How does the laser beam travel to the moon ?

PHY204 - Final Project

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Objective

Investigate how a laser beam propagates from to the Earth to the Moon.

- **Goal**: Figure out what parameters are optimal for delivering detectable and useful power to lunar surfaces.
- **Model assumptions**: Clear atmospheric conditions, neglect absorption and scattering, focus only on divergence
- **Challenges**: Extreme distances, beam divergence

Theoretical Background

- **Goal:** Understand how a laser beam evolves during propagation.
- **Monochromatic wave:**

$$E(\vec{r}, t) = \mathscr{E}(\vec{r}, t)\, e^{i(k_0 z - \omega_0 t)}$$

  - $\mathscr{E}(\vec{r}, t)$: slowly varying envelope.
  - $e^{i(k_0 z - \omega_0 t)}$: rapidly oscillating carrier.
- **Scalar wave equation**

$$\boxed{\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0}$$

Theoretical Background

Assumptions used to simplify the wave equation:

- Envelope varies slowly.
- Beam diverges only slightly —mostly forward-propagating.
- Temporal effects are neglected, we assume a **stationary envelope**.

Paraxial wave equation

$$\boxed{\nabla_\perp^2 \mathscr{E} + 2ik_0 \frac{\partial \mathscr{E}}{\partial z} = 0}$$

- $\nabla_\perp^2 \mathscr{E}$: transverse curvature of the beam.
- $2ik_0 \partial \mathscr{E} / \partial z$: describes evolution along the propagation axis.

Gaussian Beam Characteristics (1/2)

Analytical solution to the paraxial wave equation

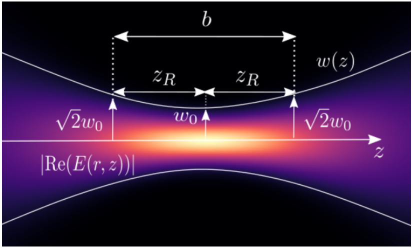
$$\boxed{\mathscr{E}(r,z) = \mathscr{E}_0 \frac{w_0}{w(z)} \exp\left(-\frac{r^2}{w^2(z)}\right) \exp\left(i\frac{k_0 r^2}{2R(z)} - i\zeta(z)\right)}$$

- Models a beam that is:
  - Narrowest at its **waist** (w_0)
  - Symmetrical in the transverse plane
  - Spreads due to diffraction as it propagates

Gaussian Beam Characteristics (2/2)

Key physical parameters:

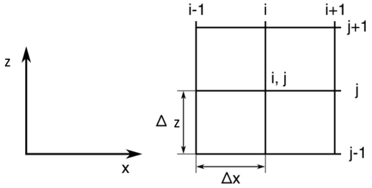
- w_0 –Beam waist
  $w(z) = w_0\sqrt{1 + \left(\frac{z}{z_R}\right)^2}$
- $z_R = \frac{\pi w_0^2}{\lambda}$ –Rayleigh range
- $R(z) = z\left[1 + \left(\frac{z_R}{z}\right)^2\right]$
  Radius of curvature of the phase front
- $\zeta(z) = \tan^{-1}\left(\frac{z}{z_R}\right)$
  Gouy phase shift



Gaussian Beam.

Discretization and moving window

- Divide the computational window in Δx and Δz steps.



Space discretization.

- Diffraction over large distances :
  $\rightarrow$ computational memory waste on empty space
  $\rightarrow$ reflections at the boundary if the box is too small

Discretization and moving window

- Change of variable for a fixed size simulation box :
  Transverse component : tracks the beam as it spreads

$$\xi = \frac{x}{w_0} \Rightarrow \frac{\partial}{\partial x} = \frac{1}{w_0}\frac{\partial}{\partial \xi}$$

  Longitudinal component : stretches the propagation steps

$$\zeta(z) = \arctan\left(\frac{z}{z_R}\right) \Leftrightarrow z(\zeta) = z_R \tan\zeta$$

- Transformed equation :

$$\frac{\partial C}{\partial \zeta} = \frac{i}{4}\nabla_{\xi}^{2} C + i(1-\xi^2)C$$

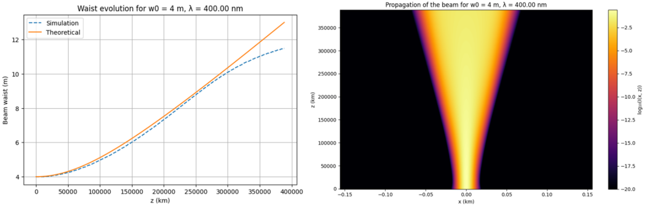
$\rightarrow$ artificial potential term : soft restoring force

Split step FFT

- Outline of the algorithm :

$$
\begin{aligned}
C(\xi, \zeta + \Delta\zeta) = \mathscr{F}^{-1}\left[e^{-\frac{i}{4}k_\xi^2\frac{\Delta\zeta}{2}}\mathscr{F}[C(\xi,\zeta)]\right] \quad & \text{(1/2 diffraction)} \\
\times\, e^{i(1-\xi^2)\Delta\zeta} \quad & \text{(Full potential step)} \\
\times\, \mathscr{F}^{-1}\left[e^{-\frac{i}{4}k_\xi^2\frac{\Delta\zeta}{2}}\mathscr{F}[\cdot]\right] \quad & \text{(Half-step diffraction)}
\end{aligned}
$$

- Preliminary results :



Crank–Nicolson Matrix Formulation

This scheme can be expressed compactly in matrix form as:

$$A\mathbf{U}^{n+1} = B\mathbf{U}^{n},$$

where $\mathbf{U}^n$ contains discretized field values at step n, and matrices A and B are tridiagonal with elements derived from centered differences. Neumann boundary conditions are enforced at the edges.

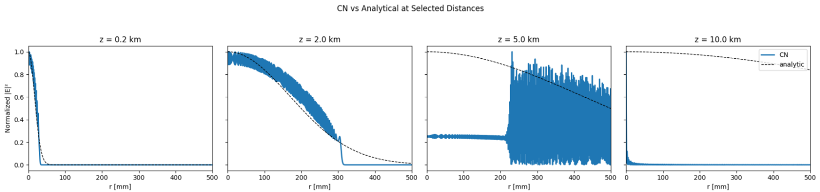
Matrix A

$$\begin{bmatrix} 1+4\sigma & -4\sigma & 0 & \cdots & 0 & 0 \\ -\sigma(1-\frac{1}{2}) & 1+2\sigma & -\sigma(1+\frac{1}{2}) & \cdots & 0 & 0 \\ 0 & -\sigma(1-\frac{1}{3}) & 1+2\sigma & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\sigma(1-\frac{1}{2j}) & 1+2\sigma & -\sigma(1+\frac{1}{2j}) \\ 0 & 0 & \cdots & 0 & -4\sigma & 1+4\sigma \end{bmatrix}$$

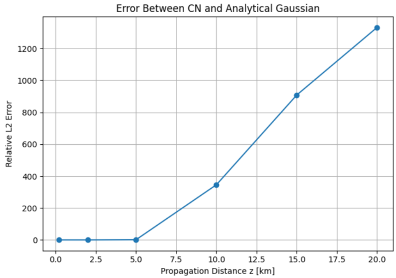
Matrix B

$$\begin{bmatrix} 1-4\sigma & 4\sigma & 0 & \cdots & 0 & 0 \\ \sigma(1-\frac{1}{2}) & 1-2\sigma & \sigma(1+\frac{1}{2}) & \cdots & 0 & 0 \\ 0 & \sigma(1-\frac{1}{3}) & 1-2\sigma & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \sigma(1-\frac{1}{2j}) & 1-2\sigma & \sigma(1+\frac{1}{2j}) \\ 0 & 0 & \cdots & 0 & 4\sigma & 1-4\sigma \end{bmatrix}$$

Crank–Nicolson in Physical Space



Comparison of CN simulation vs analytic beam profiles at selected distances.

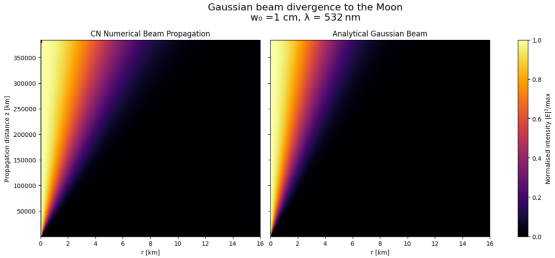


Evolution of error of CN method in a physical space model.

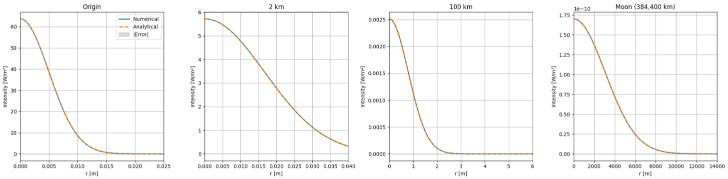
Crank–Nicolson in Computational Space

- **Coordinate transformation :**
  - Transverse normalization : $\xi = r/w(z)$
  - Propagation variable : $\zeta = \arctan(z/z_R)$
- **Absorption at edges :**
  - Absorption mask $\alpha(\xi) = exp(-(\xi/\xi_{max})^8)$ applied near boundaries
  - Eliminates artificial reflections
- **Centering :**
  - $i(1 - \xi^2)$ is the potential term
  - Keeps the beam guided and centered

Validation of the model



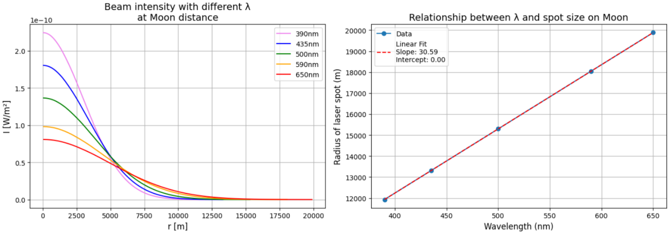
Left: CN simulation. Right: analytical solution.



Radial intensity profiles at selected distances (origin, 2 km, 100 km, Moon).

Parameter Effects (1/2)

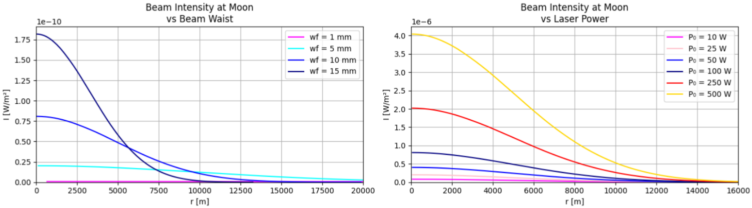
- **Wavelength:** shorter λ result in tighter focusing and less spreading.



Effects of wavelength on intensity profile and final spot size on the Moon

Parameter Effects (2/2)

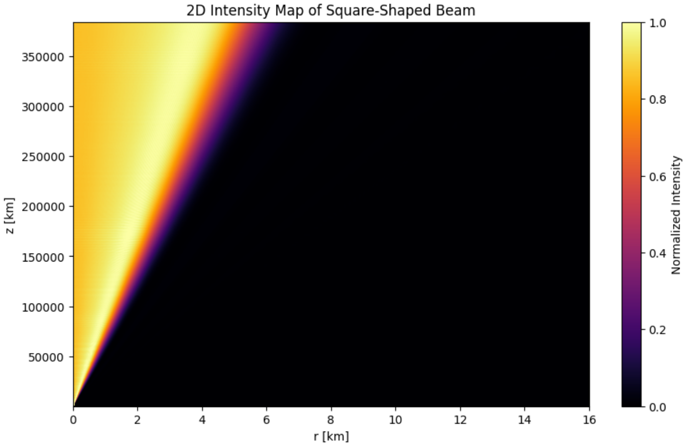
- **Beam waist:** led to stronger confinement of the beam.
- **Power:** scaled linearily the intensity. No effect on beam shape nor on divergence.



Left: effect of beam width on intensity profile at Moon. Right: effect of beam power on intensity profile at Moon

Square Beam Profile

- Sharp edges cause severe diffraction causing "lobes"
- Poor confinement, low efficiency.



Square shaped beam propagation to the Moon

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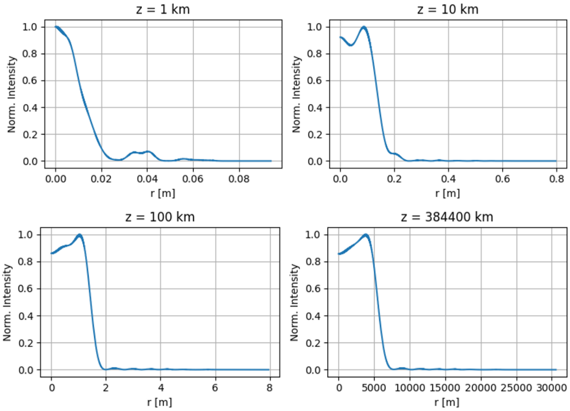
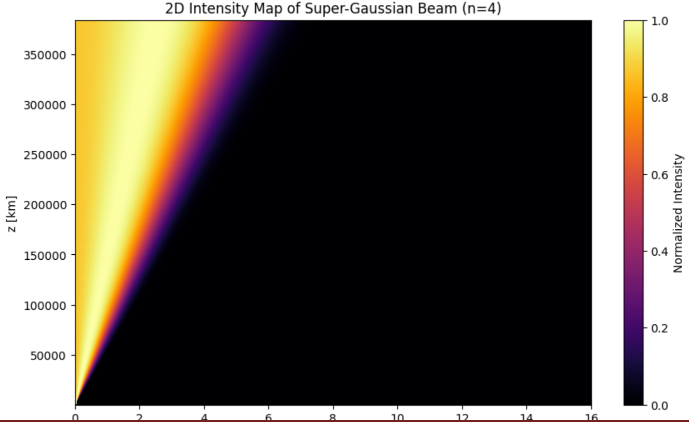


Square shaped beam profile propagation to the Moon

Super-Gaussian Beams

- Order-n beams: flatter center, sharper edges.
- Orders $n = 4$ provide optimal spot size at Moon.
- Higher n increases divergence due to strong edge diffraction.



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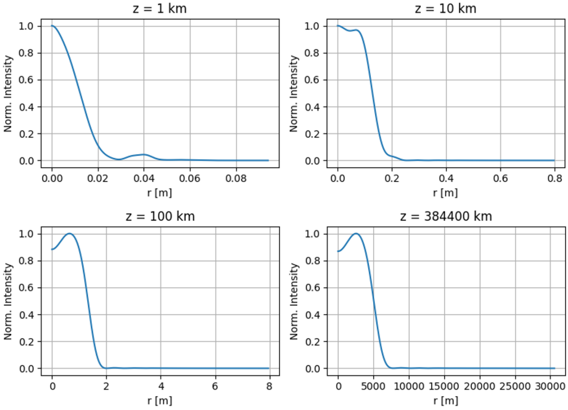
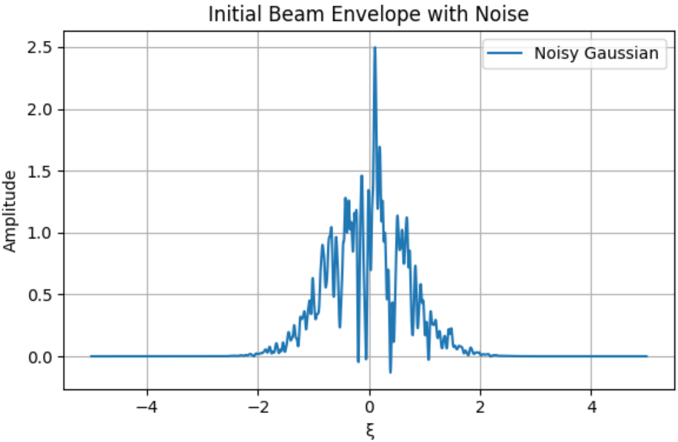


Noisy Gaussian Propagation

- Initial noise is smoothed by diffraction.
- Approximates real-world atmospheric imperfections.



Noisy Gaussian beam propagation to the Moon

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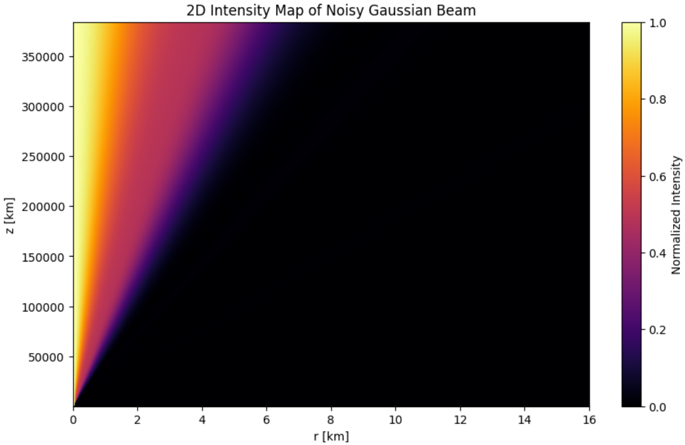

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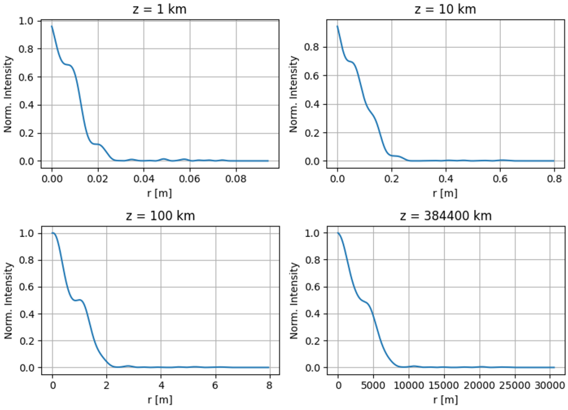

Noisy Gaussian beam profile propagation to the Moon

Key Takeaways

- Simulating lunar laser propagation requires adaptive grids.
- Crank-Nicolson in computational space is robust and accurate.
- Key parameters:
  Super-Gaussian beam (order $n = 4$)
  Shorter wavelength ($\lambda = 400$nm)
  Larger beam waist (up to 0.5m)
  Varying power depending on application

Thank You!