List of Relevant Courses
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Bachelor of Science in Mathematics
Minor in Data Science

Abstract

This is a comprehensive list of my coursework at Constructor University, encompassed in **180 Credit Points**. This figure does not take credits for internships, language and extracurricular modules into account, which add up to make **215 Credit Points** in total.

Spring 2025 – 45 CP

- Topology & Differential Geometry
- Stochastic Financial Mathematics
- Applied Machine Learning
- Discrete Mathematics
- Stochastic Processes
- Bachelor Thesis & Seminar

Fall 2024 – 15 CP

- Real Analysis
- Dynamical Systems
- Mathematics Impact Project[1]

Spring 2024 – 30 CP

- Analysis III
- Algebraic Topology
- Numerical Methods
- Undergraduate Seminar
- Mathematical Modeling

Fall 2023 – 35 CP

- Foundations of Mathematical Physics
- Probability & Random Processes
- Data Analytics & Modeling
- Introductory Algebra
- Complex Analysis
- Number Theory

Spring 2023 – 27.5 CP

- Real Linear Algebra
- Data Structures & Processing
- Applied Mathematics (Analysis II)
- Calculus & Elements of Linear Algebra II

Fall 2022 – 27.5 CP

- Analysis I
- Introduction to Data Science
- Calculus & Elements of Linear Algebra I
- Mathematical Foundations of Computer Science

[1] General-Audience Seminar.

Topology & Differential Geometry, 5 CP

Introduction

This course is an advanced introduction to the theory of differentiable manifolds, focusing on fundamental concepts, structures, and theorems in differential topology and geometry. It provides a rigorous foundation for understanding smooth manifolds, tangent spaces, vector fields, differential forms, and Riemannian geometry. Key topics include Sard's theorem, the Whitney embedding theorem, Lie groups, Stokes' theorem, symplectic and Riemannian manifolds, and their applications. The course is based on material from John M. Lee's Introduction to Smooth Manifolds, with supplementary references such as Milnor's lecture notes and Spivak's Calculus on Manifolds. The material is explored through a combination of theoretical concepts, proofs, and applications.

- Review of differentiation in $\mathbb{R}^n$: Total and partial derivatives, inverse function theorem
- Topological preliminaries: Topological spaces, compactness, connectedness, product spaces
- Definition and examples of manifolds: Coordinate charts, real projective space
- Differentiable structures: Smooth maps, diffeomorphisms, and partitions of unity
- Tangent spaces: Derivations, differentials, rank theorem, submersions, immersions, and embeddings
- Sard's theorem and Whitney embedding theorem
- Lie groups: Definitions, homomorphisms, and Lie algebras
- Vector fields: Tangent bundles, integral curves, flows, and Lie derivatives
- Differential forms: Wedge product, pullbacks, exterior derivatives, and integration
- Orientation and Stokes' theorem
- Symplectic manifolds: Symplectic structures, Darboux theorem, and Hamiltonian systems
- Riemannian manifolds: Metrics, distance, volume forms, and pseudo-Riemannian geometry

Intended Learning Outcomes

By the end of the course, students will be able to:

- Demonstrate a thorough understanding of the foundational concepts of smooth manifolds and differentiable structures;
- Analyze and prove fundamental results in differential topology and geometry, such as Sard's theorem, the Whitney embedding theorem, and Stokes' theorem;
- Apply concepts from Lie groups, symplectic geometry, and Riemannian geometry to theoretical and practical problems;
- Perform computations involving tangent spaces, vector fields, differential forms, and Riemannian metrics;
- Develop an intuitive and rigorous understanding of advanced topics, such as Hamiltonian systems and integration on manifolds;
- Read and engage with mathematical literature in differential topology and geometry;
- Build a strong foundation for further study in differential geometry, mathematical physics, and related fields.

Stochastic Financial Mathematics, 7.5 CP

Introduction

This module is a first hands-on introduction to stochastic modeling. Examples will mostly come from the area of Financial Mathematics, so that this module plays a central role in the education of students interested in Quantitative Finance and Mathematical Economics. The module is taught as an integrated lecture-lab, where short theoretical units are interspersed with interactive computation and computer experiments.
Topics include a short introduction to the basic notions of financial mathematics, binomial tree models, discrete Brownian paths, stochastic integrals and ODEs, Ito's Lemma, Monte-Carlo methods, finite differences solutions, the Black-Scholes equation, and an introduction to time series analysis, parameter estimation, and calibration. Students will program and explore all basic techniques in a numerical programming environment and apply these algorithms to real data whenever possible.

Intended Learning Outcomes

By the end of the module, students will be able to:

- apply fundamental concepts of deterministic and stochastic modeling;
- design, conduct, and interpret controlled in-silico scientific experiments;
- analyze the basic concepts of financial mathematics and their role in finance;
- write computer code for basic financial calculations, binomial trees, stochastic differential equations, stochastic integrals, and time series analysis;
- compare their programs and predictions in the context of real data;
- demonstrate the usage of a version control system for collaboration and the submission of code and reports.

Note

The course is available (in an older format) at

http://math.jacobs-university.de/petrat/teaching/2022_fall_stochastic_methods/

Bachelor Thesis & Seminar, 15 CP

Introduction

The module provides a hands-on introduction to Machine Learning (ML), emphasizing practical aspects of workflows and applications. Topics include k-Means clustering, Nearest Neighbor (NN) and Naive Bayes techniques, Decision Trees, Support Vector Machines (SVMs).

Particular emphasis is on Neural Networks and Deep Learning. Theoretical concepts such as distance metrics, graphs, and networks are reviewed. Lectures are combined with Python exercises with particular emphasis on the SciKit-learn package. Disciplinary applications and case studies are immersed as bridging elements.

Intended Learning Outcomes

By the end of the module, students will be able to

- solve mathematical problems in that field;
- enumerate and describe main Machine Learning (ML) tasks and applications,
- discern and explain important ML approaches to classification and regression,
- apply Nearest Neighbor and Naive Bayes techniques to classification problems,
- employ Decision Trees and Support Vector Machines to solve data science problems,
- understand and use Neural Network and Deep Learning techniques
- address Machine Learning tasks by means of the Python library SciKit-learn (sklearn).

Relevant Courses

Data Analytics & Modeling, 7.5 CP

Discrete Mathematics, 5 CP

Introduction

This module is an introductory lecture in discrete mathematics. The lecture consists of two main components, enumerative combinatorics and graph theory. The lecture emphasizes connections of discrete mathematics with other areas of mathematics such as linear algebra and basic probability, and outlines applications to areas of computer science, cryptography, etc. where employment of ideas from discrete mathematics has proven to be fruitful.

The first part of the lecture—enumerative combinatorics—deals with several classical enumeration problems (Binomial coefficients, Stirling numbers), counting under group actions and generating function.

The second half of the lecture—graph theory—includes a discussion of basic notions such as chromatic number, planarity, matchings in graphs, Ramsey theory, and expanders, and their applications.

Intended Learning Outcomes

By the end of the module, students will be able to

- solve mathematical problems in that field;
- demonstrate their mastery of basic tools in discrete mathematics.
- develop the ability to use discrete mathematics concepts (such as graphs) to model problems in computer science.
- analyze the definition of basic combinatorial objects such as graphs, permutations, partitions, etc.
- formulate and design methods sand algorithms for solving applied problems basic on concepts from discrete mathematics.

Stochastic Processes, 5 CP

Introduction

This module serves as an introduction to the theory of stochastic processes. It starts with a review of Kolmogorov axioms for probability spaces and continues by providing a rigorous treatment of topics such as the independence of events and Borel-Cantelli Lemma, Kolmogorov's zero-one law, random variables, expected value and variance, the weak and strong laws of large numbers, and the Central limit theorem. More advanced topics that will follow include finite and countable state Markov chains, Galton-Watson trees, and the Wiener process. Several relevant applications that will be discussed are percolation on graphs, the application of Markov chains to sampling problems, and probabilistic methods in graph theory. The module also includes examples from mathematical finance.

Intended Learning Outcomes

By the end of the module, students will be able to:

- demonstrate their mastery of basic stochastic methods;
- develop ability to use stochastic processes to model real-world problems, e.g., in finance;
- analyze the definition of basic probabilistic objects, and their numerical features;
- formulate and design methods and algorithms for solving applied problems based on ideas from stochastic processes.

Introduction

This module is a mandatory graduation requirement for all undergraduate students to demonstrate their ability to deal with a problem from their respective major subject independently by means of academic/scientific methods within a set period. Although supervised, the module requires students to be able to work independently and regularly and set their own goals in exchange for the opportunity to explore a topic that excites and interests them personally and which a faculty member is interested to supervise. Within this module, students apply their acquired knowledge about the major discipline, skills, and methods to conduct research, ranging from the identification of suitable (short-term) research projects, preparatory literature searches, the realization of discipline-specific research, and the documentation, discussion, interpretation, and communication of the results.
This module consists of two components, an independent thesis and an accompanying seminar. The thesis component must be supervised by a Jacobs University faculty member and requires short-term research work, the results of which must be documented in a comprehensive written thesis including an introduction, a justification of the methods, results, a discussion of the results, and conclusions. The seminar provides students with the opportunity to present, discuss, and justify their and other students' approaches, methods, and results at various stages of their research to practice these skills, improve their academic writing, receive and reflect on formative feedback, thereby growing personally and professionally.

Intended Learning Outcomes

On completion of this module, students should be able to:

1. independently plan and organize advanced learning processes;
2. design and implement appropriate research methods taking full account of the range of alternative techniques and approaches;
3. collect, assess, and interpret relevant information;
4. draw scientifically founded conclusions that consider social, scientific, and ethical insights;
5. apply their knowledge and understanding to a context of their choice;
6. develop, formulate, and advance solutions to problems and arguments in their subject area, and defend these through argument;
7. discuss information, ideas, problems, and solutions with specialists and non-specialists.

Real Analysis, 5 CP

Introduction

This module focuses on the description, analysis, and representation of linear functionals and operators defined on general topological vector spaces, most prominently on abstract Banach and Hilbert spaces. Even though abstract in nature, the tools of Real Analysis play a central role in applied mathematics, e.g., in partial differential equations. To illustrate this strength of Real Analysis is one of the goals of this module.

Topics covered in this module include: point-set topology, Banach spaces, the Hahn-Banach theorem, weak topologies, compactness theorems (Tychnov's theorem, Banach-Alaoglu theorem, and the Arzela-Ascoli theorem), Hilbert spaces, and the Lebesgue spaces, spectral theory of compact operators.

Intended Learning Outcomes

By the end of the module, students will be able to demonstrate their mastering of advanced methods and concepts from Real Analysis to independently

- solve mathematical problems in that field;
- summarize the theory of operators on Banach and Hilbert spaces;
- analyze continuity, boundedness and compactness in the broader context of linear operators;
- apply the tools of Real Analysis in other branches of mathematics.

Dynamical Systems,

Introduction

This module is an introduction to dynamical systems. Dynamical systems naturally arise from iterations of maps or flows of vector fields on manifolds. The theory of dynamical systems has its roots in classical problems in celestial mechanics such as the three body problem or statistical physics. The aim of this module is to introduce the participants to the most basic dynamical systems and to study their properties. The module covers topics from discrete as well as continuous dynamical systems, including:

- a review of linear differential and difference equations in arbitrary dimensions
- circle maps
- toral automorphisms, horseshoes, and the solenoid
- recurrence, topological transitivity, and periodic orbits
- topological mixing as well as their measure theoretic counterparts such as ergodicity
- stability
- periodic orbits
- differential equations in the plane and the Poincarè-Bendixon theorem
- chaotic dynamics, e.g., in the Lorenz system
- asymptotic techniques
- structural stability
- bifurcation theory

Intended Learning Outcomes

By the end of the module, students will be able to

- demonstrate their mastery of advanced methods and concepts from Dynamical Systems to independently solve mathematical problems in that field;
- assess the central importance of the theory of dynamical systems in analyzing the long-term behavior of continuous processes;
- compare the qualitative behaviors of various dynamical systems;
- qualitatively and quantitatively distinguish different forms of dynamical systems.

Mathematics Impact Project,

Introduction

The Community Impact Project is designed to convey the required personal and social competencies for enabling students to finish their studies at Jacobs as socially conscious and responsible graduates (part of the Jacobs mission) and to convey social and personal abilities to the students, including a practical awareness of the societal context and relevance of their academic discipline.
By the end of this project, students should be able to:

1. understand the real-life issues of communities, organizations, and industries and relate them to concepts in their own discipline;
2. enhance problem-solving skills and develop critical faculty, create solutions to problems, and communicate these solutions appropriately to their audience;
3. apply media and communication skills in diverse and non-peer social contexts;
4. develop an awareness of the societal relevance of their own scientific actions and a sense of social responsibility for their social surroundings;
5. reflect on their own behavior critically in relation to social expectations and consequences;
6. work in a team and deal with diversity, develop cooperation and conflict skills, and strengthen their empathy and tolerance for ambiguity.

Notes

I am organizing a series of five mathematical "Thought Experiments" for school pupils. More information may be found here,

https://elshenawyom.github.io/blog/2024/gedankenexperimente/

https://www.meermint.de/angebote/gedankenexperimente-mathe-zum-begreifen-und-verstehen

Analysis III, 7.5 CP

Introduction

This module is the third module in the core Analysis education for students in Mathematics. It builds on the two independent modules “Analysis I” and “Applied Mathematics” and provides a more abstract point of view.

In the first part of the module, the Riemann integral is generalized to the Lebesgue notion of integration which requires a more involved framework, but offers powerful natural limit theorems and is also the basis for the Lebesgue function spaces that provide a natural setting for many problems in nonlinear analysis, mathematical physics, and partial differential equations. The development of the subject starts with a brief introduction to measure theory without aiming for a comprehensive treatment to arrive early at the notion of the Lebesgue integral. Emphasis is placed on the limit theorems (Fatou’s lemma, monotone convergence, and dominated convergence) and their consequences. It concludes with the introduction of Lebesgue spaces and their basic properties.

In the second part of the module the notions of gradient, curl and divergence will be discussed in terms of operations on vector fields and differential forms on manifolds, examples of which will be given in various areas of mathematics. The theory of the integration of differential forms will be provided and the Stokes’ Theorem, which is already known from the special settings in the ”Applied Mathematics” module, will be proved. Finally, basic concepts of differential geometry (connection, parallel transport, and curvature) will be introduced.

Intended Learning Outcomes

By the end of the module, students will be able to

- distinguish between the Riemann and Lebesgue integrals;
- use the central limit theorems in a variety of contexts;
- formulate and employ the central properties of Lebesgue spaces;
- explain the definition of a manifold and its tangent space;
- transform notions from elementary vector analysis into an intrinsic geometric setting.

Algebraic Topology, 5 CP

Introduction

This module is mostly concerned with the comprehensive treatment of the fundamental ideas of singular homology/cohomology theory and duality. The first part studies the definition of homology and the properties that lead to the axiomatic characterization of homology theory. Further algebraic concepts such as cohomology and the multiplicative structure in cohomology are then introduced. In the last section the duality between the homology and cohomology of manifolds is studied and a few basic elements of obstruction theory are discussed. The module provides a solid introduction to fundamental ideas and results that are used today in most areas of pure mathematics and theoretical physics.

Intended Learning Outcomes

By the end of the module, students will be able to

- demonstrate their mastery of advanced methods and concepts from Algebraic Topology to independently solve mathematical problems in that field;
- assess the central importance of homology theory and its role in mathematics;
- compare different examples of homologies and cohomologies;
- analyze different calculational tools to compute homologies.

Numerical Methods, 5 CP

Introduction

This module covers calculus-based numerical methods, in particular root finding, interpolation, approximation, numerical differentiation, numerical integration (quadrature), and a first introduction to the numerical solution of differential equations. The lecture comprises

- number representations
- Gaussian elimination
- LU decomposition
- Cholesky decomposition
- iterative methods
- bisection method
- Newton's method
- secant method
- polynomial interpolation
- Aitken's algorithm
- Lagrange interpolation
- Newton interpolation
- Hermite interpolation
- Bezier curves
- De Casteljau's algorithm
- piecewise interpolation
- Spline interpolation
- B-Splines
- Least-squares approximation
- polynomial regression
- difference schemes
- Richardson extrapolation
- Quadrature rules
- Monte Carlo integration
- time stepping schemes for ordinary differential equations
- Runge Kutta schemes
- finite difference method for partial differential equations

Intended Learning Outcomes

By the end of the module, students will be able to

- describe the basic principles of discretization used in the numerical treatment of continuous problems;
- command the methods described in the content section of this module description to the extent that they can solve standard text-book problems reliably and with confidence;
- recognize mathematical terminology used in textbooks and research papers on numerical methods in the quantitative sciences, engineering, and mathematics to the extent that they fall into the content categories covered in this module;
- implement simple numerical algorithms in a high-level programming language;
- understand the documentation of standard numerical library code and understand the potential limitations and caveats of such algorithms.

Undergraduate Seminar, 5 CP

Introduction

The Undergraduate Seminar is a module in which students give presentations on a particular area of mathematics, jointly discuss the topic of the presentation, and also discuss and reflect on presentation styles and the role of the subject topic in a broader context. The topics for the presentations are chosen by the instructor in consultation with the class and may come from a wide range of mathematical areas, typically outside of the standard first or second year math curriculum.
The goals of the module are threefold. First, it develops skills in mathematical communication: presentation, discussion, writing, and working with mathematical literature. Second, it provides a perspective on selected advanced and/or current topics in mathematics. Third, it helps students identify interesting areas of research and possible thesis subjects and advisors, as some of the suggested topics will relate to the research interests of the mathematics faculty.

Intended Learning Outcomes

By the end of the module, students will be able to:

- read and understand basic research literature in some areas of mathematics;
- employ effective strategies for self-learning in a well-defined but new subfield of mathematics;
- use common strategies, tools, and databases for literature searches;
- know the advantages and disadvantages of blackbox vs. slide-based presentations and the software tools for mathematical slides;
- communicate mathematical results in a comprehensive way in at least one chosen style of presentation;
- think critically about their own and other students' presentations;
- respond to feedback in a constructive way.

Note

This course has successfully been completed. Results may be found below,

https://elshenawyom.github.io/seminars-talks/

under talks referenced with *Mathematics Undergraduate Seminar at Constructor University.*

Mathematical Modeling, 7.5 CP

Introduction

The idea of this module is to introduce and teach mathematical methods starting with concrete scientific problems (mostly but not exclusively taken from physics). This module thus provides a first introduction to mathematical modeling, with an emphasis of the modeling of phenomena in physics, but also in other fields such as biology, economy, engineering, environmental sciences, finance, and industry. In modeling, we face two difficulties: Firstly, we have to find a good mathematical representation of the problem at hand, and secondly, we need to solve this problem either exactly, or with approximate analytical or numerical techniques. This class focuses mostly on deterministic problems, and discusses stochastic problems only briefly. The main mathematical techniques come from Analysis/Calculus, Linear Algebra, Differential Equations, and Probability. In the Mathematical Modeling Lab, the students work independently and in groups to find formulations of modeling problems and their solutions.

The following topics will be covered:

- Population Dynamics
- Fluid Mechanics
- Systems of Linear Equations
- Electrical Networks
- Linear Programming
- The Ideal Gas
- 1^{st} and 2^{nd} Laws of Thermodynamics
- Harmonic Oscillator
- ODEs and Phase Space
- Stability of Linear Systems
- Electromagnetism and Wave Equation
- Brownian Motion
- Monte-Carlo Method

The following mathematical skills will be covered and developed:

- derivatives and integration in one variable
- derivatives and integration in many variables
- integral theorems: Gauß and Stokes
- extreme value problems
- Taylor series
- Fourier series
- ODEs
- elementary introduction to PDEs
- elementary probability and stochastic processes

Intended Learning Outcomes

By the end of the module, students will be able to

- formulate mathematical models of problems from the sciences
- describe solution methods to modeling problems
- explain the usage of analysis and linear algebra techniques in modeling
- recognize different solution methods for modeling problems
- illustrate the use of ODEs and PDEs to describe phenomena in physics
- solve simple stochastic modeling problems.

Note

A project was completed for the tutorial component of this course, which was awarded with an A+.

https://elshenawyom.github.io/projects/math_modeling/

Foundations of Mathematical Physics, 5 CP

Introduction

This module is about the application of mathematics in physics. Physics and mathematics have a very intimate relationship. On the one hand, big discoveries in physics have often led to interesting new mathematics, and on the other hand, new developments in mathematics have made possible new discoveries in physics. The goal of this module is to look at some examples of that, and to gain an insight what role rigorous mathematics has played and plays today in explaining physical phenomena. This class discusses examples from the major theories of classical mechanics, quantum mechanics, electrodynamics, and statistical mechanics.

Intended Learning Outcomes

By the end of the module, students will be able to

- command the methods described in the content section of this module description to the extent that end of the module, students will be able to
- demonstrate the application of mathematics in the context of physics
- explain the mathematical foundations of classical mechanics, quantum mechanics, statistical physics, and electrodynamics
- discuss the solutions to both linear and non-linear equations in physics
- breakdown the Hamiltonian formalism in the context of classical and quantum mechanics
- apply variational methods and their role in minimization and maximization problems

Note

Professor Sören Petrat is the instructor for this course. For more details on the course structure, seek

http://math.jacobs-university.de/petrat/teaching/2023_fall_mathematical_physics/.

Probability & Random Processes, 5 CP

Introduction

The module provides students with basic skills needed for formulating real-world problems dealing with randomness and probability in mathematical language, and methods for applying a toolkit to solve these problems. The lecture comprises the following topics:

- Combinatorial probability.
- Conditional probability and Bayes' formula.
- Binomials and Poisson-Approximation
- Random Variables, distribution and density functions.
- Independence of random variables.
- Conditional Distributions and Densities.
- Transformation of random variables.
- Joint distribution of random variables and their transformations.
- Expected Values and Moments, Covariance.
- High dimensional probability: Chebyshev and Chernoff bounds.
- Moment-Generating Functions and Characteristic Functions,
- The Central limit theorem.
- Random Vectors and Moments, Covariance matrix, Decorrelation.
- Multivariate normal distribution.
- Markov chains, stationary distributions.

Intended Learning Outcomes

By the end of the module, students will be able to

- command the methods described in the content section of this module description to the extent that they can solve standard text-book problems reliably and with confidence;
- recognize the probabilistic structures in an unfamiliar context and translate them into a mathematical problem statement;
- recognize common mathematical terminology used in textbooks and research papers in the quantitative sciences, engineering, and mathematics to the extent that they fall into the content categories covered in this module.

Data Analytics & Modelling, 7.5 CP

Introduction

The module offers an introduction to the principles of data analytics and predictive data modeling and is structured into four parts.

First, essential concepts from statistics are reviewed in the data modeling context, illustrating key ideas including randomness, distributions, and confidence regions. Examples and case studies are discussed to distinguish between proper and improper uses of statistics.

Basic linear algebra is reviewed in the second part of the module, emphasizing vectors, distances, linear equations, matrices, and inversion. Key ideas such as the least squares approach are motivated with geometrical principles.

The third part of the module is concerned with matrix decompositions such as the Singular Value Decomposition (SVD) and its close relatives Principal Component Analysis (PCA) and Empirical Orthogonal Function (EOF) analysis.

The fourth part clarifies the distinction between linear and nonlinear modeling, and introduces key nonlinear techniques. Flexible educational formats (mostly online and hybrid) allow for asynchronous learning. Lectures are combined with Python exercises. Disciplinary applications and case studies are immersed as bridging elements.

Intended Learning Outcomes

By the end of the module, students will be able to

- identify important problem types and solution approaches in data analytics,
- understand how key concepts from statistics and linear algebra enter data science,
- explain matrix decompositions and their usage in data science,
- discuss regularization concepts and optimality criteria in data analytics,
- know the basics of nonlinear modeling and related computational approaches,
- convert data structures to Python/NumPy arrays for usage in data modeling
- apply Python statistics and linear algebra tools in data analytics and modeling.

Introductory Algebra, 7.5 CP

Introduction

This module is an introduction to abstract algebra, which covers a range of topics from basic notions and methods in group theory to elements of ring theory and basic field theory.

The module covers basic constructions in group theory in more detail, such as quotient groups, direct and semi-direct products, special classes of groups (e.g. matrix groups, permutation groups), specific types of groups (nilpotent, solvable, and simple), basic examples of rings (e.g., polynomial rings, integral domains), and divisibility theory in commutative rings (principal ideal domains and unique factorization domains).

The module also includes a basic introduction to the theory of fields, including field extensions, and algebraic and transcendental extensions and the existence of splitting fields for polynomials over fields.

Intended Learning Outcomes

By the end of the module, students will be able to

- give precise proofs of the basic results of the subject; demonstrate their mastery of basic methods and concepts from Algebra to independently solve problems in that field;
- assess the central importance of group theory and its applications to different areas of math;
- explain the definitions of groups, rings, ideals, fields, and modules;
- compare different examples of groups, rings, ideals, fields and modules from mathematics and physics.

Complex Analysis, 5 CP

Introduction

Complex analysis begins with the study of holomorphic functions on domains in the complex plane. Various equivalent definitions for holomorphy are proved, the simplest being that locally such a function has a convergent power series development in the standard coordinate of the complex numbers. The Local holomorphic change of coordinates reduces the local theory to the study of complex monomials and as a consequence it is proved that non-constant holomorphic functions are open maps, have discrete level sets and do not take on their local maxima.

The global theory starts with the Cauchy Integral Theorem and the resulting Integral Formula which describes holomorphic functions as boundary integrals. This also provides methods of construction for holomorphic functions and their physically relevant harmonic real parts. Other methods of construction utilize a subtle approximation theory, in the topology of uniform convergence on compact subsets, which is intertwined with the homotopy characteristics of the domain at hand.

Simply connected domains that do not coincide with the plane itself are shown to be equivalent to the unit disk (Riemann's mapping theorem). An indication of the general version of this result (the Uniformization Theorem) is sketched. In the study of more general one-dimensional complex manifolds (Riemann surfaces) which is initiated in the module, the interaction of analysis, geometry and symmetry considerations becomes more transparent.

Intended Learning Outcomes

By the end of the module, students will be able to

- give precise proofs of the basic results of the subject;
- use the theory to compute quantities, e.g., integrals, of importance;
- have intuition for the interaction of the analytic and geometric sides of the subject;
- be in a position of initiating a study of the higher-dimensional theory.

Number Theory, 5 CP

Introduction

This module is an elementary introduction to number theory, whose aim is to familiarize the audience with the classical ideas and methods of the field, as well as some of its more recent applications especially in cryptography and related technologies. Topics covered in this module include prime numbers and their distribution, the fundamental theorem of arithmetic, modular arithmetic, primitive roots, finite fields, and quadratic reciprocity.

The second part of the module is more topical and deals with more advanced topics such as Riemann Zeta function, primes in arithmetic progressions, continued fractions and diophantine approximations, Pell's equation, Minkowski's Geometry of numbers, the Gauss circle problem, and related lattice point counting problems.

Intended Learning Outcomes

By the end of the module, students will be able to

- demonstrate their mastery of basic tools of number theory;
- develop the ability to use number theoretic concepts and structures for applications in cryptographic platforms;
- analyze the definitions of basic number theoretical concepts such as primes numbers, congruences, and finite fields;
- formulate and design methods and algorithms for solving applied problems using tools from number theory.

Real Linear Algebra, 7.5 CP

Introduction

The fundamental concepts and techniques of Linear Algebra are introduced in a rigorous and more abstract way. The first half of this module covers vector spaces and linear maps, while the second half covers inner products and geometry. The following topics will be covered:

- Vector spaces
- Linear Operators
- Dual spaces
- Isomorphisms
- Connection to matrices
- Sums and direct sums
- Fundamental spaces of a linear operator
- Diagonalization of linear operators (on finite dimensional spaces)
- Cayley-Hamilton theorem
- Jordan decomposition
- Jordan normal form and its applications to linear differential equations
- Decomplexification and complexification
- Bilinear Forms and their classification
- Quadratic forms and orthogonalization
- Euclidean and unitary spaces
- Orthogonal and unitary operators
- Self-adjoint operators

Intended Learning Outcomes

By the end of the module, students will be able to

- describe the concept of a vector space and linear operator in an abstract way
- explain the connection of abstract linear algebra in the context of matrix algebra
- discuss the proofs of the major theorems from class
- illustrate the use of bilinear forms and their role in geometry
- distinguish bilinear forms in the context of Euclidean, unitary and symplectic spaces

Data Structures & Processing, 7.5 CP

Introduction

In this module, data structures and the data analysis pipeline are introduced in three parts. The first part gives an overview of the data analysis pipeline from capturing and processing to storing and analyzing data. Database concepts and management as well as the basic distinction between structured and unstructured data are reviewed, including an introduction to the relational data model, supplemented by examples of how specific disciplinary databases are handled. The second part is concerned with different types of structured data, starting with time series and images as examples of ordered data vectors and data matrices, respectively, and addressing both numeric and text data. Particular emphasis will be on tables and their higher-dimensional extensions, allowing for multivariate correlation and regression studies. The third part deals with unstructured data as obtained from web scraping and text mining. Unstructured data need to be prepared for subsequent analyses and use through operations such as merging, ordering, transforming, and resampling. Flexible educational formats (mostly online and hybrid) allow for asynchronous learning. Lectures are combined with Python exercises with particular emphasis on the Pandas package. Disciplinary applications and case studies are immersed as bridging elements.

Intended Learning Outcomes

By the end of the module, students will be able to

- enumerate and explain key operations along the data analysis pipeline,
- understand the basics of database management and important data models,
- process ordered data sets such as time series and images,
- prepare unstructured data sets for processing and analysis,
- apply the Pandas package to process and display time series, images, and tables.
- use Python tools to prepare and process unstructured data

Applied Mathematics, 7.5 CP

Introduction

This module covers advanced topics from calculus that are part of the core mathematics education of every Physicist and also forms a fundamental part of the mathematics major. It features examples and applications from the physical sciences. The module is designed to be taken with minimal pre-requisites and is tightly coordinated with the parallel module Calculus and Elements of Linear Algebra II. The style of development strives for rigor, but avoids abstraction and prefers simplicity over generality. Topics covered include:

- Taylor series, power series, uniform convergence
- Advanced concepts from multivariable differential calculus, here mainly the inverse and implicit function theorem; elementary vector calculus and Lagrange multipliers are covered in Calculus and Elements of Linear Algebra II
- Riemann integration in several variables, and line integrals
- The Gauss and Stokes integral theorems
- Change of variables and integration in polar coordinates
- Fourier integrals and distributions
- Applications to partial differential equations that are important in physics (Laplace, Poisson, diffusion, wave equations)
- Very brief introduction to complex analysis (Cauchy formula and residue theorem) 26 The lecture part is complemented by a lab course in Numerical Software (Scientific Python), which has become an essential tool for numerical computation and data analysis in many areas of mathematics, physics, and other sciences. Topics include:
- Writing vectorized code using NumPy arrays
- An introduction to SciPy for special functions and black-boxed algorithms (root solvers, quadrature, ODE solvers, and fast Fourier transform)
- Visualization using Matplotlib, including a general introduction to the effective visualization of scientific data and concepts
- The lab also includes a very brief comparative introduction to MATLAB, another standard numerical tool.

Intended Learning Outcomes

By the end of the module, students will be able to

- apply series expansions in a variety of mathematical and scientific contexts;
- solve, simplify, and transform integrals in several dimensions;
- explain the intuition behind the major theorems;
- use the major theorems in an application context;
- compute Fourier transforms and apply them to problems in Calculus and Partial Differential Equations;
- distinguish differentiability in a complex from a real variable;
- use numerical software to support simple numerical tasks and to visualize data.

Calculus & Elements of Linear Algebra II, 5 CP

Introduction

This module is the second in a sequence introducing mathematical methods at the university level in a form relevant for study and research in the quantitative natural sciences, engineering, Computer Science, and Mathematics. The emphasis in these modules is on training operational skills and recognizing mathematical structures in a problem context. Mathematical rigor is used where appropriate. However, a full axiomatic treatment of the subject is provided in the first-year modules “Analysis I” and “Linear Algebra”. The lecture comprises the following topics

- Directional derivatives, partial derivatives
- Linear maps
- The total derivative as a linear map
- Gradient and curl (elementary treatment only, for more advanced topics, in particular the connection to the Gauss and Stokes’ integral theorems, see module “Applied Mathematics”
- Optimization in several variables, Lagrange multipliers
- Elementary ordinary differential equations
- Eigenvalues and eigenvectors
- Hermitian and skew-Hermitian matrices
- First important example of eigendecompositions: Linear constant-coefficient ordinary differential equations
- Second important example of eigendecompositions: Fourier series
- Fourier integral transform
- Matrix factorizations: Singular value decomposition with applications, LU decomposition, QR decomposition

Intended Learning Outcomes

By the end of the module, students will be able to

- apply the methods described in the content section of this module description to the extent that they can solve standard text-book problems reliably and with confidence;
- recognize the mathematical structures in an unfamiliar context and translate them into a mathematical problem statement;
- recognize common mathematical terminology used in textbooks and research papers in the quantitative sciences, engineering, and mathematics to the extent that they fall into the content categories covered in this module.

Analysis I, 7.5 CP

Introduction

This module introduces fundamental concepts and techniques in a concise and rigorous way. The class conveys the pleasure of doing mathematics, and motivates mathematics concepts from problems and concrete examples, but also shows the power of abstraction and of formal reasoning. The following topics will be covered:

- Proof by induction, and elementary combinatorics
- Groups, equivalence relations, and quotients
- Natural numbers, integers, rationals, and real numbers
- Sequences and series, and convergence
- Functions of a single real variable, continuity, and the intermediate value theorem,
- Metric spaces, and the continuous functions as a metric space
- Differentiation, mean value theorem, and the inverse mapping theorem in one variable
- Riemann integral
- Fundamental theorem of Calculus, and the integration by parts with applications
- Integral mean value theorem
- Change of variables
- Taylor series with integral and Lagrange remainders
- Elementary point-set topology (neighborhoods, open and closed sets, compactness, and Heine-Borel)

Intended Learning Outcomes

By the end of the module, students will be able to

- cleanly formulate mathematical concepts and results discussed in class;
- outline proofs which have been given in the lectures ;
- independently prove results which are direct consequences of those proved in the lectures;
- understand and use fundamental mathematical terminology to communicate mathematics at a university level.

Introduction to Data Science, 7.5 CP

Introduction

The module introduces data science with an integrated presentation of three essential components, namely, (1) societal/legal implications and business opportunities, (2) technical/theoretical background and case studies, (3) an introduction to the Python coding environment. The first component entails a conceptual introduction to the opportunities and the challenges of a digitally transformed and data-driven society, presentations on industry standards and legal frameworks, and discussions of critical issues such as cybersecurity and surveillance. The second component includes topics such as data science terminology, digital data and their representations, and introductions to exploratory data analysis and prominent supervised and unsupervised learning tasks. The third component offers an introduction to the Python ecosystem of data representation, processing, analysis, and visualization, starting with Jupyter notebooks, installing suitable environments, and introductions to data science related packages such as NumPy, SciPy, Matplotlib, Seaborn, and Pandas. Fundamental data science concepts are summarized and illustrated using real-world data from various disciplines. Flexible educational formats (mostly online and hybrid) allow for asynchronous learning. Lectures are combined with an exposure to Python programming and data processing and visualization environments, including hands-on practicals, examples, and exercises.

Intended Learning Outcomes

By the end of the module, students will be able to

- explain societal implications of the digital transformation,
- understand the legal data protection framework,
- carry out basic data processing and visualization tasks,
- apply fundamental data science methods to structured data,
- understand the logic of Python scripts and functions,
- compose Python code using templates

Calculus & Elements of Linear Algebra I, 5 CP

Introduction

This module is the first in a sequence introducing mathematical methods at the university level in a form relevant for study and research in the quantitative natural sciences, engineering, Computer Science, and Mathematics. The emphasis in these modules is on training operational skills and recognizing mathematical structures in a problem context. Mathematical rigor is used where appropriate. However, a full axiomatic treatment of the subject is provided in the first-year modules "Analysis I" and "Linear Algebra".
The lecture comprises the following topics:

- Brief review of number systems, elementary functions, and their graphs
- Brief introduction to complex numbers
- Limits for sequences and functions
- Continuity
- Derivatives
- Curve sketching and applications (isoperimetric problems, optimization, error propagation)
- Introduction to Integration and the Fundamental Theorem of Calculus
- Review of elementary analytic geometry
- Vector spaces, linear independence, bases, coordinates
- Matrices and matrix algebra
- Solving linear systems by Gauss elimination, structure of general solution
- Matrix inverse

Intended Learning Outcomes

By the end of the module, students will be able to:

- apply the methods described in the content section of this module description to the extent that they can solve standard text-book problems reliably and with confidence;
- recognize the mathematical structures in an unfamiliar context and translate them into a mathematical problem statement;
- recognize common mathematical terminology used in textbooks and research papers in the quantitative sciences, engineering, and mathematics to the extent that they fall into the content categories covered in this module.

Mathematical Foundations of Computer Science, 7.5 CP

Introduction

The module introduces students to the mathematical foundations of computer science. Students learn to reason logically and clearly. They acquire the skill to formalize arguments and to prove propositions mathematically using elementary logic. Students are also introduced to fundamental concepts of graph theory and elementary graph algorithms.
After establishing the concept of algorithms, the first part covers basic elements of discrete mathematics, leading to Boolean algebra, propositional logic, and predicate logic. Students learn how to use fundamental proof techniques to prove (or disprove) simple propositions. The second part of the module introduces students to basic concepts of algebraic structures like groups, rings, and fields, and different structure-preserving maps (homomorphisms). Students study how these abstract concepts relate to problems in computer science. The last part of the module covers the basic elements of graph theory and the different representations of graphs. Elementary graph algorithms are introduced that have a wide range of applicability in computer science.

Intended Learning Outcomes

By the end of this module, students will be able to:

1. explain basic concepts and properties of algorithms;
2. understand the concept of termination and complexity metrics;
3. illustrate basic concepts of discrete math (sets, relations, functions);
4. use basic proof techniques and apply them to prove properties of algorithms;
5. summarize basic principles of Boolean algebra and propositional logic;
6. use predicate logic and outline concepts such as validity and satisfiability;
7. distinguish abstract algebraic structures such as groups, rings, and fields;
8. classify different structure-preserving maps (homomorphisms);
9. understand calculations in finite fields and their applicability to computer science;
10. explain elementary concepts of graph theory and use different graph representations;
11. outline basic graph algorithms (e.g., traversal, search, spanning trees).

Notes

A modern version of the course may be found here,

https://cnds.constructor.university/courses/mfcs-2024/