06th November

Completed Exercises from the lecture on

<Equivalent Expressions>

1. Hard, Pages 2-9 ;

Can be found below.

Hard

(1) **371cbf6b** Multiple choice · One answer only

$$(ax+3)\left(5x^2-bx+4\right)=20x^3-9x^2-2x+12$$

The equation above is true for all x, where a and b are constants. What is the value of $\boxed{ab}$?

a. 20
b. 40
[✓ c. 24]
d. 18

$(-ab+15)x^2 = -9x^2$

$\Rightarrow -ab+15=-9$

$\Rightarrow ab = 15+9 = 24$

$ax(5x^2-bx+4) + 3(5x^2-bx+4) = 20x^3-9x^2-2x+12$

$5ax^3 - abx^2 + 4ax + 15x^2 - 3bx + 12 = 20x^3-9x^2-2x+12$

$5ax^3 + (15-ab)x^2 + (4a-3b)x + \cancel{12} = 20x^3-9x^2-2x+12$

$16 - 3b = -2 \Rightarrow b = 6$

(2) **40c09d66** Short answer Case-Insensitive

If $\frac{\sqrt{x^5}}{\sqrt[3]{x^4}} = x^{\frac{a}{b}}$ for all positive values of x, what is the value of $\frac{a}{b}$?

$$x^{a-b} = \frac{x^a}{x^b}$$

$$= x^{\frac{a}{b}} = \frac{\sqrt[2]{x^5}}{\sqrt[3]{x^4}}$$

$$(x^a)^b = x^{a \cdot b}$$

$$\sqrt[a]{x} = x^{\frac{1}{a}}$$

$$= \frac{(x^5)^{\frac{1}{2}}}{(x^4)^{\frac{1}{3}}}$$

$$= \frac{x^{\frac{5}{2}}}{x^{\frac{4}{3}}}$$

$$= x^{\frac{5}{2}-\frac{4}{3}} \quad = x^{7/6}$$

(3) **34847f8a** MULTIPLE CHOICE One answer only

$$\frac{2}{x-2}+\frac{3}{x+5}=\frac{rx+t}{(x-2)(x+5)}$$

The equation above is true for all $x > 2$, where r and t are positive constants. What is the value of rt ?

a. 60
b. 15
c. 20
d. -20

$$\frac{2}{x-2}\cdot\frac{\overbrace{x+5}^{=1}}{x+5}+\frac{3}{x+5}\cdot\frac{\overbrace{x-2}^{=1}}{(x-2)}$$

$$\frac{\boxed{5}x+\boxed{4}}{(x-2)(x+5)}=\frac{rx+\boxed{t}}{(x-2)(x+5)} \Rightarrow \quad r=5 \quad t=4$$

(4) **137cc6fd** SHORT ANSWER Case-Insensitive

$$\sqrt[5]{70n}(\sqrt[6]{70n})^2$$

For what value of x is the given expression equivalent to $(70n)^{30x}$, where $n > 1$?

$$\underbrace{\sqrt[5]{70n}}\left(\sqrt[6]{70n}\right)^2$$

$$(x^a)^b = x^{a\cdot b}$$

$$= (70n)^{\frac{1}{5}} \cdot \underline{\left((70n)^{\frac{1}{6}}\right)^2}$$

$$= (70n)^{\frac{1}{5}} \cdot (70n)^{\frac{1}{3}} = (70n)^{\frac{1}{5}+\frac{1}{3}}$$

$$= (70n)^{\frac{8}{15}} = (70n)^{30x}$$

$$\Rightarrow \frac{8}{15} = 30x \Rightarrow \boxed{x = \frac{8}{15\cdot 30} = \frac{8}{450} = \frac{4}{225}}$$

(5) **ea6d05bb** Short answer Case-Insensitive

The expression $(3x-23)(19x+6)$ is equivalent to the expression ax^2+bx+c, where a, b, and c are constants. What is the value of $\boxed{b}$?

$$18x + -23 \cdot 19x = (18 + -23 \cdot 19)\, x$$

$$= (b)\, x$$

$$\Rightarrow (18 + -23 \cdot 19) = b = -419$$

(6) **d8789a4c** MULTIPLE CHOICE One answer only

$$\frac{x^2-c}{\cancel{x-b}} = (x+b)\times(x-b)$$

In the expression above, b and c are positive integers. If the expression is equivalent to $x+b$ and $x \neq b$, which of the following could be the value of c ?

- a. 6
- [✓ b. 4]
- c. 8
- d. 10

$$(x+a)(x-a) = x^2-a^2$$

$$\Rightarrow (x^2-c) = (x+b)(x-b)$$

$$= x^2 + \cancel{b}x - \cancel{b}x - b^2$$

$$\Rightarrow x^2-c = x^2-b^2$$

• $c = b^2$, b integer

⇒ only b=4 works

(7) **5355c0ef** Short answer Case-Insensitive

$$0.36x^2 + 0.63x + 1.17$$

The given expression can be rewritten as $a\,\underline{(4x^2 + 7x + 13)}$, where a is a constant. What is the value of a ?

$4ax^2 + 7ax + 13a$

$0.36x^2 + 0.63x + 1.17$

$4a = 0.36 \Rightarrow a = 0.09,$

$\boxed{0.09,\ \frac{9}{100}}$

(20) **7355b9d9** MULTIPLE CHOICE One answer only

If $k - x$ is a factor of the expression $-x^2 + \frac{1}{29}nk^2$, where n and k are constants and $k > 0$, what is the value of n ?

a. $-\frac{1}{29}$
b. $\frac{1}{29}$
c. 29
d. -29

Total of marks: 64

$$\underbrace{\frac{1}{29} n K^2 - x^2}_{\text{diff. of two squares}} = \overset{\text{factor}}{(K - x)}(\sim)$$

$$\boxed{(x+a)(x-a) = x^2 - a^2}$$

$$\sqrt{\frac{1}{29} n K^{\cancel{2}}} \qquad \sqrt{x^{\cancel{2}}}$$

$$\Downarrow$$

$$(K - x)(\sim) = \frac{1}{29} n K^2 - x^2 = \left(\sqrt{\frac{1}{29} n}\, K - x\right)\left(\sqrt{\frac{1}{29} n}\, K + x\right)$$

Then by comparison

$$(K - x) = \left(\sqrt{\frac{1}{29} n}\, K - x\right)$$

$$\Rightarrow \quad 1 = \text{Coeff } K = \text{Coeff } K = \sqrt{\frac{1}{29} n}$$

$$\Rightarrow \quad 1 = \sqrt{\frac{1}{29} n} \quad \Rightarrow \quad \frac{1}{29} n = 1 \quad \Rightarrow \boxed{n = 29}$$