Stat 140: Rules for Probabilities

Probability (Chapters 13 and 14)

General Terminology

1. Trial: a single attempt or realization of a random phenomenon
2. Outcome: an observed value in a trial
3. Event: A set of possible outcomes. Use letters like A, B, C
4. Sample Space: The set of all possible outcomes. Use S.
5. Complement: The complement of the event A is the set of all possible outcomes *not* in A. Use A^C.
6. Disjoint Events: have no outcomes in common
7. Independent Events: knowing one occurred doesn't change what you know about the chances of the other occurring.

Probability Definitions and Rules

Let A and B be events, and let S be the sample space.

Foundations

- $0 \leq P(A) \leq 1$
- $P(S) = 1$
- $P(A^c) = 1 - P(A)$

Probability of A *or* B (or both) occurring

- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- If A and B are disjoint events, then $P(A \text{ or } B) = P(A) + P(B)$.

Conditional Probability; Probability of A *and* B both occurring

- Conditional Probability of B given A: $P(B|A) = \frac{P(\text{B and } A)}{P(A)}$
- $P(A \text{ and } B) = P(A) \times P(B|A) = P(B) \times P(A|B)$
- If A and B are **independent** events, then $P(A \text{ and } B) = P(A) \times P(B)$.
- A and B are **independent** if (and only if) $P(B|A) = P(B)$, or turning that around, $P(A|B) = P(A)$.

Bayes' Rule

$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|A^c)P(A^c)}$