Agenda:

0. Homework Questions ✓
1. Polynomial Long Division
2. Drawing Polynomials
3. Binomial Coefficients
4. The Exponential Function
5. Discussion & More Questions

I) Polynomial Long Division

Usual Guesses: 0, 1, -1, 2 ...

Last time: $x^3-2x^2-5x+6=0$, what is x?

Strategy: "reduce" to $\boxed{n=2}$

how?: $(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)$ (Fundamental Thm of Alg.)

$\Rightarrow$ divide by $(x-\alpha_i)$

↳ some root.

1. Guess! $x=1$, $1^3-2\cdot1^2-5\cdot1+6=0$ $\Rightarrow$ 2. divide by $(x-1)$

$$
\begin{array}{r l}
 & x^2-x-6 \\
x-1 \,\big) & \cancel{x^3}-2x^2-5x+6 \\
 & -(\cancel{x^3}-x^2) \\
 & \quad -\cancel{x^2} \\
 & \quad -(\cancel{-x^2}+x) \\
 & \qquad -\cancel{6x+6} \\
 & \qquad -(\cancel{-6x+6})
\end{array}
$$

$x^3-2x^2-5x+6=(x-1)(x^2-x-6)$

3. solve $=(x-1)(x-3)(x+2)$

$\boxed{\text{Roots} = \{3,-2,1\}}$

why it works: FTA = division with no remainder!

Another Example: x^3+2x^2-x-2.

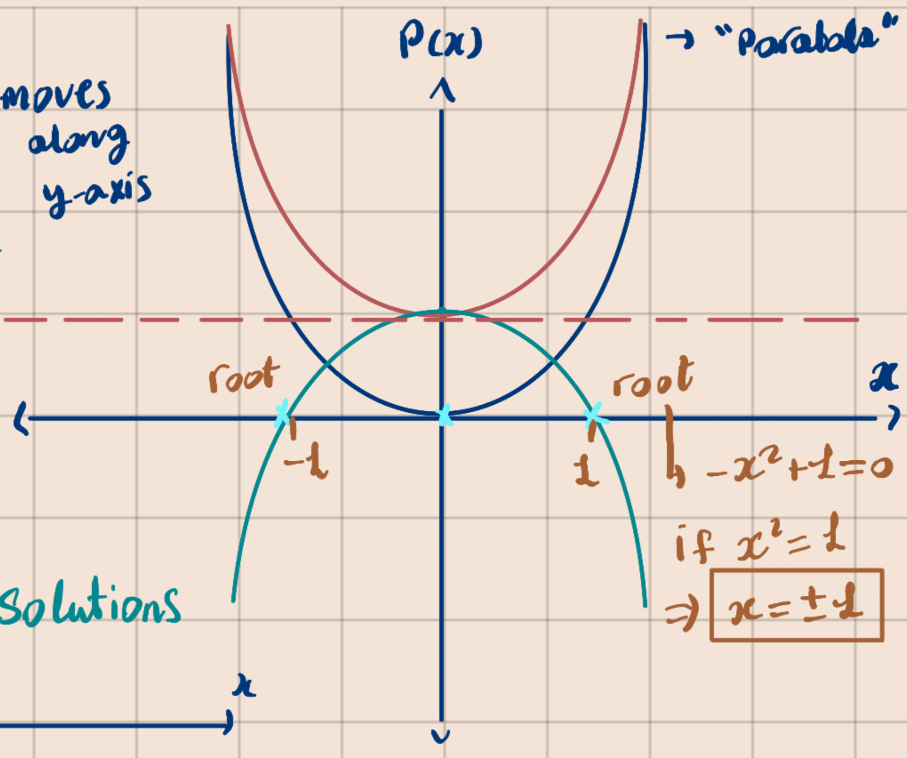
1. Guess
2. Divide
3. solve

II) Drawing Polynomials

$P(x) = x^2+1$ moves along y-axis

mirrors to the other side ← $P(x) = -x^2+1$

$P(x) = x^2$



$x^2+1=0 \Rightarrow x=i$

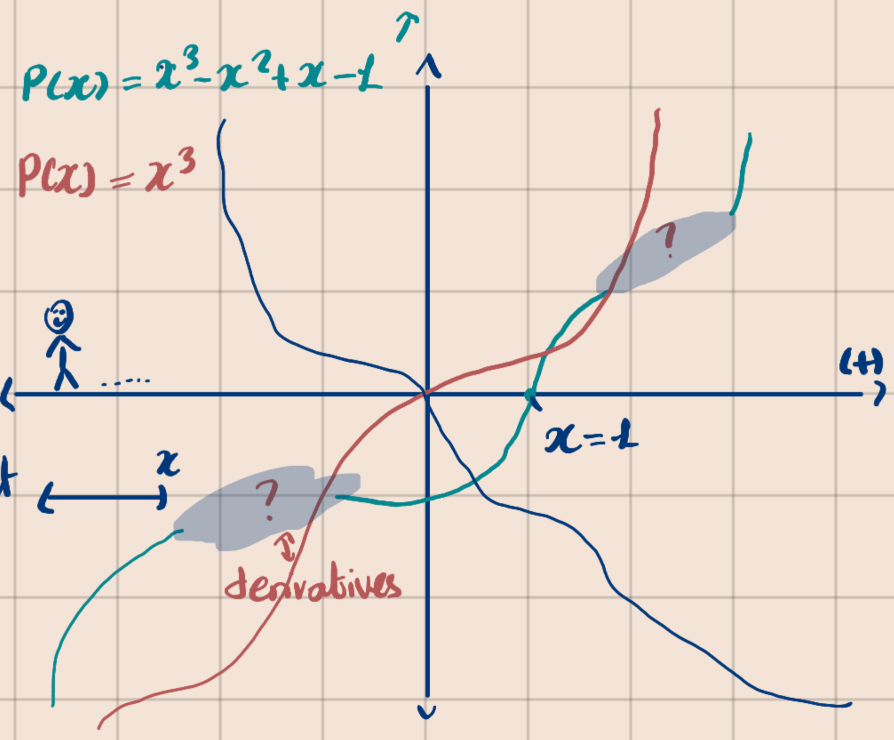
Graphically: $x^2+1=0$ has no solutions

It does not intersect

$p(x)=x^3-x^2+x-1=(x-1)(x^2+1)$

$\text{Roots}_p = \{1, i, -i\}$

$x^2+1=0 \Rightarrow x=\pm i$



Problem: not all roots intersect

Solution: derivatives... later

III) Binomial Coefficients

"n choose k"

Reminder: $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$, with $\binom{n}{k} := \dfrac{n!}{k!(n-k)!}$

$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$ "factorial"

$k! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot k$

Ex: $\binom{5}{2} = \dfrac{5!}{2!\,3!} = \dfrac{\cancel{1 \cdot 2 \cdot 3} \cdot 4 \cdot 5}{(1 \cdot 2)(\cancel{1 \cdot 2 \cdot 3})} = \dfrac{\cancel{4} \cdot 5}{\cancel{2}} \cdot 2 = 10$ ($n = 5$, $k = 2$)

Property: Pascal's Triangle

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k},$$

"(4.) made precise"

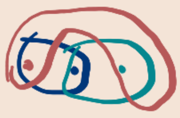
1. Start with 1
2. Always 1 on the ends
3. The n^{th} row has n elements
4. "add two elements on top"

$n=0$: $1_{k=0} \rightarrow \binom{0}{0}$

$n=1$: $1_{k=0}$ $1_{k=1}$ (4.)

$n=2$: $1_{k=0}$ $2_{k=1}$ $1_{k=2}$

$n=3$: $1_{k=0}$ $3_{k=1}$ $3_{k=2}$ $1_{k=3}$ $\rightarrow \binom{3}{2}$



$= \binom{3}{2} = 3$

$n=4$: $1_{k=0}$ $4_{k=1}$ $6_{k=2}$ $4_{k=3}$ $1_{k=4}$

$n=5$: $1_{k=0}$ $5_{k=1}$ $10_{k=2}$ $10_{k=3}$ $5_{k=4}$ $1_{k=5}$

$\vdots$

exercise: write Pascal's Triangle until $n = 10$

Proof? NEXT TIME