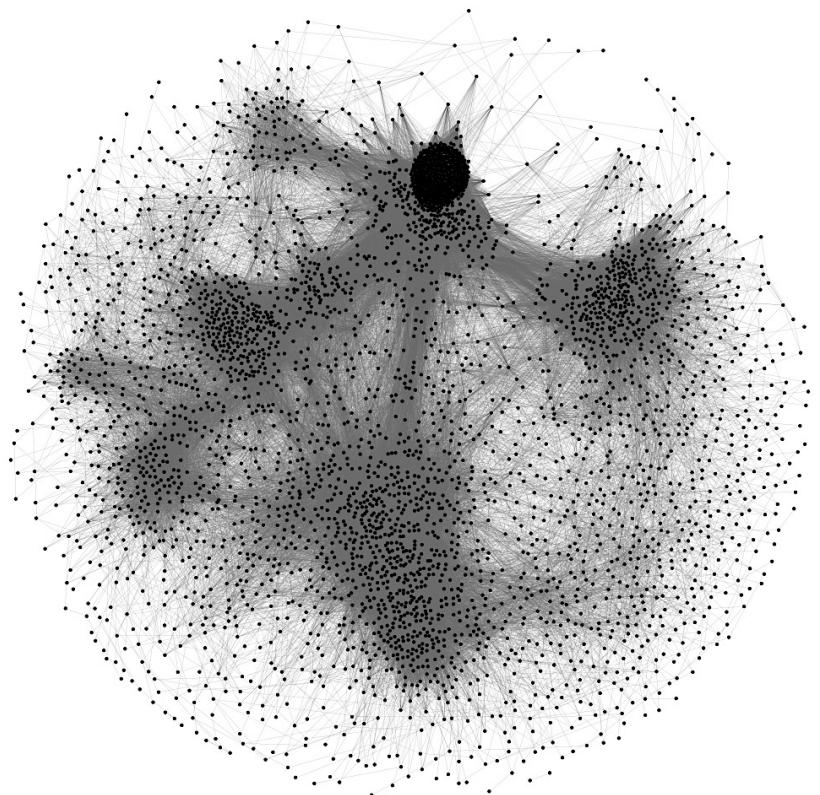
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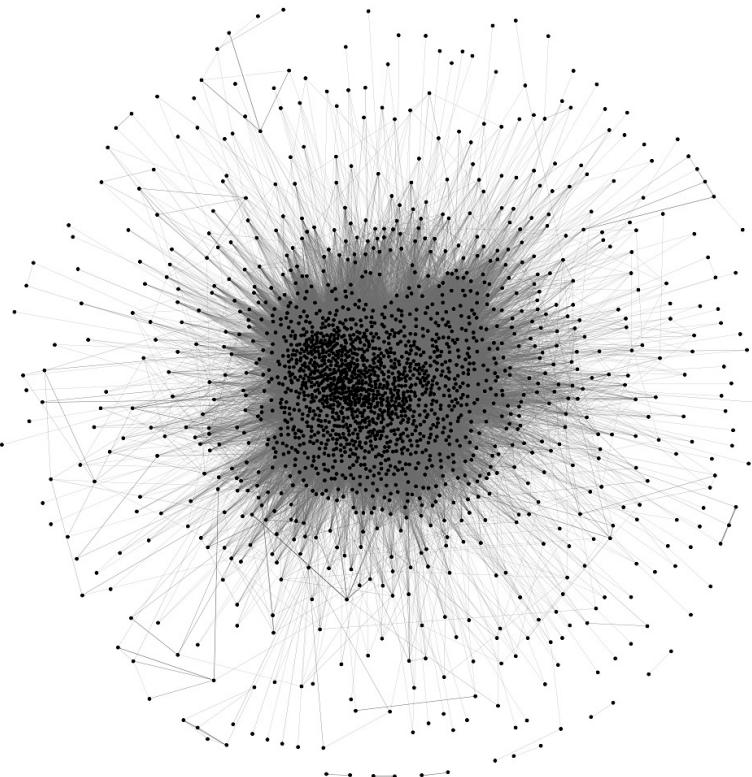
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- Why? Profits depend on $\mathbf{A}'\mathbf{A}$ and \mathbf{K} (adjacency matrices!)

A Tale of Two Networks



Product Market Similarity - $\mathbf{A}'\mathbf{A}$
based on 10-K (Hoberg & Phillips, 2016)



Common Ownership Weights – \mathbf{K}
based on (corrected) 13-F data

Data

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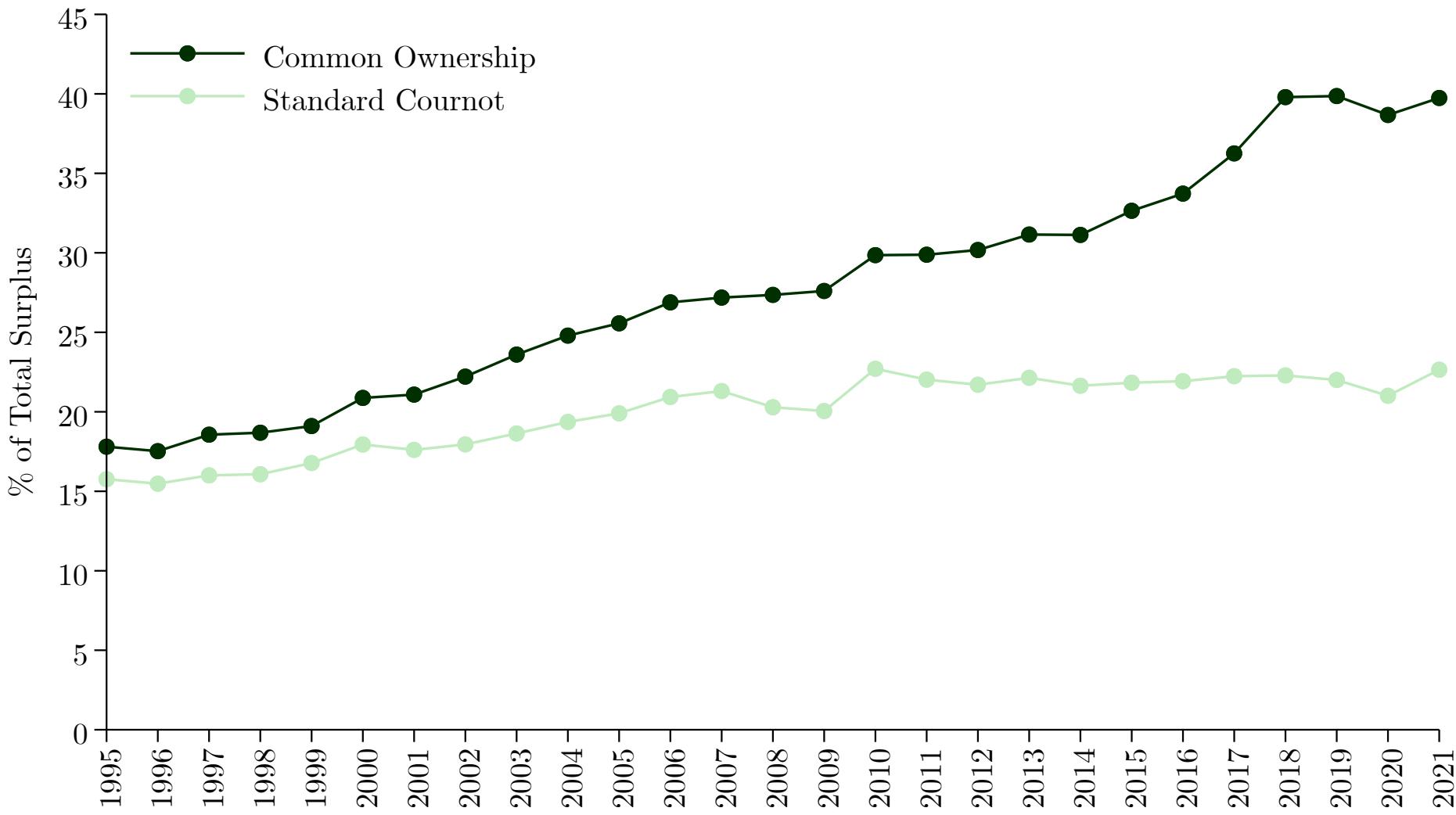
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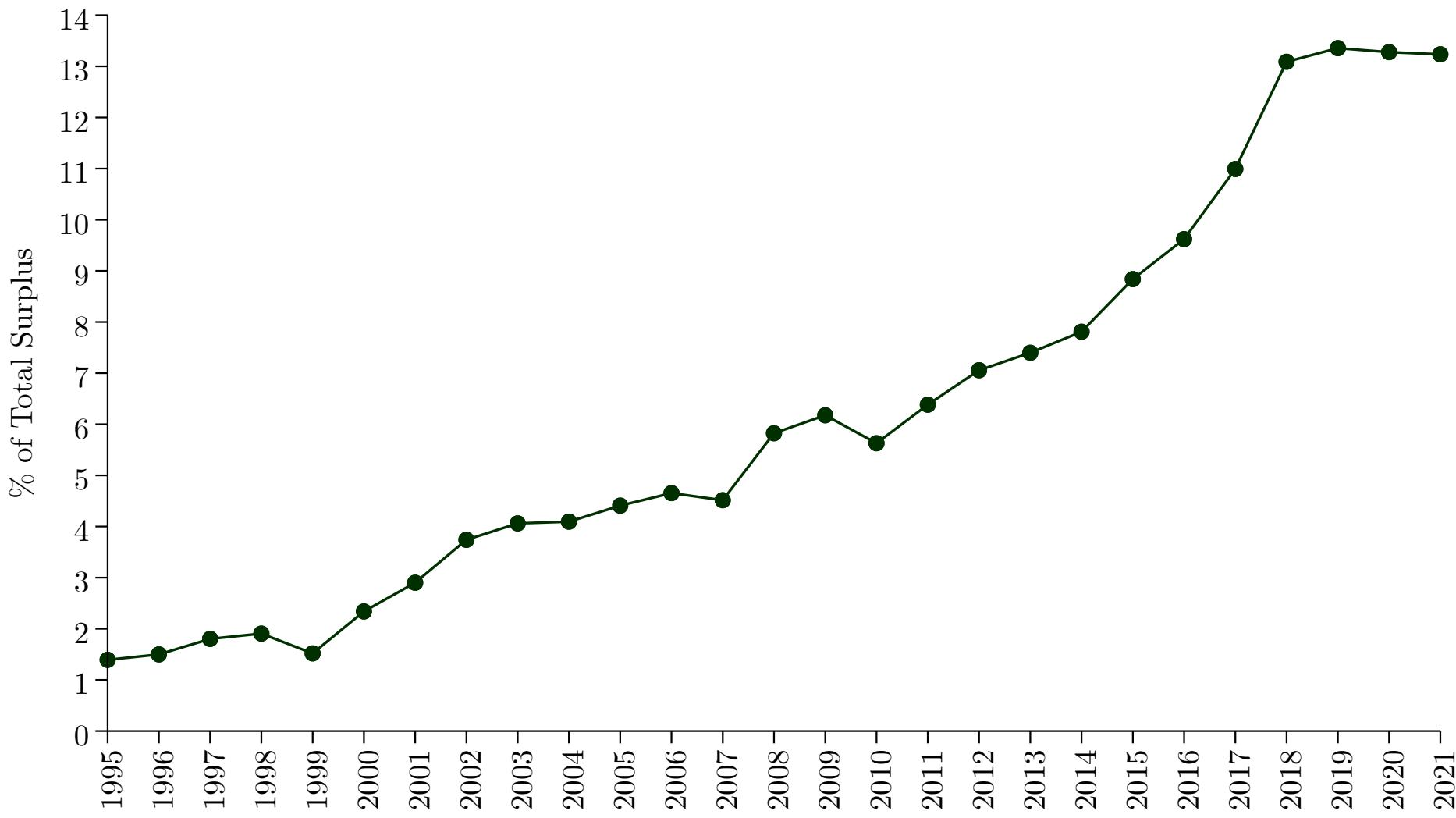
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- **Extensive Validation** in HP and Pellegrino (2019).

Empirics

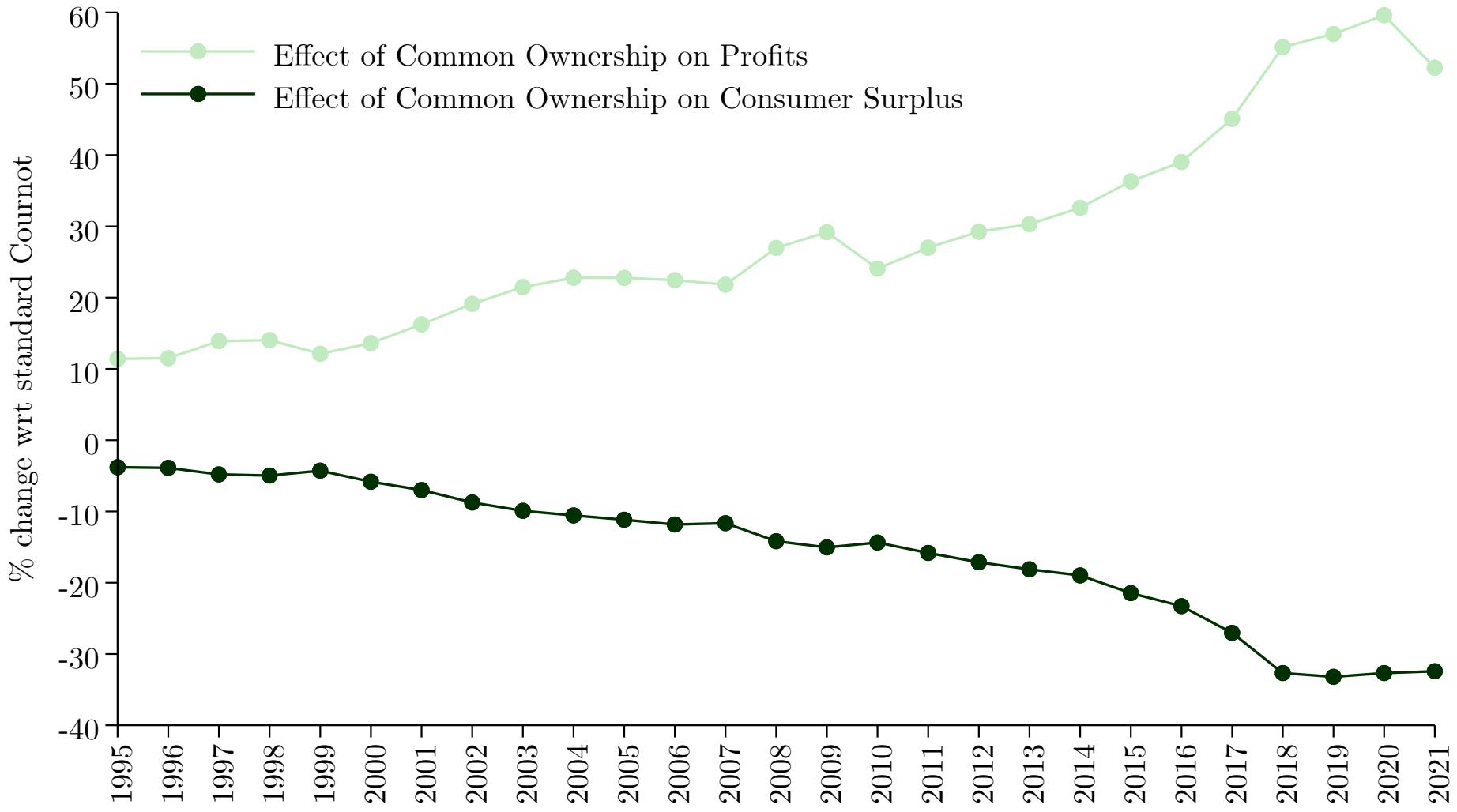
Profit Share of Total Surplus



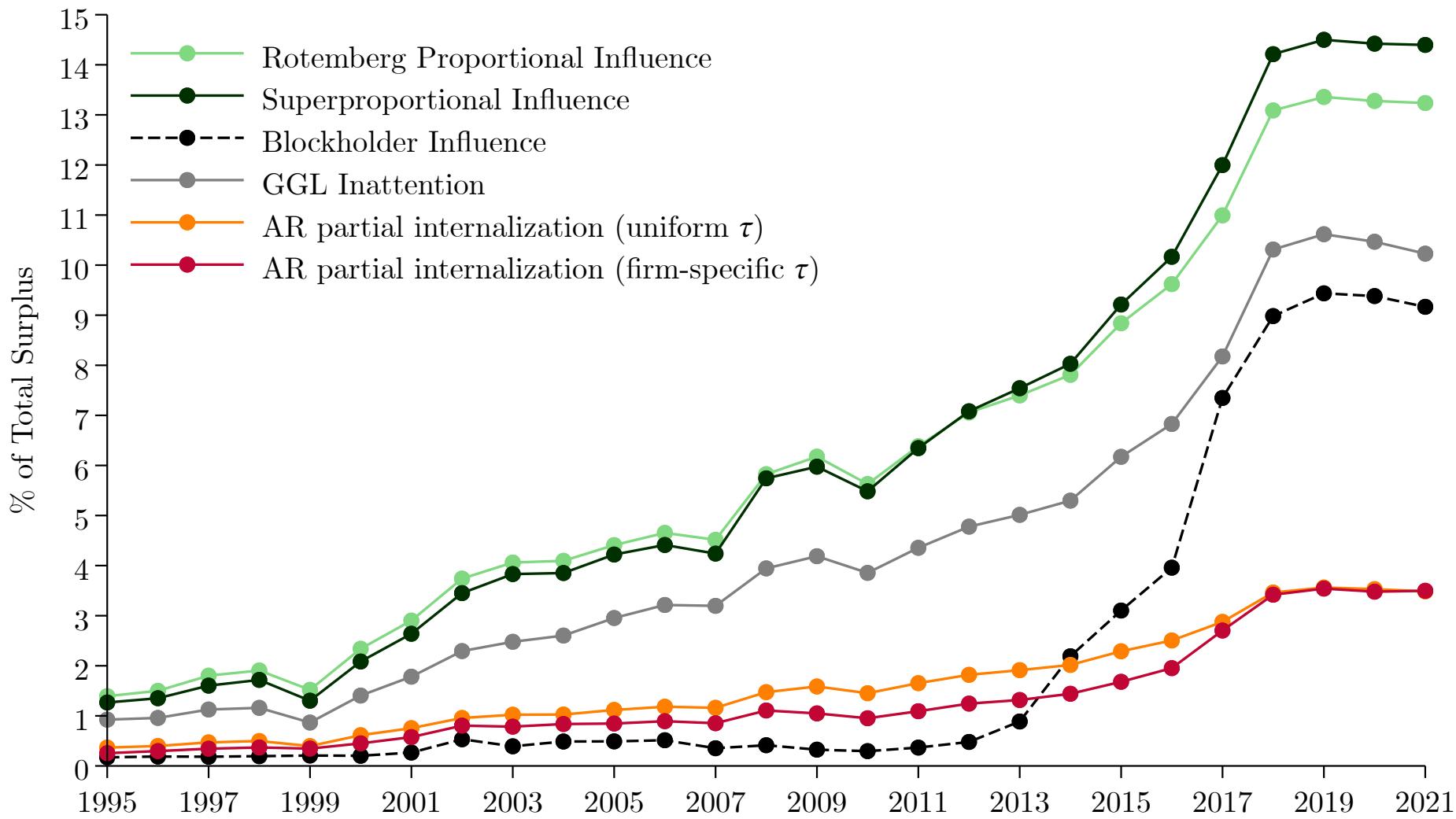
Deadweight Loss



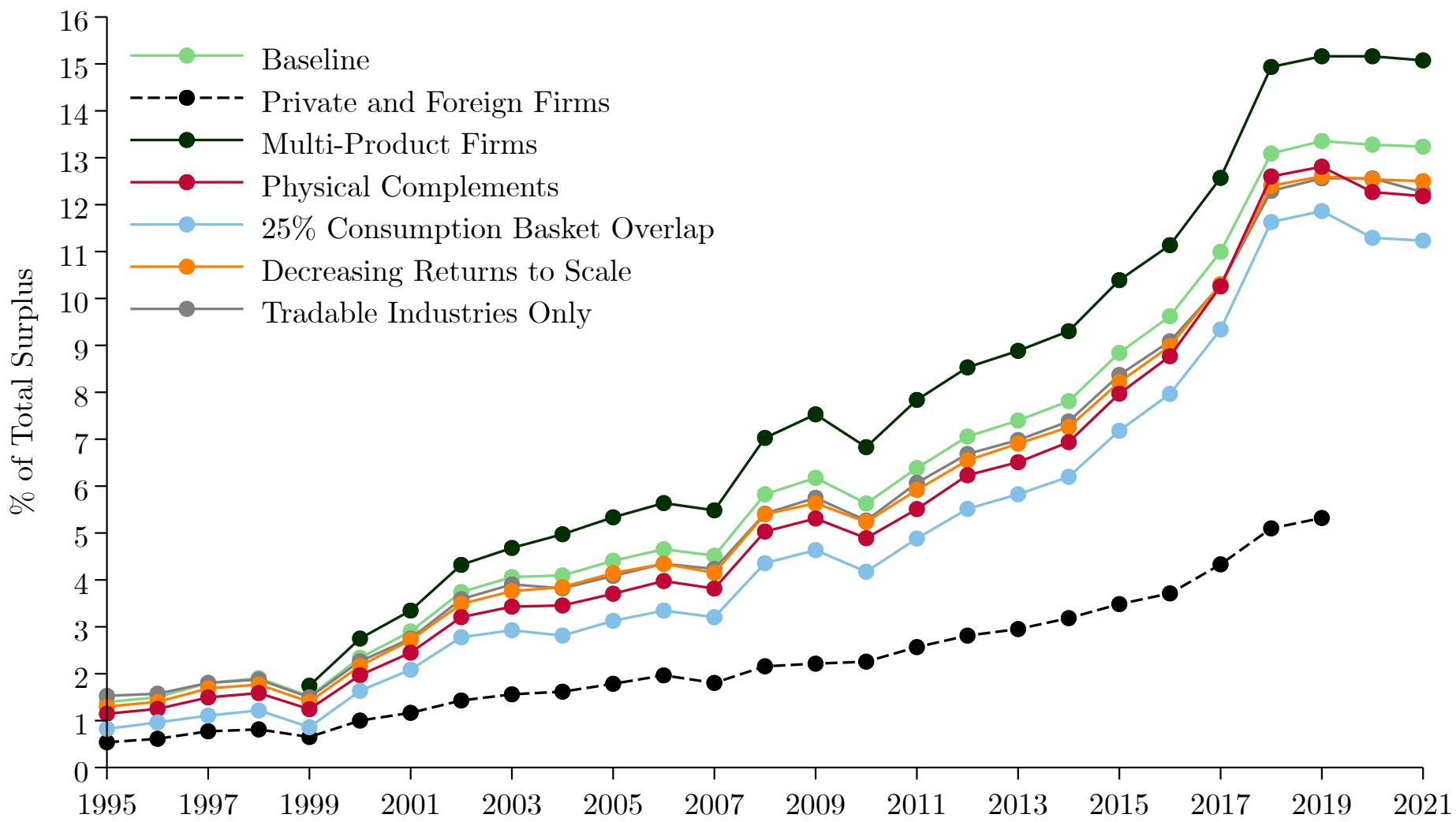
Incremental Effect of Common Ownership



DWL: Alternative Governance Assumptions



DWL: Robustness & Extensions



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- Large effects, robust in a wide battery of model variants:
 - ▶ Efficiency: deadweight loss $\approx 13\%$
 - ▶ Distributional: CO increases aggregate profits by 50%, reallocates them across firms, depresses CS by almost 30%.
- These effects were absent in the early 1990s. They have progressively appeared over time with the rise of CO.

thank you

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- There are settings however where we expect larger shareholders to exert more influence (e.g. majority shareholders).
- We thus develop 3 alternative versions of the model where firms' objective function weights investors based on size.

1) Super-Proportional Influence

$$\tilde{\kappa}_{ij} \stackrel{\text{def}}{=} \frac{\sum_{z=1}^Z s_{iz} \gamma_{iz} s_{jz}}{\sum_{z=1}^Z s_{iz} \gamma_{iz} s_{iz}}$$

$$\gamma_{iz} \stackrel{\text{def}}{=} \sqrt{s_{iz}}$$

2) Blockholders

$$\tilde{\kappa}_{ij} \stackrel{\text{def}}{=} \frac{\sum_{z=1}^Z s_{iz} b_{iz} s_{jz}}{\sum_{z=1}^Z s_{iz} s_{jz}}$$

$b_{iz} = 1$ if and only if $s_{iz} > 5\%$

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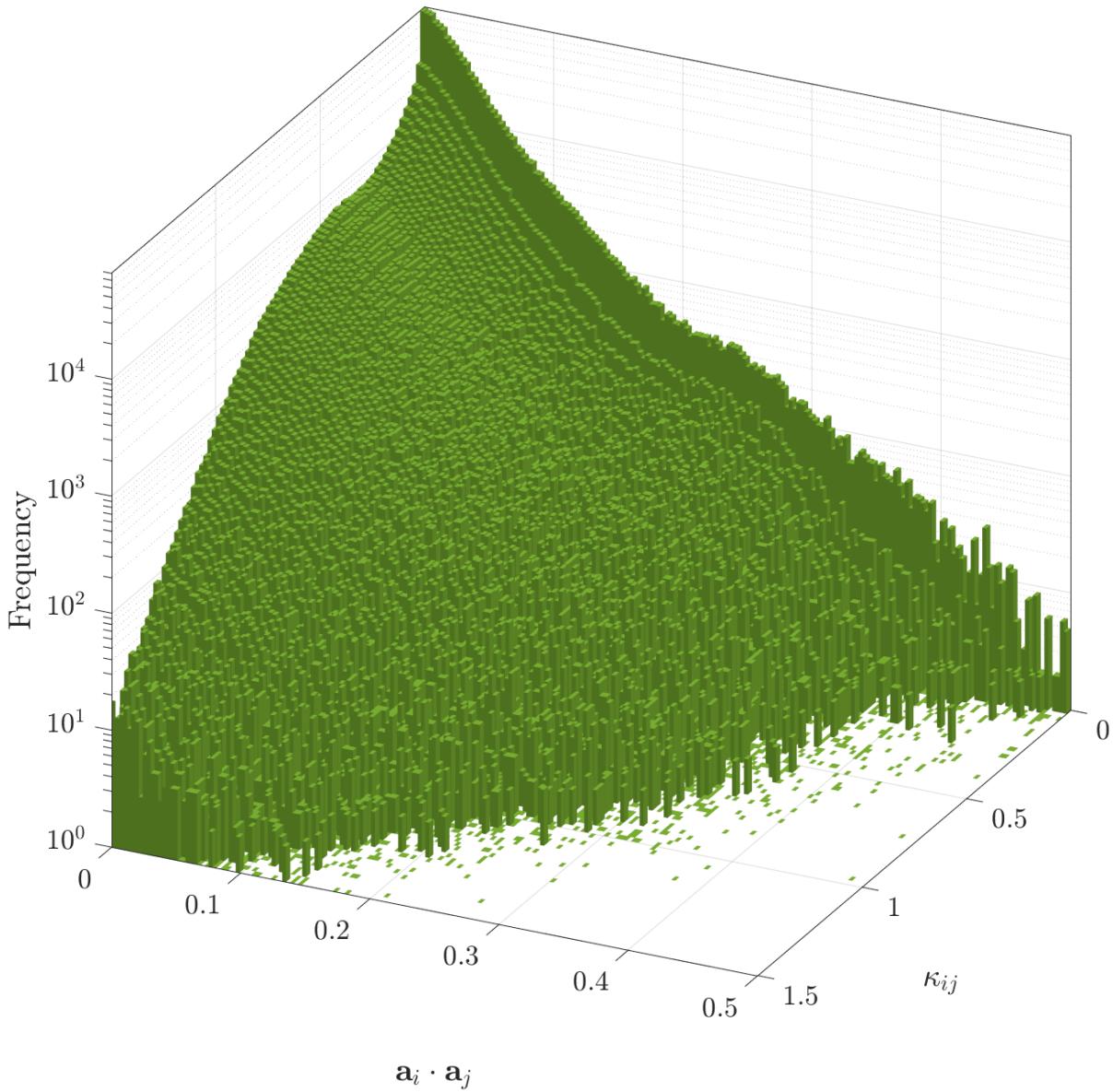
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4) Azar & Ribeiro (2022) mitigation factor

$$\phi \propto \pi + \tau_i \sum_{j \neq i} \kappa_{ij} \pi_j$$

$$\tau_i = \frac{\exp [\theta_0 + \log (\text{IHHI}_i)]}{1 + \exp [\theta_0 + \log (\text{IHHI}_i)]}$$



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- Every other object is identified in closed form (correct units).

Are Passive Investors also Passive Owners?

William McNabb III, Vanguard's chairman and CEO, said the following in a **2015** letter to board of directors of Vanguard's portfolio companies:

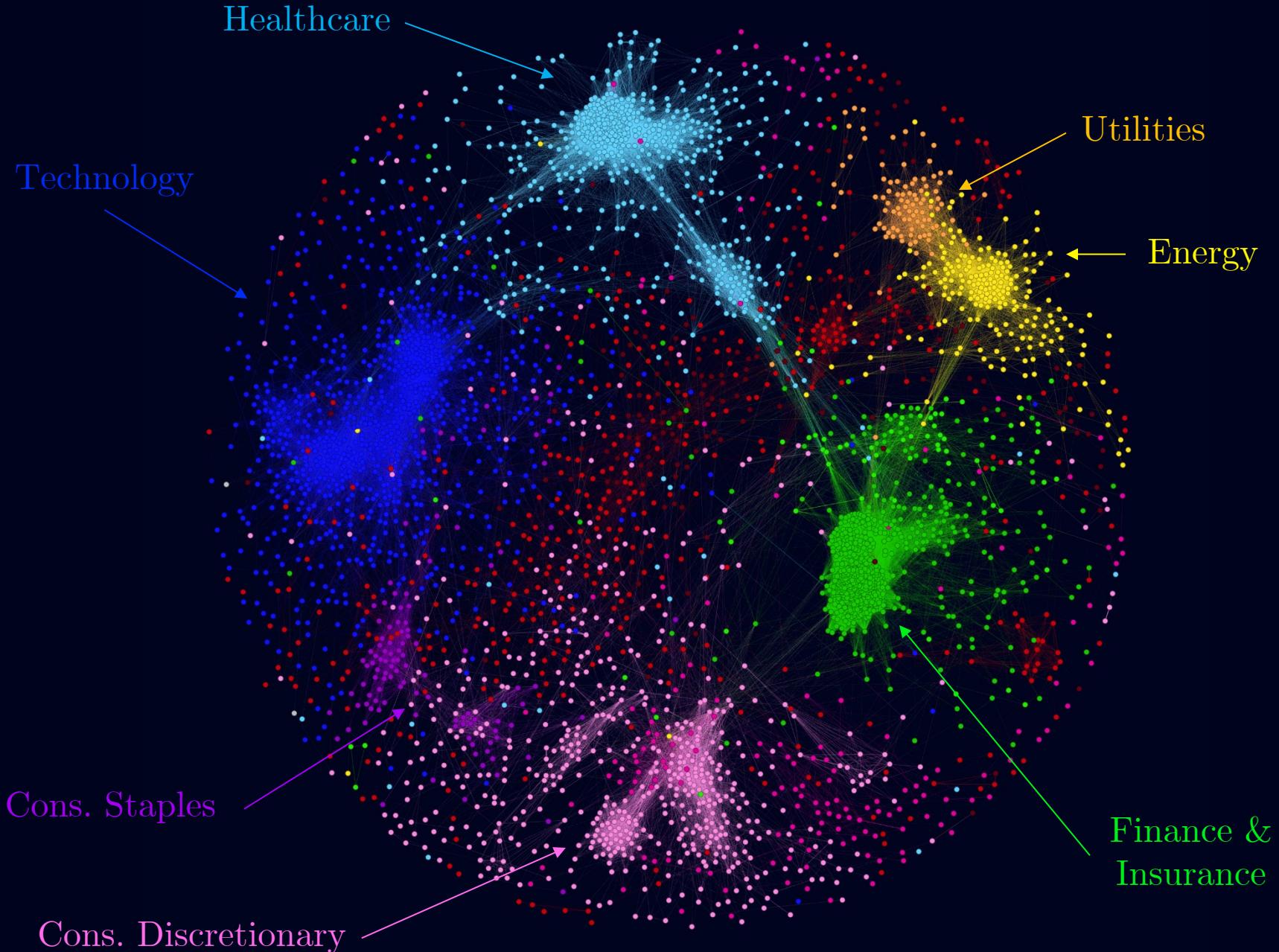
- “*In the past, some have mistakenly assumed that our predominantly passive management style suggests a passive attitude with respect to corporate governance. Nothing could be further from the truth.*”

Fact: the three largest index fund managers all offer active funds as well.

Question: why would throw away votes from passive holdings?

Appel, Gormley & Keim (JFE 2016):

- “Passive Investors, not Passive Owners: passive funds influence firms' governance [...] through their large voting blocs [...] passive ownership is associated with improvements in firms' longer-term performance.”



10-K-BASED CLASSIFICATIONS OF FIRMS IN BUSINESS SERVICES (SIC-3 = 737)

Submarket 1: Entertainment (Sample Focal Firm: **Wanderlust Interactive**)

43 rivals: Maxis, Piranha Interactive Publishing, Brilliant Digital Entertainment, Midway Games, Take Two Interactive Software, THQ, 3DO, New Frontier Media, . . .

SIC codes of rivals: computer programming and data processing [SIC-3 = 737] (24 rivals), motion picture production and allied services [SIC-3 = 781] (4 rivals), miscellaneous other (13 rivals)

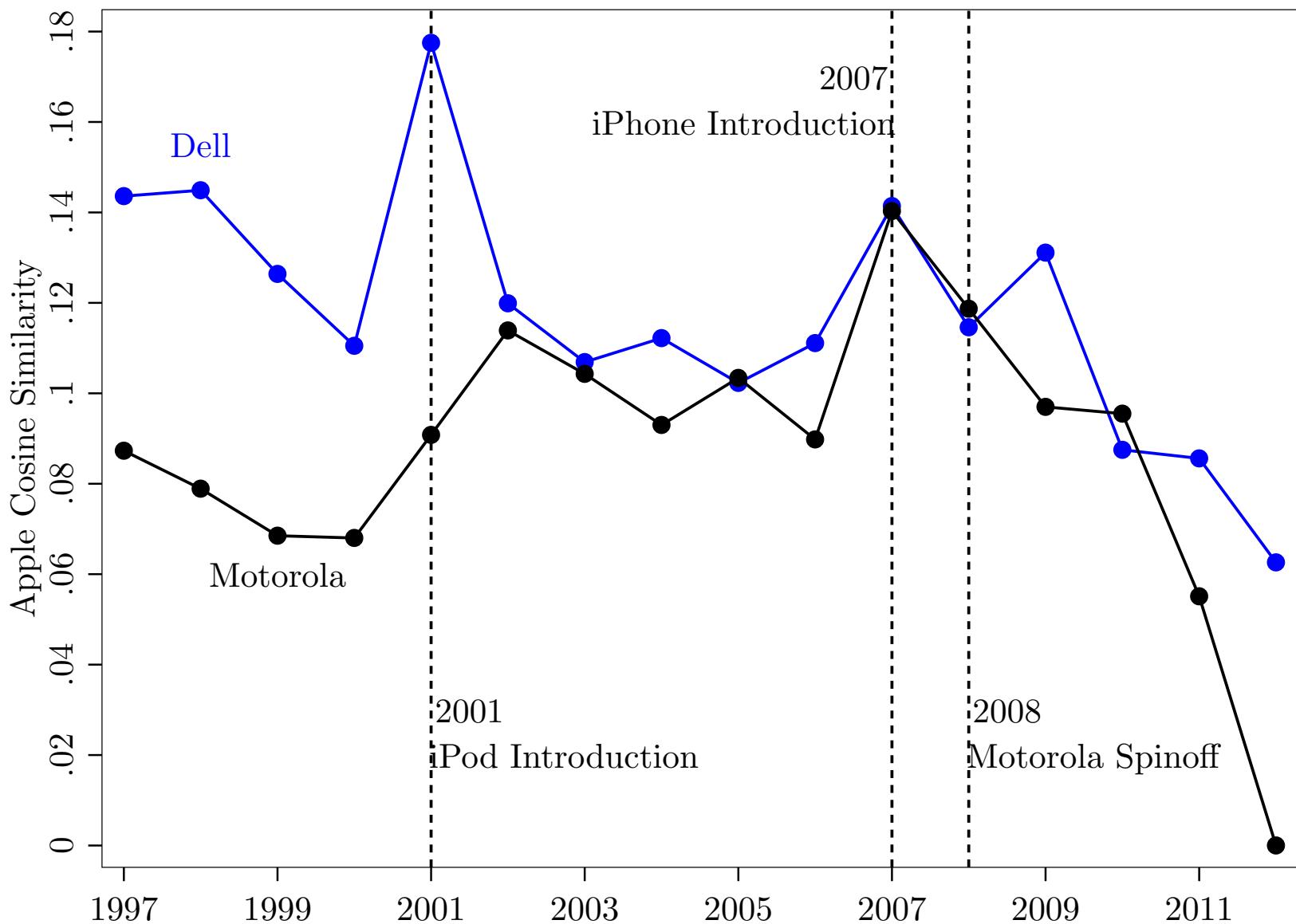
Core words: entertainment (42), video (42), television (38), royalties (35), internet (34), content (33), creative (31), promotional (31), copyright (31), game (30), sound (29), publishing (29), . . .

Submarket 2: Medical Services (Sample Focal Firm: **Quadramed Corp.**)

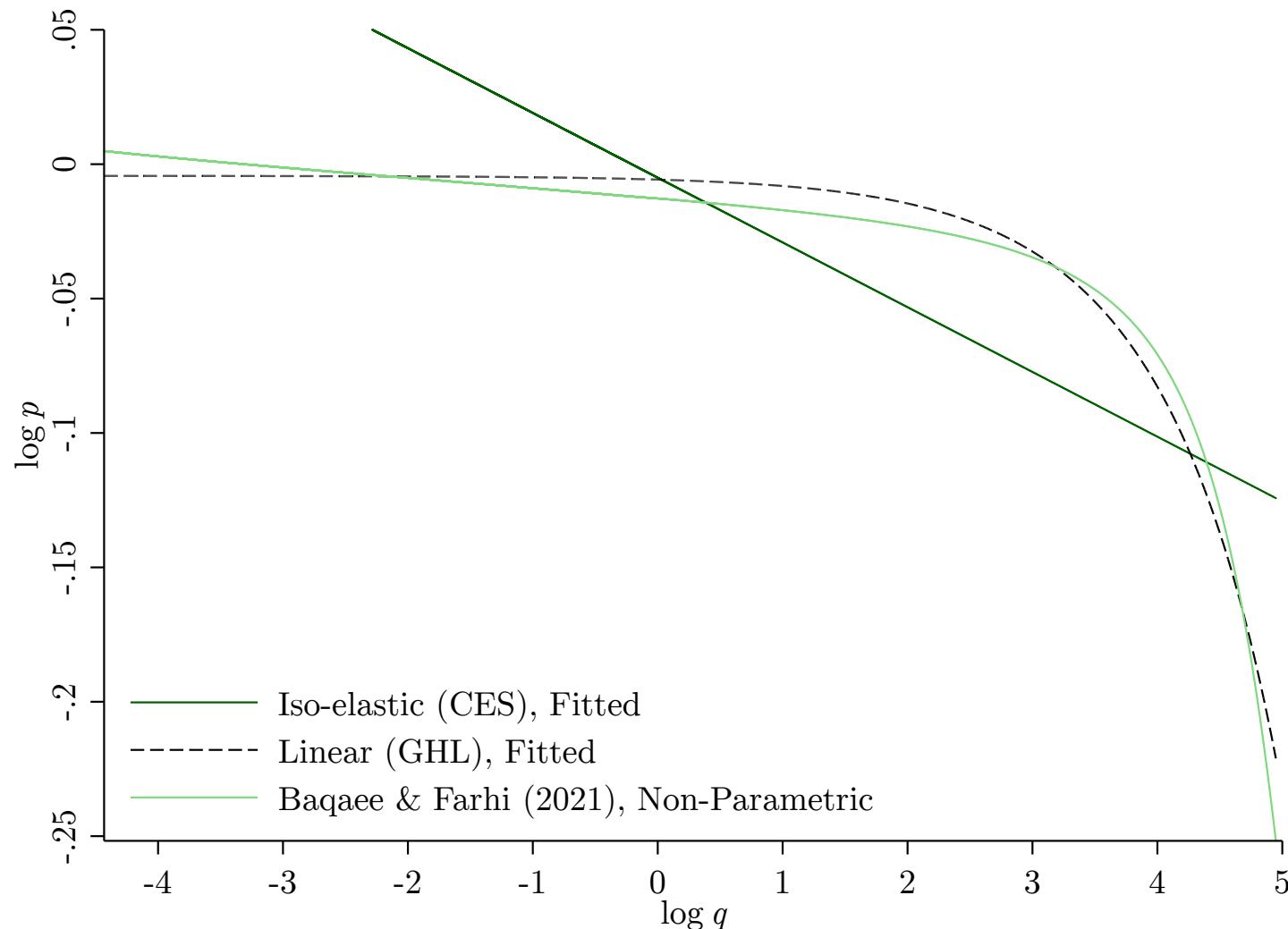
66 rivals: IDX Systems, Medicus Systems, Hpr, Simione Central Holdings, National Wireless Holdings, HCIA, Apache Medical Systems, . . .

SIC codes of rivals: computer programming and data processing [SIC-3 = 737] (45 rivals), insurance agents, brokers, and service [SIC-3 = 641] (5 rivals), miscellaneous health services [SIC-3 = 809] (4 rivals), management and public relations services [SIC-3 = 874] (3 rivals), miscellaneous other (9 rivals)

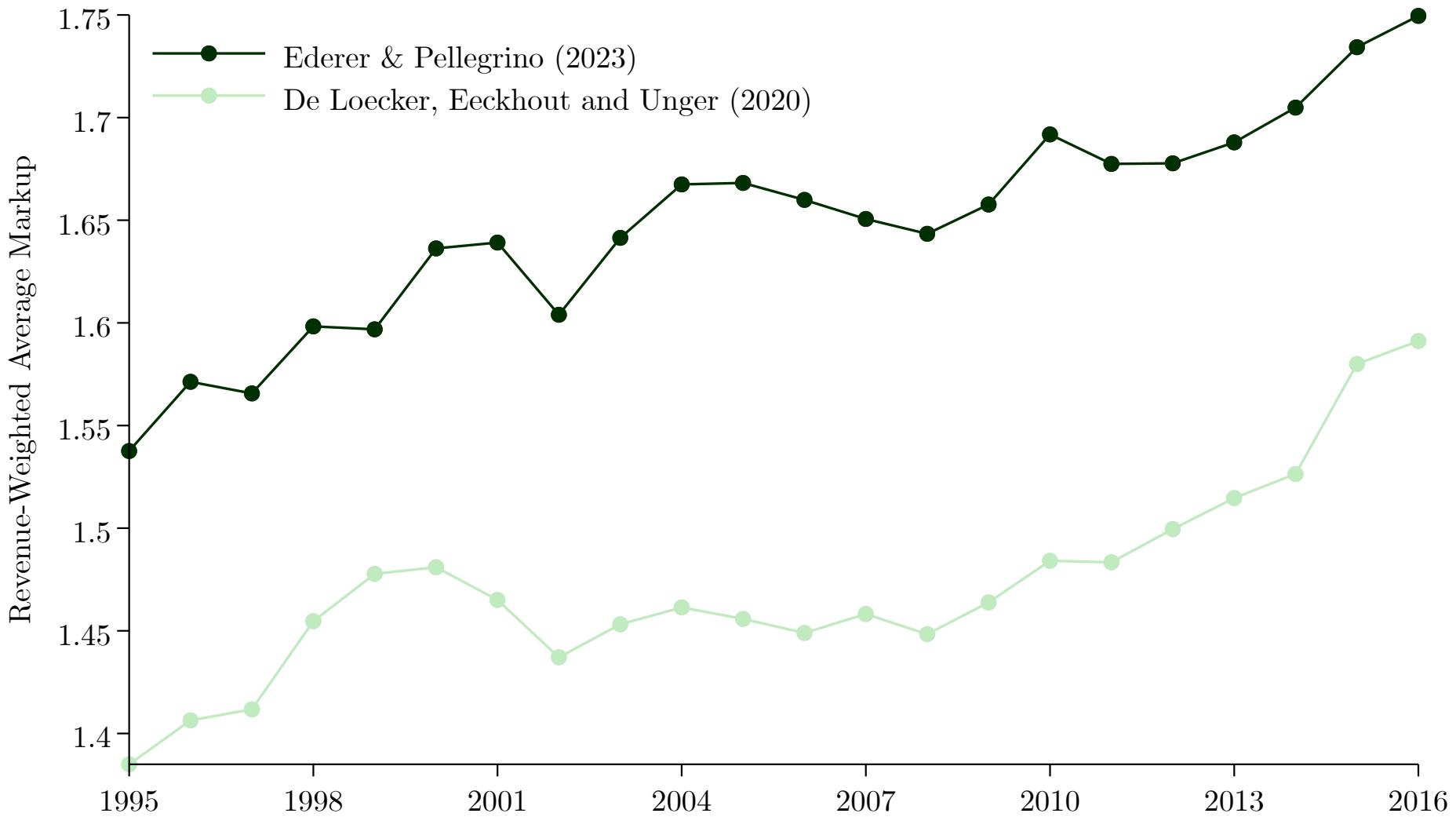
Core words: client (59), database (54), solution (49), patient (47), copyright (47), secret (47), physician (47), hospital (46), health care (46), server (45), resource (44), functionality (44), billing (44), . . .



Linear vs. Isoelastic Demand



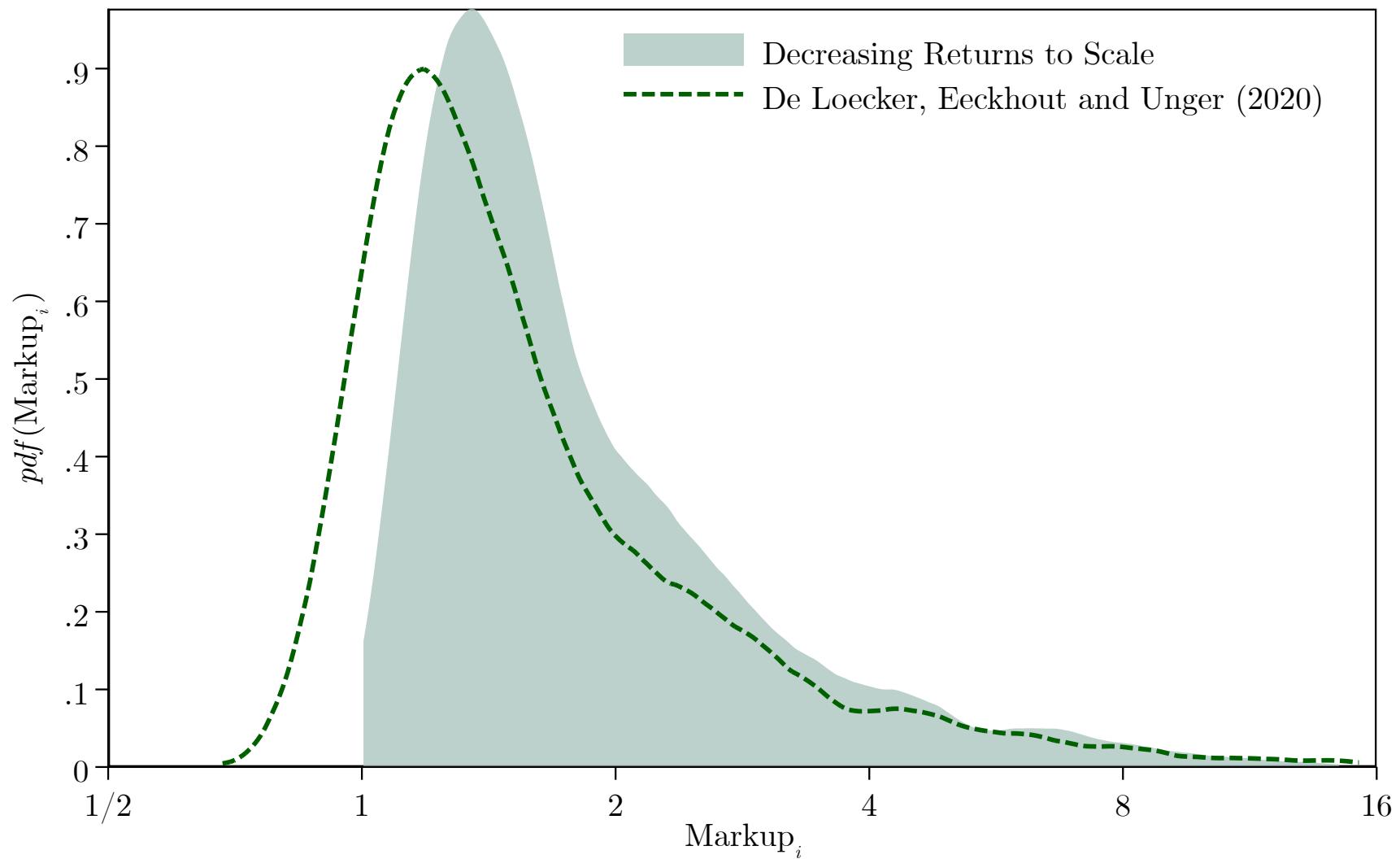
Markups: Time Series



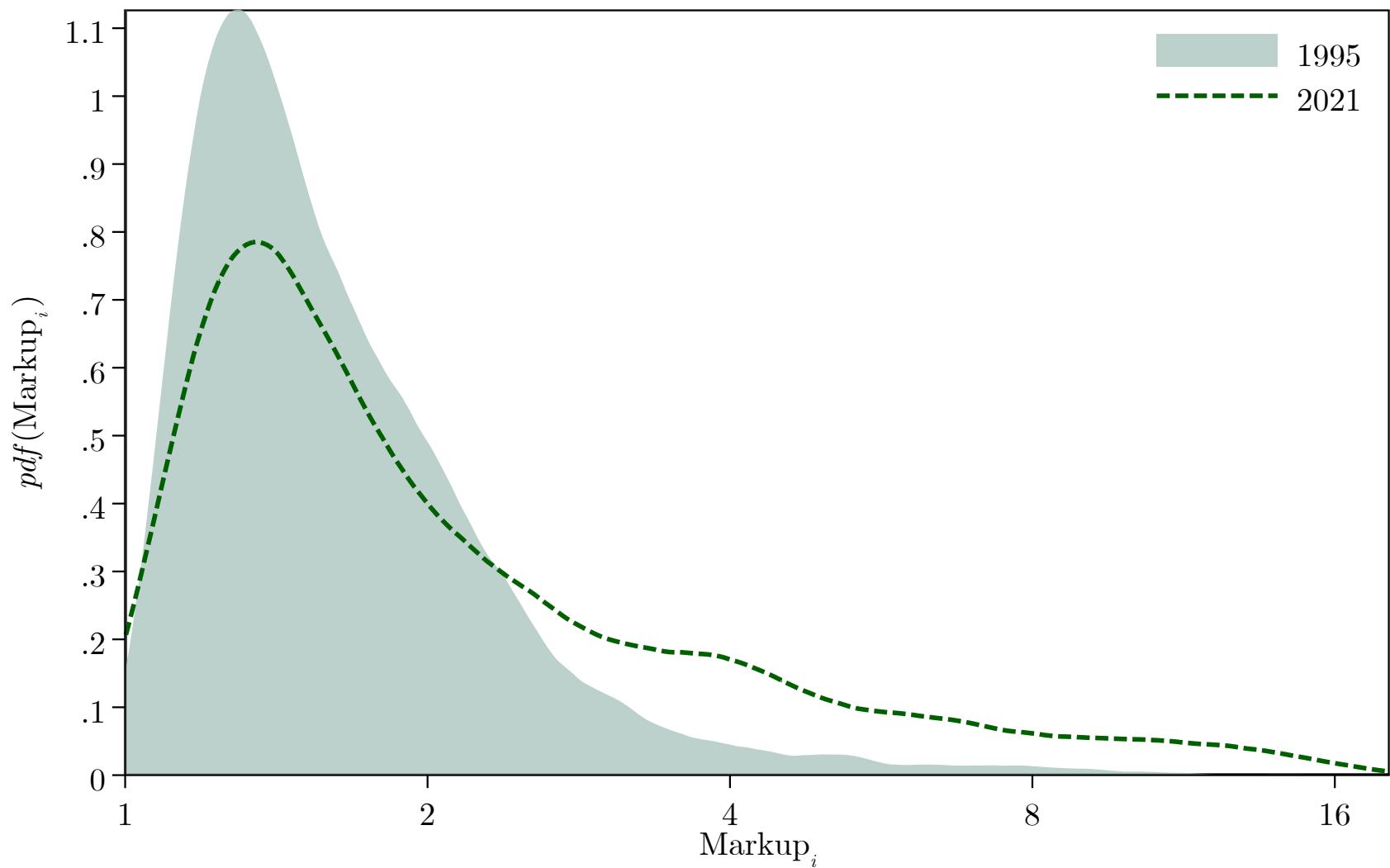
Markups: EP22 vs. DEU20

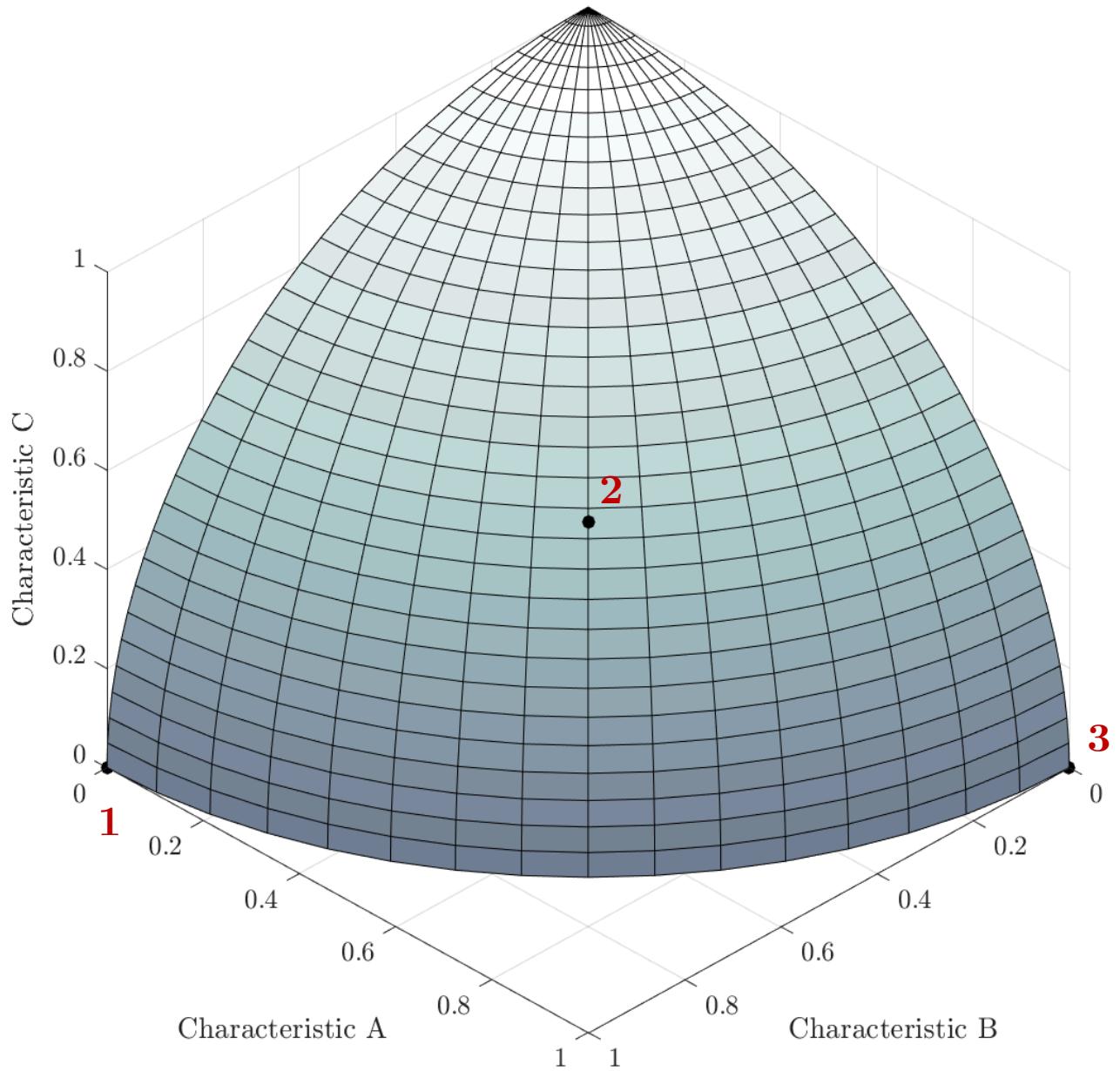


Markup Distribution: EP22 vs. DEU20



Distribution of Markups





$$\begin{aligned} \frac{\partial \mathbf{p}}{\partial \mathbf{q}} &\equiv -(\mathbf{I} + \boldsymbol{\Sigma}) \\ &= \begin{bmatrix} -1 & -.58 & 0 \\ -.58 & -1 & -.58 \\ 0 & -.58 & -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \frac{\partial \mathbf{q}}{\partial \mathbf{p}} &\equiv -(\mathbf{I} + \boldsymbol{\Sigma})^{-1} \\ &= \begin{bmatrix} -2 & 1.73 & -1 \\ 1.73 & -3 & 1.73 \\ -1 & 1.73 & -2 \end{bmatrix} \end{aligned}$$