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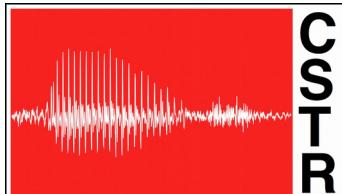
SpeechWave



Contrastive Representation Learning

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Outline

- Contrastive Learning
- Unsupervised Contrastive Learning
 - CPC
 - SimCLR
- Supervised Contrastive Learning
- Conclusion



Self-Supervised Learning (SSL)

- **Goal:** Learning universal transferable representation
- **Paradigms:**
 - Generative
 - Focus on sample-level reconstruction + Independent assumption
 - Lower ability in modelling correlation & structure
 - Contrastive
 - Learn by contrasting *positive* & *negatives* in a *latent space*
 - ...

Contrastive Learning

- Learn an encoder, $f(x)$, such that ...

$$\text{score}(f(x), f(x^+)) \gg \text{score}(f(x), f(x^-))$$

- x : **Anchor** (reference or baseline)
- x^+ : **Positive** (similar) \rightarrow data augmentation (view)
- x^- : **Negative** (dissimilar) \rightarrow sampling (???)
- **Score**: a similarity/agreement measure
- Contrastive Loss ... *distance-based (metric learning)* ... NOT *error-prediction*

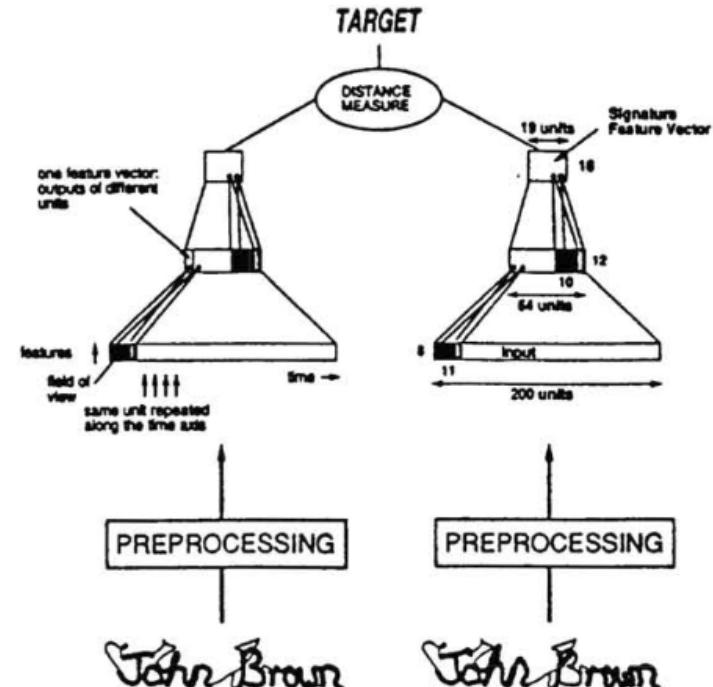
Siamese Neural Network (SNN)

Advances in Neural Information Processing, 1994

Signature Verification using a “Siamese” Time Delay Neural Network

Jane Bromley, Isabelle Guyon, Yann LeCun,
Eduard Säckinger and Roopak Shah
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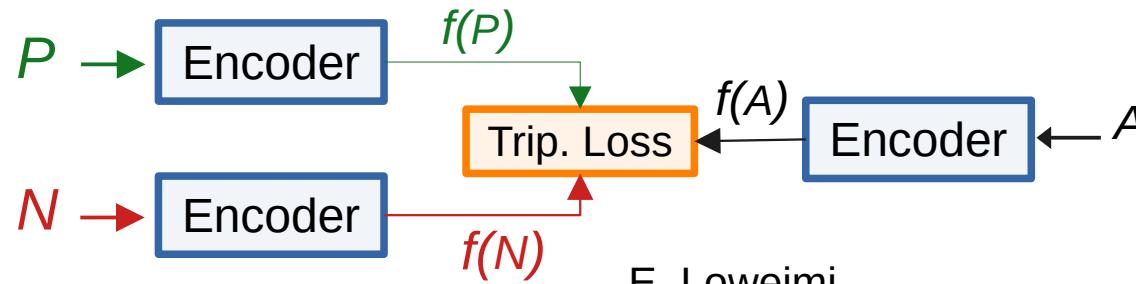


Siamese Neural Network (SNN)

- Returns embedding (similar \leftrightarrow close), NOT $p(y|x)$
- Contains two identical subnets (encoder)
- Loss: $L = L^+ + L^-$ or *Triplet loss*: $L(A, P, N)$
- Hard negative mining required ($f(N)$ nearby $f(A)$)

α : margin

$$\mathcal{L}_{\text{Triplet}}(A, P, N) = \max(\| f(A) - f(P) \|^2 - \| f(A) - f(N) \|^2 + \alpha, 0)$$



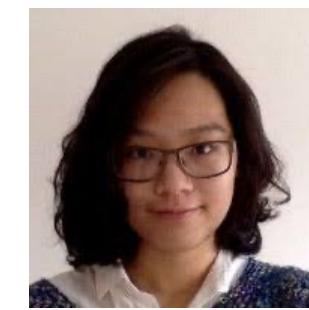
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Representation Learning with Contrastive Predictive Coding

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Contrastive Predictive Coding (CPC)

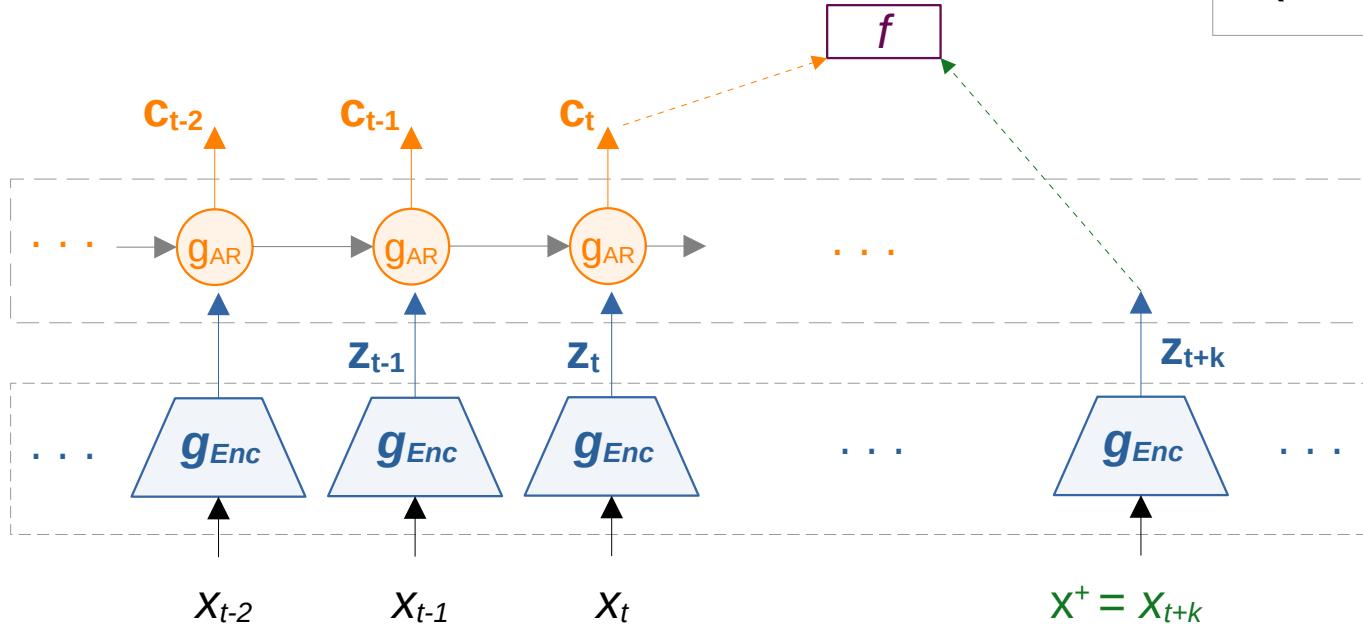
- **Coding:** Representation Learning
- **Predictive:**
 - Models correlation between “*history+current*” & “*future*”
 - Requires learning global/local structure & shared info between parts ... beyond local smoothness
- **Contrastive:** Learning paradigm ... Loss

CPC Components

- Architecture: Encoder + AutoRegressive
- Model: Log-Bilinear (similarity measure)

$$Z_t = g_{\text{Enc}}(x_t) \quad C_t = g_{\text{AR}}(Z_t)$$

$$f_k(x_{t+k}, C_t) = \exp(Z_{t+k}^T W_k C_t)$$



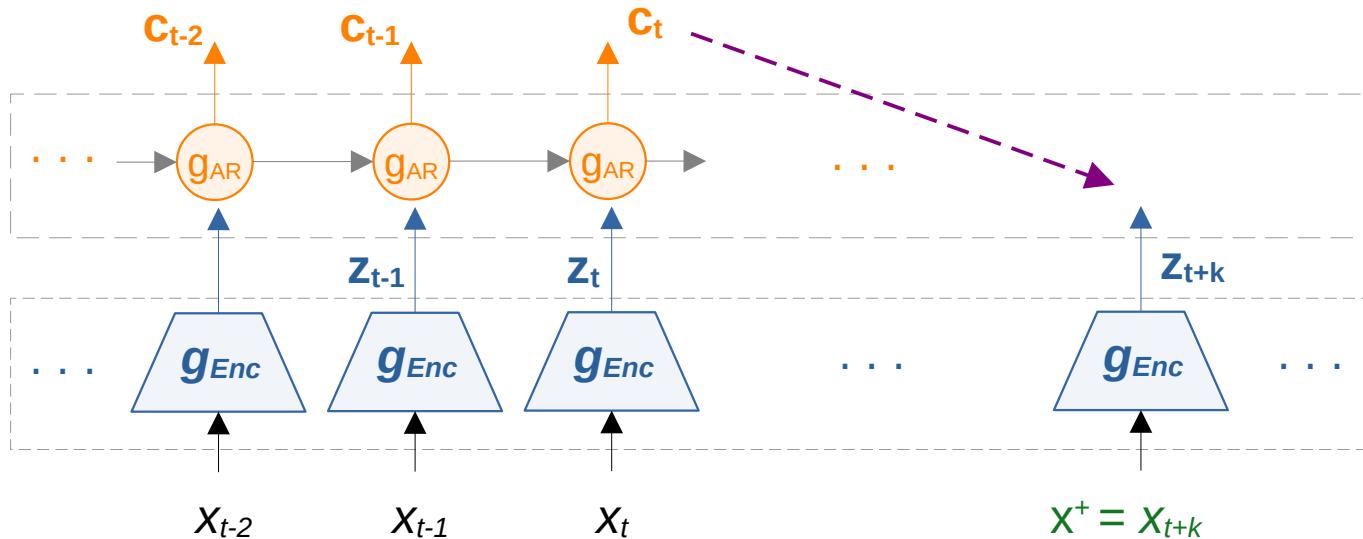
 g_{AR} : RNN-ish
 g_{enc} : CNN-ish

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CPC Goal

- * Goal: using c_t , predict z_{t+k}
- * Question: Predict “ $t+k$ ” or “ $t+1 \leq t \leq t+k$ ” ???

c_t : “current + history”
 x_{t+k} : “future”

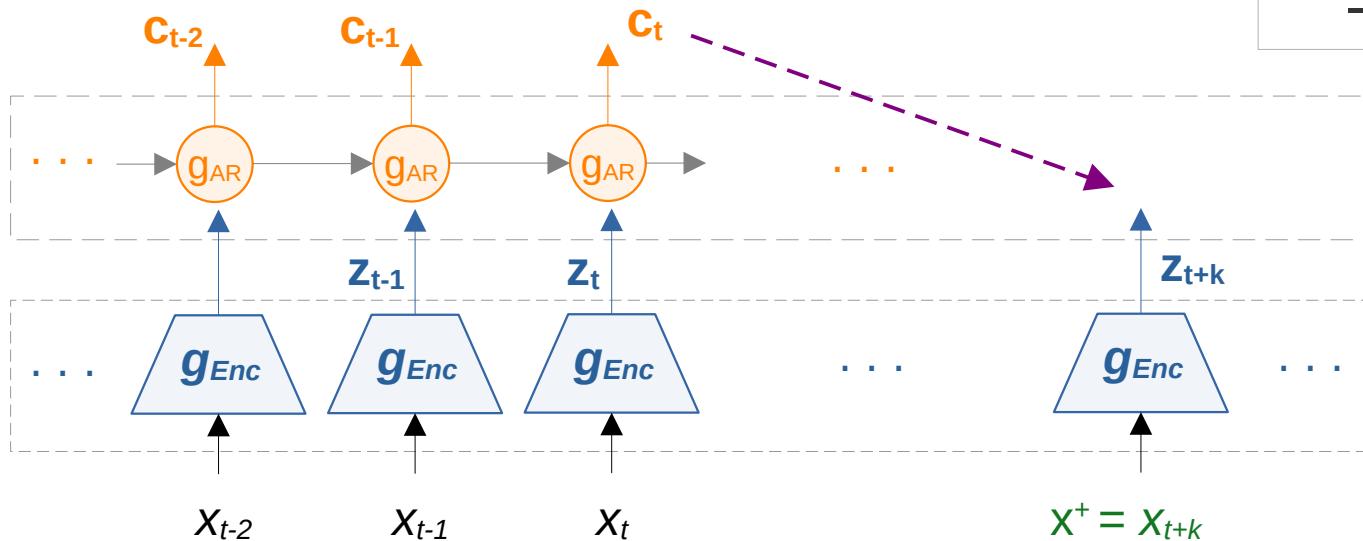


 g_{AR} : RNN-ish
 g_{enc} : CNN-ish

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CPC Goal

- * Goal: using c_t , predict z_{t+k}
- * How: Contrastive Loss



Triplet ...

- Anchor: c_t
- $x^+ \sim P(x|c_t) \leftrightarrow x^+ = x_{t+k}$
- $x^- \sim P(x_{t+k})$

Proposal distribution

CPC Loss: InfoNCE

- Given $X = \{x_1, x_2, \dots, x_N\}$, $x^+ = x^{t+k}$; $\#x^- = N-1$
- InfoNCE* is different from NCE** loss used in Word2Vec

$$\begin{aligned}\mathcal{L}_X(\text{anchor}) &= -\log \frac{\text{sim}(x^+, \text{anchor})}{\sum_{x_j \in X} \text{sim}(x_j, \text{anchor})} && \text{General form} \\ &= -\log \frac{\text{sim}(x^+, \text{anchor})}{\text{sim}(x^+, \text{anchor}) + \sum_{x_j \in X_{neg}} \text{sim}(x_j, \text{anchor})}\end{aligned}$$

$$\mathcal{L}^{\text{InfoNCE}} = \mathbb{E}_X [\mathcal{L}_X]$$

* NCE: Noise Contrastive Estimation

** Actually, negative sampling (simpler version) is used.

CPC Loss: InfoNCE

- Given $X = \{x_1, x_2, \dots, x_N\}$, $\textcolor{green}{X^+} = X^{t+k}$; $\#\textcolor{red}{X^-} = N-1$
- InfoNCE* is different from NCE** loss used in Word2Vec

$$\begin{aligned}\mathcal{L}_X &= -\log \frac{f_k(x_{t+k}, c_t)}{\sum_{x_j \in X} f_k(x_j, c_t)} \\ &= -\log \frac{f_k(x_{t+k}, c_t)}{f_k(x_{t+k}, c_t) + \sum_{x_j \in X_{neg}} f_k(x_j, c_t)}\end{aligned}$$

In CPC context
←

$$\mathcal{L}^{\text{InfoNCE}} = \mathbb{E}_X [\mathcal{L}_X]$$

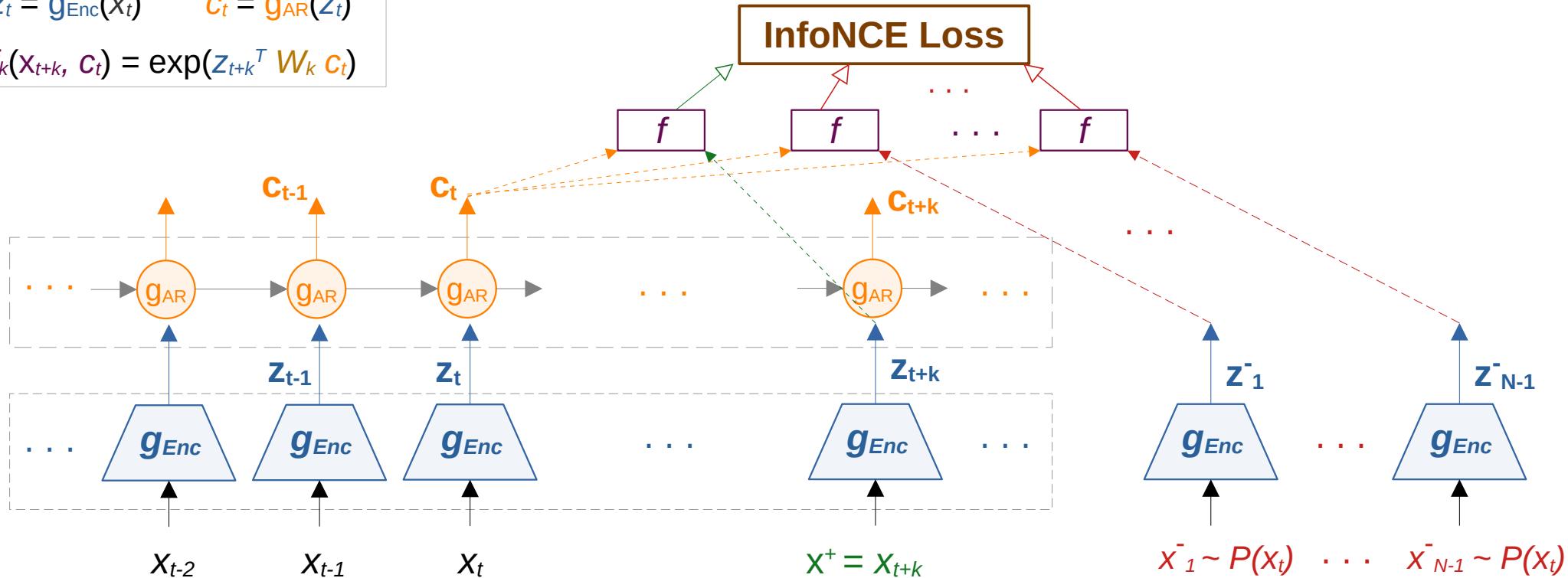
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CPC Framework

$$z_t = g_{\text{Enc}}(x_t) \quad c_t = g_{\text{AR}}(z_t)$$

$$f_k(x_{t+k}, c_t) = \exp(z_{t+k}^T W_k c_t)$$

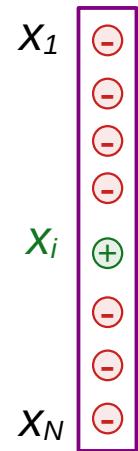



 g_{AR} : RNN-ish
 g_{enc} : CNN-ish

InfoNCE Interpretation (1)

- Given $X = \{x_1, x_2, \dots, x_N\}$, $x^+ = x^{t+k}$; $\#x^- = N-1$
- InfoNCE Loss \equiv Categorical CE Loss

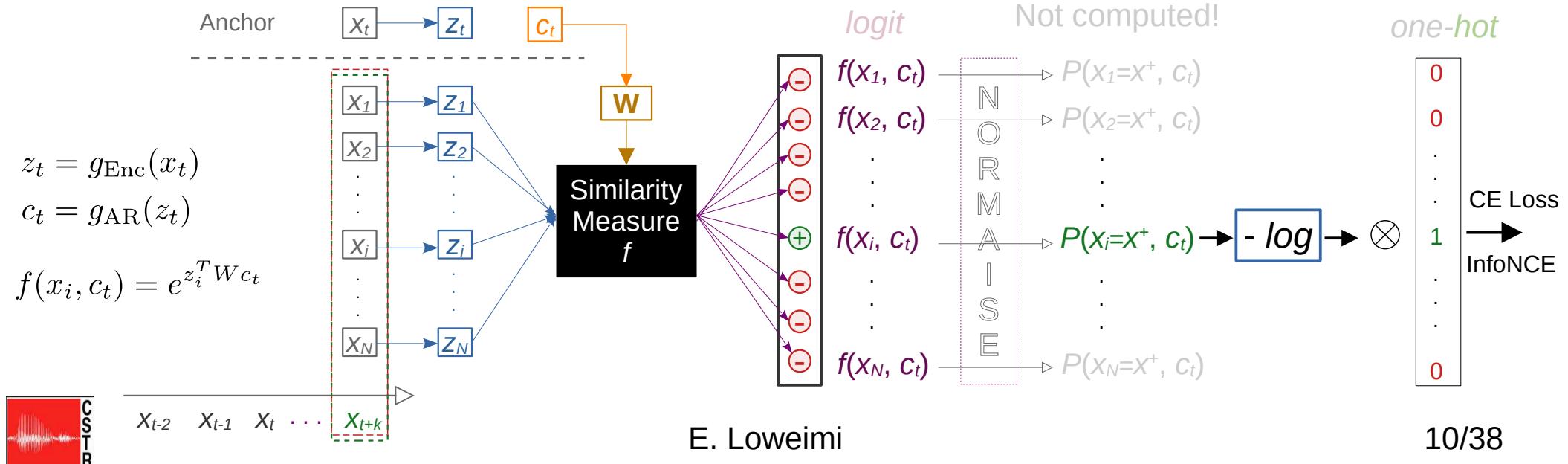
$$\begin{aligned}\mathcal{L}_X &= -\log \frac{f_k(x_{t+k}, c_t)}{\sum_{x_j \in X} f_k(x_j, c_t)} & P(x_i = x^+ | X, c_t) \\ &= -\log \frac{f_k(x_{t+k}, c_t)}{f_k(x_{t+k}, c_t) + \sum_{x_j \in X_{neg}} f_k(x_j, c_t)}\end{aligned}$$



$$\mathcal{L}^{\text{InfoNCE}} = \mathbb{E}_X [\mathcal{L}_X]$$

InfoNCE Interpretation (1)

- Given $X = \{x_1, x_2, \dots, x_N\}$, $x^+ = x^{t+k}$; $\#x^- = N-1$
- InfoNCE \equiv Categorical CE Loss \equiv Models $P(x_i = x^+ | X, c_t)$



InfoNCE Interpretation (1)

- Given $X = \{x_1, x_2, \dots, x_N\}$, $x^+ = x^{t+k}$; $\#x^- = N-1$
- Minimise InfoNCE Loss \equiv Maximise $P(x_i = x^+ | X, c_t)$

$$P(x_i = x^+ | X, c_t) = \frac{f_k(x_i = x^+, c_t)}{\sum_{x_j \in X} f_k(x_j, c_t)} = \frac{f_k(x_i = x^+, c_t)}{f_k(x = x^+, c_t) + \sum_{x_j \in X_{neg}} f_k(x_j, c_t)}$$

$$\mathcal{L}_{\text{InfoNCE}} = -\mathbb{E}_X [\log P(x_i = x^+ | X, c_t)]$$

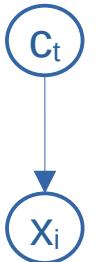
InfoNCE Loss Interpretation (2)

- Maximising $P(x_i = x^+ | X, c_t) \equiv$ Minimising $L_{InfoNCE}$
- Maximising $P(x_i = x^+ | X, c_t)$ RELATED to maximising $I(x; c)$

Proof in Appendix A

$$P(x_i = x^+ | X, c_t) = \dots = \frac{\frac{P(x_i | c_t)}{P(x_i)}}{\sum_j \frac{P(x_j | c_t)}{P(x_j)}} \propto \frac{P(x_i | c_t)}{P(x_i)}$$

$$\mathbb{I}(x; c) = \sum_{(x,c)} P(x, c) \log \frac{P(x, c)}{P(x)P(c)} = \sum_{(x,c)} P(x, c) \log \frac{P(x|c)}{P(x)} \propto \frac{P(x|c)}{P(x)}$$



Mutual Information (MI)



InfoNCE Loss Interpretation (2)

- Minimising $L_{\text{InfoNCE}} \equiv$ Maximising $P(x_i = x^+ | X, c_t) \dots$
- ... is *RELATED, but NOT EQUIVALENT*, to maximising $I(x; c) \dots$

$$P(x_i = x^+ | X, c_t) = \dots = \frac{\frac{P(x_i | c_t)}{P(x_i)}}{\sum_j \frac{P(x_j | c_t)}{P(x_j)}} \propto \frac{P(x_i | c_t)}{P(x_i)}$$

$$\mathbb{I}(x; c) = \sum_{(x, c)} P(x, c) \log \frac{P(x, c)}{P(x)P(c)} = \sum_{(x, c)} P(x, c) \log \frac{P(x|c)}{P(x)} \propto \frac{P(x|c)}{P(x)}$$



$\mathbb{I}(x; c) \geq \log N - \mathcal{L}_{\text{InfoNCE}}$

InfoNCE Loss Interpretation (3)

- Minimising L_{InfoNCE} \equiv maximising the lower bound of $I(x; c)$
- Effect of Larger N
 - Tighter lower bound + better approximations
 - Implicit hard mining

$$I(x; c) = \sum_{x, c} P(x, c) \log \frac{P(x, c)}{P(x)} = \dots = \mathbb{E}_X \left[\log \frac{P(x_{t+k}, c_t)}{P(x_{t+k})} \right]$$

$I(x; c) \geq \log N - \mathcal{L}_{\text{InfoNCE}}$

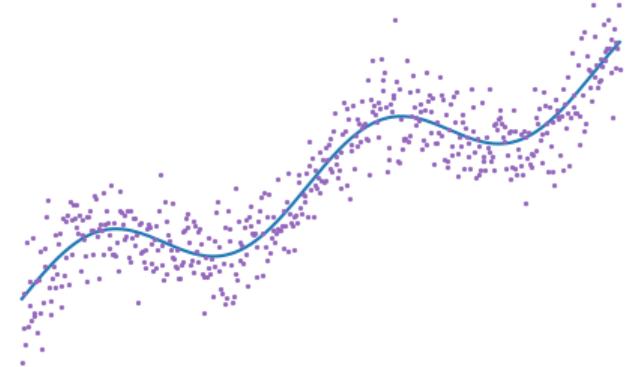
$X = \{\text{positive, negatives}\}$



Quiz Time

$$z_t = g_{\text{Enc}}(x_t)$$
$$c_t = g_{\text{AR}}(z_t)$$

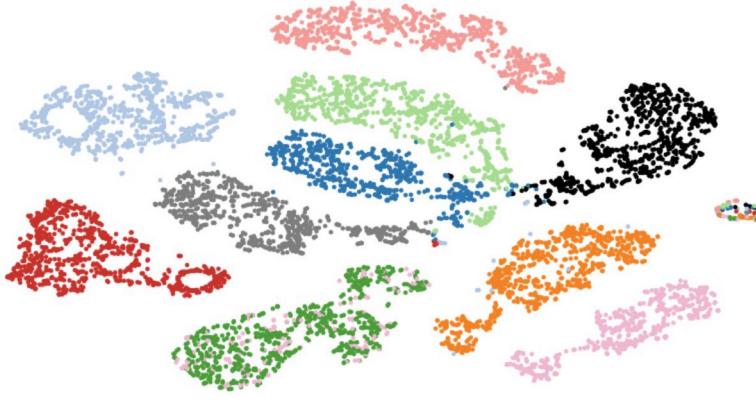
- Recall $f(x_{t+k}, c_t) = \exp(z_{t+k}^T (W c_t))$; Compare h_i s with f ...
 - Q1: $h_1(x_{t+k}, c_t) = \exp(z_{t+k}^T (W z_t))$
 - Q2: $h_2(x_{t+k}, c_t) = \exp(c_{t+k}^T (W c_t))$
 - Q3: $h_3(x_{t+k}, c_t) = \exp((z_{t+k}^T W) c_t)$
 - **Q4:** $h_4(x_{t+k}, c_t) = \exp(x_{t+k}^T (W c_t))$
- Q5: Larger N is better, here $N=8$; What upperbounds N ?
- Q6: Compare CPC with LPC



Experimental Setting

- Data: 100h LibriSpeech, 16 kHz
- Task: phone (41 classes) & speaker (251 classes) classification
- g_{Enc} : ResNet; 5-layer CNN + FC (512 nodes) $\rightarrow z_t$
 - Input: raw wave; Strides: [5,4,2,2,2] \rightarrow frame shift: 160 samples = 10 ms
- g_{AR} : GRU with 256 nodes $\rightarrow c_t$
- Optimiser: Adam
- Size of mini-batch: N=8; Prediction target: k=12
- Classifier: Multi-class linear logistic regression

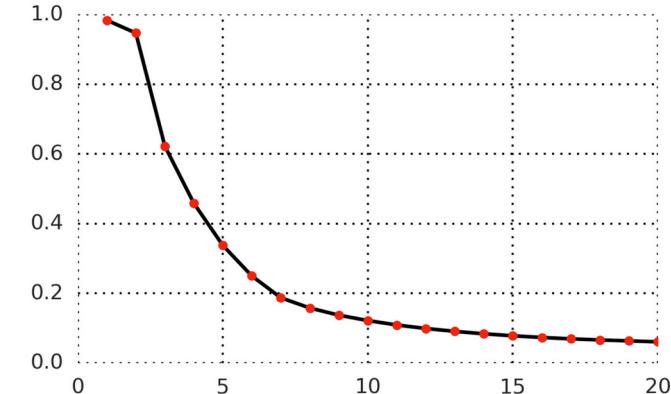
Experimental Results (1)



* t-SNE visualisation

- $k=12$, $N=8$
- Each colour \leftrightarrow a speaker

Average accuracy of predicting x^+



* Larger $k \rightarrow$ lower accuracy in predicting x^+ [$N=8$]

- Recall InfoNCE \equiv Categorical CE

Experimental Results (2)

Train a linear classifier on top of ...

Method	ACC
Phone classification	
Random initialization	27.6
MFCC features	39.7
CPC	64.6
Supervised	72.5
Speaker classification	
Random initialization	1.87
MFCC features	17.6
CPC	97.4
Supervised	98.5

* Using non-linear classifier (single hidden layer): $64.6 \rightarrow \mathbf{72.5}$

==> CPC representation is NOT perfectly linearly separable

* Random Init.: g_{Enc} and g_{AR} untrained

* Supervised: train E2E with the same architecture

Experimental Results (3)

- * Optimal k for phone classification is 12.
- * Optimal k for speaker classification is ???

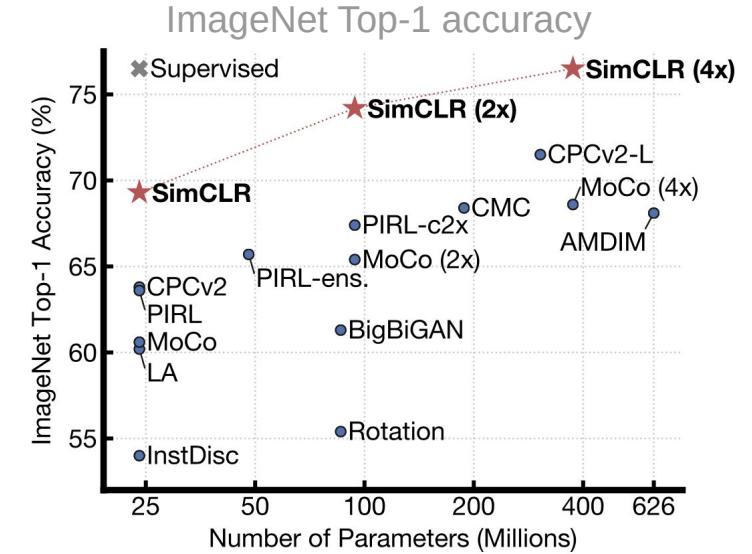
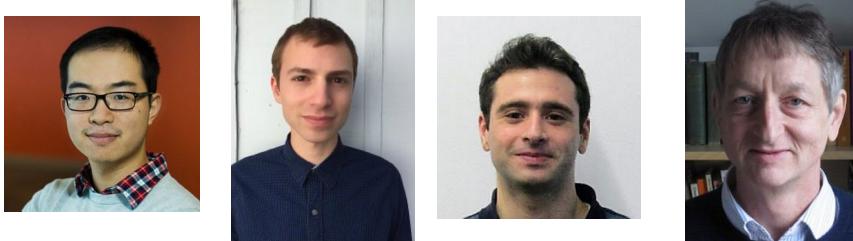
- * Best negative samples from
- * Same spk → Harder negatives than Mixed spk

* Hard negative: a negative that is closer or as close as the positive to the anchor (in the embedding space).

Method	ACC
#steps predicted	
2 steps	28.5
4 steps	57.6
8 steps	63.6
12 steps	64.6
16 steps	63.8
Negative samples from	
Mixed speaker	64.6
Same speaker	65.5
Mixed speaker (excl.)	57.3
Same speaker (excl.)	64.6
Current sequence only	65.2

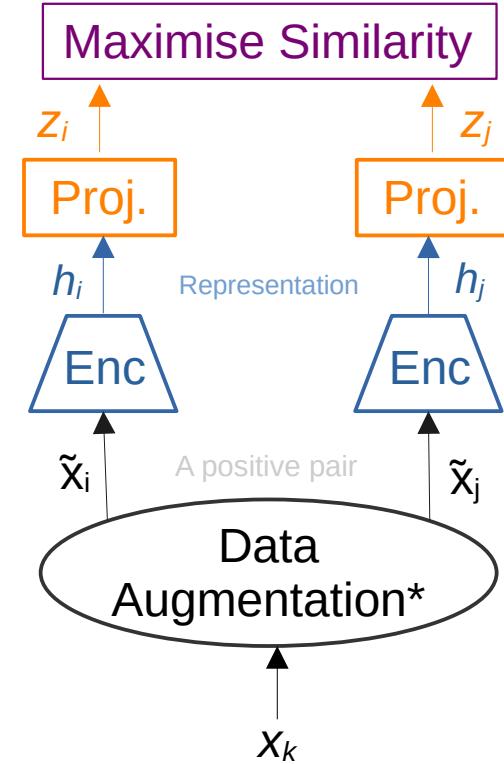
A Simple Framework for Contrastive Learning of Visual Representation

Ting Chen¹ Simon Kornblith¹ Mohammad Norouzi¹ Geoffrey Hinton¹



Framework's Modules

- Data Augmentation*, $\tilde{x} = \text{Aug}(x)$
 - Two correlated views per anchor
- Encoder: ResNet (no RNN)
 - $h = \text{Enc}(x) \rightarrow$ downstream tasks
- Projection Head
 - $z = g(h) = W_2 \text{ReLU}(W_1 h)$ rather than $W_1 h$



Contrastive Loss: NT-Xent

- Given *batch*: $\{x_k, y_k\}, k=1, \dots, N$
- Data Aug returns: $\{\tilde{x}_l, \tilde{y}_l\}, l = \{1, \dots, 2N\} \leftarrow \text{multiviewed batch}$
 - Consists of N positive pairs: $(\tilde{x}_{2k-1}, \tilde{x}_{2k}); \tilde{x}_{2k-1} \sim T \text{ & } \tilde{x}_{2k} \sim T$
- Sim is cosine similarity (L_2 -Normed)

Normalised
Temperature-scale
Cross Entropy

$$\mathcal{L} = \sum_{i \in I} \mathcal{L}_i = - \sum_{i=1}^{2N} \log \frac{\exp(\text{sim}(z_i, z_{j(i)})/\tau)}{\sum_{k=1}^{2N} 1_{[k \neq i]} \exp(\text{sim}(z_i, z_k)/\tau)}$$

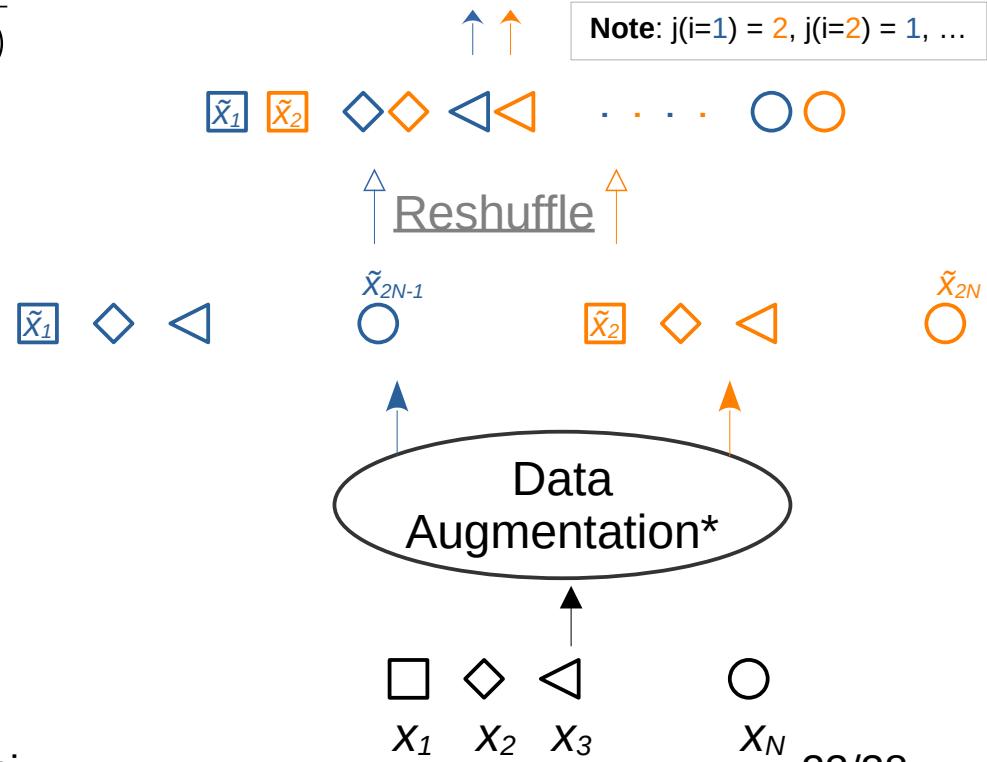
NT-Xent vs Multi-class N-pair Loss

- Multi-class N-pair loss (N-pair-mc) → Extension of Triplet
 - instead of one negative sample, involves $N-1$ negative pairs
- NT-Xent & N-pair-mc r identical equation-wise, except temperature scaling
- Both include N pairs BUT generated differently ...
 - NT-Xent \leftrightarrow Data Aug*; N-pair-mc \leftrightarrow class labels

$$\begin{aligned} \mathcal{L}_{\text{N-pair-mc}} &= \log \left(1 + \sum_{k=1}^{2N} 1_{[k \neq i, j]} \exp(\mathbf{z}_i^T \mathbf{z}_k - \mathbf{z}_i^T \mathbf{z}_j) \right) && \text{Positive pair: } (x_i, x_j) \\ &= -\log \frac{\exp(\mathbf{z}_i^T \mathbf{z}_j)}{\exp(\mathbf{z}_i^T \mathbf{z}_j) + \sum_{k=1}^{2N} 1_{[k \neq i, j]} \exp(\mathbf{z}_i^T \mathbf{z}_k)} \end{aligned}$$

Understanding Data Augmentation*

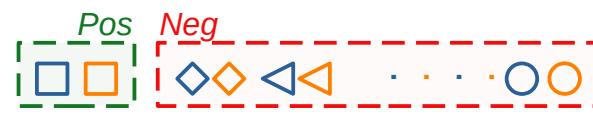
$$\mathcal{L} = \sum_{i \in I} \mathcal{L}_i = - \sum_{i=1}^{2N} \log \frac{\exp(\text{sim}(z_i, z_{j(i)})/\tau)}{\sum_{k=1}^{2N} 1_{[k \neq i]} \exp(\text{sim}(z_i, z_k)/\tau)}$$



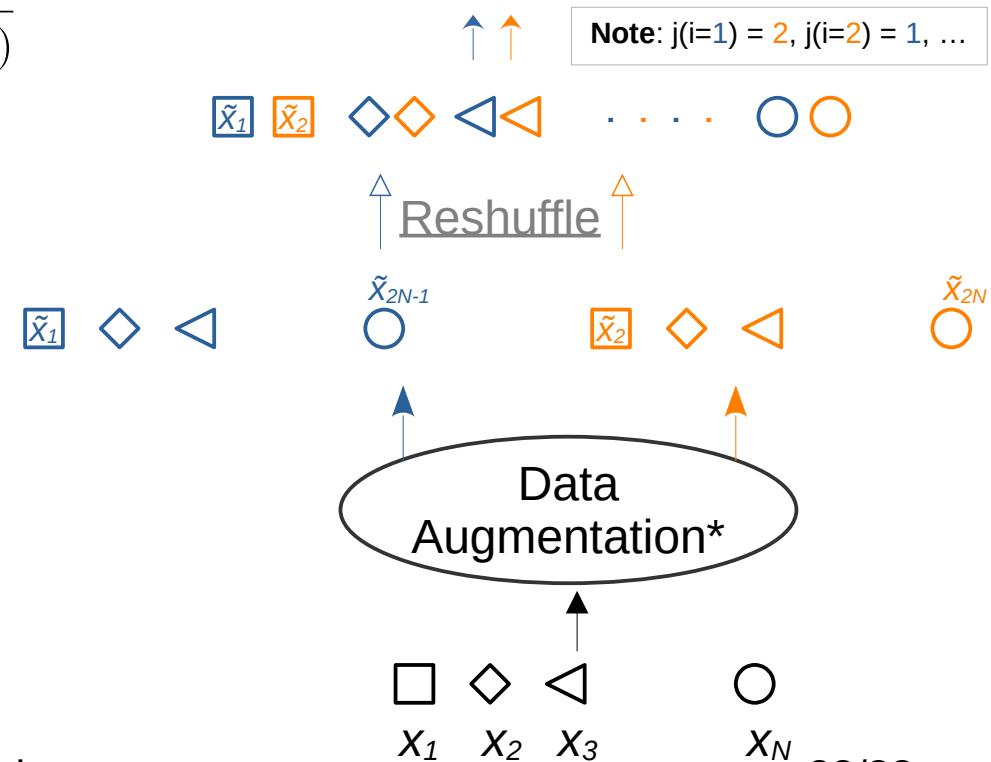
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i=1, anchor: \tilde{x}_1 , x^+ : \tilde{x}_2



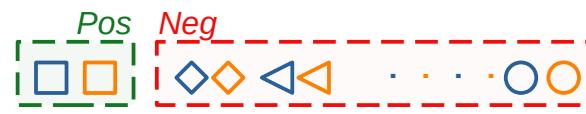
E. Loweimi



Understanding Data Augmentation*

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i=1, anchor: \tilde{x}_1 , X^+ : \tilde{x}_2

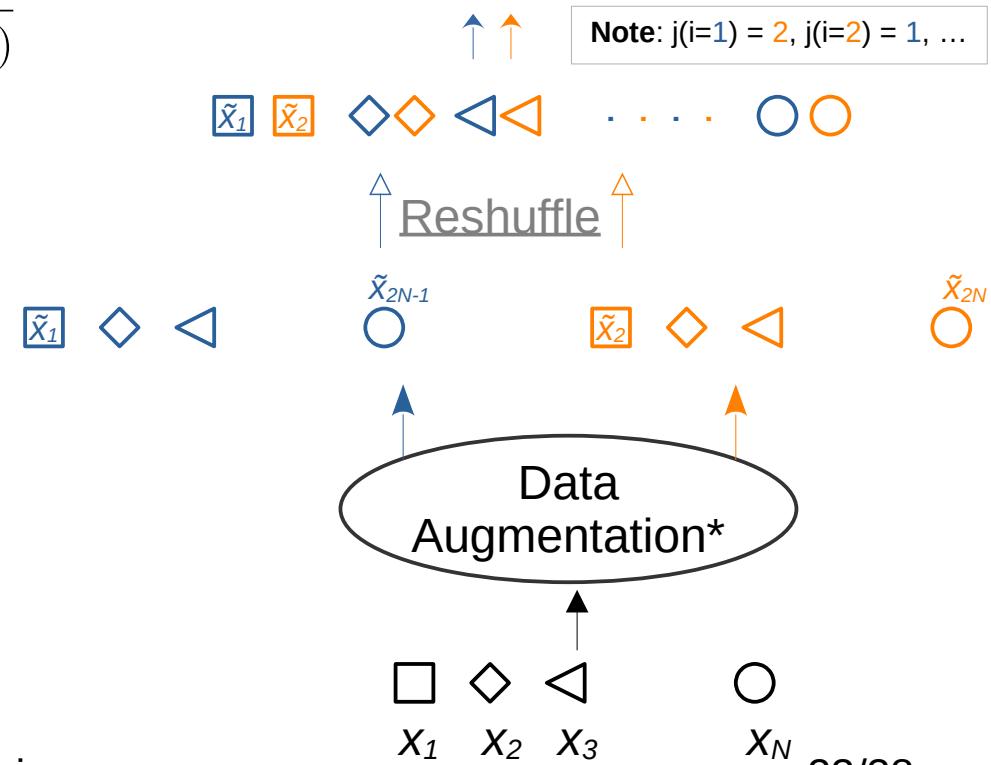


i=2, anchor: \tilde{x}_2 , X^+ : \tilde{x}_1



Obviously $\mathcal{L}_1 \neq \mathcal{L}_2$

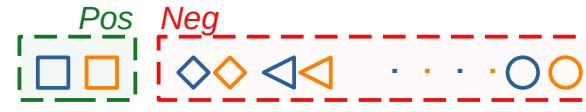
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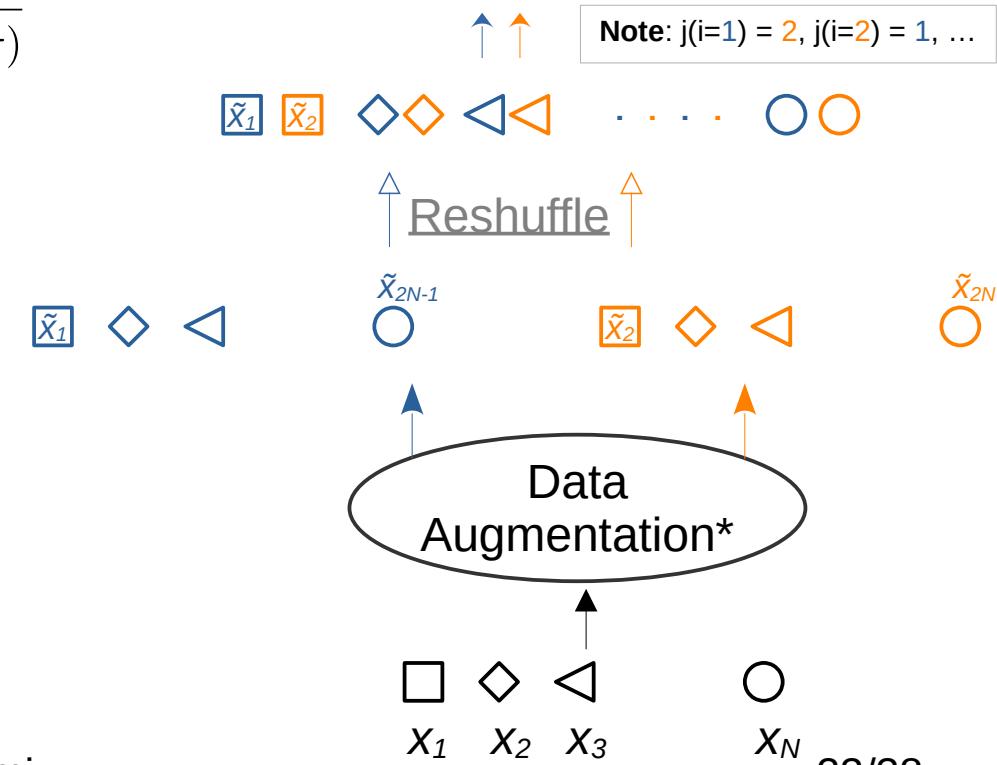
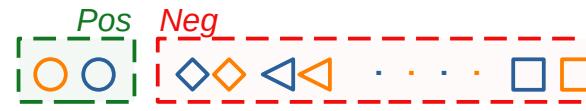
Understanding Data Augmentation*

$$\mathcal{L} = \sum_{i \in I} \mathcal{L}_i = - \sum_{i=1}^{2N} \log \frac{\exp(\text{sim}(z_i, z_{j(i)})/\tau)}{\sum_{k=1}^{2N} 1_{[k \neq i]} \exp(\text{sim}(z_i, z_k)/\tau)}$$

i=1, anchor: \tilde{x}_1 , X^+ : \tilde{x}_2



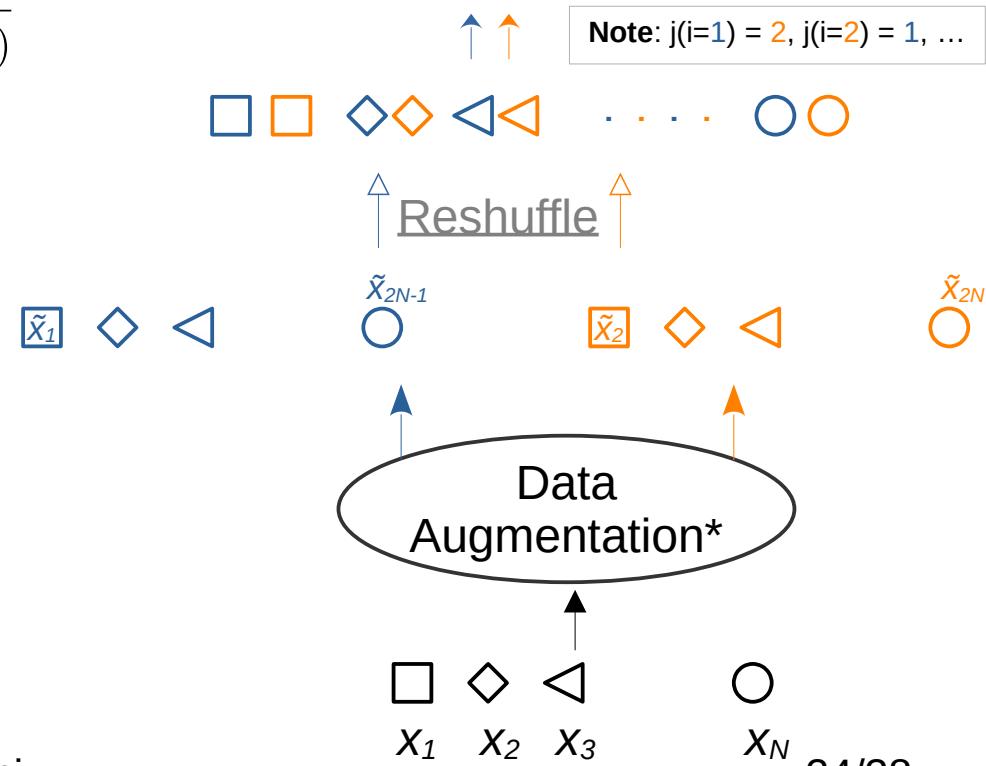
i=2N, anchor: \tilde{x}_{2N} , X^+ : \tilde{x}_{2N}



Data Augmentation* ... Advantage

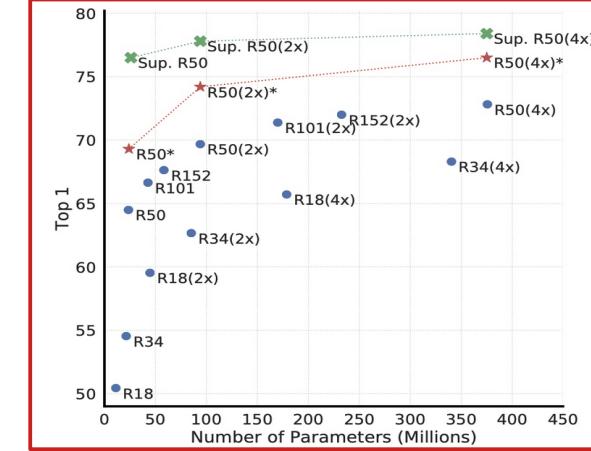
$$\mathcal{L} = \sum_{i \in I} \mathcal{L}_i = - \sum_{i=1}^{2N} \log \frac{\exp(\text{sim}(z_i, z_{j(i)})/\tau)}{\sum_{k=1}^{2N} 1_{[k \neq i]} \exp(\text{sim}(z_i, z_k)/\tau)}$$

- * Anchor is not directly used in loss
- * More general framework than CPC
- * CPC was applicable to sequential data



Experimental Results (1)

- Train a Linear classifier on top of learned representation →
- Semi-supervised
 - Sample n% (class balanced)
 - fine-tune



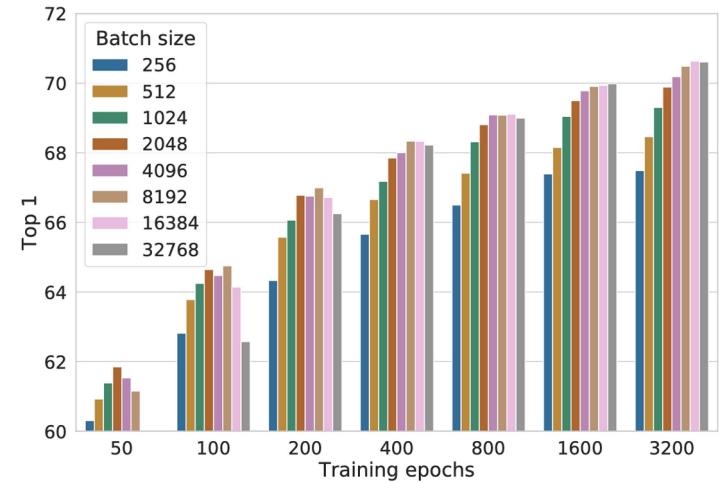
Architecture	Label fraction					
	1%		10%		100%	
	Top 1	Top 5	Top 1	Top 5	Top 1	Top 5
ResNet-50	49.4	76.6	66.1	88.1	76.0	93.1
ResNet-50 (2x)	59.4	83.7	71.8	91.2	79.1	94.8
ResNet-50 (4x)	64.1	86.6	74.8	92.8	80.4	95.4

2x: 2 times wider ResNet-50

Experimental Results – #Epochs & Batch size

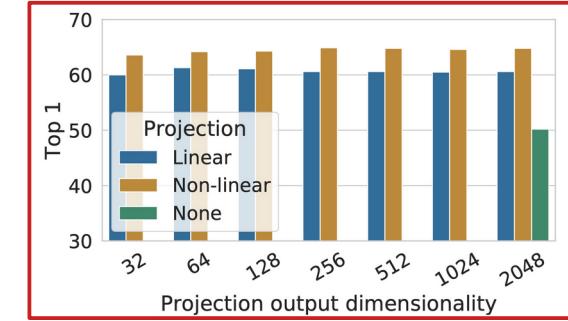
- Compared with supervised paradigm we require ...
 - More epochs ($> \times 100$)
 - Larger batch ($> \times 1000$)

* ALMOST one order of magnitude larger

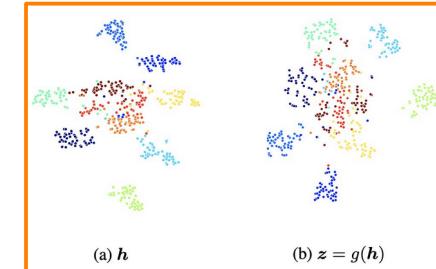


Experimental Results – Projection Head

- Role: map to a loss space
- Effect on overall **accuracy**:
 - Non-linear > Linear > None
- Its dimension is not critical!
- Useful for computing loss,
NOT as a representation!
 - Accuracy & cluster separation



What to predict?	Random guess	Representation h	Representation $g(h)$
Color vs grayscale	80	99.3	97.4
Rotation	25	67.6	25.6
Orig. vs corrupted	50	99.5	59.6
Orig. vs Sobel filtered	50	96.6	56.3

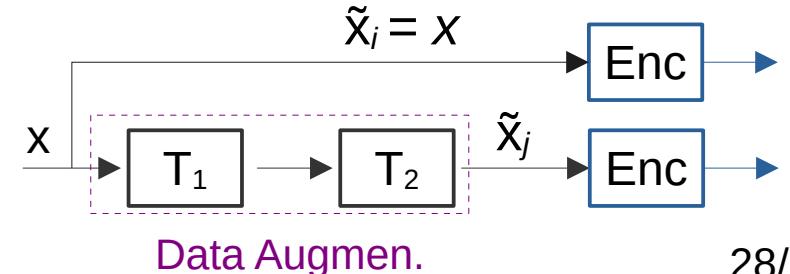
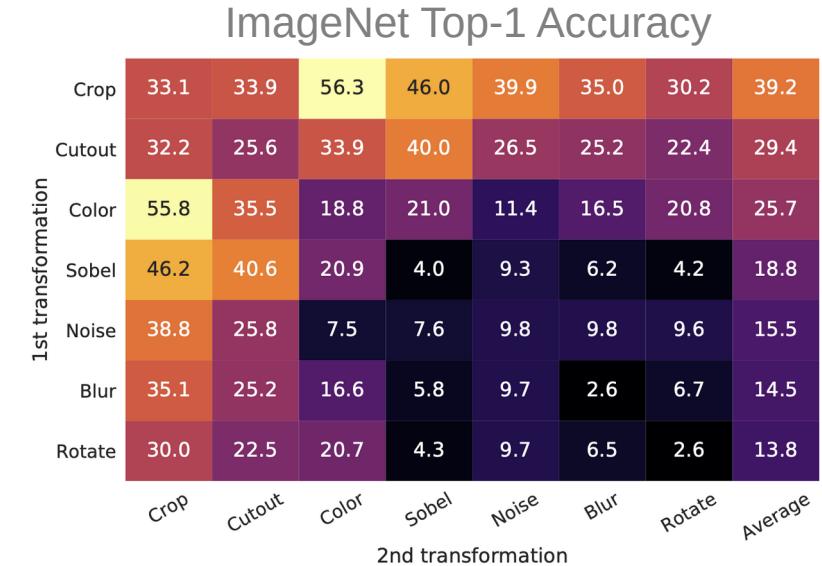


$h = \text{Enc}(x); z = \text{Proj}(h)$

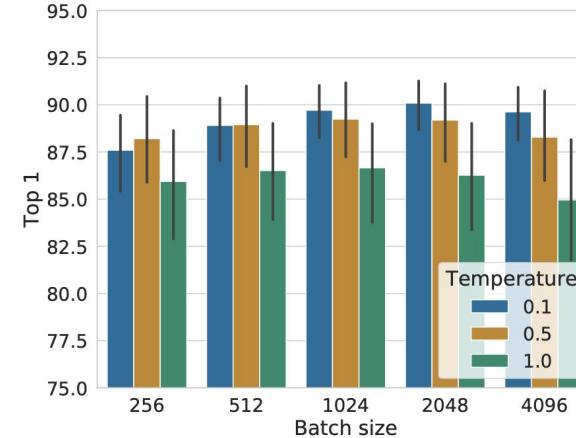
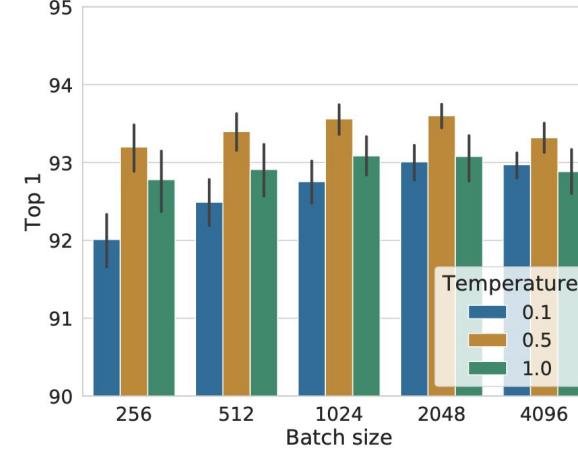
27/38

Experimental Results – Data Augmen.

- Composition of Data Augmentation is important!
- Best (here):
 - Random crop + colour distortion
- Note: low performance because of asymmetric augmentation
 - Recall $(\tilde{x}_i, \tilde{x}_j) \sim T$



Experimental Results – Temperature

(a) Training epochs ≤ 300 (b) Training epochs > 300

- * Temperature adjustment is helpful for any batch size or #epochs.
 - [HERE] $\tau_{optimal}$ usually < 1
- * More discussion on τ role in the next paper ...



NIPS 2020

Supervised Contrastive Learning

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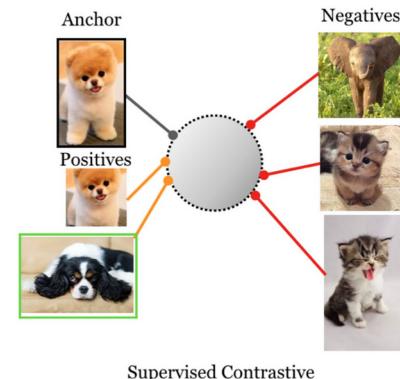
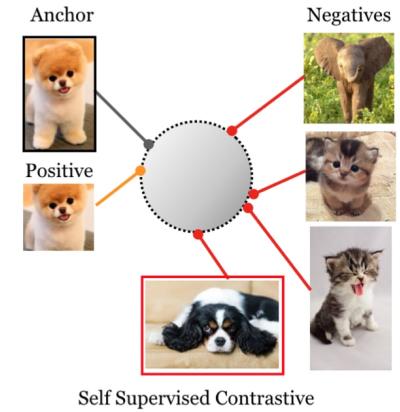
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Dilip Krishnan
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Motivation

- Unsupervised:
 - Triplet: (*anchor*, x^+ , x^-)
- Challenge:
 - x^- & anchor ... same class
 - Multiple x^+ 's in X
- Solution: use class labels
- How: *supervised* contrastive



Recall NT-Xent ...

- Given batch: $\{x_k, y_k\}, k=1, \dots, N$
 - Data Aug returns: $\{\tilde{x}_l, \tilde{y}_l\}, l = \{1, \dots, 2N\} \leftarrow \text{multiviewed batch}$
 - $\tilde{x}_{2k-1} \& \tilde{x}_{2k} \leftarrow$ two views of x_k
 - Triplet $\rightarrow \tilde{x}_i$: anchor; $\tilde{x}_{j(i)}$: x^+ ; \tilde{x}_m : x^- where $m = l - \{i, j(i)\}$
 - $Z_i = \text{Proj}(\text{Enc}(\tilde{x}_i))$

$$\mathcal{L}^{self} = \sum_{i \in I} \mathcal{L}_i^{self} = - \sum_{i \in I} \log \frac{\exp(z_i^T z_{j(i)}/\tau)}{\sum_{a \in A(i)} \exp(z_i^T z_a/\tau)}$$

$$A(l) = l - \{l\}$$

Supervised Contrastive Loss (SupCon)

$$\mathcal{L}^{self} = \sum_{i \in I} \mathcal{L}_i^{self} = - \sum_{i \in I} \log \frac{\exp(z_i^T z_{j(i)}/\tau)}{\sum_{a \in A(i)} \exp(z_i^T z_a/\tau)}$$

Simply ...
Average
over batch
positives!

$$\mathcal{L}_{out}^{sup} = \sum_{i \in I} \mathcal{L}_{out,i}^{sup} = - \sum_{i \in I} \left[\frac{1}{|P(i)|} \sum_{p \in P(i)} \log \frac{\exp(z_i^T z_p/\tau)}{\sum_{a \in A(i)} \exp(z_i^T z_a/\tau)} \right]$$

$$\mathcal{L}_{in}^{sup} = \sum_{i \in I} \mathcal{L}_{in,i}^{sup} = - \sum_{i \in I} \log \left\{ \left[\frac{1}{|P(i)|} \sum_{p \in P(i)} \frac{\exp(z_i^T z_p/\tau)}{\sum_{a \in A(i)} \exp(z_i^T z_a/\tau)} \right] \right\}$$

Avg inside log

- * $P(i)$: set of x^+ in $A(i)$ for anchor x_i
- * $|P(i)|$: cardinality

Learning a Nonlinear Embedding by Preserving Class Neighbourhood Structure

SupCon vs NCA

Ruslan Salakhutdinov and Geoffrey Hinton
 Department of Computer Science
 University of Toronto

$$\mathcal{L}_{out}^{sup} = \sum_{i \in I} \mathcal{L}_{out,i}^{sup} = - \sum_{i \in I} \left[\frac{1}{|P(i)|} \sum_{p \in P(i)} \log \frac{\exp(z_i^T z_p / \tau)}{\sum_{a \in A(i)} \exp(z_i^T z_a / \tau)} \right]$$

SupCon includes log, NCA does not.

$$\mathcal{L}_{in}^{sup} = \sum_{i \in I} \mathcal{L}_{in,i}^{sup} = - \sum_{i \in I} \log \left\{ \left[\frac{1}{|P(i)|} \sum_{p \in P(i)} \frac{\exp(z_i^T z_p / \tau)}{\sum_{a \in A(i)} \exp(z_i^T z_a / \tau)} \right] \right\}$$

NCA: Neighbouring Component Analysis

The NCA objective (as in [9]) is to maximize the expected number of correctly classified points on the training data:

$$O_{NCA} = \sum_{a=1}^N \sum_{b: c^a = c^b} p_{ab} \quad (5)$$

One could alternatively maximize the sum of the log probabilities of correct classification:

$$O_{ML} = \sum_{a=1}^N \log \left(\sum_{b: c^a = c^b} p_{ab} \right) \quad (6)$$

SupCon: L_{in} vs L_{out}

- Jensen's Inequality
 - Loss ($-\log$) is convex
 - A **typo** ...

$$\mathcal{L}_{in}^{sup} \leq \mathcal{L}_{out}^{sup}$$

$$\mathcal{L}_{out}^{sup} \leq \mathcal{L}_{in}^{sup}$$

cause \log is a concave function, Jensen's Inequality [23] implies that $\mathcal{L}_{out}^{sup} \leq \mathcal{L}_{in}^{sup}$. One might thus be tempted to conclude that \mathcal{L}_{in}^{sup} is the superior supervised loss function (since it bounds \mathcal{L}_{out}^{sup}). However, this conclusion is *not* supported

Got it
right here

SupCon: L_{in} vs L_{out}

- Although $L_{in} \leq L_{out}$, L_{out} is better; Y?
 - Grad L_{in} does not see $1 / |P(i)|$
 - Susceptible to bias in $|P(i)|$
 - I think ... $\log \Sigma$ vs. $\Sigma \log$
 - $\Sigma \log$... closer to Gaussian
 - Mean \leftrightarrow better representative

Loss	Top-1
\mathcal{L}_{out}^{sup}	78.7%
\mathcal{L}_{in}^{sup}	67.4%

ImageNet Top-1

Experimental Results (1)

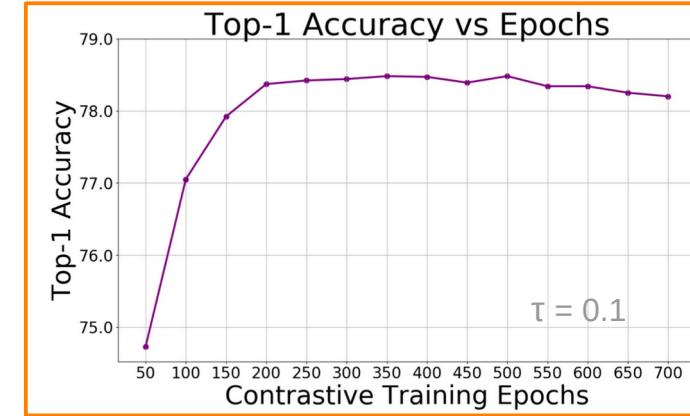
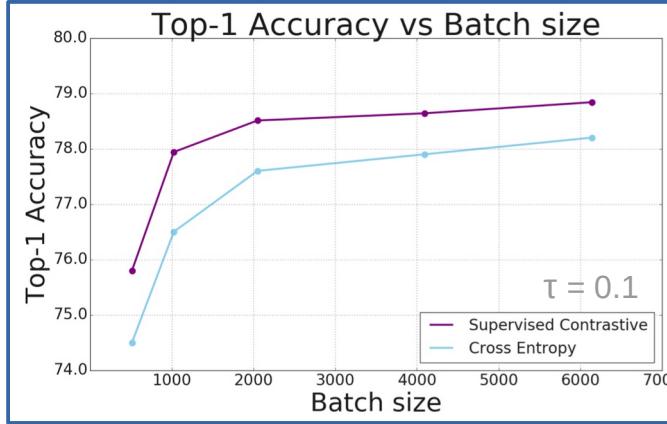
Dataset	SimCLR[3]	Cross-Entropy	Max-Margin [32]	SupCon
CIFAR10	93.6	95.0	92.4	96.0
CIFAR100	70.7	75.3	70.5	76.5
ImageNet	70.2	78.2	78.0	78.7

* *SupCon* outperforms *SimCLR*; is it fair?

1 [3]	3	5	7	9	No cap (13)
69.3	76.6	78.0	78.4	78.3	78.5

* Effect of # x^+ in mini-batch (N): Diminishing return after 7.

Experimental Results (2)



- * **Batch size** is important: The larger the N, the better
- * Contrastive learning requires more **epochs** than supervised

Experimental Results (3)

- * **Temperature** (τ) adjustment is helpful ...
- * OPTIMAL [here]: 0.1

- * Temperature Effects ...

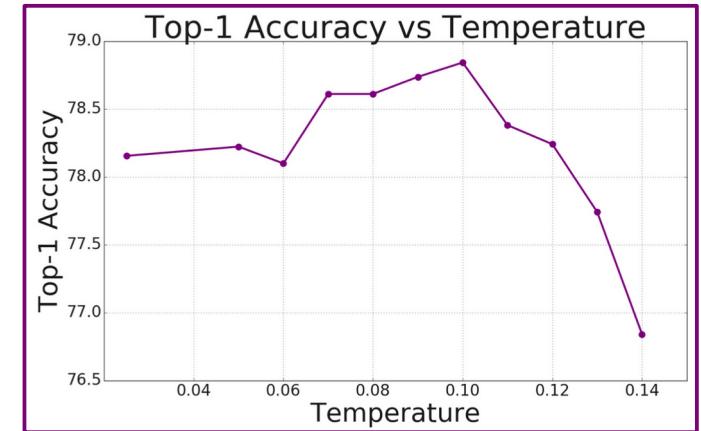
1) Higher $\tau \rightarrow$ Smaller Grad-Loss $\rightarrow \| \nabla \mathcal{L} \| \propto \frac{1}{\tau}$

2) Higher $\tau \rightarrow$ Smoother (softer/flatter) distribution

3) Lower $\tau \rightarrow$ Makes the hard negatives harder (Y?)

$$e^{\text{sim}(\mathbf{e}_a, \mathbf{e}_n)/\tau} \stackrel{\tau < 1}{\gg} e^{\text{sim}(\mathbf{e}_a, \mathbf{e}_n)}$$

4) Very Low $\tau \rightarrow$ Numerical instability



Conclusion – Contrastive Learning

- Goal: universal transferable representation learning
- Paradigms: unsupervised (CPC, SimCLR) & supervised
- Loss function: InfoNCE (CPC), NT-Xent (SimCLR), SupCon
- Modules: Data Aug, Encoder^{+RNN}, Projection
- Influential factors:
 - (Composite) Data Aug, Encoder, non-linear projection, #epochs, batch size, temperature, etc.



That's It!

- Thanks for Your Attention!
- Q/A
- Appendices:
 - A: Density Ratio Proof



Appendix A: Density Ratio Proof

$$\begin{aligned}
 P(x_i = x^+ \mid X, c_t) &= \frac{P(x_i = x^+, X \mid c_t)}{\sum_{j=1}^N P(x_j = x^+, X \mid c_t)} = \dots = \frac{\frac{P(x_i \mid c_t)}{P(x_i)}}{\sum_j \frac{P(x_j \mid c_t)}{P(x_j)}} \\
 &= \frac{1}{Z(c_t)} \frac{P(x_i \mid c_t)}{P(x_i)} \propto \frac{P(x_i \mid c_t)}{P(x_i)}
 \end{aligned}$$

$$\begin{aligned}
 P(x_i = x^+, X \mid c_t) &= \prod_{k=1}^N P(x_k, x_i = x^+ \mid c_t) \\
 &= \dots = P(x_i \mid c_t) \prod_{k \neq i} P(x_k) = P(x_i \mid c_t) \frac{\prod_{k=1}^N P(x_k)}{P(x_i)}
 \end{aligned}$$

$$P(x_j \mid c_t) = \begin{cases} P(x_j \mid c_t), & x_j = x^+ \\ P(x_j), & x_j = x^- \end{cases}$$