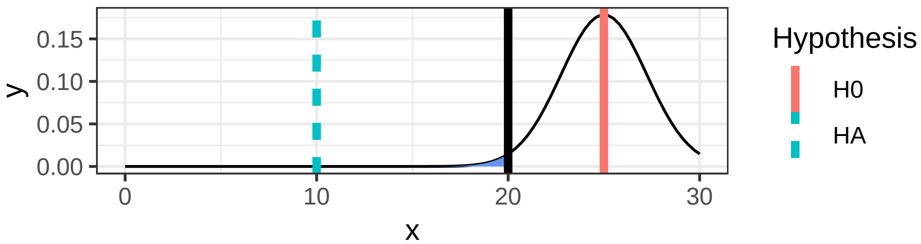
Hypothesis Testing - Finishing the Paint Drying Example

- Data Model: $X_1, \dots, X_5 \overset{\text{i.i.d.}}{\sim} \text{Normal}(\theta, 5^2)$
- We know that θ is either 10 (quick dry) or 25 (regular).
- We have a vague feeling that batch 1 is quick-dry; we will calculate a p-value to see how strong the evidence is that the batch we used is *not* 25 minutes.

$H_0: \theta = 25$ vs. $H_A: \theta = 10$

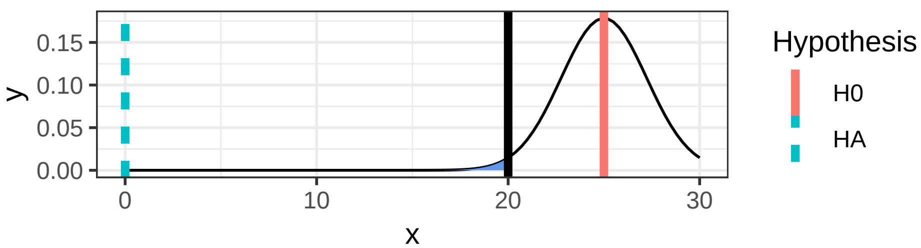
- After collecting data, we observe a sample mean of $\bar{x} = 20$ minutes.
- We saw that the likelihood ratio test is *equivalent* to a test based on $\bar{x}$
- The p-value is $P(\bar{X} \leq 20 | \theta = 25)$ ("extreme" values of $\bar{x}$ are those that are at least as small as $\bar{x} = 20$)
- If H_0 is true, then $\bar{X} \sim \text{Normal}(25, 5^2/5)$




```r
pnorm(20, mean = 25, sd = 5)
```

```
## [1] 0.1586553
```

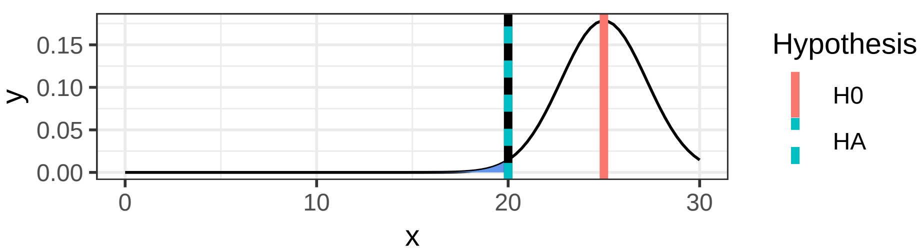
What about a test of $H_0: \theta = 25$ vs. $H_A: \theta = 0$?




```r
pnorm(20, mean = 25, sd = 5)
```

```
## [1] 0.1586553
```

What about a test of $H_0: \theta = 25$ vs. $H_A: \theta = 20$?




```r
pnorm(20, mean = 25, sd = 5)
```

```
## [1] 0.1586553
```

Main point: In this framework the null hypothesis is "privileged"

- Everything is framed in terms of strength of evidence against the null hypothesis.
- p-value calculations done assuming H_0 is true
- In many examples (like this one), details of H_A are relatively unimportant (here, only determines direction of "more extreme")