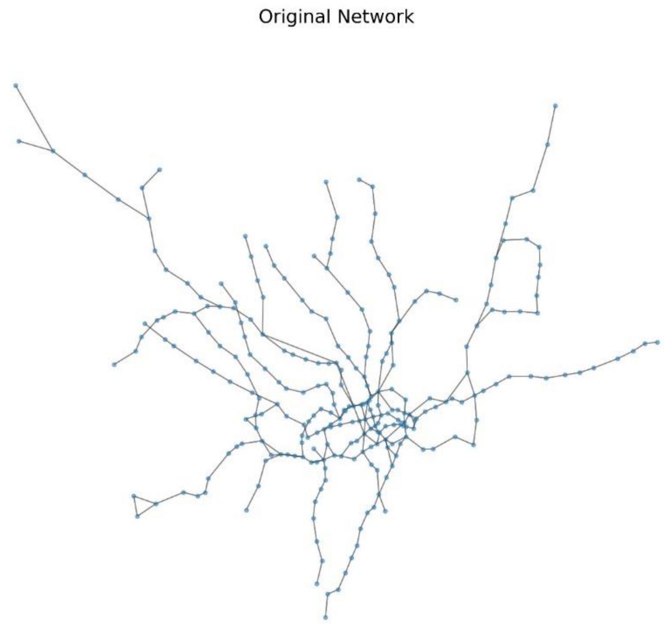
Centrality of the London Underground Network following Random and Targeted Attacks

Before Attack:

Important nodes are those that are extensively linked with other nodes, with high centrality generally indicating high importance. This analysis uses two centrality measures: Degree Centrality (DC) and Betweenness Centrality (BC).

The network comprises 266 stations, with 73% having a DC of 0.00755 (Figures 1, 2). The station with the highest DC value, 0.02642, is Baker Street, which has a degree value of 7. Additionally, Baker Street has the highest BC value, 0.34222. Green Park and King's Cross St. Pancras also have high BC (Figure 4).



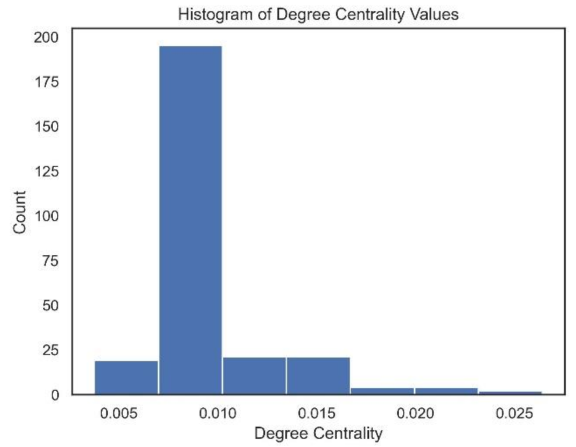
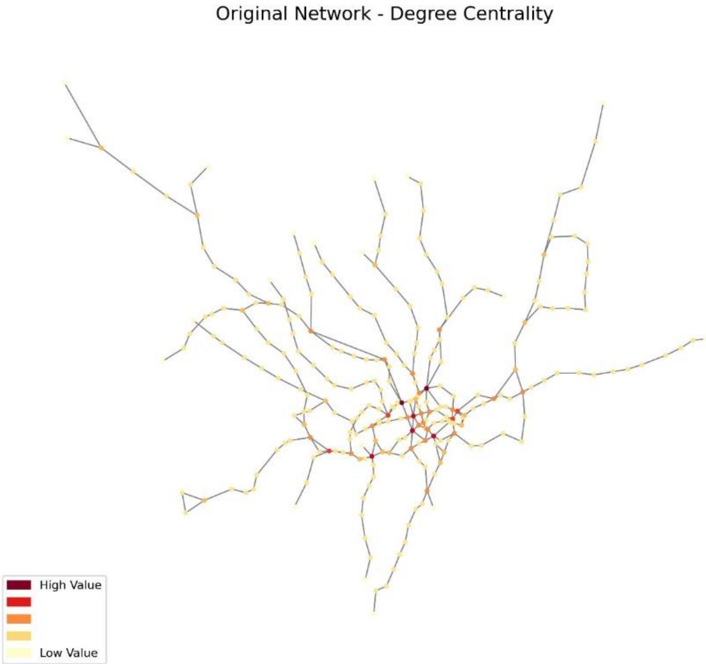


Figure 2: Histogram of Degree Centrality Values on the London Underground Network. Created using Networkx and Pandas in Python.

Figure 1: London Underground Network, created using Networkx in Python.

Stations with high centrality, like Waterloo, act as connectors between different lines (Figure 3). Notably, Bethnal Green and Stratford have high BC, indicating their importance despite their low degree of direct connections, with values of 2 and 3 respectively (Figure 4).



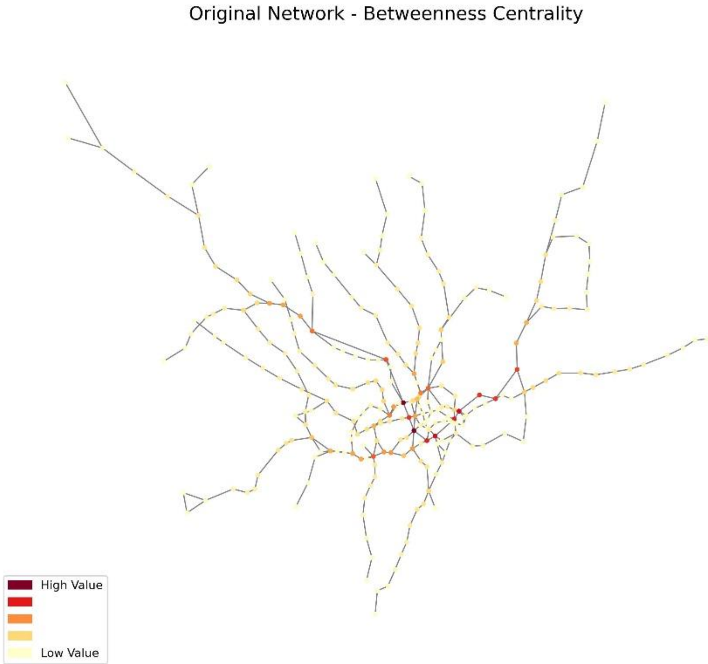


Figure 3: Centrality Graphs of the London Underground Network, created using Networkx in Python.

	name	degree	betw_centrality	degree_centrality
0	Baker Street	7	0.34222	0.02642
92	King's Cross St. Pancras	7	0.20009	0.02642
4	Oxford Circus	6	0.14634	0.02264
22	Waterloo	6	0.27436	0.02264
115	Earl's Court	6	0.20716	0.02264
14	Green Park	6	0.33773	0.02264
23	Bank	5	0.25450	0.01887
137	Turnham Green	5	0.12984	0.01887
48	Liverpool Street	5	0.27736	0.01887
17	Paddington	5	0.18005	0.01887

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0	Baker Street	7	0.34222	0.02642
14	Green Park	6	0.33773	0.02264
48	Liverpool Street	5	0.27736	0.01887
22	Waterloo	6	0.27436	0.02264
23	Bank	5	0.25450	0.01887
27	Westminster	4	0.25367	0.01509
49	Bethnal Green	2	0.23785	0.00755
63	Mile End	4	0.23748	0.01509
8	Bond Street	4	0.23554	0.01509
99	Stratford	3	0.22668	0.01132

Figure 4: Tables displaying the ten highest stations base on Degree Centrality (left) and Betweenness Centrality (right). Created using Networkx and Pandas in Python.

Random attack:

While nearly all stations retained their degree value following a random attack, DC increases slightly due to the reduction in the total number of nodes; 263 compared to 266 (Figure 5). For instance, Baker Street's DC increased from 0.02642 to 0.02672. The top ten stations with the highest BC, although now in a different order, remained unchanged (Figure 6). These findings suggest that the removal of random stations has minimal impact on the importance of key stations within the network.

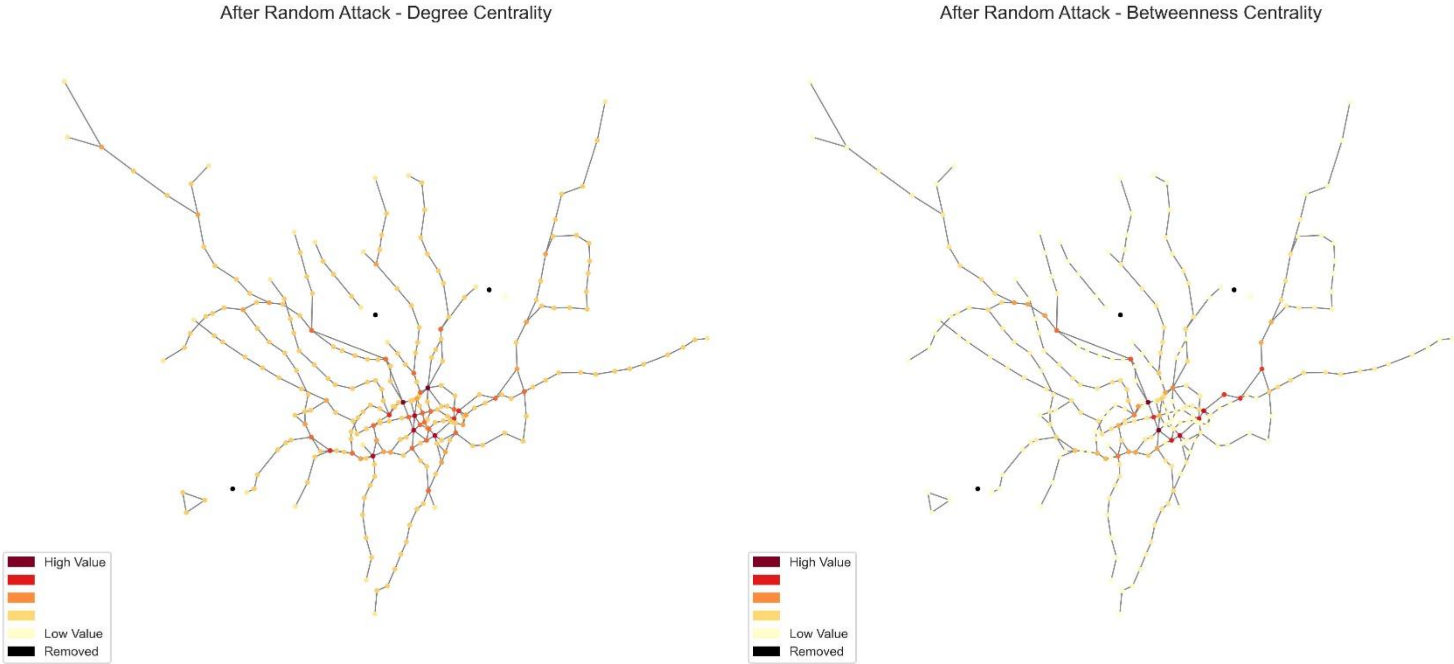

Figure 5: Centrality Graphs of the London Underground Network following a random attack and the removal of three stations: Blackhorse Road, Golders Green, and Hounslow West. Created using Networkx in Python.

	name	degree	betw_centrality	degree_centrality
0	Baker Street	7	0.33104	0.02672
92	King's Cross St. Pancras	7	0.17029	0.02672
22	Waterloo	6	0.27197	0.02290
4	Oxford Circus	6	0.11956	0.02290
115	Earl's Court	6	0.18211	0.02290
14	Green Park	6	0.32177	0.02290
48	Liverpool Street	5	0.26789	0.01908
17	Paddington	5	0.17363	0.01908
23	Bank	5	0.25203	0.01908
137	Turnham Green	5	0.10482	0.01908

	name	degree	betw_centrality	degree_centrality
0	Baker Street	7	0.33104	0.02672
14	Green Park	6	0.32177	0.02290
22	Waterloo	6	0.27197	0.02290
48	Liverpool Street	5	0.26789	0.01908
23	Bank	5	0.25203	0.01908
27	Westminster	4	0.25010	0.01527
8	Bond Street	4	0.23395	0.01527
49	Bethnal Green	2	0.22991	0.00763
63	Mile End	4	0.22987	0.01527
99	Stratford	3	0.22006	0.01145

Figure 6: Tables displaying the top ten stations base on Degree Centrality (left) and Betweenness Centrality (right) following a random attack. Created using Networkx and Pandas in Python.

Targeted Attack:

Following a targeted attack and the removal of three stations based on passenger entry counts, there is a shift in the locations of important nodes based on centrality values (Figure 7). Furthermore, the most significant stations based on BC changed, with the appearance of stations such as Tottenham Court Road, Finchley Road, Holborn, London Bridge and Earl's Court (Figure 8).

BC values of the most important stations decreased. For example, Baker Street on the original network has a BC of 0.34222 and following a random attack a value of 0.33104. However, following a targeted attack, BC is 0.29993.

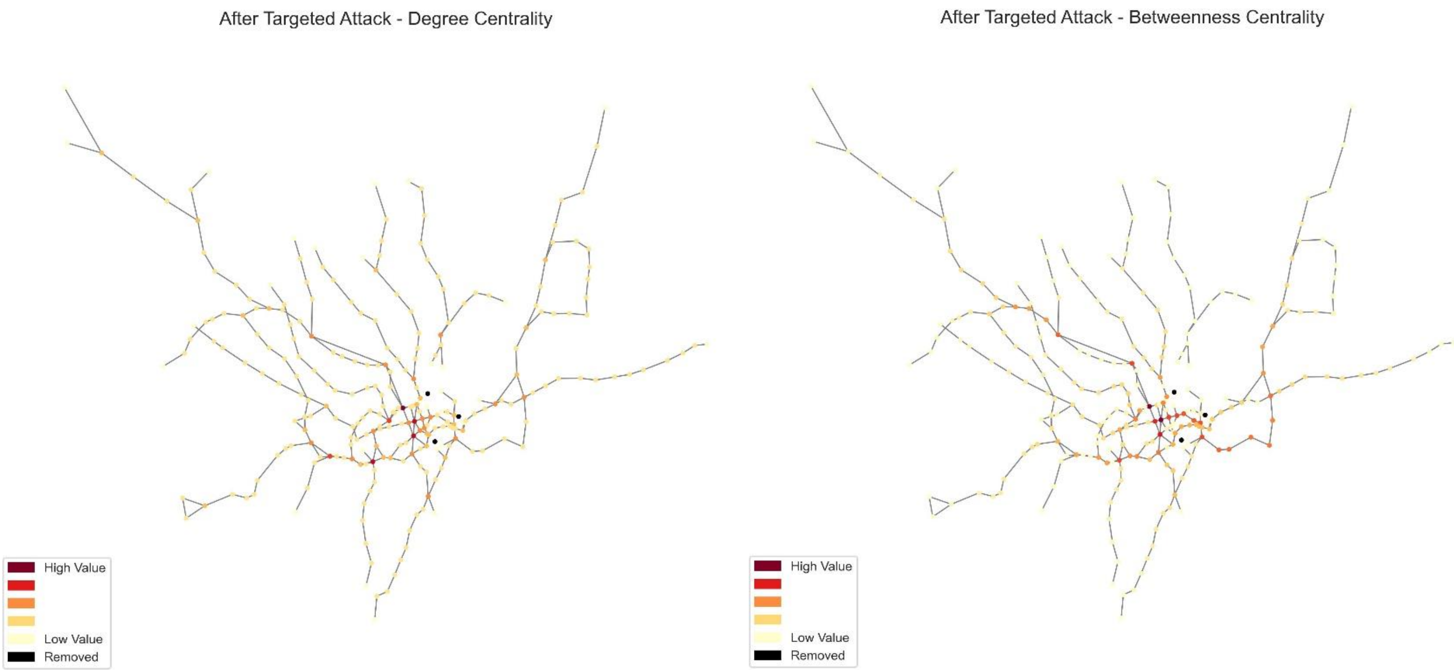

Figure 7: Centrality Graphs of the London Underground Network following a targeted attack and the removal of three stations: King's Cross St. Pancras, Liverpool Street and Waterloo. Created using Networkx in Python.

	name	degree	betw_centrality	degree_centrality
0	Baker Street	7	0.29993	0.02672
14	Green Park	6	0.23046	0.02290
112	Earl's Court	6	0.19188	0.02290
4	Oxford Circus	6	0.29727	0.02290
134	Turnham Green	5	0.12187	0.01908
17	Paddington	5	0.15618	0.01908
61	Mile End	4	0.07761	0.01527
172	Wembley Park	4	0.17604	0.01527
220	Stockwell	4	0.10056	0.01527
210	Camden Town	4	0.13451	0.01527

	name	degree	betw_centrality	degree_centrality
0	Baker Street	7	0.29993	0.02672
4	Oxford Circus	6	0.29727	0.02290
14	Green Park	6	0.23046	0.02290
8	Bond Street	4	0.21693	0.01527
41	Tottenham Court Road	4	0.19631	0.01527
10	Finchley Road	4	0.19554	0.01527
42	Holborn	4	0.19403	0.01527
52	London Bridge	4	0.19245	0.01527
112	Earl's Court	6	0.19188	0.02290
22	Bank	3	0.18735	0.01145

Figure 8: Tables displaying the top ten stations base on Degree Centrality (left) and Betweenness Centrality (right) following a targeted attack. Created using Networkx and Pandas in Python.

Discussion:

The median BC is lowest following a random attack. Additionally, low centrality nodes increased, with a greater number of stations having a BC less than 0.04 (Figure 9). This is partly due to the positioning of the randomly removed stations, which resulted in isolating sections of the network, visible in the north and southwest (Figure 5).

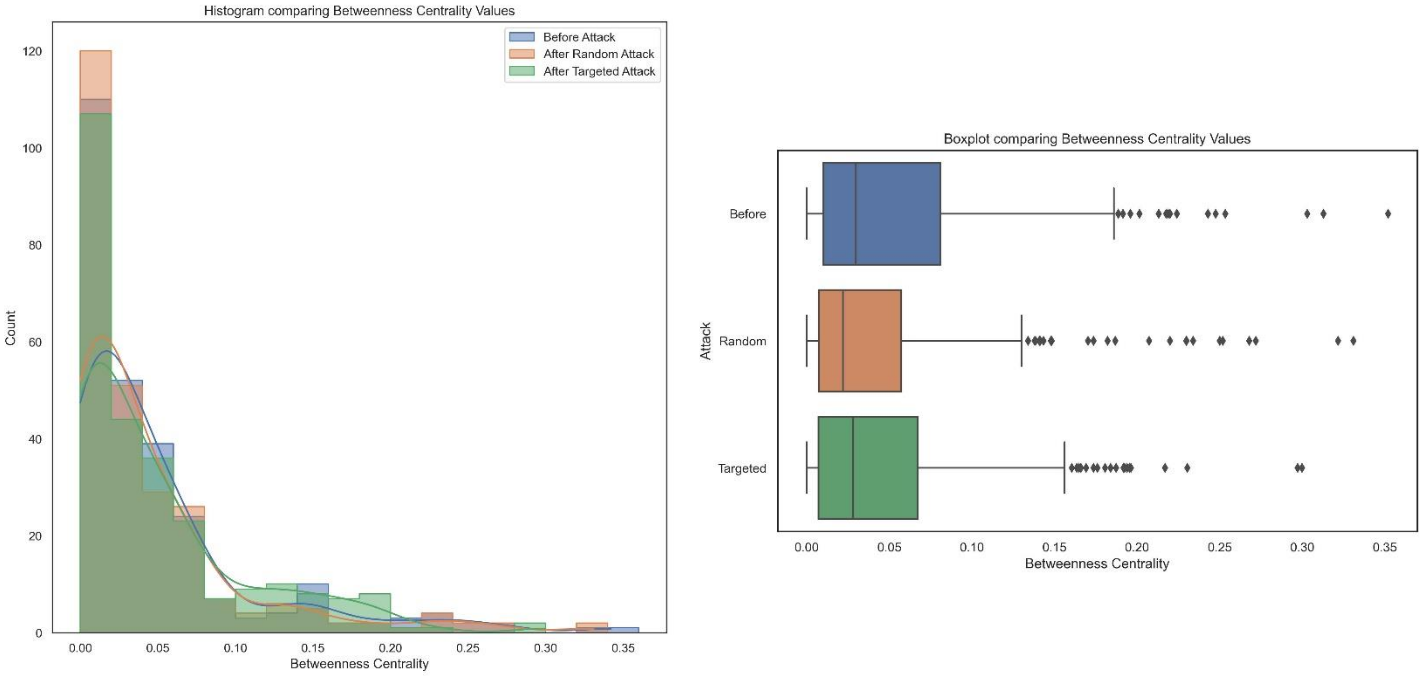

Figure 9: Histogram and Boxplot comparing Betweenness Centrality Values Before an Attack, After a Random Attack and After a Targeted Attack. Created using Networkx, Pandas and Seaborn in Python.

Overall, a targeted attack would be the most damaging, with station removal causing major disruption. When random stations are removed, particularly if they have low centrality, it has little effect on the network, and stations with high centrality remain the most important. However, when a targeted attack is conducted, there is a greater impact on the network's makeup, and the locations of high centrality shifts.

Future analysis may wish to investigate other network measures, such as closeness centrality, to determine the most damaging attack strategy.