Roots of Quadratic Equations.

$f(x) = ax^2 + bx + c$ is a quadratic (degree two) function, for $a \neq 0$.

If x_0 is a root of $f(x)$, then plugging $f(x_0)$ gives zero. The quadratic formula below describes roots x_0 of $f(x)$.

$$x_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Not convinced?

Pf

$$x_0 = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \iff 2ax_0 = -b \pm \sqrt{b^2-4ac}$$

$$\iff 2ax_0 + b = \pm\sqrt{b^2-4ac}$$

$$\iff (2ax_0+b)^2 = (\pm\sqrt{b^2-4ac})^2$$

$$\iff 4a^2x_0^2 + 4ax_0b + b^2 = b^2 - 4ac$$

$a \neq 0!$

$$\iff 4a^2x_0^2 + 4ax_0b + 4ac = 0$$

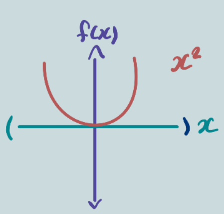
$$\iff ax_0^2 + bx_0 + c = 0$$

Good. We can apply this in an example.

Example. Find the roots of $f(x) = x^2 + 3x + 4$.

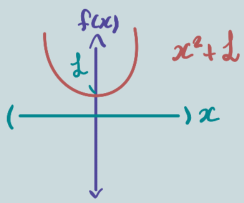
$$\boxed{x_0 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

A quadratic equation looks like this...

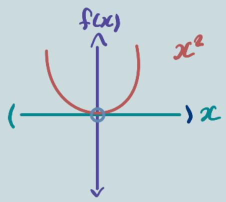


A root exists when the graph intersects the x-axis, where the height $f(x) = 0$. This gives us three cases,

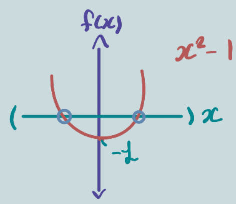
Examples.



No solution.



One Solution...



Two solutions!

we can (and should!) link this to $x_0 = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$. Notice that there is an interesting object, that is $\pm\sqrt{b^2-4ac}$. Maybe more examples will help.

Example. The roots x_0 of $f(x) = x^2+4x+4$ are

$$\boxed{x_0 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

Its vertex (probably) lies on the x-axis...