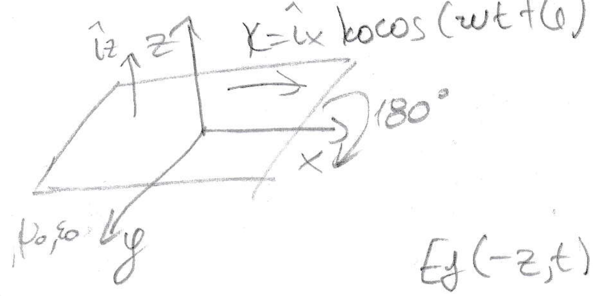
Πεδία Α 3η σειρά

8) $\vec{K} = \hat{i}_x K_0 \cos(\omega t + \varphi)$



$\vec{E} = \vec{E}(z,t) = E_x(z,t)\hat{i}_x + E_y(z,t)\hat{i}_y + E_z(z,t)\hat{i}_z$

$\vec{H} = \vec{H}(z,t) = H_x(z,t)\hat{i}_x + H_y(z,t)\hat{i}_y + H_z(z,t)\hat{i}_z$

$E_y(-z,t) = -E_y(z,t)$

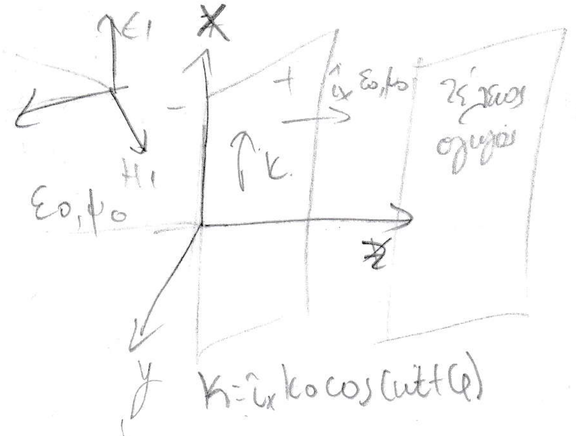
$E_x(-z,t) = E_x(z,t)$

$E_z(-z,t) = -E_z(z,t)$

$H_y(-z,t) = -H_y(z,t)$

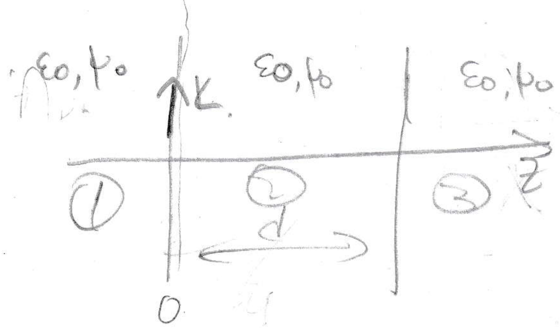
$H_x(-z,t) = H_x(z,t)$

$H_z(-z,t) = -H_z(z,t)$



Το E βρίσκεται

(Το k είναι στον z, H ⊥ E, E στην διεύθυνση ρεύματος → x & H στον y.



Θεωρώ ότι το E σε κάθε περιοχή θα είναι σε φορέα ισοδιάθλεων. Άρα

$\vec{E}_1 = (A_1 e^{-jkz} + B_1 e^{jkz})\hat{x}$

$\vec{E}_2 = (A_2 e^{-jkz} + B_2 e^{jkz})\hat{x}$

$\vec{E}_3 = (A_3 e^{-jkz} + B_3 e^{jkz})\hat{x}$

Αφού το δεξιότερο μέσο είναι τέλειος αγωγός, τότε $E_3 = 0$

Επίσης για το E_1 $A_1 e^{-jkz} = 0$, αφού τα κύματα δεν προέρχονται από το άπειρο, αλλά από το φύλλο κύματος.

Άρα $\vec{E}_1 = B_1 e^{jkz}\hat{x}$

$\vec{E}_2 = (A_2 e^{-jkz} + B_2 e^{jkz})\hat{x}$

$\vec{E}_3 = 0$

Εφαρμόζοντας συνθήκη για τον αγωγό

$E(z=d) = A_2 e^{-jkd} + B_2 e^{jkd} = 0$ (1)

Οριακές συνθήκες στην πηγή

$\hat{i}_x \times (E_1(z=0^+) - E_1(z=0^-)) = \vec{0} \Rightarrow E_1(z=0^+) = E_1(z=0^-)$ (2)

$\Rightarrow A_1 = A_2 + B_2$

Οριακές συνθήκες για H: (Έχω $Z_0=\sqrt{\frac{\varepsilon_0}{\mu_0}}$)

$$\vec{H}_1=-\frac{1}{Z_0}A_1e^{jkz}\hat{i}_y$$

$$\vec{H}_2=\frac{1}{Z_0}(A_2e^{-jkz}+C_2e^{jkz})\hat{i}_y$$

$$\Rightarrow \hat{i}_z\times\left(H_2(z=0^+)-H_1(z=0^-)\right)=\vec{K}$$

$$\Rightarrow (A_1+B_2-A_2)\cdot\hat{i}_x=(-Z_0k_0e^{j\varphi})\hat{i}_x$$

$$\Rightarrow A_1+B_2-A_2=-Z_0k_0e^{j\varphi} \quad (3)$$

Συνδυάζοντας τις οριακές

$$\left.\begin{aligned}A_1&=A_1+A_2\\A_1+B_2-A_2&=-Z_0k_0e^{j\varphi}\end{aligned}\right\}\Rightarrow B_2=-Z_0k_0e^{j\varphi}$$

$$\Rightarrow B_2=-\frac{1}{2}Z_0k_0e^{j\varphi}.$$

$$(2)\Rightarrow A_2=-B_2e^{-j2kd}\Rightarrow C_2=\frac{Z_0}{2}k_0e^{j(\varphi-2kd)}$$

Άρα $\boxed{A_1=-\frac{Z_0}{2}k_0e^{j\varphi}(1-e^{-j2kd})}$

Άρα βρίσκω αντίστοιχα το H (αντικατάσταση)

$$H=\begin{cases}H_1=\frac{k_0}{2}\left(e^{j\varphi}(1-e^{-j2kd})\right)\hat{i}_y e^{jkz}\\ H_2=\left(-\frac{k_0}{2}e^{j(\varphi-kz)}-\frac{k_0}{2}e^{j(\varphi(-2kd+kz))}\right)\hat{i}_y\\ H_3=0\end{cases}$$

(β) Παίρνω οριακές συνθήκες

$$K_{αγωγού}=\hat{i}_z\times(\vec{0}-H_2(z=d^-))=\hat{i}_z\times\left(-\left(-\frac{k_0}{2}e^{j\varphi-jkd}-\frac{k_0}{2}e^{j(\varphi-2kd+kd)}\right)\hat{i}_y\right)$$

Άρα $K_{αγωγού}=\hat{i}_z\times(k_0e^{j(\varphi-kd)}\hat{i}_y)=-k_0e^{j(\varphi-kd)}\hat{x}$

$$\vec{\nabla}\cdot\vec{K}+\hat{i}_z(J_3^+-J_2^+)+j\omega\sigma=0\Rightarrow \sigma=0$$

(γ) $K_{αγ}(t)=-k_0\cos(\omega t+\varphi-2kd)\hat{i}_x$ & $H_2(z=d,t)=-k_0\cos(\omega t+\varphi-kd)\hat{i}_y$.

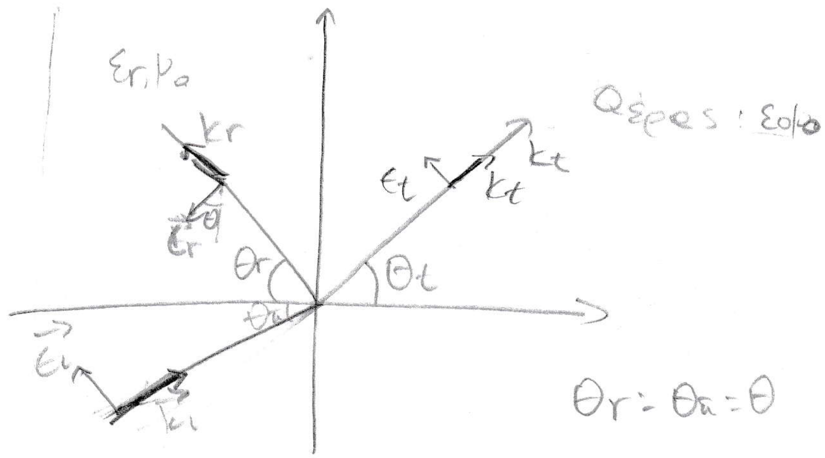
$$F_m=K_{αγ}\times\frac{B_{αγ}+B_2(z=d^-)}{2}=\frac{\mu_0 K_{αγ}\times\vec{H}_2}{2}=\frac{\mu_0}{2}\begin{vmatrix}\hat{i}_x&\hat{i}_y&\hat{i}_z\\0&H_2&0\\ \cdot&&\end{vmatrix}$$

$$\vec{F}_m=\frac{\mu_0}{2}k_0^2\cos^2(\omega t+\varphi-kd)\hat{i}_z$$

$$\langle F_m\rangle=\frac{\mu_0k_0^2}{2}\langle\cos^2(\omega t+\varphi-kd)\rangle\hat{i}_z=\frac{\mu_0k_0^2}{2}\cdot\frac{1}{2}\hat{i}_z=\frac{\mu_0k_0^2}{4}\hat{i}_z\Rightarrow\langle\vec{F}_m\rangle=\frac{\mu_0k_0^2}{4}\hat{i}_z$$

$$\ldots\frac{1}{T}\left(\int^T\frac{1}{2}+\frac{1}{2}\cos(2\omega t+2\varphi-2kd)dt\right)=$$

Άσκηση 9



Λόγω της ΤΜ πόλωσης θα ισχύουν

$k_i = k_1(\cos\theta\,\hat{i}_x + \sin\theta\,\hat{i}_y)$

$k_r = k_i(-\cos\theta\,\hat{i}_x + \sin\theta\,\hat{i}_y)$

(α) Με βάση τα παραπάνω είναι:

$\vec{E}_i = E_0 e^{-jk_i(\cos\theta x + \sin\theta y)}(-\sin\theta\,\hat{i}_x + \cos\theta\,\hat{i}_y)$

Όμως $\lambda_0 = 1\mu \Rightarrow f_0 = \frac{c}{\lambda_0} \Rightarrow f = \frac{3\cdot 10^8}{10^{-6}}\,Hz = 3\cdot 10^{14}\,Hz$ $\quad \omega = 6\pi\, 10^{14}$ rad/s

Άρα $k_1 = \omega\sqrt{\varepsilon_r\mu_r} = \omega\sqrt{\varepsilon_r\mu_0} \Rightarrow k_1 = 3\pi 10^6\, m^{-1}$

για $\theta = 55^\circ$ παίρνουμε $E_i = E_0 e^{-j3\pi 10^6(0,5736x + 0,8192y)}(0,5736\hat{x} - 0,8192\hat{y})$

οπότε στο πεδίο του χρόνου:

$$\boxed{E_i(t) = E_0\cos(2\pi\cdot 3\cdot 10^{14}t - 3\pi\cdot 10^6(0,5736\hat{x} + 0,8192\hat{y}))(0,5736\hat{x} - 0,8192\hat{y})}$$

Από Fensel:

$$\Gamma_{TM} = \frac{E_r}{E_i} = \frac{Z_1\cos\theta - Z_2\cos\theta_t}{Z_1\cos\theta + Z_2\cos\theta_t} =$$

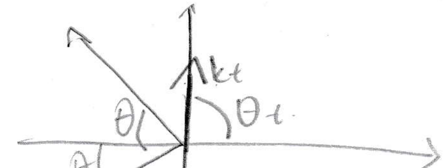
Αρκεί να υπολογίσουμε το θ_t. Όμως από νόμο Snell

$n_1\sin\theta = n_2\sin\theta_t \Rightarrow \sin\theta_t = \sin\theta\frac{n_1}{n_2} = \sin\theta\sqrt{\frac{\varepsilon_r}{\varepsilon_0}} = \frac{3}{2}\sin\theta > 1$

Αυτό σημαίνει πως θα έχουμε ολική ανάκλαση.

$\theta > \theta_{crit} = \arcsin\left(\frac{n_2}{n_1}\right) \simeq 42^\circ$

$\left.\begin{array}{l}\cos^2\theta_t + \sin^2\theta_t = 1 \\ \sin\theta_t = \sin\theta\sqrt{\varepsilon_2}\end{array}\right\}$ $\boxed{\cos\theta_t = \pm j\sqrt{\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta - 1}}$



Το πεδίο φθίνει καθώς $x \to \infty$

$$\cos\theta_t = -j\sqrt{\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta - 1}$$

$$\vec{E}_t = E_{to}e^{-jk_t r} \Rightarrow = \vec{u}\, e^{-jk_0(\sin\theta_t x + \cos\theta_t y)}$$

Frensel:

$$\Gamma_{TM} = \frac{Z_1\cos\theta - Z_2\cos\theta_t}{Z_1\cos\theta + Z_2\cos\theta_t} = \frac{Z_1\cos\theta + jZ_2\sqrt{\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta - 1}}{Z_1\cos\theta - jZ_2\sqrt{\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta - 1}}$$

Έστω $\tan\varphi = \dfrac{Z_2\sqrt{\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta - 1}}{Z_1\cos\theta}$ (1)

Άρα $\Gamma_{TM} = \dfrac{1+j\tan\varphi}{1-j\tan\varphi} \overset{\text{πολ. με το συζυγές}}{=} \dfrac{(1+j\tan\varphi)(1-j\tan\varphi)}{(1+\tan\varphi)} \xrightarrow{\cos^2\varphi = \frac{1}{1+\tan^2\varphi}} \cos^2\varphi(1+j\tan\varphi)(1-j\tan\varphi)$

$$= \cos^2\varphi(1-\tan^2\varphi + 2j\tan\varphi) = \underbrace{\cos^2\varphi - \sin^2\varphi}_{\cos(2\varphi)} + \underbrace{2\cos\varphi\sin\varphi\, j}_{j\sin 2\varphi}$$

$$= \cos(2\varphi) + \sin(2\varphi)j = e^{j2\varphi}$$

Από (1) $\varphi = \arctan\left(\dfrac{Z_2\sqrt{\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta - 1}}{Z_1\cos\theta}\right)$

$n = \dfrac{c_0}{c} = \sqrt{\mu\varepsilon}\, c_0 = \sqrt{\dfrac{\mu_0\varepsilon}{\mu_0\varepsilon_0}} = \sqrt{\dfrac{\varepsilon}{\varepsilon_0}}$

$Z = \sqrt{\dfrac{\mu}{\varepsilon}} \Rightarrow Z n = \mu_0 c_0$

$$\Rightarrow \varphi = \frac{\frac{\mu_0 c_0}{n_2}\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta - 1}}{\frac{\cancel{\mu_0 c_0}}{n_1}\cos\theta}$$

δ) $\vec{S} = \dfrac{1}{2}\vec{E}\times\vec{H}^*$

Faraday: $-j\omega\mu_0\vec{H}_t = \nabla\times\vec{E}_t$

$= (|t_{TM}|E_0 e^{j(\varphi - k_y y)} e^{-k_2|\cos\theta_t|x}\sin\theta_t \cdot jk_1\sin\theta_t + j|\cos\theta_t||t_{TM}|E_0(-k_2|\cos\theta_t|)e^{j(\varphi - k_y y)}e^{-k_2|\cos\theta_t|x}\hat{i}_z$

Παρατηρώ ότι και οι δύο όροι έχουν κοινούς όρους. Άρα

Θέτω $\alpha = |t_{TM}|E_0 e^{-k_2|\cos\theta_t|x} e^{j(\varphi - k_1\sin\theta y)}$

$\Rightarrow H_t = \dfrac{\alpha}{Z_2}(|\cos\theta_t|^2 - \sin^2\theta_t)\hat{i}_z \Rightarrow H_t^* = \dfrac{\alpha^*}{Z_2}(|\cos\theta_t|^2 - \sin^2\theta_t)\hat{i}_z$

αλλάζουν πρόσημους

$\Rightarrow S = \dfrac{1}{2}E\times\vec{H}_t^* = \dfrac{\alpha}{2}\cdot\dfrac{\alpha^*}{Z_2}(|\cos\theta_t|^2 - \sin^2\theta_t)(-j\cos\theta_t\hat{i}_y - \sin\theta_t\hat{i}_x)\times\hat{z}$

$$= \frac{|a|^2}{2Z_2}\left(-\sin\theta_t \hat{i}_y + j|\cos\theta_t| \hat{i}_y\right)$$

$$= \frac{|a^2|}{2Z_2}\left(-\sin\theta_t \hat{i}_y + j|\cos\theta_t| \hat{i}_x\right)$$

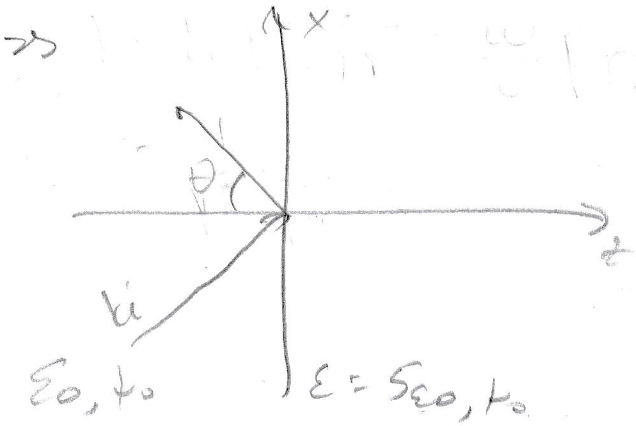
$$\Rightarrow S = \frac{|E_0|^2 |t_{TM}|^2}{2Z_2}\left(\cos\theta_t \hat{i}_x + \sin\theta_t \hat{i}_y\right) e^{-2kz|\cos\theta_t| x}$$

$$\langle \vec{N} \rangle = Re\{\vec{S}\} = \langle \vec{N}(x=0)\rangle e^{-\xi x}, \; \xi > 0 \quad \text{άρα}$$

$P_{αυξ x} = 0$

(ε) Προφανώς $\frac{P_{ref}}{P_i} = 1$ & $\frac{P_t}{P_i} = 1$.

(10)(α) $k r = k(\hat{i}_x + \hat{i}_z)$ $\quad E = [2\cos\theta\, \hat{i}_x - j3\hat{i}_y - 2\sin\theta]\, e^{j(\vec{k}\vec{r})}$



Υπάρχει γωνία Brewster άρα η πόλωση είναι TM

$\sin^2\theta_B = \dfrac{\varepsilon_2}{\varepsilon_1+\varepsilon_2} = \dfrac{5}{6} \Rightarrow \sin\theta_B = 0{,}9129$, $\cos\theta_B = 0{,}4082$ $\leftarrow \sin\theta_B = \sqrt{\dfrac{5}{6}}$

Άρα αφού η y είναι 0 τότε

$k_i = k_0(\sin\theta_B, 0, \cos\theta_B) = 2\pi 10^6 (0{,}9129\,\hat{i}_x + 0{,}4082\,\hat{i}_z)$

(β) $r_{\parallel B} = \dfrac{Z_2\cos\theta_1 - Z_1\cos\theta_2}{Z_2\cos\theta_1 + Z_1\cos\theta_2} \underset{Z_i = \frac{Z_0}{n_i}}{\overset{\theta_1+\theta_2=\pi/2}{=}} \dfrac{n_1\cos\theta_1 - n_2\sin\theta_2}{n_1\cos\theta_1 + n_2\sin\theta_1} = -\dfrac{2}{3}$

Αφού υπάρχει μηδενικός τότε το Er θα διαδοθεί κάθετα στο $y=0$

$E_r = r_{TE}\cdot E_{i,y}\, e^{-j2\pi 10^6(0{,}9129\hat{i}_x - 0{,}4082\hat{i}_z)(x\hat{i}_x + y\hat{i}_y + z\hat{i}_z)}$

$r_{TE} = \dfrac{E_r}{E_i} = -\dfrac{2}{3}(\cancel{3}j)e^{-j2\pi 10^6(0{,}9129x - 0{,}4082z)}$ $\quad \hat{i}_y = \hat{i}_y\, 2j\, e^{-jk_0(0{,}9129x - 0{,}4082z)}$

$r_{TM} = \dfrac{2Z_2\cos\theta_1}{Z_1\cos\theta_1 + Z_2\cos\theta_2} = \dfrac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\sin\theta_1} = \dfrac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_2\cos\theta_1} = \dfrac{1}{\sqrt{5}}$ $\quad t_{TE} = 1$

$E_T = (t_{TM}\,\hat{e}_T\, E_i + t_{TE}\,\hat{e}_{TE}\, E_r)\, e^{-jk_2 \vec{r}}$
$= [0{,}8944(0{,}9129x - 0{,}4082z) - j\hat{i}_y]\, e^{-jk_0\sqrt{5}(0{,}4082x + 0{,}9129z)}$

(γ) $\vec{H}_i = \dfrac{1}{Z_0}(\vec{k}_i \times \vec{E}_i) + \dfrac{1}{Z_0}(\vec{k}_r \times \vec{E}_r) = \dfrac{1}{377}\Big[\left(\dfrac{\sqrt{5}}{\sqrt{6}}, 0, \dfrac{1}{\sqrt{6}}\right)\times(2\cos\theta_1, -3j, 2\sin\theta_1)\, e^{jkr}$

$+ \left(\dfrac{\sqrt{5}}{\sqrt{6}}, 0, -\dfrac{1}{\sqrt{6}}\right)\times(0, 2j, 0)\, e^{-j\vec{k}_r\vec{r}}$

$= \dfrac{1}{377}\left[-3j(0{,}9129\,\hat{i}_z - 0{,}4082\,\hat{i}_x) - 2\hat{i}_y\right] e^{-jk_0(0{,}9129x + 0{,}4082z)} + [2j(0{,}4082\,\hat{i}_x + 0{,}9129\,\hat{i}_z)]$

δ) $r \cdot k_2 = 0 \rightarrow \cos\theta_1 x + \sin\theta_1 \cdot z = 0$

Άρα για το E_i:

$E_x = 2\cos[\omega t - k_0(0,9129x + 0,4082z)]$

$E_y = 3\cos(\omega t - k_0(0,9129x + 0,4082 + \pi/2]$

$= 3\sin(\omega t - k_0(0,9129x + 0,4082]$

Άρα $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{E_x}{2}\right)^2 + \left(\frac{E_y}{3}\right)^2 = 1 \rightarrow \tan\theta = \frac{\cos(\omega t - \vec{k}\vec{r} + \pi/2)}{\cos(\omega t - \vec{k}\vec{r}} = \tan(\omega t - \vec{k}\vec{r})$

Για το E_r:

$\left.\begin{array}{l} E_{0y} = 2j \\ E_{0x} = 0 \end{array}\right\}$ Γραμμική πόλωση κάθετη στο επ. ί, άρα είναι πόλωση ΤΕ.

Για το E_i:

$\frac{5E_x^2}{4} + E_y^2 = 1$ & $\tan\theta = \tan(\omega t - \vec{k}\vec{r})$

ε) $\frac{P_r}{P_i} = \frac{1/z_0 |E_r|^2}{1/z_0 |E_i|^2} = \frac{2}{2^2(\cos^2\theta + \sin^2\theta) + 3^2} = \frac{4}{13} = 0,3077$

$\frac{P_t}{P_i} = 1 - \frac{P_r}{P_i} = \frac{9}{13} = 0,6923$