A Comparison of Two Wavelet Denoising Methods: SURE and UniversalThreshold

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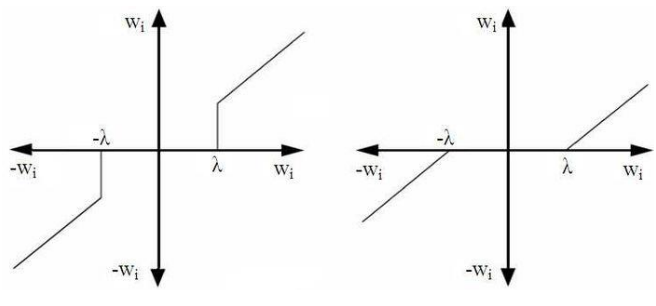
***Abstract*—Wavelet Denoising is known to be the best method to denoise signals from gaussian white noise [1]. However, they are several Wavelet Denoising technics. In this paper, we compare two of them: SURE (Stein Unbiased Estimate of Risk) and UniversalThreshold. We will see that SURE is more efficient in most of the cases. We compare performances for different image types, noise variance and with using hard and soft threshold.**

I. Introduction

Gaussian White Noise seems especially hard to denoise, because it touches equally all the frequencies of the signal. Although, Wavelet Denoising is very efficient against it.

Wavelet Denoising works in three main steps:

- Wavelet Transform is a correlation between the signal and a set of wavelets e.g. biorthogonal (*bior* in matlab), Daubechies, Haar, Mexican Hat, etc. The output of the transform is a set of correlation coefficients. In this paper, we will only use the default wavelet set proposed in MATLAB: *bior4.4*.
- Coefficient filtering: we want to keep only higher coefficients. The problem here is to setup a threshold. If a coefficient is lower than the threshold, it will be deleted. The threshold can be *soft* or *hard*, as represented in figure 1. Soft threshold gives a smoother result, but blurs the edges.
- Inversed Wavelet Transform from filtered set of coefficients, gives us the denoised signal.



(a) *(b)*

Fig. 1. (a) Hard Threshold and (b) Soft Threshold. W_i the value of Wavelet Transform coefficients and λ the threshold value.

UniversalThreshold and Stein Unbiased Estimate of Risk (SURE) are used as pre-implemented methods in MATLAB, using:

wdenoise2()

The technics used here are different by they way to define the threshold. UniversalThreshold has a fixed threshold, computed using:

$$\lambda = \sigma\sqrt{2\log(n)} \quad (1)$$

With σ the standard deviation of the noise level and n the sample size.

In the other hand, SURE adapts the threshold depending of the smoothness of the signal, as described in [2].

(a) *(b)*

Fig. 2. (a) crowd image, with a high level of details. (b) bank image, with fewer edges.

II. Experience

A. Setup

We use MATLAB 2019.b. As said before, the used wavelet denoising methods are pre-implemented in MATLAB. The two images used for the tests are shown in figure 2. We have chosen those two images because one (crowd) has a high level of details, which means a lot of edges. The other (bank) has a low level of details, with less and sharp edges.

Our testbench is simple: adding noise to the images, denoising them and comparing noised and denoised images to the original one.

B. Tested parameters

To compare the two denoising methods, we choose to play on the following parameters:

- Image type, as explained before.
- Noise variance, which can be seen as the noise strength.
- Threshold type.

C. *Comparing method*

To compare the two methods, we use Signal Noise Ratio (SNR), which is the most used metric to measure the noise in a signal. SNR is computed using:

$$SNR = \frac{Signal}{Noise} \qquad (2)$$

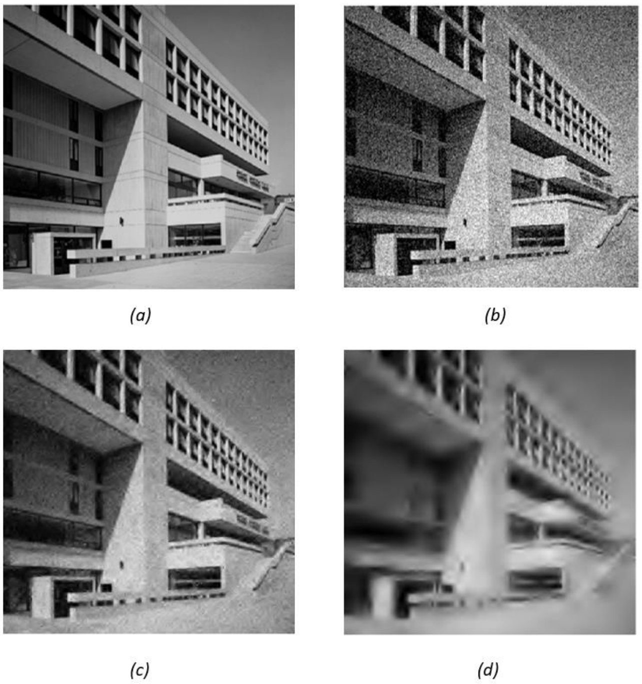


Fig. 3. (a) shows the original bank image, (b) has some gaussian white noise with variance 0.01. (c) is the denoised image using SURE and (d) is the denoised image using UniversalThreshold. (c) and (d) both use soft threshold here.

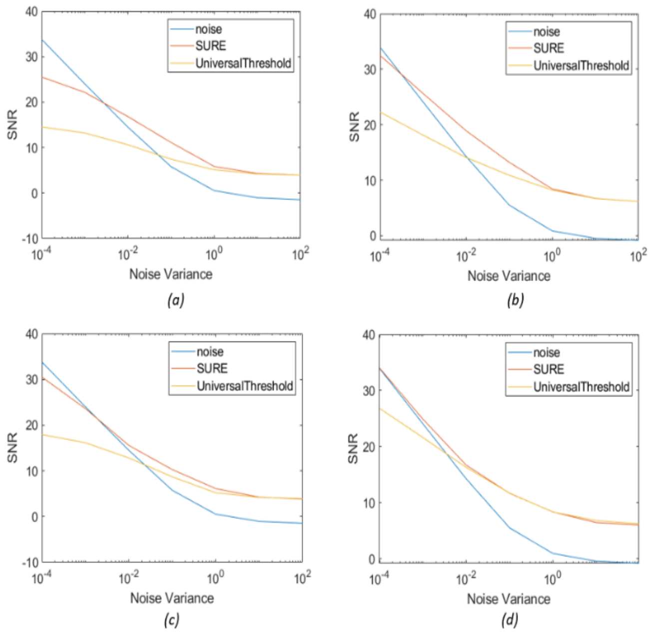


Fig. 4. Each graphic shows the SNR for raw noise, and for each denoising method. (a) is for the crowd image, in case of a soft threshold. (b) the bank image for soft threshold. (c) the crowd image for hard threshold and (d) the bank image for hard threshold.

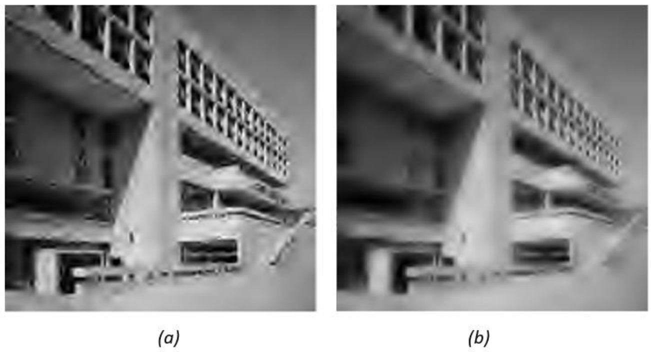


Fig. 5. Denoised images using UniversalThreshold, with hard threshold (a) and soft threshold (b)

III. Results

A. *Visual differences*

Figure 3(c) and 3(d), show respectively the results of method SURE and UniversalThreshold. We can see that the methods are different, as UniversalThreshold gives a much smoother result than SURE. In the other hand, SURE seems to keep more noise. So, we can say that UniversalThreshold removes more noise than SURE, but it also adds blur. So finally, SURE result looks better.

B. *SNR differences*

A gaussian white noise variance of 10^{-4} shows a very low noise, almost invisible for humans (high SNR). In the other hand, a variance of 10^{2} is so high that's it only remains noise, the image is imperceptible. Figure 4 shows us that SURE always has a better SNR compared to UniversalThreshold.

Figure 4 also shows that Wavelet Denoising is globally more efficient on bank image, with few details, as shown in figure 4(b) and 4(d). Also, it's not worth to use wavelet denoising if the noise variance is too low e.g. in figure 4(b), it's worth to use UniversalThreshold for a noise variance greater than 0.01.

Finally, the threshold type makes some great differences, especially for UniversalThreshold, and Hard threshold improves its denoising performance, as it reduces the blurring effect, as shown is figure 5.

However, SNR gives us results about noise, but it's important not to forget that it doesn't necessarily matches with human perception.

Conclusion

SURE method has very satisfying visual results and has always a better SNR compared to UniversalThreshold, especially for high detailed images. However, UniversalThreshold may be more efficient for images with low level of details and smooth edges. Also, we can improve UniversalThreshold performances on edges using a hard threshold.

However, in most of the cases, SURE will be a better denoising method.

To go further in this work, we could compare the two denoising methods on an image with smooth edges. We could also compare different sets of wavelets.

REFERENCES

[1] Kaur, S., & Singh, N. (2014). Image Denoising Techniques: A Review. *International Journal of Innovative Research in Computer and Communication Engineering*, *2*(6).J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68–73.

[2] Xiao-Ping Zhang, & Desai, M. D. (1998). *Adaptive denoising based on SURE risk. IEEE Signal Processing Letters, 5(10), 265–267*. doi:10.1109/97.720560