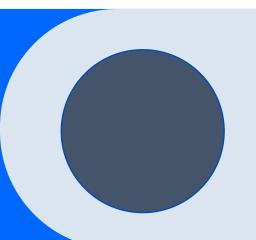


# **QR** Decomposition



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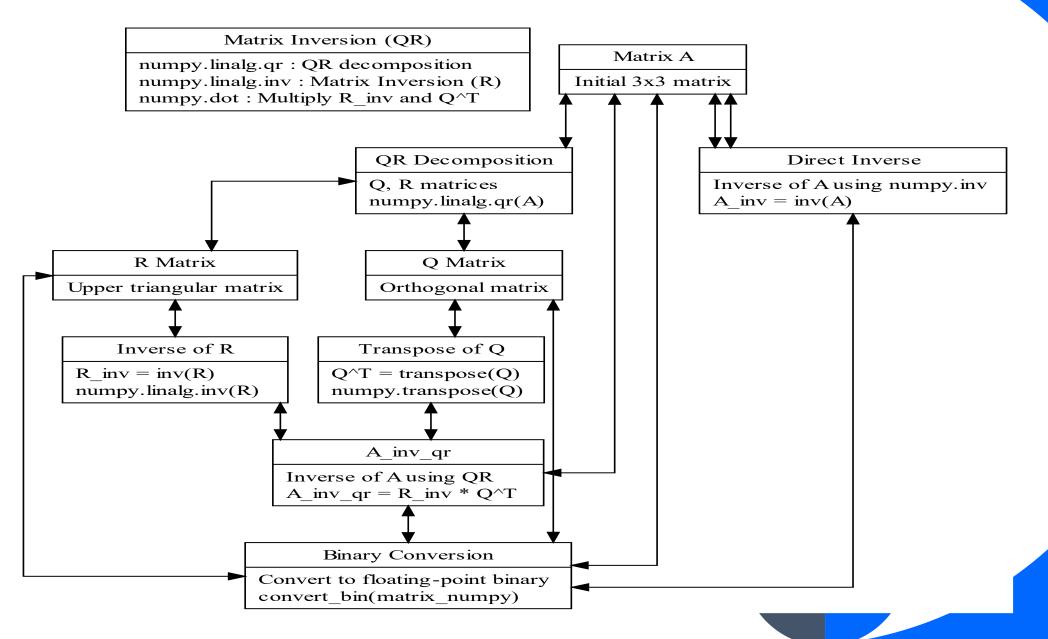
# **Agenda**

- Modeling
- Architecture
- Code
- Simulation
- FPGA

### **Cordic UML**

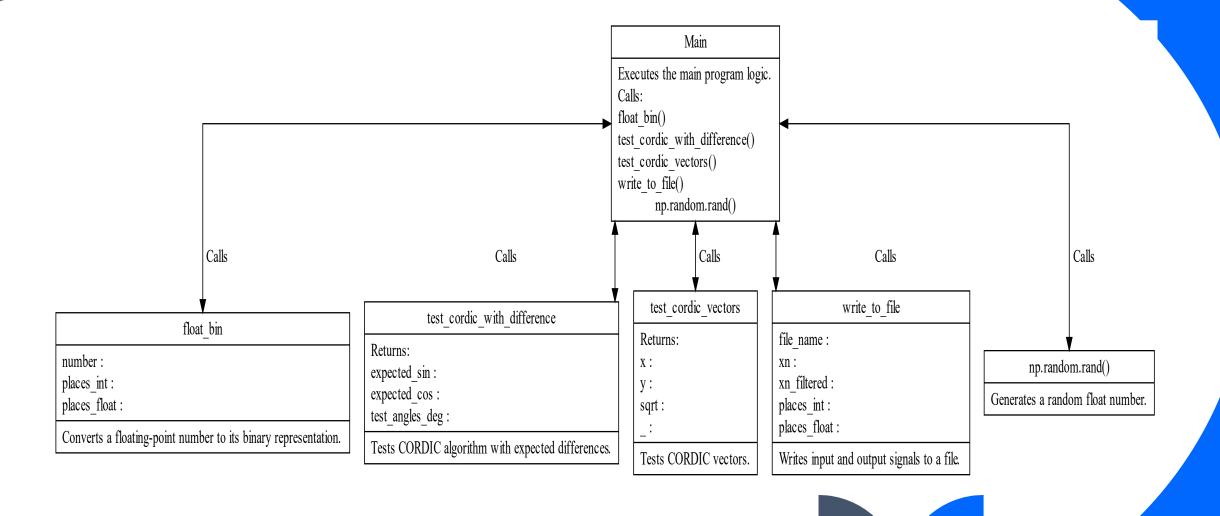
```
python_modeling.cordic_modeling.CORDIC
tan_array:
change:
scaling_factor :
inverse_y:
iterations:
inverse_x:
  _init__(self, iterations):
_generate_tan_array(self):
_initialize_arrays(self, x_in, y_in, sqrt, theta):
generate_sqrt_sin_cos(self, x_in=1, y_in=0, sqrt=False, theta=0):
generate_K(self) :
```

### Matix inverse using QR

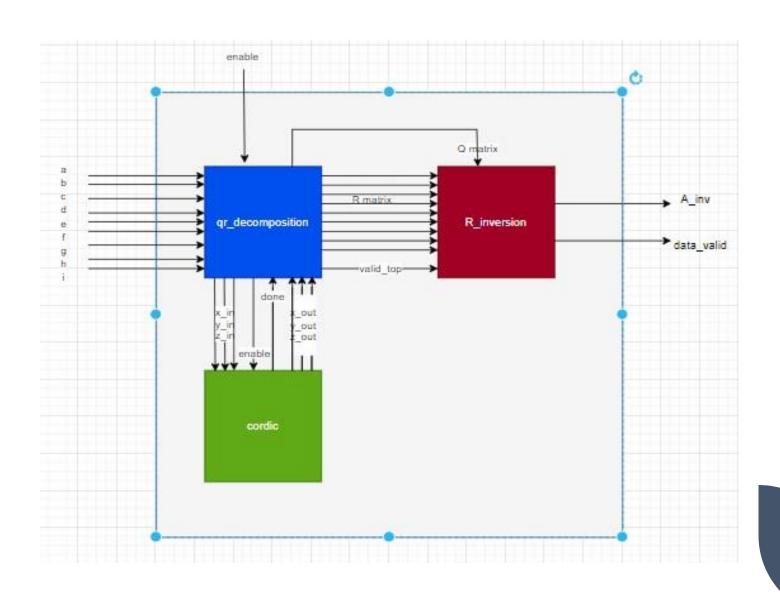


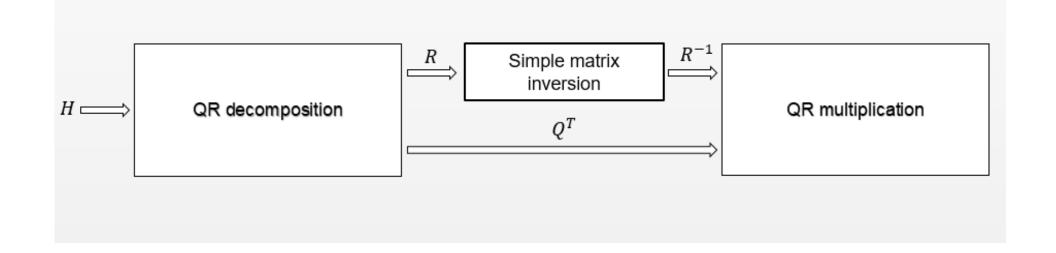
# **Binary conversion**

### **Main test**



### **Architecture**





# Code

```
module cordic (
     input logic clk,
     input logic rst n,
     input logic select,
     input logic enable,
     input logic signed [31:0] x in,
     input logic signed [31:0] y in,
     input logic signed [31:0] z in,
     output logic signed [31:0] x out,
     output logic signed [31:0] y out,
     output logic signed [31:0] z_out,
     output logic done
L);
     // Parameters for CORDIC
     parameter ITER = 15;
     // Internal registers and wires
     logic signed [31:0] x [0:ITER-1];
     logic signed [31:0] v [0:ITER-1];
     logic signed [31:0] z [0:ITER-1];
     logic done 1;
     // Arctangent lookup table
     // q8.24 fixed-point format
     logic signed [31:0] arctan table [0:14] = '{
         32'sb000000001100100100000111111011010,
         32'sb00000000011101101011000110011100,
         32'sb0000000000111110101101101111101011,
         32'sb00000000000111111101010110111010,
         32'sb000000000000111111111101010101101,
         32'sb0000000000000111111111111101010101,
         32'sb0000000000000011111111111111101010,
         32'sb000000000000000001111111111111111,
         32'sb00000000000000000011111111111111,
         32'sb00000000000000000011111111111111,
         32'sb0000000000000000000111111111111,
         32'sb00000000000000000000011111111111,
         32'sb00000000000000000000001111111111
     logic signed [31:0] scaling factor = 32'b000000001001101101101101101; // Adjust as needed
     logic signed [63:0] x_out_64,y_out_64;
```

Cordic block

```
always comb begin
   if (enable) begin
       // Initialize inputs
       x[0] = x in;
        y[0] = y in;
        z[0] = z in;
        for (int i = 0; i < ITER-1; i++) begin
            if (!select) begin
                // Rotational Mode
                if (z[i] >= 0) begin
                    x[i+1] = x[i] - (y[i] >>> i);
                   y[i+1] = y[i] + (x[i] >>> i);
                    z[i+1] = z[i] - arctan_table[i];
                end else begin
                    x[i+1] = x[i] + (y[i] >>> i);
                    y[i+1] = y[i] - (x[i] >>> i);
                    z[i+1] = z[i] + arctan_table[i];
                end
            end else begin
                // Vectoring Mode
                if (v[i] >= 0) begin
                   x[i+1] = x[i] + (y[i] >>> i);
                   y[i+1] = y[i] - (x[i] >>> i);
                    z[i+1] = z[i] + arctan table[i];
                end else begin
                   x[i+1] = x[i] - (y[i] >>> i);
                   y[i+1] = y[i] + (x[i] >>> i);
                   z[i+1] = z[i] - \arctan table[i];
                end
            end
        // Assert done after last iteration
        done 1 = 1;
        x out 64=x[ITER-1] * scaling factor;
        y out 64=y[ITER-1] * scaling factor;
   end else begin
        done 1 = 0;
```

```
// Output the final iteration values
    always_ff @(posedge clk or negedge rst_n) begin
        if (!rst n) begin
            // Reset state
            done \leq 0;
            x out \leq 0;
            y_out <= 0;</pre>
            z out <= 0;
        end else begin
            // Scaling factor to convert the final result back to proper format
            // Multiply by scaling factor, considering fixed-point format (q8.24)
            x out <= x out 64[55:24]; // Scaling down to correct fractional bits
            y out <= y out 64[55:24];</pre>
            z out <= z[ITER-1];</pre>
            done <= done 1;
endmodule
```

```
module gr decomposition (
                                                                                      // State transition logic
    input logic clk,
                                                                                   □ always ff @(posedge clk or negedge rst n) begin
   input logic rst n,
   input logic enable,
                                                                                          if (!rst n) begin
   input logic done1, // the cordic out is ready
                                                                                                current state <= IDLE;
   input logic signed [31:0] a, b, c,
                                                                                           end else begin
   input logic signed [31:0] d, e, f,
                                                                                               current_state <= next state;
   input logic signed [31:0] g, h, i,
   input logic signed [31:0] cor_x_out,
                                                                                           end
   input logic signed [31:0] cor y out,
   input logic signed [31:0] cor z out,
   output logic
   output logic signed [31:0] cor_x_in,
                                                                                      // Next state logic
   output logic signed [31:0] cor_y_in,
                                                                                   always comb begin
   output logic signed [31:0] cor_s_in,
                                                                                          case (current state)
   output logic
                           select out, // for kind of cordic
   output logic signed [31:0] a out, b out, c out,
                                                                                                IDLE: begin
   output logic signed [31:0] d_out, e_out, f_out,
                                                                                                    if (enable)
   output logic signed [31:0] g_out, h_out, i_out,
                                                                                                          next state = phase1 1 in;
   output logic signed [31:0] Q [2:0][2:0],
   output logic
                           data valid
                                                                                                    else
                                                                                                         next state = IDLE;
                                                                                                end
logic [5:0] current_state, next_state;
                                                                                               phase1 1 in: next state = phase1 1 out;
parameter IDLE
                   = 'b000000.
                                                                                               phase1 1 out: begin
         phase1_1_in = 'b00001,
         phase1 1 out = 'b00010,
                                                                                                    if (done1)
         phase1_2_in = 'b00011,
                                                                                                         next state = phase1 2 in;
         phase1 2 out = 'b00100,
                                                                                                    else
         phase1 3 in = 'b00101.
         phase1_3_out = 'b00110,
                                                                                                         next state = phase1 1 out;
         phase2 1 in = 'b00111,
         phase2_1_out = 'b01000,
                                                                                               phase1 2 in: next state = phase1 2 out;
         phase2 2 in = 'b01001,
                                                                                               phase1 2 out: begin
         phase2 2 out = 'b01010,
         phase2_3_in = 'b01011,
                                                                                                    if (done1)
         phase2 3 out = 'b01100,
                                                                                                         next state = phase1 3 in;
         phase3 1 in = 'b01101.
                                                                                                     else
         phase3_1_out = 'b01110,
         phase3 2 in = 'b01111,
                                                                                                         next state = phase1 2 out;
         phase3_2_out = 'b10000,
         multiply = 'b10001;
                                                                                                phase1 3 in: next state = phase1 3 out;
                                                                                               phase1 3 out: begin
logic signed [31:0] R [2:0][2:0];
logic signed [31:0] a prime, b prime, c prime, d prime, e prime, f prime;
                                                                                                    if (done1)
logic signed [31:0] g prime, h prime, i prime, thetal;
                                                                                                          next state = phase2 1 in;
logic signed [31:0] phay1 [2:0][2:0], phay2 [2:0][2:0], phay3 [2:0][2:0];
logic signed [31:0] temp [2:0][2:0]; // Intermediate matrix after phay1 * phay2
                                                                                                         next state = phase1 3 out;
```

```
phase2_1_in: next_state = phase2_1_out;
   phase2 1 out: begin
       if (done1)
           next state = phase2 2 in;
       else
           next_state = phase2_1_out;
   phase2_2_in: next_state = phase2_2_out;
   phase2 2 out: begin
       if (done1)
           next_state = phase2_3_in;
           next_state = phase2_2_out;
   phase2_3_in: next_state = phase2_3_out;
   phase2_3_out: begin
       if (done1)
           next state = phase3 1 in;
       else
           next_state = phase2_3_out;
   phase3_1_in: next_state = phase3_1_out;
   phase3 1 out: begin
       if (done1)
           next_state = phase3_2_in;
       else
           next_state = phase3_1_out;
    phase3_2_in: next_state = phase3_2_out;
    phase3 2 out: begin
       if (done1)
           next_state = multiply;
           next_state = phase3_2_out;
   multiply: begin
       if (data valid)
           next_state = IDLE;
       else
           next_state = multiply;
    default: next_state = IDLE;
endcase
```

```
// Output logic
always comb begin
     case (current state)
          IDLE: begin
             cor_x_in = 0;
              cor_y_in = 0;
              cor s in = 0;
         end
         phase1 1 in: begin
             enable_out = 1;
             cor x in = a;
             cor_y_in = d;
              cor s in = 0;
              select out = 1;
          end
         phase1 1 out: begin
              enable_out = 0;
              a prime = cor x out;
              theta1 = cor_y_out;
          phase1_2_in: begin
             enable out = 1;
             cor_x_in = b;
             cor y in = e;
             cor_s_in = theta1;
              select_out = 0;
          end
          phase1_2_out: begin
             enable out = 0;
             b_prime = cor_x_out;
             e_prime = cor_y_out;
          end
          phase1 3 in: begin
             enable_out = 1;
             cor x in = c;
             cor_y_in = f;
             cor s in = theta1;
             select_out = 0;
```

```
phase1_3_out: begin
                                                            phase2_3_out: begin
   enable out = 0;
                                                               enable out = 0;
    c prime = cor x out;
    f_prime = cor_y_out;
                                                                c prime = cor x out;
    R[0][0] = a_prime;
                                                                i prime = cor y out;
    R[0][1] = b_prime;
                                                                R[0][0] = a_prime;
    R[0][2] = c_prime;
    R[1][0] = 0:
                                                                R[0][1] = b_prime;
    R[1][1] = e prime;
                                                                R[0][2] = c prime;
    R[1][2] = f_prime;
                                                               R[2][0] = 0;
    R[2][0] = q;
    R[2][1] = h:
                                                                R[2][1] = h prime;
    R[2][2] = i;
                                                               R[2][2] = i prime;
    phay1[0][0] = c_prime;
    phay1[0][1] = f_prime;
                                                                phay2[0][0] = c prime;
    phay1[0][2] = 0;
                                                                phay2[0][1] = i prime;;
    phay1[1][0] = -f_prime;
                                                                phav2[0][2] = 0;
    phay1[1][1] = c_prime;
    phay1[1][2] = 0;
                                                                phay2[1][0] = 0;
    phay1[2][0] = 0;
                                                                phay2[1][1] = 1;
    phav1[2][1] = 0;
    phay1[2][2] = 1;
                                                                phay2[1][2] = 0;
end
                                                                phay2[2][0] = - i_prime;
phase2_1_in: begin
                                                                phay2[2][1] = c_prime;
   enable out = 1;
    cor_x_in = R[0][0];
                                                                phay2[2][2] = 1;
    cor_y_in = R[2][0];
                                                            end
    cor = in = 0;
                                                           phase3_1_in: begin
    select out = 1;
end
                                                               enable_out = 1;
phase2_1_out: begin
                                                                cor_x in = R[1][1];
   enable out = 0;
    a_prime = cor_x_out;
                                                                cor y in = R[2][1];
    theta1 = cor_y_out;
                                                                cor_s_in = 0;
                                                                select_out = 1;
phase2_2_in: begin
  enable_out = 1;
                                                            end
   cor_x_in = R[0][1];
                                                            phase3 1 out: begin
    cor v in = R[2][1];
    cor s_in = theta1;
                                                               enable out = 0;
    select_out = 0;
                                                                e_prime = cor_x_out;
end
                                                                theta1 = cor y out;
phase2 2 out: begin
   enable_out = 0;
                                                            end
   b_prime = cor_x_out;
                                                           phase3_2_in: begin
    h prime = cor y out;
                                                               enable_out = 1;
end
phase2_3_in: begin
                                                                cor_x_in = R[1][2];
   enable out = 1;
                                                                cor y in = R[2][2];
    cor_x_{in} = R[0][2];
    cor_y_in = R[2][2];
                                                                cor z in = theta1;
    cor_s_in = theta1;
                                                                select out = 0;
    select out = 0;
                                                            end
```

```
phase3_2_out: begin
             enable_out = 0;
              f_prime = cor_x_out;
              i_prime = cor_y_out;
              R[1][1] = e prime;
              R[1][2] = f prime;
              R[2][1] = 0;
              R[2][2] = i_prime;
              phay3[0][0] = 1;
              phay3[0][1] = 0;
              phay3[0][2] = 0;
              phay3[1][0] = 0;
              phay3[1][1] = f_prime;
              phay3[1][2] = i_prime;
              phay3[2][0] = 0;
              phay3[2][1] = - i_prime;
             phay3[2][2] = f_prime;
1999-9
         multiply: begin
              for (int i = 0; i < 3; i++) begin
                for (int j = 0; j < 3; j++) begin
                  temp[i][j] = 0;
                     for (int k = 0; k < 3; k++) begin
                       temp[i][j] = temp[i][j] + (phay1[i][k] * phay2[k][j]);
                end
              end
               for (int i = 0; i < 3; i++) begin
                  for (int j = 0; j < 3; j++) begin
                    Q[i][j] = 0;
                    for (int k = 0; k < 3; k++) begin
                      Q[i][j] = Q[i][j] + (temp[i][k] * phay3[k][j]);
                 end
              end
              data_valid = 1;
      endcase
palways_comb begin
    a_out= R[0][0];
    b_out= R[0][1];
     c_out= R[0][2];
     d_out= R[1][0];
     e_out= R[1][1];
    f_out= R[1][2];
     g_out= R[2][0];
     h_out= R[2][1];
     i_out= R[2][2];
  endmodule
```

```
module R_inversion (
     input logic clk,
     input logic rst_n,
     input logic signed [31:0] a_in, b_in, c_in,
     input logic signed [31:0] d_in, e_in, f_in,
     input logic signed [31:0] g_in, h_in, i_in,
     input logic signed [31:0] Q [2:0][2:0],
     imput logic
     output logic signed [31:0] A_inv [2:0][2:0],
     output logic data valid
     logic [1:0] current state, next state;
     logic signed [31:0] r11_inv, r22_inv, r33_inv; // Diagonal inverses
     logic signed [31:0] R [2:0][2:0];
     logic signed [31:0] R_inv [2:0][2:0]; // Inverse of R matrix (3x3)
     logic signed [31:0] Q_T [2:0][2:0];
     logic done;
     typedef enum logic [1:0] {
         IDLE.
         INVERT DIAGONAL,
         INVERT OFF DIAGONAL,
     } state t;
always comb begin
     R[0][0] = a in;
     R[0][1] = b_{in};
     R[0][2] = c_{in};
     R[1][0] = d_{in};
     R[1][1] = e_in;
     R[1][2] = f_in;
     R[2][0] = g_{in};
     R[2][1] = h in;
     R[2][2] = i_{in};
     Q T[0][0] = Q[0][0];
     Q_T[0][1] = Q[1][0];
     Q_T[0][2] = Q[2][0];
     Q_T[1][0] = Q[0][1];
     Q_T[1][1] = Q[1][1];
     Q_T[1][2] = Q[2][1];
     Q_T[2][0] = Q[0][2];
     Q_T[2][1] = Q[1][2];
     Q_T[2][2] = Q[2][2];
```

```
// State Transition
always ff @(posedge clk or negedge rst_n) begin
    if (!rst n)
        current_state <= IDLE;
   else
        current state <= next state;
end
// Next State Logic
always comb begin
    case (current state)
        IDLE:
            if (start)
                next_state = INVERT_DIAGONAL;
            else
                next_state = IDLE;
        INVERT_DIAGONAL:
            next_state = INVERT_OFF_DIAGONAL;
        INVERT_OFF_DIAGONAL:
            next state = DONE;
        DONE:
            next state = IDLE;
        default:
            next state = IDLE;
   endcase
end
```

#### R\_inversion block

```
// Diagonal Inversion
always ff @(posedge clk or negedge rst n) begin
   if (!rst n) begin
       r11 inv <= 0;
        r22 inv <= 0;
        r33 inv <= 0;
       R inv[0][0] \le 0;
        R inv[1][1] \leftarrow 0;
        R inv[2][2] \le 0;
   else if (current_state = INVERT_DIAGONAL) begin
       // Invert diagonal elements (if non-zero)
        r11 inv \leftarrow (R[0][0] != 0) ? (1 \leftarrow 16) / R[0][0] : 0; // Fixed-point inversion (1/R)
       r22 inv <= (R[1][1] != 0) ? (1 << 16) / R[1][1] : 0;
        r33 inv \leftarrow (R[2][2] != 0) ? (1 \leftarrow 16) / R[2][2] : 0;
                                                                                                          assign done = (current state == DONE);
        // Store the inverted diagonal elements
                                                                                                  always comb begin
        R inv[0][0] <= r11 inv;
                                                                                                        if (done) begin
       R_inv[1][1] <= r22_inv;
                                                                                                             for (int i = 0; i < 3; i++) begin
        R inv[2][2] <= r33 inv;
                                                                                                                        for (int j = 0; j < 3; j++) begin
                                                                                                                          A inv[i][j] = 0;
end
                                                                                                                              for (int k = 0; k < 3; k++) begin
                                                                                                                                A_{inv[i][j]} = A_{inv[i][j]} + (R_{inv[i][k]} * Q_{T[k][j]});
// Off-diagonal Inversion using Back Substitution
always ff @(posedge clk or negedge rst n) begin
                                                                                                                        end
   if (!rst n) begin
       R_{inv[0][1]} \leftarrow 0;
                                                                                                           data valid = 1;
       R inv[0][2] \le 0;
                                                                                                         end else begin
        R inv[1][2] \leftarrow 0;
                                                                                                           data valid = 0;
                                                                                                           end
    else if (current state = INVERT OFF DIAGONAL) begin
       // Invert the off-diagonal elements using back substitution
                                                                                                     endmodule
       // R_inv[1,2] = R[1,2] / (R[1,1] * R[2,2])
       R \text{ inv}[1][2] \le (R[1][2] * r22 \text{ inv} * r33 \text{ inv}) >> 16); // Fixed-point multiplication
       // R_{inv}[0,1] = -R[0,1] / (R[0,0] * R[1,1])
       R_{inv}[0][1] \leftarrow -((R[0][1] * r11_inv * r22_inv) >> 16); // Fixed-point multiplication
       // R_{inv}[0,2] = -R[0,2] / R[0,0] - (R[0,1] * R[1,2]) / (R[0,0] * R[1,1] * R[2,2])
       R inv[0][2] <= -(((R[0][2] * r11 inv * r33 inv) >> 16) +
          ((R[0][1] * R[1][2] * r11_inv * r22_inv * r33_inv) >> 32)); // Second-order fixed-point multiplication
```

```
module top (
     input logic clk,
     input logic rst n,
     input logic enable,
     input logic signed [31:0] a, b, c,
    input logic signed [31:0] d, e, f,
    input logic signed [31:0] g, h, i,
    output logic signed [31:0] A inv [2:0][2:0],
    output logic
                                data valid
  );
     logic done1 top;
     logic enable top;
    logic signed [31:0] cor x out top;
    logic signed [31:0] cor y out top;
    logic signed [31:0] cor_z_out_top;
    logic signed [31:0] cor_x_in_top;
    logic signed [31:0] cor_y_in_top;
     logic signed [31:0] cor z in top;
     logic
                         select out top;
     logic
                         valid top;
    logic signed [31:0] Q top [2:0][2:0];
    logic signed [31:0] a out, b out, c out;
    logic signed [31:0] d out, e out, f out;
    logic signed [31:0] g out, h out, i out;
   cordic u0 cordic (
 .clk(clk),
 .rst n(rst n),
 .select(select_out_top),
 .enable(enable top),
 .x in(cor x in top),
 .y in(cor y in top),
 .z in(cor z in top),
 .x_out(cor_x_out_top),
 .y out(cor y out top),
 .z out(cor z out top),
 .done(done1 top)
```

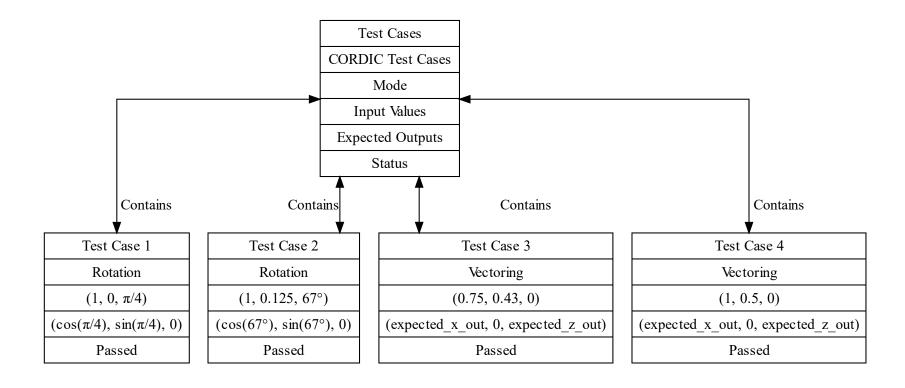
```
gr decomposition u0 gr decomposition (
  .clk(clk),
  .rst_n(rst_n),
  .enable(enable),
  .done1(done1 top),
  .a(a).
  .b(b),
  .c(c),
  .d(d).
  .e(e),
  .f(f),
  .g(g),
  .h(h).
  .i(i),
  .enable out(enable top),
  .cor_x_out(cor_x_out_top),
  .cor y out(cor y out top),
  .cor_s_out(cor_s_out_top),
  .cor_x_in(cor_x_in_top),
  .cor_y_in(cor_y_in_top),
  .cor z in(cor z in top),
  .select_out(select_out_top),
  .a out(a out),
  .b out(b out),
  .c out(c out).
  .d out(d out),
  .e_out(e_out),
  .f out(f out),
  .g_out(g_out),
  .h_out(h_out),
  .i out(i out),
  .Q(Q_top),
  .data valid(valid top)
R inversion u0 R inversion(
  .clk(clk).
  .rst_n(rst_n),
  .a in(a out),
  .b in(b out),
  .c_in(c_out),
  .d_in(d_out),
  .e_in(e_out),
  .f in(f out).
  .g in(g out),
  .h_in(h_out),
  .i_in(i_out),
  .Q(Q top),
  .start(valid_top),
  .A_inv(A_inv),
  .data_valid(data_valid)
);
```

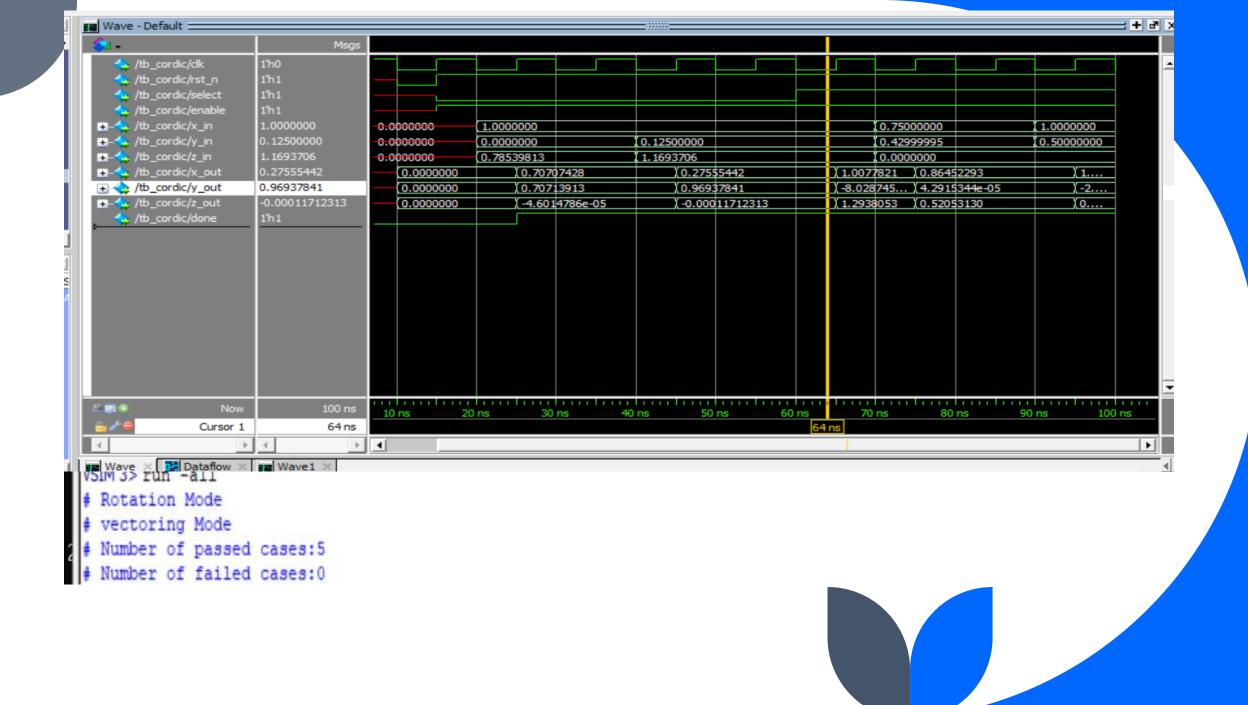
endmodule

TOP module

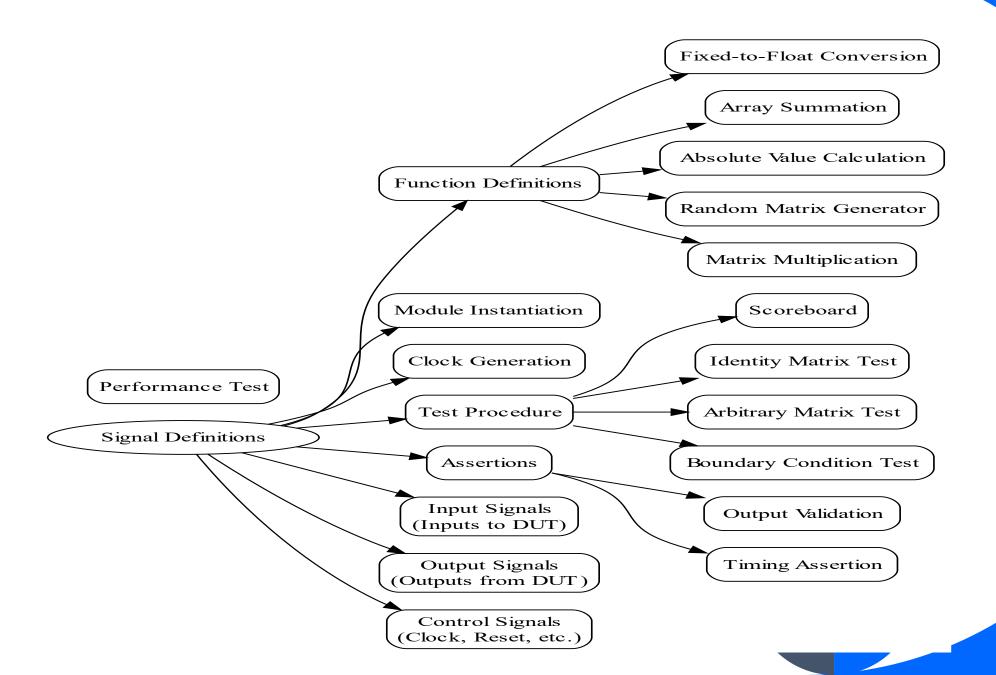
# Simulation

### **Cordic tests**





### System test



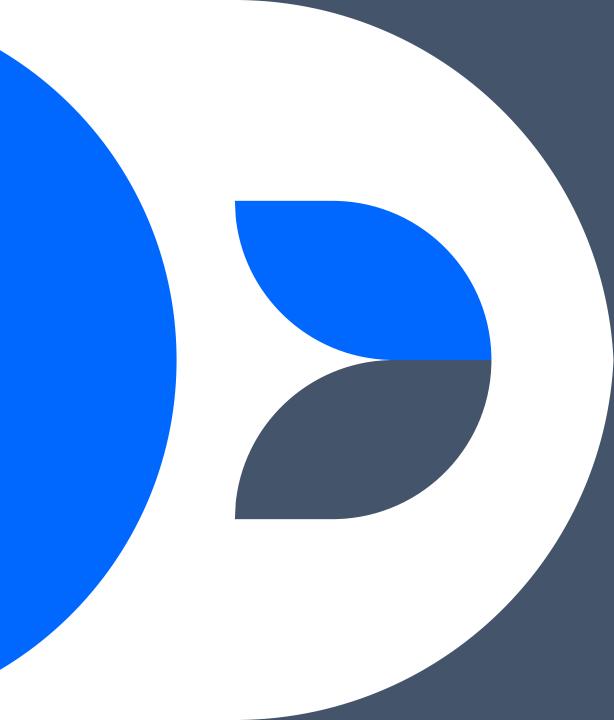
#### Results

```
Test Case 1: Identity matrix
input Matrix
a=1.000000, b=0.000000, c=0.000000
d=0.000000, e_=1.000000, f=0.000000
g=0.000000, h=0.000000, i=1.000000
```

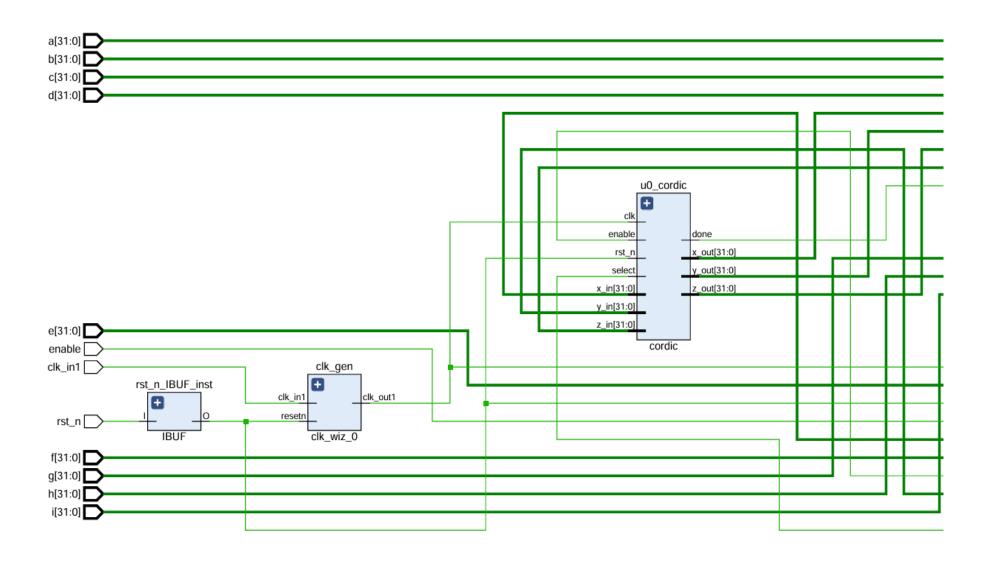
```
# Result (R):
# a_out_r=1.000000, b_out_r=0.000074, c_out_r=0.000074
# d_out_r=0.000000, e_out_r=1.000000, f_out_r=-0.000074
# g_out_r=0.000000, h_out_r=0.000000, i_out_r=1.000000
# Result (R expected):
# a_out_r=1.000000, b_out_r=0.000000, c_out_r=0.000000
# d_out_r=0.000000, e_out_r=1.000000, f_out_r=0.000000
# g_out_r=0.000000, h_out_r=0.000000, i_out_r=1.000000
```

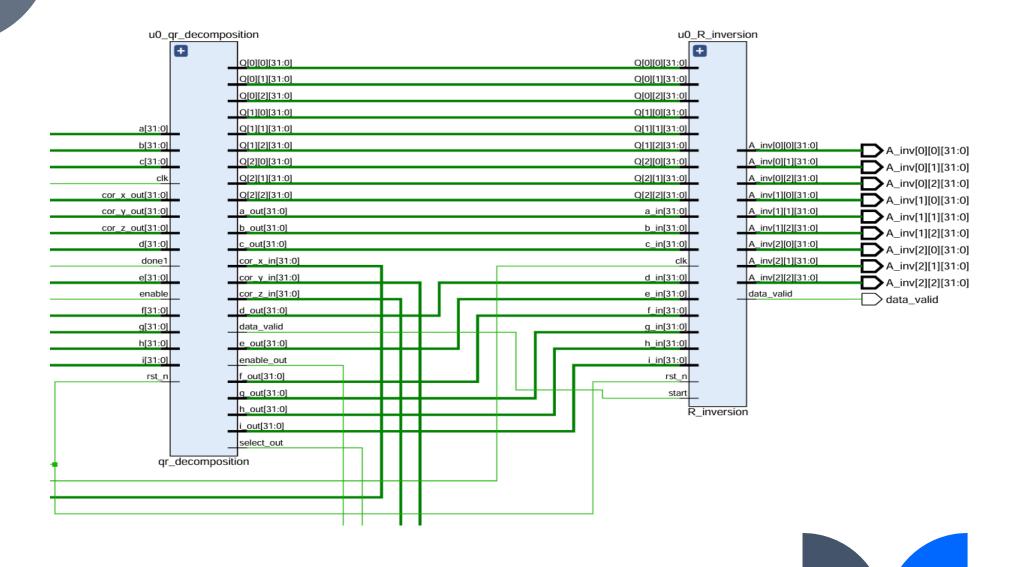
```
input Matrix
a=1.000000, b=2.000000, c=3.000000
d=2.000000, e_=1.000000, f=1.000000
g=6.000000, h=0.000000, i=1.000000
Result (Q):
a_out_q=0.000000, b_out_q=0.000000, c_out_q=0.000000
d_out_q=0.000000, e_out_q=0.000000, f_out_q=0.000000
g_out_q=0.000000, h_out_q=0.000000, i_out_q=0.000000
```

# **FPGA**

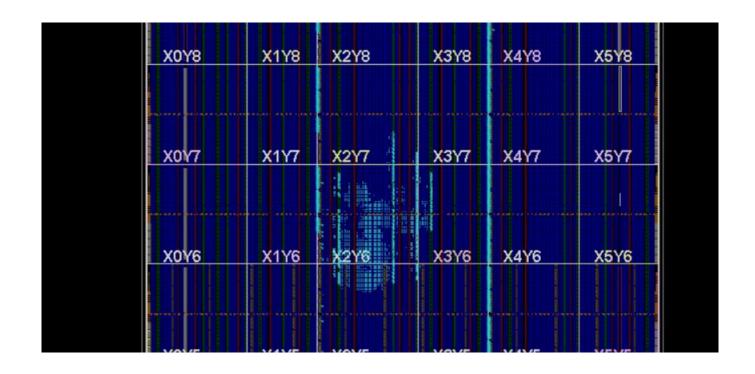


## RTL diagram

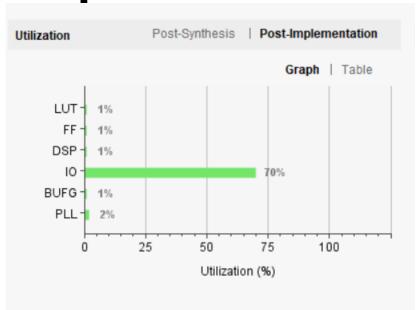




## implementation snippet



### Reports





Power	Summary   On-Chip			
Total On-Chip Power:	2.971 W			
Junction Temperature:	26.6 ℃			
Thermal Margin:	73.4 °C (126.1 W)			
Effective 3JA:	0.5 °C/W			
Power supplied to off-chip devices:	0 W			
Confidence level:	Low			
Implemented Power Report				

Failed Routes	LUT	FF	BRAMs	URAM	DSP	Start	Elapsed	Run Strategy	
	7496	235	0.00	0	100	10/2/24 9:59 PM	00:02:46	Vivado Synthesis Defaults (Vivado Synthesis 2018)	
								Vivado Implementation Defaults (Vivado Implementation 2018)	
0	7495	235	0.00	0	100	10/2/24 10:04 PM	00:25:00	Vivado Implementation Defaults (Vivado Implementation 2018)	
						10/2/24 9:58 PM	00:01:13	Vivado Synthesis Defaults (Vivado Synthesis 2018)	

# Thank you