

Term Paper Data Science 1

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Chosen Project Topic:

T4 - DISTANCE MEASURES AND CLUSTERING

Abstract

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1 Definition of Distance Measure

A distance measure is a function $d(x, y)$ that calculates a real value between two points in a space, containing two sets of points. This function must satisfy the four following axioms:

1. No negative distances:

$$d(x, y) \geq 0$$

2. Identity of indiscernibles:

$$d(x, y) = 0, \text{ iff } x = y$$

3. Symmetry:

$$d(x, y) = d(y, x)$$

4. Triangle inequality:

$$d(x, y) \leq d(x, z) + d(z, y)$$

The triangle-inequality impose the condition that a distance reflects the shortest path between two points. Thus, it is not possible to achieve a distance improvement by traveling via an intermediate point z . [1]

2 Different Distance Measurements

2.1 Euclidean Distances

The Euclidean distance calculates the distance of two points, represented as vectors of n real numbers, in an n -dimensional euclidean space. In general, a Euclidean distance measure d is called L_r -norm, for arbitrary constants r .

$$d([x_1, x_2, \dots, x_n], [y_1, y_2, \dots, y_n]) = \sum_{i=1}^n (|x_i - y_i|^r)^{\frac{1}{r}}$$

The typical euclidean norm refers to the L_2 -norm and is calculated as the positive square root of the sum of all squared distances in each dimension.

$$d([x_1, x_2, \dots, x_n], [y_1, y_2, \dots, y_n]) = \sqrt{\sum_{i=1}^n (|x_i - y_i|)^2}$$

It is simple to verify three of the aforementioned axioms (1).

1. No negative distances:
The nonnegative values are given by the positive square root. let $x \neq y$, then the square of any real number is always positive.
2. Identity of indiscernibles:
For $x = y$ the value is obviously 0. Let $x = y$, then $(|x - y|)^2 = 0$ and $\sqrt{0} = 0$.
3. Symmetry:
Symmetrie is cleary given by the square of each distance.
 $(x - y)^2 = (y - x)^2$.
4. Triangle inequality:
As a matter of fact, this axiom requires a more difficult proof. However, to keep it simple, the Euclidean space possesses the property that the sum of the lengths of Cathetus and Ancathetus is always longer than the length of the Hypothenuse. [1]

2.2 Cosine Distance

The Cosine distance gives the angle between two points x and y represented as vectors in an n -dimensional space. It does not make a difference between a vector and a multiple of that vector. The cosine distance can be calculated by applying the arc-cosine function to the cosine of the angle θ between x and y [1].

$$d(x, y) = \cos(\theta) = \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i^2}}$$

The requirements for a distance measure are fulfilled for the Cosine distance:

1. No negative distances:
Regardless of the dimensionality of the space the values of the cosine distance are between 0 and 180 degrees, therefore no negative distances can occur.
2. Identity of indiscernibles:
Two vectors can have a cosine distance of 0 if and only if they are

located in the same direction. (This applies also to vectors that are multiples of one another and therefore are in the same direction.)

3. Symmetry:

Symmetry is obviously given by the equality to measure an angle between x and y and an angle between y and x .

4. Triangle inequality:

A rotation from x to y can be explained by a rotation from x to z and then to y . Therefore a sum of these two rotations is always bigger or equal than the rotation directly from x to y [1].

3 Data Set Description

3.1 Solar Flare

The solar flare dataset is taken from the UCI Machine Learning Repository [2]. Each point contains data recorded for on active region of the sun. The first three attributes are the McIntosh classification of sunspot groups:

1. Z-value: modified Zurich sunspot class, $\{A, B, C, D, E, F, H\}$
2. p-value: description of the penumbra of the largest spot. A penumbra is a part of a sunspot that is darker than the suns surface. $\{x, r, s, a, h, k\}$
3. c-value: description of the distribution of sunspots in a group $\{x, o, i, c\}$

A detailed descriptions of the letter codes can be found in the original paper by McIntosh [3]. The following entries are as follows:

4. Activity (1: reduced, 2: unchanged)
5. Evolution (1: decay, 2: no growth, 3: growth)
6. Code for the previous 24h flare activity
7. Is the region historically complex? (1: yes, 2: no)
8. Todoooooo
9. Area (1: small, 2: large)
10. Area of the largest spot (1: $\geq 5 \text{ deg}^2$, 2: $> 5 \text{ deg}^2$)

The last three entries are a predicted flare classes

3.2 Wine recognition dataset

This dataset contains the chemical analysis results of Italian wines from 3 different cultivators [4]. The dataset consists of 178 instances, each of them having 13 numeric attributes according to different measurements taken for different constituents (alcohol, malic acid, ash, alcalinity of ash, magnesium, total phenols, flavanoids, nonflavanoid phenols, proanthocyanins, color intensity, hue, OD280/OD315 of diluted wines, proline). Each instance belongs to either one of three classes containing 59, 71 and 48 data points. It was created by R. A. Fisher in July 1988.

4 Clustering Algorithms

4.1 K-Means

4.2 K-Medoids

The K-Medoids clustering method is related to the well-known K-means algorithm, but uses medoids (representative points for each cluster) instead of means to define new cluster centers, which makes it more robust to outliers. It partitions the dataset by assigning each data point to the closest of k cluster centers, which are defined by the most centrally located medoids. A medoid is a point with a minimal average dissimilarity to all other data points in the same cluster. The most commonly used algorithm to solve this NP-hard problem heuristically is the PAM (Partitoning Around Medoids) algorithm, that works as following:

1. First initialize the algorithm by selecting k data points to be the medoids and assigning every data point to its closest medoid.
2. Compare the average dissimilarity coefficient of a swap of each medoid m and a non-medoid data point \bar{m} . Find a swap between m and \bar{m} that would decrease the average dissimilarity coefficient the most.
3. If no change of a medoid happened in the second step, terminate the algorithm, else re-assign the data points to the new medoids and go back to step 2.

4.3 K-Median

4.4 DBSCAN

DBSCAN was developed by Martin Ester, Hans-Peter Kriegel, Jiirg Sander and Xiaowei Xu. All following definitions and descriptions are taken from their original publication [5] or their revisit of DBSCAN [6] and only apply to this algorithm.

Contrary to the aforementioned centroid-based partitioning algorithms (k-means, k-medoids and k-median) the DBSCAN (*Density Based Spatial Clustering of Applications with Noise*) algorithm uses point densities to determine clusters.

To introduce the definition of the density of a cluster, first the Eps-neighbourhood of a point is defined:

Definition 1: *Eps-neighbourhood*

A point q is part of the Eps-neighbourhood N_{Eps} of point p if the distance between them is smaller than a threshold distance called Eps.

The Eps-neighbourhood therefore is defined as $N_{Eps} = \{q \in D \mid ||p, q|| \leq Eps\}$ with D denoting the entirety of points that are supposed to be clustered and $||p, q||$ being the distance between p and q for an arbitrary distance measure.

The Eps-neighbourhood fails at being a reliable measure for the point density if a point is located at the border of a cluster. These points are called *border point*. Points that are located on the inside of a cluster are called *core points*. Hence the following definition is made:

Definition 2: *directly density-reachable and density-reachable*

A point p is directly density-reachable from a point q when

1. $p \in N_{Eps}(q)$
2. $|N_{Eps}(q)| \geq \text{MinPts}$

with MinPts being the minimal number of points that $N_{eps}(q)$ should contain so that q is considered a core point of a cluster.

A point is *density-reachable* if there is a chain of points between p and q so that all neighbouring points in the chain are directly density reachable.

To complete the definition of what is considered part of a cluster density-connectivity is defined:

Definition 3: *density-connected*

Two points p and q are considered density-connected if there is a common point o which is density-reachable from p and q .

Now a cluster can be described as:

Definition 4: *cluster*

A cluster is a non empty subset $C \in D$ so that:

1. $\forall p, q : p \in C \wedge q \text{ is density reachable from } p \Rightarrow q \in C$
2. $\forall p, q \in C : p \text{ is density-connected to } q$

Noise is easily defined as every point that is not part of a Cluster C_i .

Using these definitions DBSCAN can begin the clustering process with given values for Eps and MinPts. In the beginning all points are not labeled. Beginning with an arbitrary point p all points are iterated in a linear fashion. For each point a **RangeQuery** function is executed finding all density-reachable neighbours of p . If **RangeQuery** finds more than MinPts neighbours then p is a core point and is labeled as such. Otherwise p is marked as Noise.

In the next step every point in the Neighbourhood excluding p is expanded. Unlabeled Points get checked for the core point condition (which equals a **RangeQuery** call). Points that got labeled as Noise before are labeled as core points. When the expansion comes to an end a cluster is yielded and the next unlabeled point is chosen as p .

Two clusters may be merged if their distance is below Eps. The distance between two clusters C_1 and C_2 is defined as $||C_1, C_2|| = \min\{||q, p|| \mid p \in C_1, q \in C_2\}$.

The runtime complexity of DBSCAN heavily depends on the runtime of the **RangeQuery** function and the distance measure. Thus the runtime can exceed $\mathcal{O}(n^2)$ depending on the chosen implementations. A detailed discussion of DBSCANs runtime can be found in [6].

5 Description of Python libraries used

Libraries:

- numpy
- pyclustering
- seaborn
- sklearn

6 Description of Evaluation Module

ANMERKUNGEN:

- What are the results and how are they measured?

7 Web Frontend and User Manual

ANMERKUNGEN:

- Describe the implementation and write a brief user manual with screenshots.

8 Conclusion

ANMERKUNGEN:

- Summarize the main points and achievements
- Add your own assessment/criticism on the topic

References

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