Algebra

1, Linear Equations in one variable.

Numbers. $1, 2, 3, \ldots$

1. Define addition $(+)$. E.g. $1+2=3=2+1$

$(+)$ identity: Let 0 be the number such that for any number a,

$$a+0=a$$

$(+)$ inverse: for any number a, let $-a$ be the number such that

$$-a+a=0.$$

2. Define multiplication $(\times)$. E.g. $a \cdot b = \overbrace{a+a+\cdots+a}^{b \text{ times.}}$

- $(\times)$ identity: Let 1 be the number such that for any number a,

$$a \cdot 1 = a.$$

- $(\times)$ inverse: for any number $a \neq 0$, let $\frac{1}{a}$ be the number such that

$$\frac{1}{a} \cdot a = 1.$$

3. The distributive property links $(\times)$, $(+)$!

$$a(b+c) = a \cdot b + a \cdot c$$

This toolset enables you to solve any equation. It provides a good change of perspective: instead of worrying about 2 more operations $(-), (\div)$, we incorporate them into our system as inverses.