A Family of Vertex Transitive Graphs

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1 Sylvester Number System

Let s_n be the nth element of Sylvester's sequence [1], defined as:

$$s_0 = 2 \tag{1}$$

$$s_{n+1} = 1 + \prod_{i=0}^{n} s_n. \tag{2}$$

A mixed-base number system can be constructed from Sylvester's sequence as follows:

Definition 1.1. A *Sylvester-radix* number a is a sequence of digits a_n such that: $a_n \in \mathbb{Z} : 0 \leq a_n < s_n$.

Lemma 1. There are $(s_n - 1)$ Sylvester-radix numbers of length n.

Proof. The Sylvester-radix numbers of length 1 are (0) and (1). $(s_1 - 1) = 2$ so the lemma holds for $n = 1$.

For $n > 1$, there are s_i possible values for each digit, with $0 \leq i < n$. The number of valid digit combinations is thus given by:

$$\prod_{i=0}^{n-1} s_i = s_n - 1 \qquad \text{(by (2))}.$$

□

Corollary 1. The place value of index i in a Sylvester-radix number is $(s_i - 1)$.

The integer value of a length-n Sylvester-radix number a is thus:

$$z(a) = \sum_{i=0}^{n-1} a_i(s_i - 1). \tag{3}$$

References

[1] James J Sylvester. On a point in the theory of vulgar fractions. *American Journal of Mathematics*, 3(4):332–335, 1880.