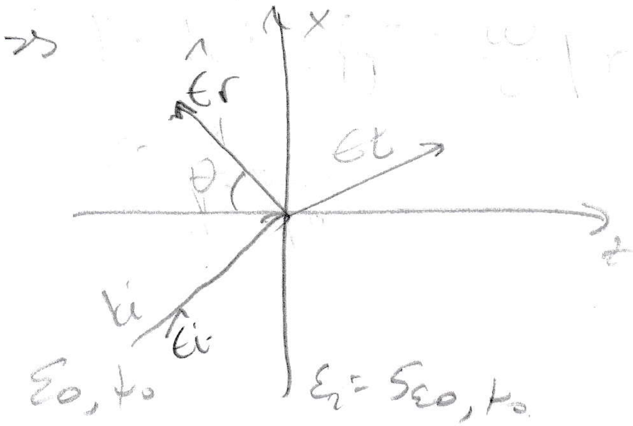
(10) (α) $k r = k(\hat{i}x + \hat{i}z)$ $E = [2\cos\theta\, \hat{i}_x - j3\hat{i}_y - 2\sin\theta]\, e^{j(\vec{k}\vec{r})}$



$n_2 = \sqrt{\frac{5\varepsilon_0}{\varepsilon_0}} = \sqrt{5}$

$n_1 = \frac{1}{\sqrt{5}}$

Υπάρχει γωνία Brewster άρα η πόλωση είναι TM

$\sin^2\theta_B = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2} = \frac{5}{6} \Rightarrow \sin\theta_B = 0{,}9129$, $\cos\theta_B = 0{,}4082$ $\leftarrow \sin\theta_B = \sqrt{\frac{5}{6}}$

Άρα αφού η y είναι 0 τότε

$k_i = k_0(\sin\theta_B, 0, \cos\theta_B) = 2\pi 10^6 (0{,}9129\,\hat{i}_x + 0{,}4082\,\hat{i}_z)$

(β) $r_{TE} = \frac{Z_2\cos\theta_1 - Z_1\cos\theta_2}{Z_2\cos\theta_1 + Z_1\cos\theta_2} \xrightarrow[Z_i = \frac{Z_0}{n_i}]{\theta_1 + \theta_2 = \pi/2} \frac{n_1\cos\theta_1 - n_2\sin\theta_1}{n_1\cos\theta_1 + n_2\sin\theta_1} = \frac{-2}{3}$

Αφού υπάρχει μηδενικός τότε το Er θα διαδοθεί κάθετα στο y=0.

$E_r = r_{TE} \cdot E_{i,y}\, e^{-j2\pi 10^6 (0{,}9129\hat{i}_x - 0{,}4082\hat{i}_z)(x\hat{i}_x + y\hat{i}_y + z\hat{i}_z)}$

$\boxed{r_{TE} = \frac{E_r}{E_i}} = -\frac{2}{3}(\cancel{3}j)e^{-j2\pi 10^6(0{,}9129x - 0{,}4082z)}$ $\quad i_y = i_y 2j\, e^{-jk_0(0{,}9129x - 0{,}4082z)}$

$t_{TM} = \frac{2Z_2\cos\theta_1}{Z_1\cos\theta_1 + Z_2\cos\theta_2} = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\sin\theta_1} = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_1} = \frac{1}{\sqrt{5}}$, $t_{TE} = 1$

$E_T = (t_{TM}\,\hat{e}_{TM} E_i + t_{TE}\,\hat{e}_{TE} E_r)\, e^{-j\vec{k}_2\vec{r}}$

$= [0{,}8944(0{,}9129x - 0{,}4082z) - j\hat{i}_y]\, e^{-jk_0\sqrt{5}(0{,}4082x + 0{,}9129z)}$

(γ) $\vec{H}_i = \frac{1}{Z_0}(\vec{k}_i \times \vec{E}_i) + \frac{1}{Z_0}(\vec{k}_r \times \vec{E}_r) = \frac{1}{377}\Big[\left(\frac{\sqrt{5}}{\sqrt{6}}, 0, \frac{1}{\sqrt{6}}\right) \times (2\cos\theta_1, -3j, 2\sin\theta_1)\, e^{-j\vec{k}\vec{r}}$

$+ \left(\sqrt{\frac{5}{6}}, 0, -\sqrt{\frac{1}{6}}\right) \times (0, 2j, 0)\, e^{-j\vec{k}_r\vec{r}}\Big)$

$= \frac{1}{377}[-3j(0{,}9129\,\hat{i}_z - 0{,}4082\,\hat{i}_x) - 2\hat{i}_y]\, e^{-jk_0(0{,}9129x + 0{,}4082z)} + \boxed{2}j(0{,}4082\hat{i}_x + 0{,}9129\hat{i}_z)$

δ) $r \cdot k_2 = 0 \to \cos\theta_1 x + \sin\theta_1 \cdot z = 0$

Άρα για το E_i:

$E_x = 2\cos[\omega t - k_0(0{,}9129x + 0{,}4082z)]$

$E_y = 3\cos(\omega t - k_0(0{,}9129x + 0{,}4082 + \pi/2)]$

$= 3\sin(\omega t - k_0(0{,}9129x + 0{,}4082]))$

Άρα $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{E_x}{2}\right)^2 + \left(\frac{E_y}{3}\right)^2 = 1 \to \tan\theta = \frac{\overset{\sin}{\cos(\omega t - \vec{k}\vec{r} + \pi/2)}}{\cos(\omega t - \vec{k}\vec{r})} = \tan(\omega t - \vec{k}\vec{r})$

Για το E_r:

$\left.\begin{array}{l} E_{oy} = 2j \\ E_{ox} = 0 \end{array}\right\}$ Γραμμική πόλωση κάθετη στο επ. ί, άρα είναι πόλωση ΤΕ.

Για το E_i:

$\frac{5E_x^2}{4} + E_y^2 = 1$ & $\tan\theta = \tan(\omega t - \vec{k}\vec{r})$

ε) $\frac{P_r}{P_i} = \frac{1/z_0 |E_r|^2}{1/z_0 |E_i|^2} = \frac{2}{2^2(\cos^2\theta + \sin^2\theta) + 3^2} = \frac{4}{13} = 0{,}3077$

$\frac{P_t}{P_i} = 1 - \frac{P_r}{P_i} = \frac{9}{13} = 0{,}6923$

ε2) $\vec{E}_i = [2\cos\theta \hat{i}_x - j3\hat{i}_y - 2\sin\theta \hat{i}_z] e^{-j\vec{k}\vec{r}}$

$E_i = [E_{TE}\hat{u}_{TE} + E_{TM}\hat{u}_{TM}] e^{-jkr}$

$E_{TE} = E_{TM} = 1$

$z_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$z_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{\mu_0}{5\varepsilon_0}}$

$\frac{z_1}{z_2} = \frac{1}{\sqrt{5}} = n_1$

$\frac{P_r}{P_i} = \frac{1}{2}\left(|r_{TE}|^2 + |r_{TM}|^2\right)$

$\frac{P_t}{P_i} = \frac{1}{2}\left(|t_{TE}|^2 + |t_{TM}|^2\right) \frac{\overset{\to 1/5}{z_1}\cos\theta_2}{z_2\cos\theta_1}$