Equivalence classes

- Given an equivalence relation $R \subseteq A \times A$, we group all equivalent objects in the same *equivalence class*:
  - For every $a \in A$, $[a]_R = \{b \in A \mid (a,b) \in R\}$
  - $A/_R = \{ [a]_R \mid b \in A \}$ is called the *quotient set* of A by R.
- For example,
  - $BankNote/_{same\_value}$ has four elements corresponding to *five_pounds, ten_pounds, twenty_pounds, fifty_pounds*
  - What is $A/_{id}$?

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Theorem 1

- Given an equivalence relation $R \subseteq A \times A$, the following statements are equivalent for all a and b of A:
  - $(a,b) \in R$
  - $[a] = [b]$
  - $[a] \cap [b] \neq \varnothing$

Theorem 2

- Given an equivalence relation $R \subseteq A \times A$, its equivalence classes form a partition of A:
  - $\bigcup_{a \in A} [a]_R = A$
  - $[a] \cap [b] = \emptyset \quad if \quad [a] \neq [b]$

Exercise

- Given a partition $A_1, \ldots, A_n$ of a set A,

$$R = \{ (a,b) \mid a \in A_i \text{ and } b \in A_i \text{ for some } i=1,\ldots n \}$$

is an equivalence relation on A.