Standard–Model Masses from Integer Baselines at a Universal RG Anchor

Jonathan Washburn

Abstract

We give a parameter–free pipeline for Standard–Model masses based on four fixed elements: a bridge–anchored energy unit $E_{\text{coh}} = \varphi^{-5}\,\text{eV}$, one frozen yardstick per sector $A_B = B_B E_{\text{coh}} \varphi^{r_0(B)}$, integer rungs $r_i = \ell_i + \tau_i$ from a deterministic word constructor with representation–independent generation torsion, and a single, sector–global residue functional encoding universal scale dressing at a common anchor $\mu_\star$. Each mass is the unique fixed point $m_i = A_B \varphi^{r_i + f_B(m_i)}$ evaluated with standard kernels (QCD 4L, QED 2L; fixed thresholds; no self-thresholding). There are no per-species knobs: scheme or input changes move an entire sector coherently and are reported as one band. We tabulate charged leptons, quarks, and $W/Z/H$ under the same locks, and supply an executable audit (CSV/CI) for PDG$\to \mu_\star$ transport and comparison.

Keywords: Standard Model particle masses renormalization group universal anchor parameter–free integer baselines

1 Introduction

Modern high–energy physics achieves extraordinary empirical accuracy, yet its core formulas still depend on externally supplied constants and per–process conventions. The last step from theory to a laboratory number often permits hidden slack: unit choices, sector–specific normalizations, or case–by–case calibrations. This paper advances a different posture: a *parameter–free* pipeline in which (i) the scale is fixed once by the Reality Bridge, (ii) the discrete content of each species is encoded by an integer hop count, (iii) the only continuous correction is a single, global scale–dressing law applied uniformly, and (iv) each reported mass is the unique fixed point of that one law. No per–particle dials are introduced at any stage.

Universal anchor (numerical).

Plain-language connection to the series. This is Part 2. Part 1 states the observation at one shared energy and shows it is not circular. Here we build the calculator: one fixed yardstick per sector, small integer baselines per species, and one shared correction rule applied the same way to every particle. Part 3 explains where those small integers come from.

Connection to the series. This paper is Part 2 of a coordinated three–paper submission. Part 1 (phenomenology) records the anchor identity and its non–circular audit. Here in Part 2 we turn that identity into a *parameter–free* pipeline with one frozen yardstick per sector and integer baselines r_i per species; every reported mass is obtained as a fixed point under a *single* sector–global residue at the same anchor. Part 3 provides the finite constructor that explains the integers and the word–charge Z. All sectors are evaluated at a single RS anchor

$$\mu_\star \; = \; 182.201 \text{ GeV},$$

fixed once (Paper 1) by a species-agnostic stationarity rule on regrouped anomalous-dimension weights. No per-sector or per-species reference scales are introduced.

Bridge and unit. The Reality Bridge sets a meter–native energy quantum

$$E_{\text{coh}} = \varphi^{-5} \text{eV},$$

forced by the golden–ratio recurrence and eight–tick closure. This quantity anchors all sectors without reference to measured particle masses. From E_{coh} each sector B receives a frozen pair of integer constants $(B_B, r_0(B))$ and a yardstick

$$A_B \; = \; B_B E_{\text{coh}} \varphi^{r_0(B)}.$$

These sector constants are chosen once (nearest integer/power–of–two factorization of the structural anchor) and then held fixed for all species in that sector.

Discrete structure. Each species i carries a structural integer

$$r_i = \ell_i + \tau_i,$$

where ℓ_i is the reduced length of the chirality–paired charge word under the eight–tick constraint and $\tau_i \in \{0, 11, 17\}$ is a single, global family torsion (first, second, third generation). These integers are part of the display; they are never adjusted post hoc.

External integers. The integers r_i are fixed by the recognition-word constructor with a single, representation-independent generation torsion. They are part of the display, not a fit knob, and are held fixed for all numerics in this paper.

Universal scale dressing. Observed masses are not bare counts; they are dressed by the universal, scale–dependent drift of fields. We encode that drift by a *single* residue functional $f(\mu)$ per sector (leptons; up– and down–type quarks; electroweak vectors and scalar). The residue uses standard running and self–energies (e.g. QED two–loop with hadronic vacuum polarization for leptons; QCD four–loop plus QED two–loop and fixed decoupling thresholds for quarks; one–loop electroweak/Higgs self–energies for $W/Z/H$), evaluated at the single universal anchor $\mu_\star$. No species–specific changes are permitted.

Fixed–point evaluation (no target on the right–hand side). For each species the reported mass is the unique solution of

$$m_i = A_B\, \varphi^{\, r_i + f(m_i)}.$$

We start from the structural value $A_B\varphi^{r_i}$ and iterate with a uniform tolerance and damping. This produces a self–consistent, meter–native number without inserting the measured m_i anywhere on the right–hand side.

Parameter–free predictions and falsifiers. Because the bridge constants $(E_{\mathrm{coh}}, B_B, r_0(B))$ are fixed upstream, the integers r_i are structural, and the residue law is global, the pipeline has no per–particle knobs. A change in scheme, loop order, or reference scale is applied to *all* species simultaneously and reported as a single theory band per sector. This makes the construction falsifiable: future shifts in reference values that exceed the declared bands would invalidate the corresponding sector without any possibility of species–by–species rescue.

What we show. We first derive and record the bridge unit E_{coh} and the frozen sector constants. We then list the integers r_i per species, specify the global residue policy, and compute masses by fixed points. For charged leptons we present a fully parameter–free table (no lepton is used as an anchor); quark and boson results follow from the same pipeline under the declared global inputs. The ladder display and the spectral–gap+residue display are shown to be equivalent under the same locks, so the presentation is robust to framing.

Contributions.

- A bridge–anchored, parameter–free energy unit $E_{\mathrm{coh}} = \varphi^{-5}\,\mathrm{eV}$ and frozen sector constants $(B_B, r_0(B))$.

- A deterministic rule for integer hop counts $r_i = \ell_i + \tau_i$ with a single, global family torsion.

- A uniform residue functional per sector and a self–consistent fixed–point evaluation for each species.

- Parameter–free charged–lepton predictions with quantified residuals and a clear falsification policy; extension to quarks and $W/Z/H$ under the same global rules.

Kernel locks (frozen for all runs). Quarks: 4–loop QCD (β_s and γ_m) with fixed decoupling at $\mu = m_c, m_b, m_t$ and $n_f = 6$ above m_t, plus 2–loop QED γ_m with a global $\alpha(\mu)$ policy (default: frozen at M_Z; a leptonic 1–loop variant sets a small "policy band"). Leptons: 2–loop QED γ_m under the same $\alpha(\mu)$ policy; small one–loop EW terms may be quoted but are applied uniformly and do not introduce species freedom. $W/Z/H$: evaluated uniformly at one loop in the final pass; here we report the RS structural values and note that the one–loop update is global (common inputs, common conversion) and does not alter the parameter–free posture.

Kernel policy and decoupling (global, species-agnostic)

We freeze one residue functional per sector and apply it uniformly to all species. A global change (scheme, loop order, inputs) moves the entire sector coherently.

Canonical normalization (fixed, not fitted)

We **define** the display map at the anchor by

$$\mathscr{F}(Z) = \frac{1}{\lambda}\ln\!\left(1 + \frac{Z}{\kappa}\right), \qquad \lambda := \ln\varphi, \;\; \kappa := \varphi,$$

so that no scale or slope is tuned against any mass. This matches the internal formal layer where

$$\mathrm{F\_ofZ}(Z) = \frac{\ln(1 + Z/\kappa)}{\lambda}, \quad \lambda = \ln\varphi, \;\; \kappa = \varphi,$$

and is the same $\mathscr{F}$ used throughout this paper. All audits below continue to use SM kernels (QCD 4L, QED 2L) and the declared threshold and $\alpha(\mu)$ policies.

Lean interface and an anchor certificate (phenomenology use)

For phenomenology we only need a lightweight certificate abstraction. Let an *anchor certificate* $\mathscr{C}$ supply, for each integer $Z \geq 0$, a symmetric interval

$$I_Z = \left[\, c_Z - \varepsilon_Z, \; c_Z + \varepsilon_Z \,\right]$$

together with a guarantee that the anchor residues of all charged fermions with index Z lie in I_Z, and that $\mathscr{F}(Z) \in I_Z$ as well. Then for the residue map $f_i := f_i(\mu_\star, m_i)$ at the single anchor,

$$\boxed{\;|f_i - \mathscr{F}(Z_i)| \;\leq\; \varepsilon_{Z_i}, \qquad |f_i - f_j| \;\leq\; 2\,\varepsilon_{Z_i} \;\text{ if }\; Z_i = Z_j.\;}$$

When the half–widths vanish ($\varepsilon_Z = 0$ for all Z), this reduces to the exact anchor identity $f_i = \mathscr{F}(Z_i)$ and equal–Z degeneracy without tolerance. In our build we use the SM transport and kernels declared in this paper to instantiate such intervals; the numerical half–widths are at or below our quoted tolerance band.

Integer map, rung integers, and anchor mass law (Lean–aligned corollary)

The integer Z is the same piecewise charge/sector index used throughout the manuscript:

$$Z = \begin{cases} 4 + (6Q)^2 + (6Q)^4, & \text{quarks (color fundamental)}, \\ (6Q)^2 + (6Q)^4, & \text{charged leptons}, \\ 0, & \text{Dirac neutrinos}. \end{cases}$$

We adopt the fixed rung integers

$$r_e = 2,\; r_\mu = 13,\; r_\tau = 19, \qquad r_u = 4,\; r_c = 15,\; r_t = 21, \qquad r_d = 4,\; r_s = 15,\; r_b = 21,$$

and record the mass–level display at the anchor (no fits):

$$\boxed{\; m_i(\mu_\star) \;=\; M_0\, \varphi^{\, r_i - 8 + \mathscr{F}(Z_i)}, \;}$$

with M_0 a common scale that cancels in anchor ratios. In particular, for any equal–Z pair one has the *parameter–free* ratio

$$\left. \frac{m_i}{m_j} \right|_{\mu_\star} \;=\; \varphi^{\, r_i - r_j}.$$

This is exactly the rung–plus–closed–form structure already used in the paper and matches the internal formalization of the exponent $E_i = r_i + \mathscr{F}(Z_i) - 8$. *Scope:* these are *anchor–specific* display relations; off–anchor we revert to standard RG.

Self-thresholding ban (heavy quarks). When predicting a heavy quark $Q \in \{c,b,t\}$, we *do not* place a decoupling step at the unknown m_Q. Instead we use an RS-structural threshold

$$\mu_Q^{(\text{th})} = \kappa_U A_U \, \varphi^{r_Q},$$

with a *single* sector-global constant κ_U fixed *once* before any comparisons (default $\kappa_U = 1$). The other heavy thresholds are treated analogously within their sectors. Variations of $\{\alpha_s(M_Z), \alpha(M_Z), \sin^2\theta_W, m_H, v, \kappa_U, \mu_\star\}$ generate a *single* sector band. No species-specific tweaks are permitted.

Leptons and $W/Z/H$. Leptons use the same two-loop QED γ_m with a sector-global $\alpha(\mu)$ policy; $W/Z/H$ use a common one-loop EW/Higgs pass with the same inputs and a uniform pole$\leftrightarrow \overline{\text{MS}}$ conversion.

2 Reality Bridge and the Coherence Quantum

Definition. The bridge fixes a universal coherence energy

$$E_{\text{coh}} = \varphi^{-5}\,\text{eV}, \qquad \varphi = \frac{1+\sqrt{5}}{2}.$$

This is the meter–native energy tick that converts the ledger's dimensionless counts into SI energy. Displaying it in electronvolts is a convenience; any SI energy unit would carry the same dimensionless factor φ^{-5}.

Why it is forced (proof sketch in words).

- *Cost functional uniqueness.* On the log axis the only symmetric, positive, reciprocal, and convex ledger cost is the golden form; its stationary structure fixes a unique per–flip "bit" of recognition cost (no continuous freedom).

- *Golden–ratio fixed point.* The self–similar recognition recurrence admits a single positive fixed point at φ. The gap between successive balanced recognitions is therefore locked to the φ–scale; one flip carries a fixed bit–gap proportional to $\ln\varphi$.

- *Eight–tick minimality.* Closure on an eight–tick time ring quantizes admissible closed words. The shortest nontrivial closed word carries exactly five such flips before closure; that integer is geometric/topological, not adjustable. Mapping cost to energy under the bridge exponentiates the negative of this word length, yielding the factor φ^{-5}.

6

Parameter–free posture. No external measurement or sector choice enters the construction: the cost symmetry, the φ fixed point, and the eight–tick closure together determine E_{coh} once and for all. In particular, there is no knob to rescale E_{coh} without breaking one of those three structural requirements.

Numerical value. Using $\varphi = (1+\sqrt{5})/2$,

$$E_{\mathrm{coh}} \;=\; \varphi^{-5}\,\mathrm{eV} \;=\; 9.016994375 \times 10^{-2}\,\mathrm{eV}.$$

(Any rounding shown elsewhere is a display choice; the defining factor is exactly φ^{-5}.)

Interpretation. E_{coh} is the universal "energy quantum" of recognition: the smallest bridge–meaningful energy increment associated with one minimally closed recognition cycle. It plays the same structural role for this framework that c and $\hbar$ play in standard displays—set once by symmetry and composition rules, not by fitting to a corpus of data. All sector yardsticks and spectrum evaluations are built atop this unit.

3 Sector Constants $(B_B, r_0(B))$

Definition. Each sector B carries a single, frozen yardstick

$$A_B \;=\; B_B\,E_{\mathrm{coh}}\,\varphi^{\,r_0(B)},$$

with $B_B \in \{2^k : k \in \mathbb{Z}\}$ and $r_0(B) \in \mathbb{Z}$. This yardstick is used for *all* species in sector B; it never varies by particle.

Origin (nearest integer decomposition, then freeze). For a sector's structural anchor we factor

$$K_B \;=\; \frac{A_B}{E_{\mathrm{coh}}},$$

through the discrete bridge basis $\{2^k\varphi^r\}$ by choosing the *nearest* integer pair (k, r) that minimizes the multiplicative log–error $\left|\ln\!\big((2^k\varphi^r)/K_B\big)\right|$. We then *freeze*

$$(B_B, r_0(B)) \;=\; \big(2^k, r\big)$$

for the rest of the paper. Any residual sub–percent mismatch is absorbed later by the single, global dressing functional $f(\mu)$ applied uniformly to the entire sector; no per–species adjustment is permitted.

7

Frozen values (sub–percent structural mismatches). The bridge yields the following integer pairs, now fixed for all evaluations:

Sector B	B_B	$r_0(B)$	structural mismatch
Leptons	2^{-22}	62	0.19%
Up quarks	2^{-1}	35	0.09%
Down quarks	2^{23}	-5	0.03%
EW vectors (W/Z)	2^1	55	0.12%
Scalar (H)	2^{-27}	96	0.29%

Here "mismatch" denotes the relative difference between K_B and its frozen representation $2^k \varphi^{r_0(B)}$; all are $< 0.3\%$ and handled *globally* by the sector's single residue law.

No per–species freedom. $(B_B, r_0(B))$ are once–per–sector constants. Changing scheme, loop order, thresholds, or reference scale must be done *globally* and moves the entire sector coherently; species–specific edits are disallowed. This policy prevents any implicit fitting while preserving the meter–native display of the bridge.

Closed forms used in numerics (up/down sectors). For quarks we use the pinned, audit–friendly closed forms

$$A_U = 2^{-1} E_{\mathrm{coh}} \varphi^{35}, \qquad A_D = 2^{23} E_{\mathrm{coh}} \varphi^{-5},$$

and the integer rungs

$$(r_u, r_c, r_t) = (4, 15, 21), \qquad (r_d, r_s, r_b) = (4, 15, 21),$$

emitted by the deterministic constructor described in Sec. 4.

4 Discrete Structure: Integer Hop Counts

Rule (fixed integers). Each species i carries a structural integer

$$r_i = \ell_i + \tau_i,$$

with two independent, *frozen* ingredients: (i) a charge–word length ℓ_i determined by a canonical constructor, and (ii) a *global* family torsion $\tau_i \in \{0, 11, 17\}$ for generations $1, 2, 3$. These integers are part of the bridge display and are never tuned after seeing data.

The charge→word constructor (what ℓ_i is). We represent the discrete gauge skeleton by the free product

$$C_3 * C_2 * \mathbb{Z} \quad \text{(color center, weak center, hypercharge lattice)}.$$

For a field with representation data (color rep, T, Y) we form a word W_8 on the eight–tick time ring:

- **Center labels.** Map the $SU(3)$ rep to $a \in \{+1, -1, 0\}$ for $\mathbf{3}$, $\bar{\mathbf{3}}$, singlet/adjoint; map $2T \bmod 2$ to $b \in \{1, 0\}$ for doublet vs. singlet/adjoint.

- **Eight–tick completion.** Impose the closure constraint

$$8Y + n_Y + \frac{n_3}{3} + \frac{n_2}{2} \in \mathbb{Z}$$

  by selecting a *minimal* triple $(n_Y, n_3, n_2) \in \mathbb{Z}^3$ with a fixed, canonical tie–break.

- **Unreduced word.** Write

$$W_8 = Y^{8+n_Y} c^{8a+n_3} w^{8b+n_2} \in C_3 * C_2 * \mathbb{Z},$$

  then reduce to free–product normal form (unique).

- **Chirality pairing (fermions).** For Dirac fermions, pair left/right factors and define the Dirac word $W_D = W_8(L) W_8(R)^{-1}$ before reduction. Bosons/scalars use the single W_8.

The *reduced length* $\ell_i := |W_D^{\text{red}}|$ (or $|W_8^{\text{red}}|$ for bosons) is an integer. It is invariant under conjugation in each factor, independent of basepoint on the time ring, and unique by free–product normal form.

Family torsion (generation splitter). Generations carry a representation–independent, discrete class on the eight–tick ring,

$$\tau(1) = 0, \qquad \tau(2) = 11, \qquad \tau(3) = 17,$$

applied *uniformly across sectors*. This is a global assignment; it is not adjusted per particle.

Fixed integers used in this work (examples).

- **Leptons (charged):**

$$r_e = 2, \qquad r_\mu = 13, \qquad r_\tau = 19.$$

- **Up–type quarks (representative):**

$$r_u = 4, \qquad r_c = 15, \qquad r_t = 21.$$

- **Down–type quarks (representative):**

$$r_d = 4, \qquad r_s = 15, \qquad r_b = 21.$$

- **Electroweak bosons and scalar:**

$$r_W = 1, \qquad r_Z = 1, \qquad r_H = 1.$$

These r_i are structural outputs of the constructor (plus the global τ) and constitute the *only* per–species integers used downstream.

Why these integers are non–tunable.

- ℓ_i follows from a deterministic reduction in $C_3 * C_2 * \mathbb{Z}$ given $(\text{color rep}, T, Y)$ and the eight–tick closure; once the constructor is fixed, ℓ_i is forced.

- τ is a single, sector–agnostic assignment (three generation classes) applied uniformly; changing it would shift *all* second/third–generation species together and is therefore a global, not per–species, operation.

Hence $r_i = \ell_i + \tau_i$ is locked before any comparison to data.

Sanity checks (constructor invariants).

- *Conjugation invariance:* replacing a rep by its conjugate flips center labels but leaves the reduced length ℓ_i unchanged for color–singlet outcomes after chirality pairing.

- *Eight–tick compatibility:* if (n_Y, n_3, n_2) and (n'_Y, n'_3, n'_2) both satisfy closure, the constructor's minimality rule yields the same ℓ_i.

- *Chiral neutrality:* species with matched left/right assignments reduce to even–length words unless center holonomies force an odd step; the listed r_i respect this parity.

Lemma (generation torsion is representation–independent). Let ℓ_i be the reduced length of the eight–tick word determined by (Y,T,color) under the canonical tie–break. Define the generation torsion $\tau : \{1,2,3\} \to \{0,11,17\}$ uniformly across sectors and set $r_i = \ell_i + \tau(\text{gen}(i))$. Then τ is independent of (Y,T,color). *Sketch.* The free–product normal form makes ℓ_i a class function of the gauge skeleton; the eight–tick closure and chirality pairing remove basepoint dependence and local completion ambiguity. The residual three–class ambiguity is a $\mathbb{Z}$–quotient of the time ring and therefore attaches to generation, not to the gauge syllables. A formal statement with the tie–break ordering is given in App. A.

Status. The lemma is formalized in App. A with the canonical tie–break ordering; it implies $r_i = \ell_i + \tau(\text{gen}(i))$ is fixed once (ℓ_i, τ) are declared and introduces no per–species freedom.

5 Universal Residues (Global Dressing Laws)

Purpose and posture. Observed masses are not bare ledger counts; they are dressed by universal scale–dependent effects from the quantum fields that permeate every sector. We encode that dressing by a *single* residue functional per sector and apply it *uniformly* to all species in that sector. No per–species tweaks are permitted; any change in scheme, loop order, thresholds, or reference inputs is a *global* change that moves the entire sector coherently.

Scheme and global inputs (frozen). We work in $\overline{\text{MS}}$ with fixed electroweak reference inputs at the Z pole:

$$\alpha_s(m_Z) = 0.1179, \qquad\qquad \alpha^{-1}(m_Z) = 127.955,$$
$$\sin^2\theta_W(m_Z) = 0.2312, \qquad\qquad m_H = 125.20 \text{ GeV},$$
$$v = 246.22 \text{ GeV}.$$

From these we set $e = \sqrt{4\pi\alpha(m_Z)}$ and $g = e/\sin\theta_W$, $g' = e/\cos\theta_W$, and $\lambda = m_H^2/(2v^2)$. These inputs are declared once and are used across all sectors without exception.

Universal anchor (frozen). All sectors share a single RS anchor

$$\mu_\star = \frac{\hbar}{\tau_{\text{rec}}\, \varphi^8}, \qquad \tau_{\text{rec}} = \frac{2\pi}{8\ln\varphi}\frac{\lambda_P}{c}, \qquad \lambda_P = \sqrt{\hbar G/c^3}.$$

Every species is evaluated at this anchor; there are no per–sector or per–species reference scales. Residuals in all tables are *non-circular*: PDG reference masses are transported to $\mu_\star$ (PDG$\to \mu_\star$) with the same RG kernels before comparison.

Lepton residue (QED two–loop; policy band). Lepton masses use a single residue functional

$$f_\ell(\mu) \;=\; \frac{1}{\ln \varphi} \int_{\ln \mu_\star^{(\ell)}}^{\ln \mu} \left[\gamma_{m,\ell}^{\mathrm{QED}}(\alpha(\mu')) \,+\, \gamma_{m,\ell}^{\mathrm{EW}}(g,g',\mu') \right] d\ln \mu',$$

with the following uniform rules:

- *QED running and anomalous dimension:* use the standard two–loop lepton mass anomalous dimension. The default policy freezes $\alpha(\mu)$ at M_Z for central values; a leptonic one–loop variant (e, μ, τ thresholds) defines a small *policy band* applied uniformly to the sector. A dispersion–based hadronic VP update can be enabled later as a *global* policy; it is not used for the central values here.

Quark residue (QCD four–loop + QED two–loop; fixed thresholds). Quark masses use a single residue functional

$$f_q(\mu) \;=\; \frac{1}{\ln \varphi} \int_{\ln \mu_\star^{(q)}}^{\ln \mu} \left[\gamma_{m}^{\mathrm{QCD}}(\alpha_s(\mu')) \,+\, \gamma_{m,q}^{\mathrm{QED}}(\alpha(\mu'),Q_q) \right] d\ln \mu',$$

with these uniform rules:

- *QCD running and anomalous dimension:* four–loop β_s and four–loop quark mass anomalous dimension.

- *Decoupling thresholds:* $\overline{\mathrm{MS}}$ decoupling at $\mu = m_c, m_b, m_t$; the same thresholds are used for all quarks. **Threshold policy and matching.** We step n_f at $\mu = m_c, m_b, m_t$ (so $n_f = 3 \to 4 \to 5 \to 6$ above m_t). At the thresholds we impose continuity for α_s (tree–level matching); the missing higher–order decoupling constants are *bracketed* inside the global sector band via the Monte–Carlo variation of (m_c, m_b, m_t) and $\alpha_s(M_Z)$.

- *QED piece:* two–loop QED contribution with quark electric charge Q_q; $\alpha(\mu)$ run as above (hadronic VP included once, *globally*).

Vector and scalar boson dressing ($W/Z/H$). For (W, Z, H) we evaluate one–loop transverse self–energies at the common reference $\mu_\star^{(V,H)} = m_Z$ using the fixed (g, g', λ, v), compute $\overline{\text{MS}}$ masses at $\mu_\star^{(V,H)}$, and apply the standard one–loop pole$\leftrightarrow \overline{\text{MS}}$ conversion. The same formula and inputs are applied to all three bosons; there are no per–boson adjustments.

Single functional per sector, no species tweaks. For any species i in sector B we use

$$f_i(\mu) \equiv f_B(\mu) \quad \text{with the appropriate kernel set for } B \in \{\ell, q, V/H\}.$$

This is a *global* functional: if any element (loop order, kernel, threshold set, reference inputs) changes, it changes *for the entire sector*. No species–specific modification is allowed.

Numerical evaluation policy. All residues are evaluated by integrating on a uniform $\ln \mu$ grid with a fixed base step and automatic halving near thresholds; the same grid policy is used for every species in the sector. The integral tolerance is fixed once. Changing step size or tolerance is a global change.

Uncertainties (global bands). We propagate a single uncertainty band per sector by varying the global inputs within their reported errors (e.g. $\alpha_s(m_Z)$, $\alpha(m_Z)$, $\sin^2 \theta_W(m_Z)$, m_H, v) and rerunning the *entire* sector. Species–specific variations are not performed.

Appendix kernels (verbatim forms). Appendix B records the explicit β and γ_m kernels used (QCD 4L and QED 2L), the heavy–flavor threshold stepping ($n_f = 3 \to 4 \to 5 \to 6$ above m_t), the $\alpha(\mu)$ policy alternatives (frozen and leptonic 1L), and the one–loop $W/Z/H$ self–energies with the pole$\leftrightarrow \overline{\text{MS}}$ conversion applied *uniformly*. Any future policy change is sector–global and reported as a single band.

Methods (RG pipeline and audit). We use a single, uniform RG pipeline for all quarks: four–loop QCD (β and quark–mass anomalous dimension) in $\overline{\text{MS}}$ with decoupling at $\mu = m_c, m_b, m_t$ (so $n_f = 3 \to 4 \to 5 \to 6$ above m_t), plus a two–loop QED mass anomalous dimension with an α_{em} policy (default "frozen"; a policy band is quoted from one–loop leptonic running). All species are evaluated at one universal matching scale $\mu_\star$ fixed by the bridge (or as explicitly stated), with global uncertainties obtained by Monte Carlo variations of $\{\alpha_s(m_Z), m_c, m_b, m_t, \mu_\star, \alpha_{\text{em}} \text{ policy}\}$. Residuals in the consolidated tables are

non–circular: PDG quark masses are transported to the same $\mu_\star$ (PDG$\rightarrow \mu_\star$) before comparison. No species is fit; an anti–fit guard flags any accidental equality ("Predicted = Reference") and omits such rows from residual statistics.

6 Fixed–Point Evaluation

Display equation (same for every species). For a species i in sector B the reported, meter–native mass is the fixed point of

$$m_i = A_B\, \varphi^{\, r_i + f(m_i)} \qquad \text{with} \quad A_B = B_B\, E_{\text{coh}}\, \varphi^{\, r_0(B)}.$$

Here r_i is the frozen integer from the constructor, and $f(\mu)$ is the sector's single, global residue functional.

Numerical procedure (uniform across the sector). We solve the fixed point with one policy for all species:

1. **Initialize (structural guess).** Set $m_i^{(0)} \leftarrow A_B\, \varphi^{\, r_i}$ (no dressing).

2. **Evaluate the residue.** On each iteration k, compute $f(m_i^{(k)})$ using the sector's global kernel on the uniform $\ln \mu$ grid (same grid and thresholds for every species in the sector).

3. **Update.** Form the undamped update
$$\tilde{m}_i^{(k+1)} = A_B\, \varphi^{\, r_i + f(m_i^{(k)})}.$$

4. **Damping (fixed, global).** Apply the same damping factor η to all species:
$$m_i^{(k+1)} \leftarrow (1-\eta)\, m_i^{(k)} + \eta\, \tilde{m}_i^{(k+1)}, \qquad \eta = 0.5.$$

5. **Convergence test.** Stop when the relative change is below a uniform tolerance
$$\frac{\left| m_i^{(k+1)} - m_i^{(k)} \right|}{m_i^{(k)}} < 10^{-8}.$$
   If the tolerance is tightened or loosened, it is done *globally* for the entire sector.

Outcome and posture. This iteration yields a unique, self–consistent solution for each species under the same global residue and thresholds. No measured mass appears on the right–hand side; the procedure never inserts m_i as a target. Any change to scheme, loop order, reference inputs, grid step, damping, or tolerance is a *global* change applied to all species simultaneously; species–specific adjustments are disallowed.

Non-circularity (proposition and audit guarantee)

Proposition. For each species i in sector B the reported mass is the unique fixed point

$$m_i = A_B \, \varphi^{r_i + f_B(m_i)},$$

with $A_B = B_B E_{\text{coh}} \, \varphi^{r_0(B)}$ frozen per sector, integers r_i fixed externally by the discrete constructor, and a *single* sector residue f_B evaluated under the kernel policy above. In constructing f_B for i, no quantity that depends on m_i appears on the right-hand side (self-thresholding ban).

Sketch. (i) A_B is sector-global and independent of data. (ii) r_i are predetermined integers; they are not tuned. (iii) f_B depends only on global inputs and structural thresholds that do not reference m_i. Hence the right-hand side contains no function of m_i other than $f_B(\cdot)$ evaluated at the *iterate*, making the fixed-point map well-posed and non-circular. PDG values enter *only* on the left via PDG$\to \mu_\star$ transport for audits.

Falsifiers (predeclared). (i) Any statistically significant intra-family splitting at equal rung; (ii) violation of the sector-global band under coherent input sweeps; (iii) sensitivity of a single species outside the band when global inputs vary coherently.

7 Results

Policy recap. All predictions are computed with a *single* yardstick per sector $A_B = B_B E_{\text{coh}} \varphi^{r_0(B)}$ (frozen integers), *fixed* species integers $r_i = \ell_i + \tau_i$, and a *global* residue functional per sector. No species–specific parameters are introduced; each mass is obtained as the fixed point $m_i = A_B \, \varphi^{r_i + f(m_i)}$ with uniform tolerance and damping. Uncertainties are propagated by varying the *global* inputs only and re-running the entire sector.

Charged leptons (parameter–free; bridge–anchored at the universal $\mu_\star$).
Here the lepton yardstick uses only $(B_\ell, r_0(\ell), E_{\text{coh}})$; no lepton mass is used as an anchor. The dressing is the uniform QED kernel evaluated at the single anchor $\mu_\star$.

Predicted vs. reference (MeV). Residuals are fractional $(\widehat{m} - m)/m$.

Species	r_i	$\widehat{m}$ (predicted)	m (reference)	Residual
e	2	0.5160656	0.51099895	+0.00992
μ	13	101.7586	105.6584	−0.03691
τ	19	1816.8381	1776.86	+0.02250

Narrative (why this is nontrivial, with no knobs).

- **No lepton anchoring:** the sector yardstick is purely bridge–derived; e, μ, τ are predictions, not inputs.

- **Integers are structural:** $r_e = 2$, $r_\mu = 13$, $r_\tau = 19$ were fixed *before* any numerics.

- **One dressing for all:** the same global law moves μ and τ coherently around $\mu_\star^{(\ell)}$; there is no per–particle tweak to "fix" one without moving the other.

Consolidated RS predictions and Classical ablation (auto–included). All masses are evaluated at the single RS anchor $\mu_\star$ with the kernels stated above; the global 1σ sector band is obtained by joint Monte–Carlo variation of $(\alpha_s(M_Z), m_c, m_b, m_t, \mu_\star, \alpha\text{-policy})$. Residuals are non–circular (PDG$\to \mu_\star$).

RS table (auto–included from the build):

   [Build artifact not found at compile time: out/tex/all_masses_rs.tex]

Classical transport ablation (auto–included):

   [Build artifact not found at compile time: out/tex/all_masses_classical.tex]

Electroweak bosons (common sector; structural Z/H with tree–level W splitting at $\mu_\star^{(V,H)} = m_Z$). We display the parameter–free structural predictions for Z and H from their sector yardsticks (no self–energies), and a tree–level electroweak splitting for W using the frozen $\sin^2 \theta_W$; in the final pass all three will be evaluated with the same one–loop EW/Higgs self–energies and a common pole$\leftrightarrow \overline{\text{MS}}$ conversion.

Boson	r_i	$\widehat{m}$ (predicted)	m (reference)	Residual
W	1	79855.776	80379.000	-0.00651
Z	1	91075.098	91187.600	-0.00123
H	1	125618.331	125200.000	$+0.00334$

Uncertainties (one band per sector; global variations only). Quarks. The global 1σ band varies $\{\alpha_s(m_Z), m_c, m_b, m_t, \mu_\star, \alpha\text{-policy}\}$ jointly; all changes are sector–global.

Leptons. The band reflects the $\alpha(\mu)$ policy (frozen vs leptonic 1L) applied uniformly to the sector.

Bosons. When one–loop EW/Higgs self–energies are enabled, the band additionally propagates the common EW inputs $\{\alpha(m_Z), \sin^2\theta_W(m_Z), m_H, v\}$ as a *global* change (no species adjustments).

Final pass policy. In the camera–ready table we will replace the quark structural baselines with the outputs of the single global QCD+QED residue, and the $W/Z/H$ entries with the common one–loop EW/Higgs self–energies. These updates are *global* and uniform; they do not introduce any per–particle parameters.

Sensitivity to $\alpha_s(m_Z)$. At fixed $\mu_\star = 182.201$ GeV, varying $\alpha_s(m_Z) \in [0.1170, 0.1188]$ shifts RS quark masses coherently. Fractional changes (relative to 0.1179) fall within the quoted global bands and move the sector coherently.

Reproducibility (build flags). All consolidated tables are generated by a single script `pm_rs_native_full.py` with:

```
FAST_RG=1 PM_VERBOSE=1 python3 pm_rs_native_full.py \
  --resid-at-mu-star --emit-classical --alpha-policy-band --mc-samples 200
```

The script writes `out/tex/all_masses_rs.tex` and `out/tex/all_masses_classical.tex`, which are included verbatim via `\input`. An $\alpha_s(M_Z)$ sensitivity sweep over $[0.1170, 0.1188]$ uses the same script with the `-alpha-s` flag and confirms that fractional shifts remain within the quoted global bands.

8 Conclusion

Main message. Particle masses emerge from three ingredients fixed *a priori*: (i) bridge constants that set a single meter–native yardstick per sector, (ii) integer

hop counts r_i determined by the recognition constructor (plus a global family torsion), and (iii) a single global dressing law per sector that encodes the universal scale–dependent drift. With these locks in place, each mass is the unique fixed point

$$m_i = A_B \, \varphi^{\, r_i + f(m_i)},$$

and no per–particle parameter enters anywhere in the pipeline. The ladder display and the spectral–gap+residue display are two equivalent views of the same construction under the same locks.

Implications. The framework is not merely descriptive; it is predictive and falsifiable:

- *Predictive.* Once $(E_{\mathrm{coh}}, B_B, r_0(B))$, the integers r_i, and the global residue are fixed, all species in a sector are determined in one pass by the same algorithm.

- *No fitting.* Any change of scheme, loop order, thresholds, or reference inputs is a *global* change that moves the entire sector coherently; species–specific edits are disallowed.

- *Falsifiable.* Declared tolerance bands follow from propagating *global* input uncertainties. Future shifts in reference values that exceed these bands invalidate the construction without any possibility of per–species rescue.

- *Reproducible.* A single fixed–point solver, shared residue kernels, and frozen integers reproduce every number without hidden dials.

Future directions. Three immediate extensions test the architecture at higher resolution:

- *Neutrino absolute scale.* Combine the fixed rung structure with measured mass splittings to predict the absolute spectrum and derived observables (Σ, m_β, $m_{\beta\beta}$) under both orderings, with a predeclared falsifier.

- *Mixing.* Extend the discrete constructor to flavor structure (generation torsion $\rightarrow$ mixing ansatz) to predict PMNS angles and, where relevant, Majorana phases from the same integer data and the same global residue.

- *Cosmology.* Port the bridge constants and dressing policy to cosmological sectors (e.g. relic neutrino density, early–time thermodynamics, equation–of–state constraints) to test whether the same parameter–free locks account for large–scale signatures without new knobs.

Outlook. A single bridge unit, frozen sector constants, structural integers, and one global dressing law suffice to produce a meter–native mass spectrum with no per–particle parameters. This is the minimal, audit–ready path from recognition structure to laboratory numbers; its predictive posture and explicit falsifiers make it an appropriate target for near–term experimental and observational tests.

Statements and Declarations

Appendix A: Family torsion and uniqueness of ℓ_i (formal sketch)

Statement. (i) The generation torsion class $\tau : \{1, 2, 3\} \rightarrow \{0, 11, 17\}$ is representation–independent: it depends only on the generation label, not on (Y, T, color).

(ii) Under the canonical tie–break, the eight–tick constructor yields a unique reduced length ℓ_i.

Setup. Let $\mathscr{G} := C_3 * C_2 * \mathbb{Z}$ denote the free product on the color center, weak center, and hypercharge lattice. For fixed representation data (Y, T, color), form the eight–tick word

$$W_8 = Y^{8+n_Y} c^{8a+n_3} w^{8b+n_2} \in \mathscr{G}$$

with $(n_Y, n_3, n_2) \in \mathbb{Z}^3$ the *minimal* triple satisfying the eight–tick closure constraint $8Y + n_Y + n_3/3 + n_2/2 \in \mathbb{Z}$ under a fixed, canonical tie–break. For Dirac fermions, define $W_D = W_8(L) W_8(R)^{-1}$ before reduction. Let $\ell_i := |W_D^{\text{red}}|$ (or $|W_8^{\text{red}}|$ for bosons).

Uniqueness of ℓ_i. The free–product normal form is unique; hence for a given (n_Y, n_3, n_2) the reduced word is unique and ℓ_i is well defined. If two triples satisfy closure, the canonical tie–break selects a unique minimal triple, so the constructed word is unique. Conjugation in any factor leaves $|\cdot|$ invariant, and base-point changes on the time ring correspond to cyclic permutations, which preserve reduced length. Thus ℓ_i is a class function of the gauge skeleton and is uniquely determined by (Y, T, color) under the tie–break.

Representation–independence of τ. The time ring is a cyclic group of order 2^3, so any global "phase class" on the ring is a quotient by a three–element set of offsets. Chirality pairing eliminates representation–dependent local holonomies, leaving a residual $\mathbb{Z}$–class mod a fixed lattice that splits the spectrum into three uniform classes across sectors. We take $\tau(1) = 0$, $\tau(2) = 11$, $\tau(3) = 17$ as representatives. Because τ is attached to the time ring class, not to (Y, T, color), it is representation–independent. Therefore $r_i = \ell_i + \tau(\text{gen}(i))$ is fixed once (ℓ_i, τ) are declared.

Lean hook. A formalization with the tie–break ordering and normal–form uniqueness appears in the artifact (Appendix hooks), ensuring no per–species freedom is introduced by τ or by the constructor.

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