Νευρο-Ασαφής Υπολογιστική
Χειμερινό Εξάμηνο 2018-2019
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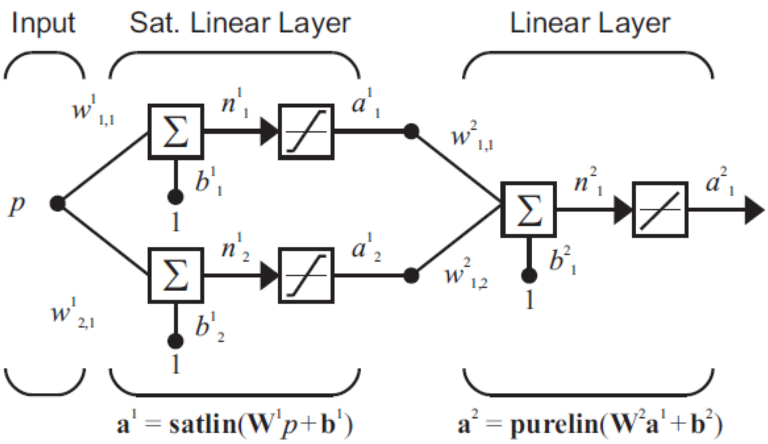
Σειρά προβλημάτων: 1η: ΟΜΑΔΙΚΕΣ (2-ΑΤΟΜΩΝ) ΕΡΓΑΣΙΕΣ

Ημέρα ανακοίνωσης: Monday, October 08, 2018
Προθεσμία παράδοσης: Κυριακή, Νοέμβριος 04, 2018

Problem-01
Consider the following neural network



$$w^1_{1,1} = 2,\ w^1_{2,1} = 1,\ b^1_1 = 2,\ b^1_2 = -1,\ w^2_{1,1} = 1,\ w^2_{1,2} = -1,\ b^2_1 = 0$$

Sketch the following responses (plot the indicated variable versus p for -3 < p < 3).

- **i.** n^1_1.
- **ii.** a^1_1.
- **iii.** n^1_2
- **iv.** a^1_2.
- **v.** n^2_1.
- **vi.** a^2_1.

Problem -02
The symmetric hard limit function is sometimes used in perceptron networks, instead of the hard limit function. Target values are then taken from the set [-1, 1] instead of [0, 1].

- ➢ Write a simple expression that maps numbers in the ordered set [0, 1] into the ordered set [-1, 1]. Write the expression that performs the inverse mapping.
- ➢ Consider two single-neuron perceptrons with the same weight and bias values. The first network uses the hard limit function ([0, 1] values), and the second network uses the symmetric hard limit function. If the two networks are given the same input

p, and updated with the perceptron learning rule, will their weights continue to have the same value?

- If the changes to the weights of the two neurons are different, how do they differ? Why?
- Given initial weight and bias values for a standard hard limit perceptron, create a method for initializing a symmetric hard limit perceptron so that the two neurons will always respond identically when trained on identical data.

Problem-03

The vectors in the ordered set defined below were obtained by measuring the weight and ear lengths of toy rabbits and bears. The target values indicate whether the respective input vector was taken from a rabbit (0) or a bear (1). The first element of the input vector is the weight of the toy, and the second element is the ear length.

$$\left\{\mathbf{p}_1 = \begin{bmatrix}1\\4\end{bmatrix}, t_1 = 0\right\}\left\{\mathbf{p}_2 = \begin{bmatrix}1\\5\end{bmatrix}, t_2 = 0\right\}\left\{\mathbf{p}_3 = \begin{bmatrix}2\\4\end{bmatrix}, t_3 = 0\right\}\left\{\mathbf{p}_4 = \begin{bmatrix}2\\5\end{bmatrix}, t_4 = 0\right\}$$

$$\left\{\mathbf{p}_5 = \begin{bmatrix}3\\1\end{bmatrix}, t_5 = 1\right\}\left\{\mathbf{p}_6 = \begin{bmatrix}3\\2\end{bmatrix}, t_6 = 1\right\}\left\{\mathbf{p}_7 = \begin{bmatrix}4\\1\end{bmatrix}, t_7 = 1\right\}\left\{\mathbf{p}_8 = \begin{bmatrix}4\\2\end{bmatrix}, t_8 = 1\right\}$$

- Use MATLAB (or Python) to initialize and train a network to solve this "practical" problem.
- Use MATLAB (or Python) to test the resulting weight and bias values against the input vectors.
- Add input vectors to the training set to ensure that the decision boundary of any solution will not intersect one of the original input vectors (i.e., to ensure only robust solutions are found). Then retrain the network. Your method for adding the input vectors should be general purpose (not designed specifically for this problem).

Problem-04

Let t : **S**$\rightarrow$[0,1] be a continuous or "fuzzy" truth function on the set **S** of statements. Define the implication operator as the truth function $t_L(A\rightarrow B)=\min(1, 1-t(A)+t(B))$ for statements A and B. Then prove the following generalized fuzzy *modus ponens* inference rule:

	$t_L(A\rightarrow B)$	$=$	c
	$t(A)$	$\geq$	α
Therefore	$t(B)$	$\geq$	$\max(0, \alpha+c-1)$

Hence, if $t(A)= t_L(A\rightarrow B)= 1$, then $t(B)=1$, which generalizes classical bivalent *modus ponens*.

Problem-05

Use the multivalued logic operations of the previous problem to prove the following generalized *modus tollens* inference rule:

	$t_L(A\rightarrow B)$	$=$	c
	$t(B)$	$\leq$	b
Therefore	$t(A)$	$\leq$	$\min(1, 1-c+b)$

Hence, if $t_L(A\rightarrow B)= 1$ and $t(B)=0$, then $t(A)=0$, which generalizes classical bivalent *modus tollens*.

Problem-06

Set X contains *n* elements $x_1, \ldots, x_n$. So, X contains 2^n nonfuzzy subsets. Define the bivalent indicator function I_A of nonfuzzy set A as follows:

$$I_A(x_i) = \begin{cases} 1, \text{ if } x_i \in A \\ \\ 0, \text{ if } x_i \notin A. \end{cases}$$

So, I_A defines the mapping $I_A: X \rightarrow \{0,1\}$.

Suppose we extend I_A to a multivalued mapping by augmenting its range from $\{0,1\}$ to $\{y_1,\ldots,y_m\}$, where $y_1=0$, $y_m=1$, and $0< y_j<1$ if $1<j<m$. Then, I_A defines the mapping I_A: $X \rightarrow \{ y_1,\ldots,y_m \}$.

- How many multivalued subsets does X have?
- In the two-dimensional case, $X=\{x_1,x_2\}$, draw the planar lattice that describes the multi-dimensional powerset of X, when m=3, and when m=5.

Problem-07

Consider the discrete dynamical system

$$\begin{aligned} x_{k+1} &= f(x_k) \\ &= c*x_k(1-x_k) \end{aligned}$$

for x values in (0,1) and $0<c\leq 4$. Many dynamical systems transition into chaos as we increase a control or gain parameter such as c. Select c=3.5 and use the two choices of initial conditions, $x_0=0.5$ and $x_0=0.51$, to generate $x_1,\ldots,x_{20}$. Plot the two trajectories. Are they aperiodic (chaotic) or periodic? Does a difference in 0.01 in initial condition significantly affect the overall shape of the discrete trajectory?

Now repeat the above experiment but use the gain parameter c=3.9 (or c=4). No matter how close two initial conditions, in a chaotic dynamical system they always produce divergent trajectories. Does c=3.9 produce chaos?

Problem-08

Show that the following signal functions S have activation derivatives dS/dx, denoted S', of the stated form:

1.
$$\text{If } S = \frac{1}{1+e^{-cx}}, \text{ then } S' = cS(1-S).$$

2.
$$\text{If } S = e^{-cx^2}, \text{ then } S' = -2cxe^{-cx^2}.$$

3.
$$\text{If } S = tanh(cx), \text{ then } S' = c(1-S^2).$$

4.
$$\text{If } S = e^{cx}, \text{ then } \frac{d^n S}{dx^n} = c^n e^{cx}.$$

5.
$$\text{If } S = 1 - e^{-cx}, \text{ then } S'' = -c^2 e^{-cx}.$$

6.
$$\text{If } S = \frac{x^n}{c+x^n}, \text{ then } S' = \frac{cnx^{n-1}}{(c+x^n)^2}.$$

Problem-09

Consider the logistic signal function:

$$S(x) = \frac{1}{1+e^{-cx}}, \quad c > 0.$$

- Solve the logistic signal function S(x) for the activation x.
- Show that x strictly increases with logistic S.
- Explain why in general the "inverse function" x increases with S, if S'>0.

Problem-10

In the respective class lecture we proved Perceptron's converge when T > 0. Prove the same, for the special case when T=0.

<u>Χρηστικές πληροφορίες</u>:
Η προθεσμία παράδοσης είναι αυστηρή. Είναι δυνατή η παροχή παράτασης (μέχρι 2 ημέρες), αλλά μόνο αφού δώσει ο διδάσκων την έγκρισή του και αυτή η παράταση στοιχίζει 10% ποινή στον τελικό βαθμό της συγκεκριμένης Σειράς Προβλημάτων. Η παράδοση γίνεται με email του αρχείου λύσεων σε μορφή pdf (typeset). Το subject του μηνύματος αυστηρά πρέπει να είναι: CE418-Problem set 01: AEM1-AEM2

<u>Ερμηνεία συμβόλων</u>:


Δεν απαιτεί την χρήση υπολογιστή ή/και την ανάπτυξη κώδικα.

Απαιτεί την χρήση του Web για ανεύρεση πληροφοριών ή διεξαγωγή πειράματος.

Απαιτεί την ανάπτυξη κώδικα σε όποια γλώσσα προγραμμαστιμού ή Matlab. Το παραδοτέο θα περιέχει:

- ❖ Την λύση της άσκησης
- ❖ Τον πηγαίο κώδικα υλοποίησης