Νευρο-Ασαφής Υπολογιστική

ΣΕΤ 3

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Πρόβλημα - 01

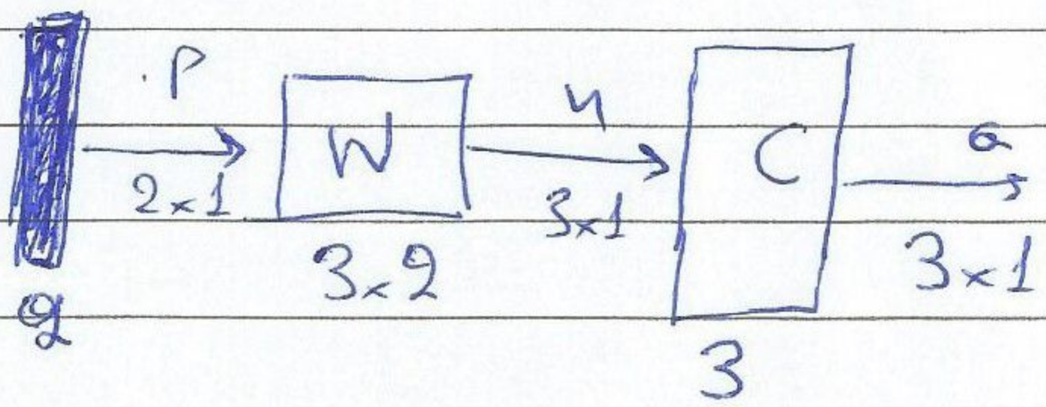
$$P_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad P_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad t_1 = 1, \; t_2 = -1$$

$$W = t_1 P_1^T + t_2 P_2^T = 1\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} + (-1)\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow W = \begin{bmatrix} 0 & -1 & 0 & 0 & 1 & -1 \end{bmatrix}$$

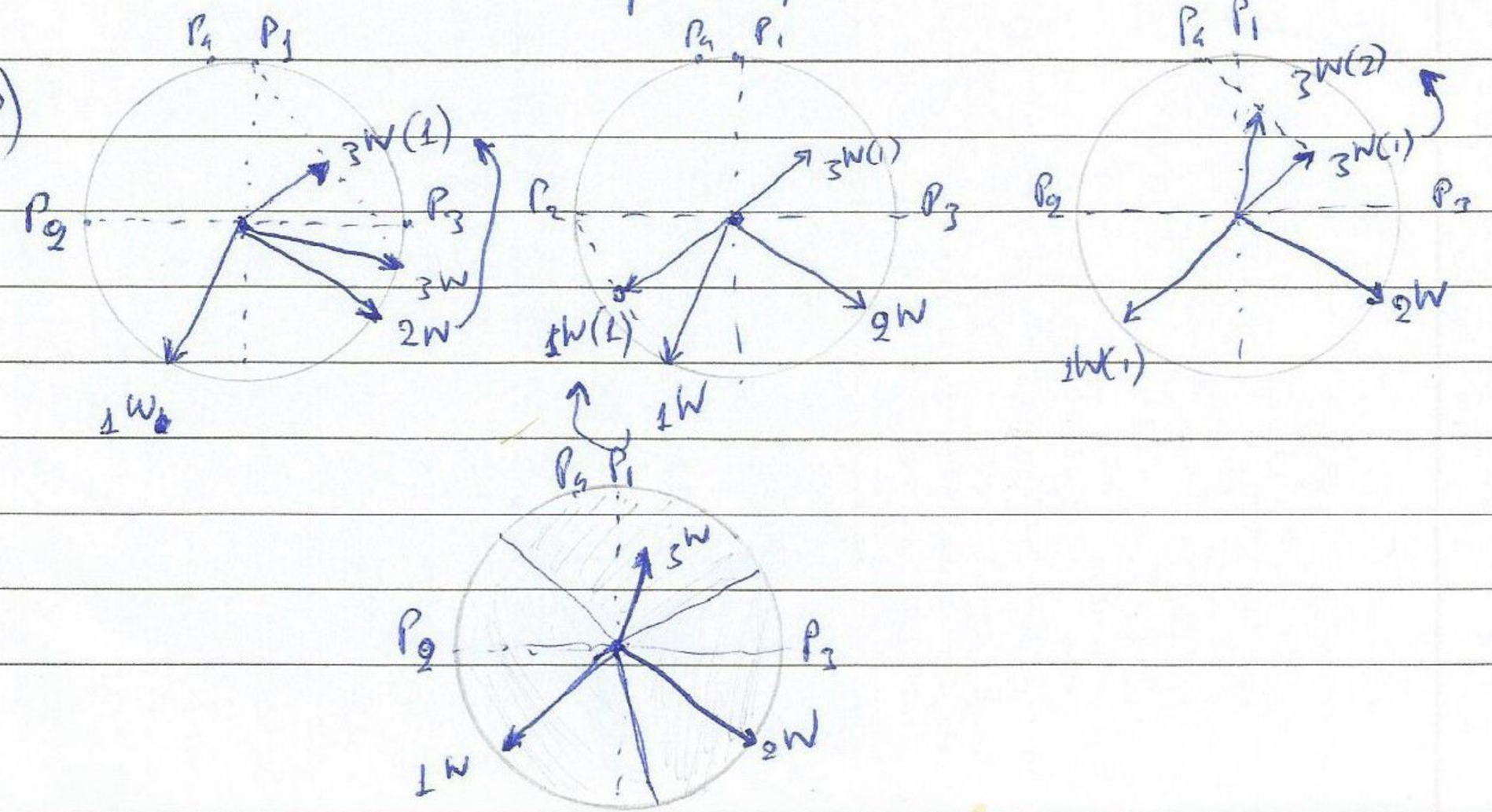
Πρόβλημα - 02

1)



$$a = \text{compet}(W_p)$$

2)



Приведба - 03

$U = \{2, 3, 4, 5, 8, 10\}$ $X = \{1, 2, 4, 6, 9\}$

$I_1 = \{1, 2, 3\}$
$I_2 = \{1, 2\}$
$I_3 = \{1, 2, 3\}$
$I_4 = \{1, 2\}$
$I_5 = \{1\}$
$I_6 = \{1, 2, 3\}$
$I_7 = \{1, 2, 3\}$
$I_8 = \{1, 2\}$
$I_9 = \{1\}$
$I_{10} = \{1, 2, 3, 4, 5, 6\}$

$DI_{1,1} = \{0\}$
$DI_{2,1} = DI_{3,1} = \dots = DI_{10,1} = 0$
$DL_{2,2} = \{1\}$, $DL_{4,3} = \{1\}$, $DL_{4,4} = \{1\}$, $DL_{6,5} = \{0, 1\}$, $DL_{9,8} = \{1\}$
$L_1^t = L_2^t = L_3^t = L_4^t = L_5^t = L_8^t = 0$
$L_1^b = L_3^b = L_5^b = L_6^b = L_7^b = L_8^b = L_9^b = L_{10}^b = 0$
$L_2^b = \{2\}$ $L_4^b = \{4, 7\}$
$E_{LW}^U(x) = \{2, 4\}$
$E_{LX}^U(U) = \{2, 4\}$

Πρόβλημα - 04

$$\frac{\partial a^u(t)}{\partial x^T} = \frac{\partial^e a^u(t)}{\partial x^T} + \sum_{x \in E_s^x(u)} s^{u,x}(t)$$

$$\sum_{u' \in E^u_{LW(x)}} \sum_{d \in Dl_{x,u'}} LW^{x,u'(d)} \times \frac{\partial a^{u'}(t-d)}{\partial x^T}$$

για $v = 2$

$$\frac{\partial a^2(t)}{\partial x^T} = \frac{\partial^e a^2(t)}{\partial x^T} + s^{2,1}(t) \left[\sum_{d=1}^{D} LW^{1,1}(d) \times \frac{\partial a^1(t-d)}{\partial x^T} \right]$$

$(s^1)^2$ πράξεις

για κάθε Q και D

Άρα $O((s^1)^2 \cdot D(Q))$

Πρόβλημα - 05

$$h_{11} = x_{11} \cdot w_{11} + x_{12} w_{12} + x_{21} w_{21} + x_{22} w_{22}$$
$$h_{12} = x_{12} w_{11} + x_{13} w_{12} + x_{22} w_{21} + x_{23} w_{23}$$
$$h_{21} = x_{21} w_{11} + x_{22} w_{12} + x_{31} w_{21} + x_{32} w_{23}$$
$$h_{22} = x_{22} w_{11} + x_{23} w_{12} + x_{32} w_{21} + x_{33} w_{22}$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial h_{11}} \cdot \frac{\partial h_{11}}{\partial w_{11}} + \frac{\partial L}{\partial h_{12}} \cdot \frac{\partial h_{12}}{\partial w_{11}} + \frac{\partial L}{\partial h_{21}} \frac{\partial h_{21}}{\partial w_{11}} + \frac{\partial L}{\partial h_{22}} \cdot \frac{\partial h_{22}}{\partial w_{11}}$$

$$= x_{11} \frac{\partial L}{\partial h_{11}} + x_{12} \frac{\partial L}{\partial h_{12}} + x_{21} \frac{\partial L}{\partial h_{21}} + x_{22} \frac{\partial L}{\partial h_{22}}$$

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial h_{11}} \cdot \frac{\partial h_{11}}{\partial w_{12}} + \frac{\partial L}{\partial h_{12}} \cdot \frac{\partial h_{12}}{\partial w_{12}} + \frac{\partial L}{\partial h_{21}} \cdot \frac{\partial h_{21}}{\partial w_{12}} + \frac{\partial L}{\partial h_{22}} \cdot \frac{\partial h_{22}}{\partial w_{12}}$$

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial h_{11}} \cdot \frac{\partial h_{11}}{\partial w_{21}} + \frac{\partial L}{\partial h_{12}} \cdot \frac{\partial h_{12}}{\partial w_{21}} + \frac{\partial L}{\partial h_{21}} \cdot \frac{\partial h_{21}}{\partial w_{21}} + \frac{\partial L}{\partial h_{22}} \cdot \frac{\partial h_{22}}{\partial w_{21}}$$

$$= x_{21} \frac{\partial L}{\partial h_{11}} + x_{22} \frac{\partial L}{\partial h_{12}} + x_{31} \frac{\partial L}{\partial h_{21}} + x_{32} \frac{\partial L}{\partial h_{22}}$$

$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial h_{11}} \cdot \frac{\partial h_{11}}{\partial w_{22}} + \frac{\partial L}{\partial h_{12}} \cdot \frac{\partial h_{12}}{\partial w_{22}} + \frac{\partial L}{\partial h_{21}} \cdot \frac{\partial h_{21}}{\partial w_{12}} + \frac{\partial L}{\partial h_{22}} \cdot \frac{\partial h_{22}}{\partial w_{12}}$$

$$= x_{22} \frac{\partial L}{\partial h_{11}} + x_{23} \frac{\partial L}{\partial h_{12}} + x_{32} \frac{\partial L}{\partial h_{21}} + x_{33} \frac{\partial L}{\partial h_{22}}$$

Πρόβλημα - 06

kernel dims # filters

Layer 1: $5 \cdot 5 \cdot 1 \cdot 100 = 2500$ weights

Layer 2: $5 \cdot 5 \cdot 100 = 250.000$ -//-

Layer 3: 0 -//-

Layer 4: $100 \cdot 100 \cdot 50 = 500.000$ -//-

Layer 5: $100 \cdot 50 = 500$ -//-

Σύνολο: 757.500 weights

(Δεν είμαστε τελείως σίγουροι για τα layers 4 & 5).

Πρόβλημα - 07

Input: [0 0 0 | 1 1 1 | 0 0 0]

convolve with: [1 0 -1] valid padding

Output:

0	0	1	1	0	0	-1	-1	0	0
0	0	-1	-1	0	0	1	1	0	0

Max pooling
stride = 2
f.s. = 2

0	1	0	-1	0
0	-1	0	1	0

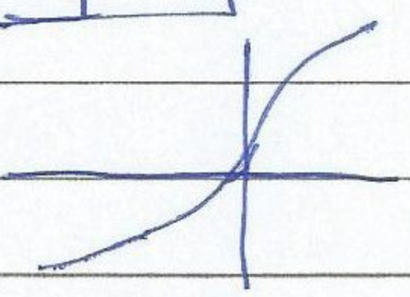
convolve with

-1	0	1
1	0	-1

Output [0 | -4 | 0]

fully connected [4 | -4]

Sigmoid activation



Output: [0.99 | 0.1]

β) Φαίνεται πως είναι διαφορικά input ως προς το (α) ερώτημα με την έννοια πως το μόνο που αλλάζει είναι πως ανεί για +1 έχουμε -1 και το αντίστροφο.

Το ίδιο συμβαίνει για το F.h. επίπεδο όπου έχουμε -4, +4.

Άρα ως output θα έχουμε | 0.01 | 0.99 |

Примјер 08

$A = \{(A|0), (B|0.3), (C|0.7), (D|1), (E|0), (f|0.2), (G|0.6)\}$

$B = \{(A|0.3), (B|1), (C|0.5), (D|0.8), (E|1), (F|0.5), (G|0.6)\}$

$C = \{(A|1), (B|0.5), (C|0.5), (D|0.2), (E|0), (F|0.2), (G|0.9)\}$

1. $A \cap B = \{(A|0), (B|0.3), (C|0.5), (D|0.8), (E|0), (F|0.2), (G|0.6)\}$

2. $A \cup B = \{(A|0.3), (B|1), (C|0.7), (D|1), (E|1), (F|0.5), (G|0.6)\}$

3. $B^c = \{(A|0.7), (B|0), (C|0.5), (D|0.2), (E|0), (F|0.5), (G|0.4)\}$

   $A \cap B^c = \{(A|0), (B|0), (C|0.5), (D|0.2), (E|0), (F|0.2), (G|0.4)\}$

4. $A \cup B^c = \{(A|0.7), (B|0.3), (C|0.7), (D|1), (E|0), (F|0.5), (G|0.6)\}$

   $(A \cup B^c) \cap C = \{(A|0.7), (B|0.3), (C|0.5), (D|0.2), (E|0), (F|0.2), (G|0.6)\}$

5. $(A \cap B)^c = \{(A|1), (B|0.7), (C|0.5), (D|0.2), (E|1), (F|0.8), (G|0.4)\}$

   ~~$(A \cap B)^c \cap C$~~

   $C^c = \{(A|0), (B|0.5), (C|0.5), (D|0.8), (E|1), (F|0.8), (G|0.1)\}$

   $(A \cap B)^c \cup C^c = \{(A|1), (B|0.7), (C|0.5), (D|0.8), (E|1), (F|0.8), (G|0.4)\}$

6. $(A \cap A^c) \cup A = A$!

Πρόβλημα - σ 9

1. $X \cap Y = (X^c \cup Y^c)^c$

Έστω $A = X \cap Y$ και $B = (X^c \cup Y^c)^c$

Εάν $x \in A \Rightarrow x \in X \cap Y \Rightarrow x \in X$ και $x \in Y \Rightarrow$

$\Rightarrow x \notin X^c$ και $x \notin Y^c \Rightarrow$

$\Rightarrow x \notin (X^c \cup Y^c) \Rightarrow$

$\Rightarrow x \in (X^c \cup Y^c)^c \Rightarrow x \in B \Rightarrow A \subseteq B$ (1)

Έστω Εάν $x \in B \Rightarrow x \in (X^c \cup Y^c)^c \Rightarrow x \notin (X^c \cup Y^c) \Rightarrow$

$\Rightarrow x \notin X^c$ και $x \notin Y^c \Rightarrow x \in X$ και $x \in Y \Rightarrow$

$\Rightarrow x \in X \cap Y \Rightarrow x \in A \Rightarrow B \subseteq A$ (2)

Από (1), (2) $\Rightarrow A = B \Rightarrow X \cap Y = (X^c \cup Y^c)^c$

2. $X \cup Y = (X^c \cap Y^c)^c$

Έστω $A = X \cup Y$ και $B = (X^c \cap Y^c)^c$

Εάν $x \in A \Rightarrow x \in (X \cup Y) \Rightarrow x \in X$ ή $x \in Y \Rightarrow$

$\Rightarrow x \notin X^c$ ή $x \notin Y^c \Rightarrow x \notin (X^c \cap Y^c)$

$\Rightarrow x \in (X^c \cap Y^c)^c \Rightarrow x \in B \Rightarrow A \subseteq B$ (1)

Εάν $x \in B \Rightarrow x \in (X^c \cap Y^c)^c \Rightarrow x \notin (X^c \cap Y^c) \Rightarrow$

$\Rightarrow x \notin X^c$ ή $x \notin Y^c \Rightarrow x \in X$ ή $x \in Y \Rightarrow$

$\Rightarrow x \in (X \cup Y) \Rightarrow x \in A \Rightarrow B \subseteq A$ (2)

Από (1), (2) $\Rightarrow A = B \Rightarrow X \cup Y = (X^c \cap Y^c)^c$

Πρόβλημα - 10

$$N_a(N_a(x)) = x \quad , \quad N_a(x) = \frac{1-x}{1+ax} \quad , \quad a>1, \; 0 \le x < 1$$

- $N_a(N_a(x)) = \dfrac{1-\left(\dfrac{1-x}{1+ax}\right)}{1+a\cdot\left(\dfrac{1-x}{1+ax}\right)} = \dfrac{1+ax-(1-x)}{1+ax+a(1-x)} =$

$$= \frac{ax+x}{1+\cancel{ax}+a} = \frac{x(a+1)}{1+a} \Rightarrow N_a(N_a(x)) = x$$