* oelshinawy@constructor.university $\rightarrow$ Teams

Numbers.

$\{0, 1, 2 \dots\}$ (with zero!)

$\mathbb{N} \subset \mathbb{N}_0 \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$

$\mathbb{N} \rightarrow \{1, 2, 3, \dots\}$

$\mathbb{Z} \rightarrow \{\dots, -1, 0, 1, \dots\}$

$\mathbb{Q} = \{\text{rational numbers}\} = \{\frac{a}{b} : a \in \mathbb{Z},\ b \in \mathbb{N}\}$ — $b \in \mathbb{N}$: avoids zero!

$:$ "such that"

$\frac{a}{b}$, $b \neq 0$ $\quad \boxed{\frac{1}{0}}$

$\boxed{\text{The Empty Set: } \emptyset = \{\}}$

$\Leftrightarrow$ "is equivalent to"

$\boxed{\text{For all : } \forall}$ $\quad A \subset B \Leftrightarrow \boxed{\forall a \in A : a \in B}$

ex: $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 10\}$

"any element in A is in B"

$\boxed{\text{There exists: } \exists}$

$\boxed{\text{Prime Factorization}}$ $\quad 20 = 10 \cdot 2 = 5 \cdot 2 \cdot 2$

... more generally, $a = \underbrace{P_1 \cdot P_2 \cdot \dots \cdot P_K}_{\text{Primes}}$ is a prime factorization.

Numbers... but Complex.

Equation: $x^2 + 1 = 0$: what is the solution? / what x satisfies this?

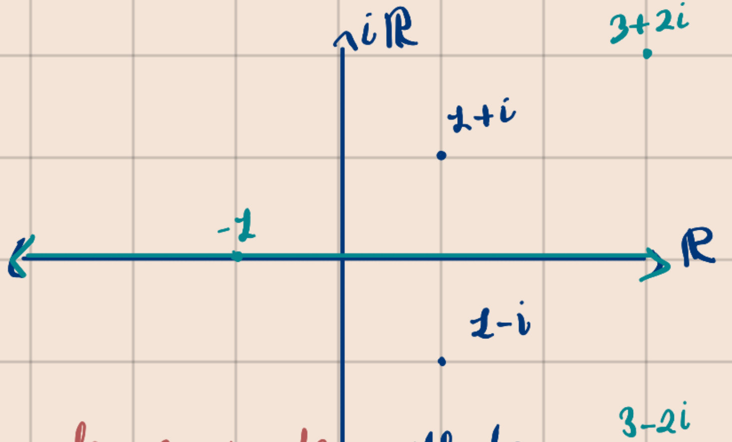
$\Leftrightarrow x^2 = -1$

No solution in $\mathbb{R}$, but we can "cook up" a solution!

Introduce i such that $i^2 = -1$.

$\mathbb{C} = i\mathbb{R} + \mathbb{R}$

$\{x + yi : x, y \in \mathbb{R}\}$



Let's consider $z = x + yi$. Its complex conjugate will be

$\bar{z} = x - yi$

"z bar"

(-1)

Question: what is the complex conjugate of a, where $a \in \mathbb{R}$?

This is just $\underset{(-1)}{a}$! $a = a + 0i \Rightarrow \bar{a} = a - 0i = a$.

"therefore"

"implies"

"star"

NB: $(x+iy)^* = (x-iy)$

Multiplying Complex Numbers:

$$(x+iy)(a+bi) = xa + x(ib) + (iy)a + \overbrace{(iy)(ib)}^{i^2=-1} =$$

$$= xa + i\,xb + i\,ya + \underbrace{i^2}_{-1} yb = (xa - yb) + i(xb + ya)$$

exercise: $(1+2i)(3+4i)$

Dividing Complex Numbers:

$\frac{x+iy}{a+ib}$, problem: we don't know how to divide by i!

solution: get rid of i - in the denominator.

$$\frac{x+iy}{a+ib} \cdot 1 = \frac{x+iy}{a+ib} \cdot \overbrace{\frac{a-ib}{a-ib}}^{=1} = \frac{xa+yb+i(-xb+ya)}{a^2+b^2}$$

$(a+ib)(a-ib)$
$= a^2 - \cancel{iba} + \cancel{iba} - \underbrace{i^2}_{} b^2$
$1 = -(-1)$
$= a^2 + b^2$

Real!

Solution (once again): multiply by conjugate

$$= \frac{\overbrace{xa+yb}^{\in \mathbb{R}}}{a^2+b^2} + i\,\frac{\overbrace{(-xb+ya)}^{\in i\mathbb{R}}}{a^2+b^2}$$

Exercise: $\frac{2+3i}{1+i}$

Key: $z = x+yi$. Then $z \cdot \bar{z} = x^2+y^2 \in \mathbb{R}$

Roots of Quadratic Equations:

$\boxed{ax^2+bx+c=0}$ is a quadratic equation.

If x is a root, then x is the value that "makes this equation $=0$".

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Path?

Path: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$\Leftrightarrow \quad 2ax = -b \pm \sqrt{b^2-4ac}$$
$$\Leftrightarrow \quad 2ax + b = \pm\sqrt{b^2-4ac}$$
$$\Leftrightarrow \quad 4a^2x^2 + 4axb + \cancel{b^2} = (2ax+b)^2 = (\cancel{\pm}\sqrt{b^2-4ac})^{\cancel{2}} = \cancel{b^2} - 4ac$$
$$\Leftrightarrow \quad \frac{\cancel{4}a^2x^2 + \cancel{4}axb + \cancel{4}ac}{4} = 0 \quad \Leftrightarrow \quad \frac{a^{\cancel{2}}x^2 + \cancel{a}bx + \cancel{a}c}{\cancel{a}} = 0 \qquad \boxed{a \neq 0!}$$
$$\Leftrightarrow \quad ax^2 + bx + c = 0.$$

necessary, otherwise $\underbrace{\cancel{a}}_{=0}x^2+bx+c$ is not quadratic!

Ex: $\begin{cases} x^2+3x-1=0 \\ x^2+4x+4=0 \\ x^2+2x+5=0 \end{cases}$

Note: in the special case of $a=1$, x^2+bx+c, we can use
$$x_\pm = -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c}$$

Ex: "derive the path"!

Roots of Polynomials of degree $n>2$

There are formulas for $n=3$, $n=4$, but $n>5$ no formulas exist.

sometimes we can guess: $x^3-2x^2-5x+6=0$, $x=1$ is a root.

$$\underbrace{(x-1)(x^2+ax+b)}_{}$$

Fundamental Theorem of Algebra. $\longrightarrow$ Next Time!