Statistical Modelling and Design of Experiments
- Third Assignment -

Albert Cerezo - acerezo@ac.upc.edu
Eloy Gil - eloy@ac.upc.edu
Universitat Politècnica de Catalunya

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1 Introduction

The main objective of this assignment is to compare the results of a G/G/1 model with the Allen-Cuneen's Approximation formula and analyze the performance of queueing systems by simulations when service times correspond to a long-tailed distribution of probability. The distribution parameters previously assigned for this work are shown in Table 1. Also, there is a second part where it is developed a small script using R language to evaluate a G/G/1 system that is included for validation purposes.

Parameter	**Value**
Parameter a (service)	0.6135
Inter-arrival times τ (arrivals)	Erlang
k	4.00
E(stage)	19.75
E(τ)	79.00

Table 1: Theoretical values versus simulation values

2 Analysis of theoretical versus simulation values

First of all, we are going to compare the theoretical values with simulated ones. In order to test it, it will be used an *R* script in order to perform the calculation for the theoretical values and the

SimQueue.jar script provided for the simulated ones. The *R* script includes the next formulas in order to get $E[x]$, $Var[x]$ and the Coefficient of variance.

$$E[x] = b\Gamma(\frac{a+1}{a})$$

$$Var[x] = b^2(\Gamma(\frac{a+2}{a}) - \Gamma^2(\frac{a+1}{a}))$$

$$Coef.ofVariation = \frac{\sqrt{Var[x]}}{E[x]}$$

Statistic	**Theoretical**	**Simulation**	**Error**
E[x]	5.85	6.0157	2.83%
Var[x]	100.106	110.796	9.64%
Std. Deviation	10.0053	10.526	4.94%
Coef. of variation	1.7103	1.7497	2.30%

Table 2: Distribution and parameter assigned values

As shown in the previous table, most errors are bounded below 5%. Therefore, we can conclude that the theoretical and simulated values are very close and either using one or another method would be enough in order to evaluate a G/G/1.

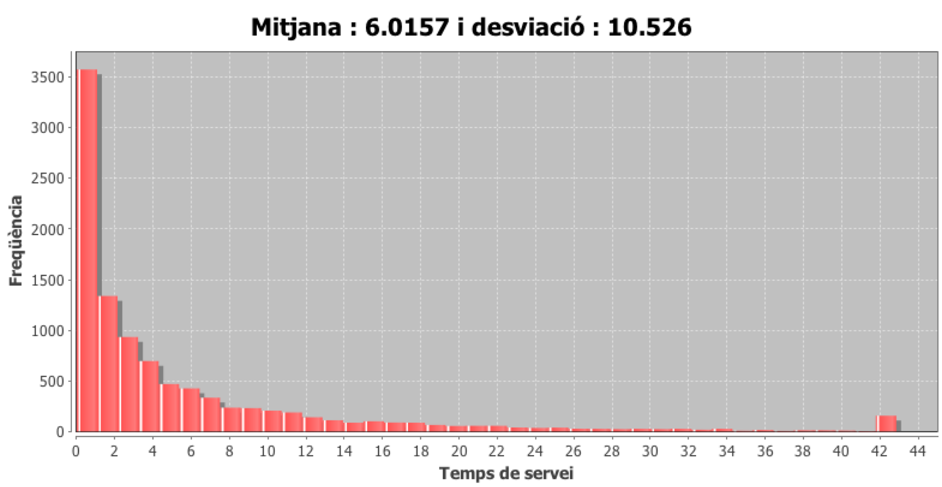

Figure 1: Frequency vs service time

3 Evaluating G/G/1

In the *R* script it has been performed and evaluation of the average waiting time according to the sequence factors $\rho = \{0.4, 0.7, 0.85, 0.925\}$

Loading factor	0.4	0.7	0.85	0.925
b	21.606	37.811	45.914	49.965
Average waiting time (W_q)	33.444	204.849	604.096	1430.811

Table 3: Average waiting times for different loading factors

Once we have the theoretical average waiting times and its corresponding b values, it is important to test these b values using a simulator. In order to have more representative results, we are going to run a simulation of each b ten times and average the results. The simulator follows the five step process proposed in the statement. Finally, the confidence interval is obtained using the t student with 9 degrees of freedom.

b	21.606	37.811	45.914	49.965
W_q	26.289	192.00	594.187	1465.827
W_q^{lower}	25.859	187.27	562.06	1312.909
W_q^{upper}	26.719	196.73	626.314	1618.745
L_q	0.332	2.430	7.521	18.553
L_q^{lower}	0.326	2.368	7.110	16.604
L_q^{upper}	0.338	2.492	7.932	20.502

Table 4: Average W_q and L_q with their CIs for different loading factors

If we compare the theoretical values from the Allen-Cuneen's Approximation formula and the simulation using queues, we can observe that for greater values of b the waiting time lays into the confidence interval whereas for lower values it lays outside.

Finally, some plots for the queue occupation versus time. As we can see, it converges in the time as we would expect in long-tailed services.

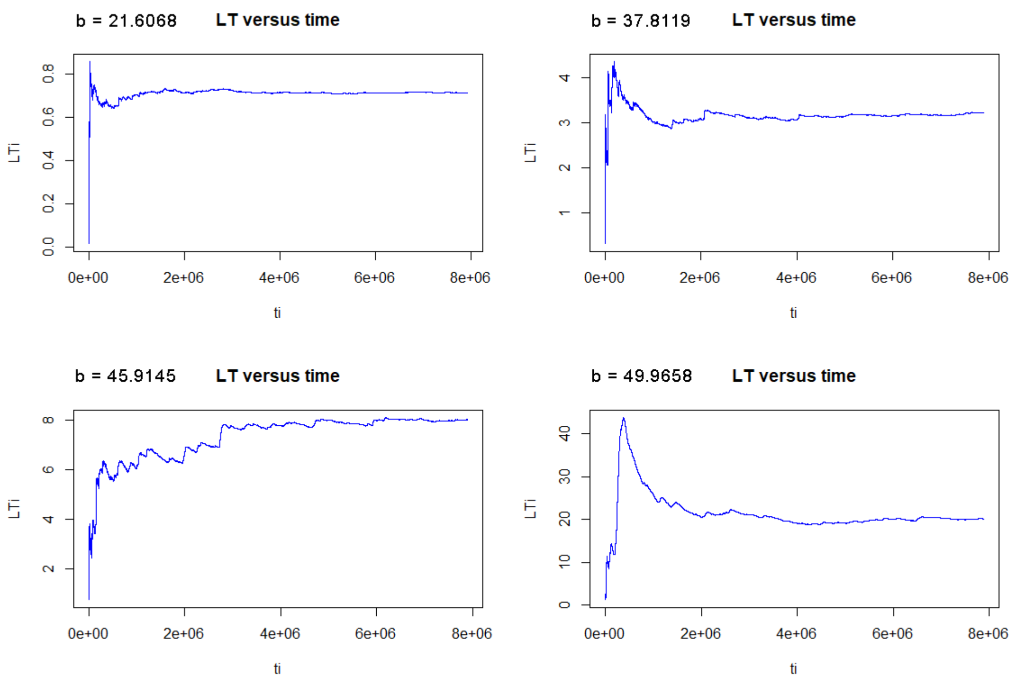

Figure 2: L_t versus time plots for different scale b