1 Perceptrons

Perceptrons are the simplest kind of neural net. Their architecture is as follows:

1. a set of **input nodes**
2. a set of **output nodes**
3. a bias node
4. each output node is connected to *every* input node and the bias node.

Three input nodes, two output nodes, and a bias node.

Recall:

$$\mathbf{o}(x) = f(\sum_{i=1}^{n} w_i x_i)$$

In a perceptron, we use the vectors of input values to calculate the output values. An output node value is the sum of the input values multiplied by the weights of the connections to the output plus the **bias** value.

$$\mathbf{o}(x) = \mathbf{y} = f(b + \sum_{i=1}^{n} w_i x_i)$$

Practice

2 Activation Functions: Overview

2.1 ReLU

$$\text{relu} = max(0, z) = \begin{cases} z, & \text{if } z >= 0 \\ 0, & \text{if } 0 < 0 \end{cases}$$

3 Perceptrons and Regression:

Question: What is regression? Give examples?
Recall: that regression relates a dependent variable to one (or more) independent explanatory variables.

Question: What is linear regression? and What is logistic regression?
Recall that *linear regression* allows us to understand relationship between two continuous variables `y = xw + b` and *logistic regression* is a type of regression analysis to predict the outcome of a dependent variable based on previous observations $f(Z) = \frac{1}{1+e^{-z}}$

Question: What is Exclusive Or (XOR)?

XOR from scratch in case students do not know.
In python, we have *bitwise operators* which are used to perform bitwise operations on integers.

- & `(AND)` $\rightarrow a\&b$
- | `(OR)` $\rightarrow a|b$
- ^ `(XOR)` $\rightarrow a\hat{}b$
- ...

To perform operations on integers:

1. Integers are converted to binary (0,1). *Note:* to convert an integer into a binary, divide by 2 specifying quotient and remainder. The binary is the remainder reversed. Example: $4/2 = 2 + \mathbf{0}$, $2/2 = 1 + \mathbf{0}$, $1/2 = 0 + \mathbf{1}$. By reversing the remainder, the binary is **100**. In python, `"0:b".format(4)`
2. Operations are performed on each bit of the pair of bits.

For example:

```
a = 8
b = 4
AND
print(a&b)
OR
print(a|b)
XOR
print(a^b)
```

In XOR, If the bits are the same, the result is 0. If the bits are different, the result is 1.

Perceptrons can only used with linear functions. The data must be *linearly separable.*

If we look at *sentential logic* for a and b, **it is true if either or the candidate is true** while in *Exclusive Or*, **it is true if only one of the candidates is true**

This means *sentential logic* is linearly separable while *Exclusive Or* is not linearly separable.

4 XOR and Linear Regression

References