30th October & 04th November

Completed Exercises from the lectures on

< Nonlinear Functions >

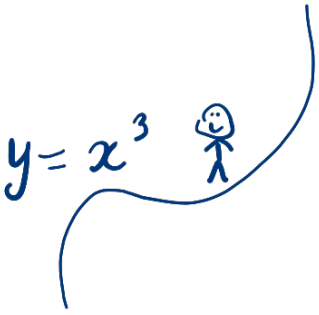
1. Medium, Page 2;

2. Hard, Pages 3-15;

Can be found below.

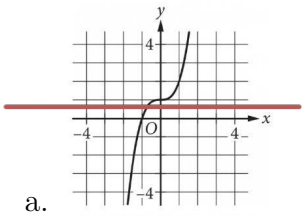
(4) **b39d74a0** MULTIPLE CHOICE One answer only

x	y
0	0
1	1
2	8
3	27

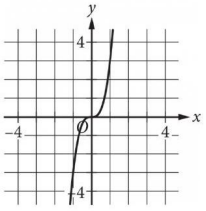


The table shown includes some values of x and their corresponding values of y. Which of the following graphs in the xy-plane could represent the relationship between x and y ?

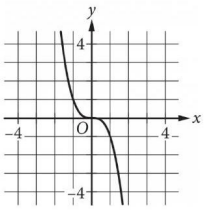
a.



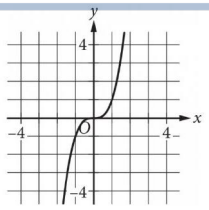
b.



c.



d.



0 0 → crosses origin → ~~✗~~

1 1 → crosses (1,1) → ~~✗~~ ~~✗~~

2 8

3 27

increasing

↓

~~✗~~

Hard

(1) **301faf80** SHORT ANSWER Case-Insensitive

The product of two positive integers is 462 . If the first integer is 5 greater than twice the second integer, what is the smaller of the two integers?

$x_1 \cdot x_2 = 462 \Rightarrow (2 \cdot x_2 + 5) \cdot x_2 = 462 \Rightarrow 2x_2^2 + 5x_2 = 462 \quad (-462)$

$x_1 = 2 \cdot x_2 + 5$

x_1 is

$\Rightarrow f(x_2) = 2x_2^2 + 5x_2 - 462 = 0$

$= ax_2^2 + bx_2 + c$

(14) **270cf326** MULTIPLE CHOICE One answer only

Which of the following functions has(have) a minimum value at -3 ?

$$f(x) = -6(3)^x - 3 \qquad g(x) = -3(6)^x$$

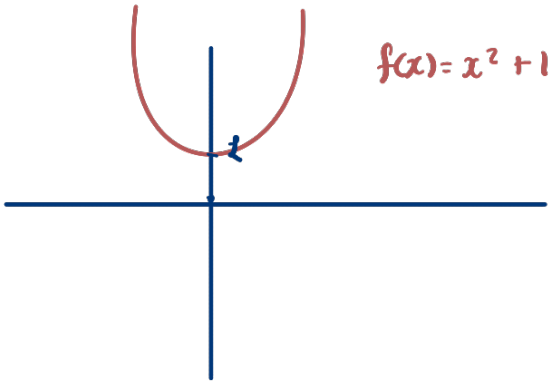
a. Neither I nor II
b. II only
c. I only
d. I and II

$f(x) = -6(3)^x - 3 \Rightarrow f(-3) = -6(3)^{-3} - 3$

$g(x) = -3(6)^x \Rightarrow g(-3) = -3(6)^{-3}$

$a \neq 0, \quad \left(\frac{1}{a}\right)^7 = a^{-7}$

$-6(3)^{-n} = -6\frac{1}{3^n}$



(15) **0121a235** MULTIPLE CHOICE One answer only

x	$p(x)$
-2	5
-1	0
0	-3
1	-1
2	0

$x=-1$

$x=2$

The table above gives selected values of a polynomial function p. Based on the values in the table, which of the following must be a factor of p ?

a. $(x-3)$
b. $(x-1)(x+2)$
c. $(x+3)$
d. $(x+1)(x-2)$

idea:

$f_1 \cdot f_2 \cdot f_3 \cdots f_n = 0$

$\Rightarrow$ one of them is zero!

f_i

$p(x) = (x-r_1)\cdot(x-r_2)\cdots(x-r_n) = 0$

roots of the Polynomial.

(19) **59d1f4b5** MULTIPLE CHOICE One answer only

$M = 1,800(1.02)^t$ t is in years

The equation above models the number of members, M, of a gym t years after the gym opens. Of the following, which equation models the number of members of the gym q quarter years after the gym opens?

$M = 1,800(1.02)^{\frac{q}{4}}$

a. $M = 1,800(1.082)^q$
b.
c. $M = 1,800(1.005)^{4q}$
d. $M = 1,800(1.02)^{4q}$

$M = 1800(1.02)^t$

$q = 4$

$t = 0 \Rightarrow 1800$

$t = 1 \Rightarrow 1800 \cdot (1 + 2\%)$

$t = 2 \Rightarrow [1800 \cdot (1+2\%)](1+2\%)$

$1800 \cdot (1+2\%)^2$

(20) **95eeeb5b** MULTIPLE CHOICE One answer only

The function f is defined by $f(x) = ax^2 + bx + c$, where a, b, and c are constants. The graph of $y = f(x)$ in the $xy-$ plane passes through the points $(7, 0)$ and $(-3, 0)$. If a is an integer greater than 1 , which of the following could be the value of $a + b$?

~~a. -3~~
b. -6
~~c. -5~~
~~d. 4~~

$f(7) = 0 = a \cdot 49 + b \cdot 7 + c$

$f(-3) = 0 = a \cdot 9 + -3b + c$

$a \cdot 49 + b \cdot 7 + \cancel{c} = 9a + -3b + \cancel{c}$

$40a + 10b = 0$

$40a = -10b \Rightarrow$ $a = \frac{-1}{4} \cdot b > 1$

- b negative
- $b < -4$

$a + b = -3$

$1^+ + -4^- = -3^+$

$a = 1$ $\quad b = -4$

$a > 1$, $\quad b < -4$

(31) **6f5540a5** MULTIPLE CHOICE One answer only

Kao measured the temperature of a cup of hot chocolate placed in a room with a constant temperature of 70 degrees Fahrenheit (°F). The temperature of the hot chocolate was 185°F at 6:00 p.m. when it started cooling. The temperature of the hot chocolate was 156°F at 6:05 p.m. and 135°F at 6:10 p.m. The hot chocolate's temperature continued to decrease. Of the following functions, which best models the temperature $T(m)$, in degrees Fahrenheit, of Kao's hot chocolate m minutes after it started cooling?

a. $185(0.85)^m$
b. $T(m) = 70 + 115(0.75)^{\frac{m}{5}}$
c. $T(m) = (185 - 70)(0.75)^{\frac{m}{5}}$
d. $T(m) = 185(1.25)^m$

$m = 0$ at 6:00 $\longrightarrow$ 185°

$m = 5$ at 6:05 $\longrightarrow$ 156°

$m = 10$ at 6:10 $\longrightarrow$ 135°

which $T(m)$ satisfies

$T(0) = 185$ ~~✗~~

$T(5) = 156$ ~~✗~~ → growing ⇒ bigger than 185

$T(10) = 135°$ ~~✗~~ → gives ≈ 36

(33) **b73ee6cf** MULTIPLE CHOICE One answer only

The population of a town is currently 50,000, and the population is estimated to increase each year by 3% from the previous year. Which of the following equations can be used to estimate the number of years, t, it will take for the population of the town to reach 60,000?

a. $50,000 = 60,000(0.03)^t$
b. $60,000 = 50,000(0.03)^t$
c. $60,000 = 50,000(1.03)^t$
d. $50,000 = 60,000(3)^t$

$a_t = a_0 \cdot (1 + r\%)^t$ → time period

Starting value

growth / decay

$y_0 = 50000$

$y_1 = y_0 (1 + 0.03)$

$y_2 = y_1 (1 + 0.03) = y_0 \cdot (1 + 0.03)^2$

$\vdots$

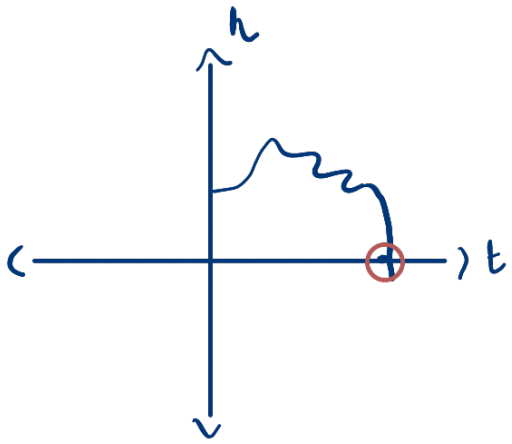
$60,000 = y_t = 50000 \cdot (1.03)^t$

(34) **7eed640d** MULTIPLE CHOICE One answer only

$$h(x) = -16x^2 + 100x + 10$$

The quadratic function above models the height above the ground h, in feet, of a projectile x seconds after it had been launched vertically. If $y = h(x)$ is graphed in the xy-plane, which of the following represents the real-life meaning of the positive x-intercept of the graph?

a. The initial height of the projectile
b. The maximum height of the projectile
c. The time at which the projectile reaches its maximum height
d. The time at which the projectile hits the ground



(35) **43926bd9** SHORT ANSWER Case-Insensitive

x	$f(x)$
1	a^1
2	a^5
3	a^9

For the exponential function f, the table above shows several values of x and their corresponding values of $f(x)$, where a is a constant greater than 1 . If k is a constant and $f(k) = a^{29}$, what is the value of k ?

$1 : a^{1+4\cdot 0} = a^1$

$2 : a^{1+4\cdot 1} = a^5$

$3 : a^{1+4\cdot 2} = a^9$

$\vdots$

$k : a^{29} = a^{1+4\cdot 7} = a^{29}$ ($k-1$)

always minus one

$k = 8$

alternatively...

$29 = 1 + 4\cdot t \Rightarrow t = 7$

$f(x) = a^x$

(36) **161126cf** MULTIPLE CHOICE One answer only

$$f(x) = (1.84)^{\frac{x}{4}}$$

The function f is defined by the given equation. The equation can be rewritten as $f(x) = \left(1 + \frac{p}{100}\right)^x$, where p is a constant. Which of the following is closest to the value of p ?

a. 46
b. 96
c. 21
d. 16

Start: $(1.84)^{\frac{x}{4}} = f(x) = \left(1 + \frac{p}{100}\right)^x$

$(b)^{x \cdot y} = (b^x)^y$

$(\text{something})^x$

$\left((1.84)^{\frac{1}{4}}\right)^x = (1.84)^{\frac{1}{4} \cdot x(t)} = \left(1 + \frac{84}{100}\right)^{\frac{x}{4}} = f(x) = \left(1 + \frac{p}{100}\right)^x$

$\left(1 + \frac{16.4}{100}\ldots\right)^x = \left(1 + \frac{p}{100}\right)^x$

(37) **a7711fe8** MULTIPLE CHOICE One answer only

What is the minimum ~~value~~ Point of the function f defined by $f(x) = (x - 2)^2 - 4$?

a. -2
b. 4
c. -4
d. 2

$f(x) = \underbrace{(x-2)^2}_{\geq 0} - 4$

$\min f$ attained at $0 - 4 = -4$ ← value

$x = 2$

4 Point

(38) **1a722d7d** MULTIPLE CHOICE One answer only

Let the function p be defined as $p(x) = \frac{(x-c)^2+160}{2c}$, where c is a constant.
If $p(c) = 10$, what is the value of $p(12)$?

a. 11.00
b. 10.75
c. 10.25
d. 10.00

$$\frac{160}{2c} = \frac{\overset{=0}{\cancel{(c-c)^2}} + 160}{2c} = p(c) = 10$$

$$\Rightarrow \frac{80}{c} = 10 \Rightarrow c = \frac{80}{10} = 8$$

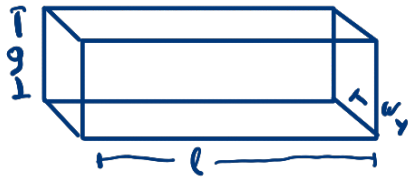
use $c = 8$

$$p(12) = \frac{(12-c)^2 + 160}{2c} = \frac{16 + 160}{16} = 1 + 10 = 11$$

(40) **48f83c34** MULTIPLE CHOICE One answer only

A right rectangular prism has a height of 9 inches. The length of the prism's base is x inches, which is 7 inches more than the width of the prism's base. Which function V gives the volume of the prism, in cubic inches, in terms of the length of the prism's base?

a. $V(x) = 9x(x-7)$
b. $V(x) = x(x+9)(x-7)$
c. $V(x) = 9x(x+7)$
d. $V(x) = x(x+9)(x+7)$



$l = 7 + w$

$w = l - 7$

Total of marks: 100

$V(l) = l \cdot w \cdot h = l \cdot w \cdot 9 = l \cdot (l-7) \cdot 9$

replace l with x...