Agenda

1. Important Consequence of the Mean Value Theorem
2. Example & sketching

1. Important Consequence of the MVT

1. $f:(a,b)\to\mathbb{R}$ differentiable
2. $f'(x)$ (≥ 0) (≤ 0)

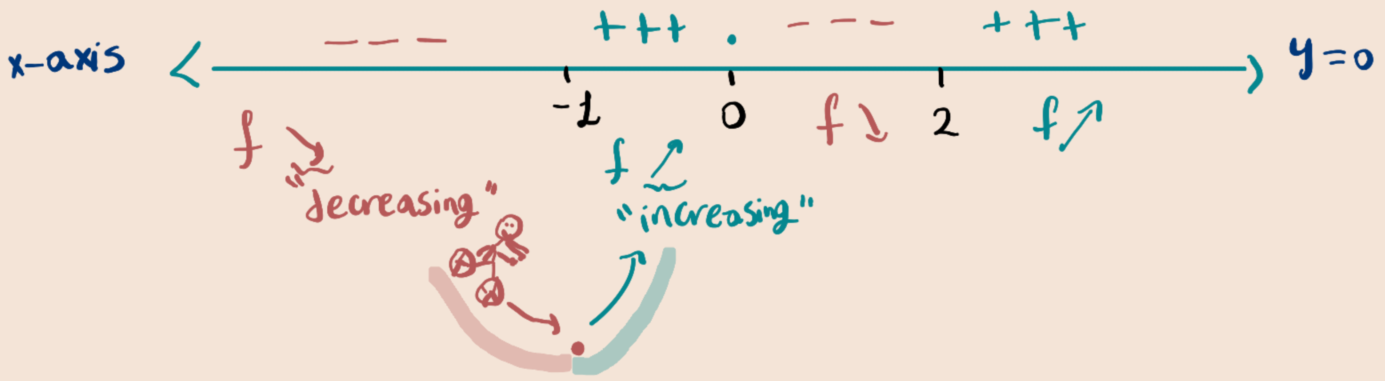
$\Rightarrow f$ (increasing) (decreasing) on (a,b)

Meaning...

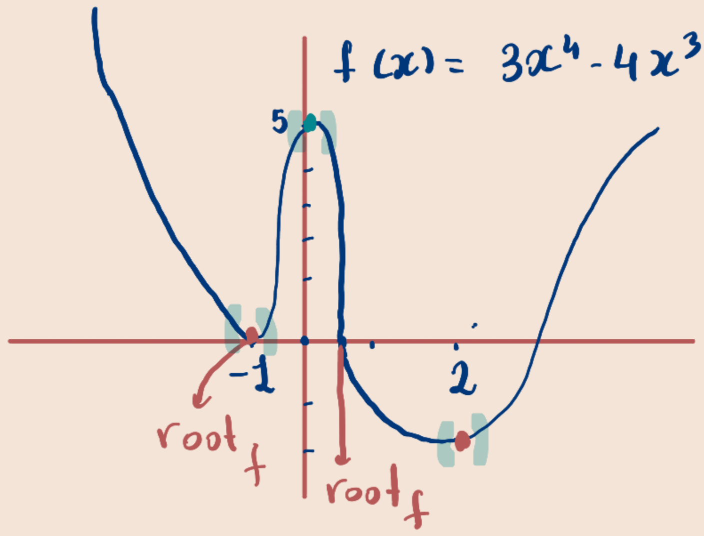
a) $f(x) = 3x^4 - 4x^3 - 12x^2 + 5 \Rightarrow f'(x) = 3\cdot 4x^3 - 4\cdot 3x^2 - 12\cdot 2x$

$12x(x-2)(x+1) = 12x(x^2 - x - 2) = 12x^3 - 12x^2 - 24x$

$f'(x) = 12x(x-2)(x+1) = (\quad)\cdot(\quad)\cdot(\quad) = (\quad)$



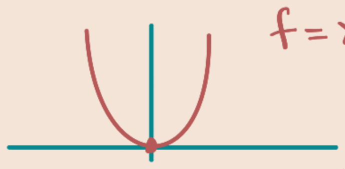
$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$



$f(-1) = 0$ local minima

$f(0) = 5$ local maxima ~~global~~

$f(2) = -27$ local minima



$f'(x) = 12x^3 - 12x^2 - 24x \Rightarrow f''(x) = 36x^2 - 24x - 24$

minimas: maximas:

$x = -1$ $\quad$ $x = 0$ $\quad$ $f''(0) = -24 < 0$

$x = 2$

$f''(-1) = 36 > 0$

$f''(2) = 74 > 0$

2nd derivative test:

$E_f = \{x : f'(x) = 0\}$. Choose $x_0 \in E$.

$f''(x_0) > 0 \Rightarrow x_0$ minimum.

$f''(x_0) < 0 \Rightarrow x_0$ maximum.

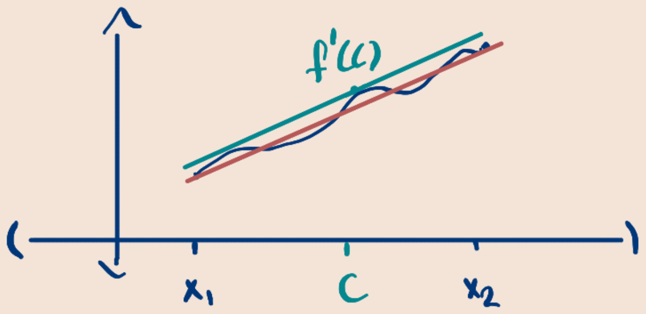
Why it Works.

Assume the contrary! $x_1 < x_2$ with $f(x_1) > f(x_2)$

Then... for some $c \in (x_1, x_2)$, $\underbrace{f'(c)}_{\geq 0} = \dfrac{\overbrace{f(x_2) - f(x_1)}^{<0}}{\underbrace{x_2 - x_1}_{>0}} < 0$

by assumption

$\cancel{f'(c) < 0}$ ≥ 0

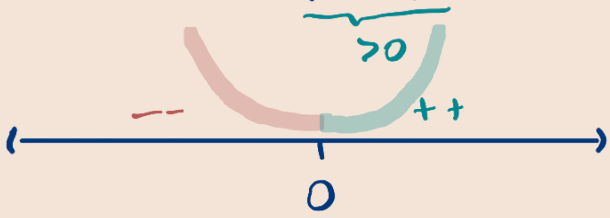


2. Example & sketching.

$$f(x) = \frac{x^2-1}{\sqrt{x^2+1}} \Rightarrow f'(x) = \frac{(\cancel{\sqrt{x^2+1}})(2x) - (x^2-1)\,x\,\cancel{(x^2+1)^{-\frac{1}{2}}}}{(x^2+1)^{3/2}} \cdot \frac{\cancel{\sqrt{x^2+1}}}{\cancel{\sqrt{x^2+1}}}$$

$$\left[\frac{f}{g}\right]' = \frac{gf' - fg'}{g^2} \qquad \left[(x^2+1)^{\frac{1}{2}}\right]' = \frac{\cancel{1}}{\cancel{2}} \cdot (x^2+1)^{-\frac{1}{2}} \cdot \cancel{2}x$$

$$f'(x) = \frac{2x(x^2+1) - (x^2-1)x}{(x^2+1)^{3/2}} = \frac{x\overbrace{(3+x^2)}^{>0}}{\underbrace{(x^2+1)^{3/2}}_{>0}} = 0 \Rightarrow x = 0$$

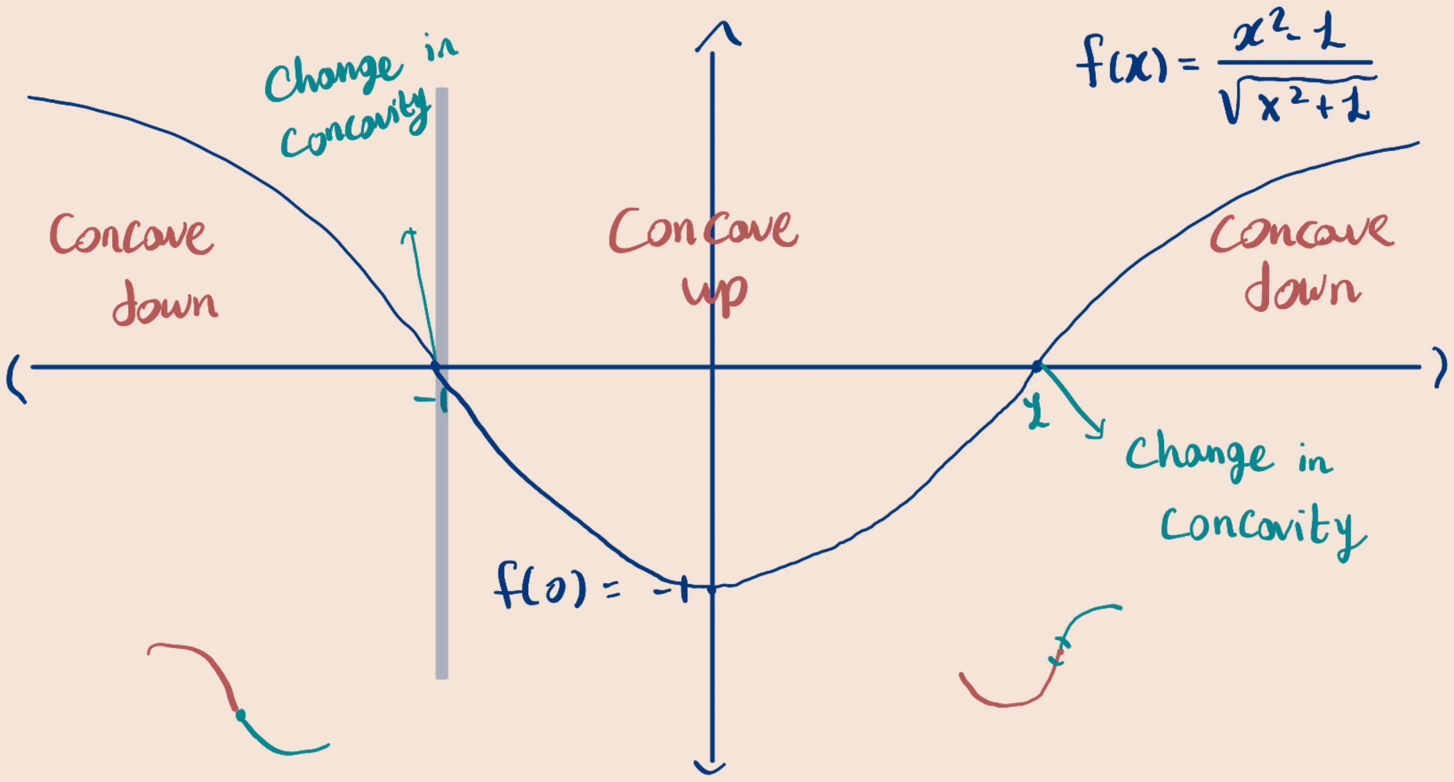


$$f'(x) = \frac{x(3+x^2)}{(x^2+1)^{3/2}} \Rightarrow f''(x) = \frac{3-3x^2}{\underbrace{(x^2+1)^{5/2}}_{>0}} = 0 \Rightarrow 1 = x^2 \Rightarrow \underbrace{x = \pm 1}$$

inflection!

Sign $(f''(0))$ = Sign $(3 - 3 \cdot 0^2)$ = ++

$f''(0) > 0 \Rightarrow 0$ is a minimum (local)



$$f(x) = \frac{x^2-1}{\sqrt{x^2+1}}$$