# A Fiscal Theory of Monetary Policy with Financial Frictions and Long-Term Debt

Elliot Spears
University of Washington
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**Abstract**

This paper introduces financial frictions into a fiscal theory of monetary policy model with long-term government debt. Incorporating a Gertler–Kiyotaki intermediary links bank leverage, credit spreads, and net worth to government bond prices, making discount rates endogenous to financial conditions. Monetary policy tightenings generate larger and more persistent inflation responses as contractions in bank credit raise firms' marginal costs. Fiscal shocks, by contrast, produce less cumulative inflation despite larger declines in expected surpluses, as intermediary balance-sheet dynamics absorb fiscal disturbances through discount-rate variation. These results show that bank balance sheets are a central transmission channel for fiscal and monetary policy under fiscal dominance, materially altering inflation and debt dynamics.

# Chapter 2: A Fiscal Theory of Monetary Policy with Financial Frictions and Long-Term Debt

Elliot Spears

## 1 Introduction

Adam Smith once wrote: "A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money."[1] In other words, a sufficient condition for a currency to hold value is its acceptance for tax payments. This simple notion lays at the heart of what is known as the fiscal theory of the price level (FTPL), which says that the price level adjusts so that the real value of government debt equals the present value of future primary surpluses.

Under the gold standard, paper currency was valuable due to its ability to be redeemed for gold. In so far as a government with a currency on a gold standard would restrict convertability (or fail to convert paper bills to gold), the currency depreciated. Without enough gold to "soak up" the currency, this would lead to inflationary pressures. In the same vein, under FTPL, inflation arises when there is an imbalance between outstanding debt and future surpluses, which ultimately entails an imbalance between the money supply and tax receipts.

The motivation for introducing an alternative theory of price level determination to what is implied by the Taylor principle is simple. The Taylor principle tells us that inflation is

[1] Smith, Adam (1776). *Wealth of Nations*, Vol. I, Book II, Chapter II.

brought under control when interest rates respond more than one-to-one to it. Our experience during the zero lower bound period casts doubt on this notion.[2] If nominal interest rates must respond more than one-for-one to inflation to avoid a situation where real interest rates drop below zero and inflation explodes, then the experience of the early to mid 2010s undermines our belief in the Taylor principle's ability to determine the price level. Therefore, it is desirable to begin exploring alternative theories of inflation — this is the purpose of FTPL.

Traces of fiscal theory can be found as far back as Sargent and Wallace (1981) and McCallum (1984), where the interactions between fiscal and monetary policy are examined to assess their effects on inflation. Sargent & Wallace argued that in an environment where fiscal policy "dominates" monetary policy (meaning the government declares its budget to the world, leaving the central bank to determine how to bridge any gaps between the government's spending and its revenue by providing seigniorage and revenue from bond sales), the central bank may not be able to control inflation.

Leeper (1991) introduces the terminology that distinguishes between an "active" versus "passive" fiscal policy regime. An active fiscal regime simply requires that government budget surpluses do not respond one-for-one to changes in the price level — fiscal theory requires active fiscal policy. The work on fiscal theory has gradually expanded, with seminal developments made by Sims (1994, 2011), Woodford (1995, 2001), and Cochrane (2005, 2022a).

The purpose of this paper is to build upon the foundation of Cochrane (2022b), where fiscal policy is active, monetary policy is passive (interest rates respond less than one-for-one to inflation), and long-term debt is incorporated in order drive realistic inflation dynamics. The contribution to the literature is the incorporation of financial frictions into the model à la Gertler and Kiyotaki (2010), which alter the dynamics of all variables in the model, especially government debt and surpluses.

[2] End of 2008 to the end of 2015.

Following the literature, I implement four experiments, which involve analyzing fiscal and monetary shocks, each of which is performed with and without monetary policy rules. Both the fiscal shock, which constitutes a shock to surpluses, and the monetary policy shock result in fundamental shifts in the dynamics of the endogenous variables brought about by the presence of financial frictions. With the exception of the case of a monetary policy shock without rules, we see credit contract in wake of the shocks.

Firms face marginal costs that are increasing in the borrowing costs incurred to pay their expenses. When a deficit shock hits, borrowing costs rise — increasing marginal costs, which leads to a stronger inflationary response than what is observed in the absence of financial frictions. In wake of this, discount rates rise over the medium term in a manner that is smaller in magnitude than the benchmark case without financial frictions, however discount rates remain above trend for longer, which has the effect of prolonging the reevaluation of debt.

Importantly, the financial frictions feed into the debt valuation equation itself through bond prices. The bank's shadow value of net worth, along with the stochastic discount factor (SDF), are among the elements that price the government debt portfolio. Bond prices pin down the discount rate, which itself is an explicit ingredient of the debt valuation equation. Therefore, we have a linkage between financial intermediaries and bond prices, which is a key gap needed filling in the FTPL literature.

In terms of the financial frictions themselves, there is a bank who provides credit to firms issued out deposits made by households, which the bank compensates households for. A key element of the bank's budget constraint is that it owns long-term government-issued debt. Hence, there are interactions between government debt and bank balance sheets. It turns out that interest rate spreads and bank net worth are major drivers of bond prices, which in turn determines discount rates and, therefore, government debt dynamics.

By introducing a Gertler–Kiyotaki intermediary into the long-term debt FTPL environment, this paper shows that the discount rates governing the valuation of government debt inherit the dynamics of intermediary net worth. Because discount rate variation is a key determinant of inflation in FTPL, fiscal and monetary disturbances now propagate through bank balance sheets. This linkage causes shocks to alter the valuation equation through movements in bond prices, leverage, and the intermediary pricing kernel, thereby altering the dynamics of inflation and debt.

## 2 Related Literature

The fiscal theory of the price level is not new. As mentioned in the introduction, the notion of a distinction between fiscal and monetary dominant regimes, along with differing consequences that flow from having one regime in place over another, goes back to at least the early 80s with Sargent and Wallace (1981). Fiscal theory has come a long way with the work discussed in the introduction, but not without resistance.

One of the principal objections raised against fiscal theory is that the government is capable of violating its budget constraint for certain price levels. Buiter (2002) states that fiscal theory mistakes what is ultimately a budget constraint for an equilibrium condition. The early 2000s saw an explosion of critiques along these lines leveled against fiscal theory — see Bassetto (2002); Marimon (2001); Kocherlakota et al. (1999), for a sample. Then Cochrane came out with "Money as Stock",[3] which answered these objections by drawing an analogy between stock price determination via the dividend discount model and the debt valuation equation that is at the heart of fiscal theory, making it a valuation equation, or a market-clearing condition and not some crude budget constraint.[4] He also showed that the theory works even when the government is forced to respect its budget constraint.

The model in this paper draws its ingredients primarily from two sources: Cochrane (2022b) and Gertler and Kiyotaki (2010), the financial frictions obviously come from the latter. A few modifications are made to the bank optimization problem, but those familiar

[3] Cochrane (2005)

[4] ibid. See page 504

with the GK[5] mechanism will easily recognize it. Overall, this paper attempts to answer a call from Cochrane (2023) to incorporate financial frictions into the FTPL framework. The motivation for doing this is that Cochrane (2022a) shows the significant effects discount rates have on inflation. Furthermore, the financial sector plays an important role in determining discount rate variation. Therefore, it's of considerable interest to add ingredients into the model that tie the financial sector to discount rates, bond prices, and, ultimately, the debt valuation equation.

The road map for this paper is: section three will introduce the core elements of the benchmark fiscal theory model. Then, I will introduce the financial frictions, showing how they alter the model. In section four we'll take a look at the IRFs produced by simulations with the model. We will compare the benchmark model from Cochrane (2022b) with the results generated by financial frictions. Section five will conduct additional policy-relevant analysis. Section six concludes.

# 3 Model

Figure 1 provides a bird's-eye view of how the model works. Households and banks acquire debt issued by the government. In an ideal world, the government repays previous debts accumulated when running deficits by running future surpluses — soaking up its currency in the form of tax revenue in the process. This fiscal backing of the currency ultimately determines the prices level. If future surpluses are expected to fall short of the amount necessary to cover the value of existing debt, then price level will rise, and vice versa. Hence the fiscal theory maxim: *the price level adjust so that the real value of nominal government debt equals the present value of future primary surpluses.*

[5] Gertler and Kiyotaki (2010)

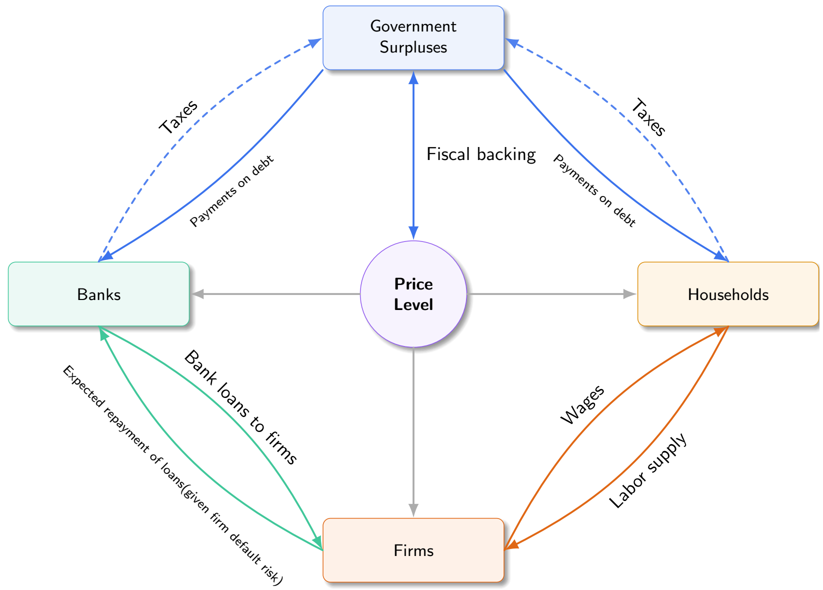


Figure 1: Model Overview

Thus, the government's surpluses affect the entire economy through its effects on the price level. Banks affect, who carry large amounts of long-term debt, can unilaterally move the price of long-term debt by their investment decisions. This, in turn, affects the value of government debt, which ultimately influences the dynamics of inflation. The health of the bank's balance sheet is tied to firm performance. Firms can default on the loans they receive from the bank.

### *3.1 Brief Remarks On Long-Term Debt*

In this model, we will be incorporating long-term debt. The reason long-term debt is useful is threefold. First, in the real world, the government issues debt that spans several decades before maturity — 10-year Treasuries, for example, are a key instrument in interbank dealings.

Second, the inclusion of long-term debt makes the dynamics of the model more realistic. For instance, it draws out the response of inflation to monetary and fiscal shocks, instead of generating a one or two period response in inflation to these disturbances, which is what you get when you only include short-term debt.

Third, restricting the government to short-term debt issuance creates unrealistic dynamics (for the U.S.)[6] in the *direction* of inflation's response to monetary disturbances. For instance, take the following simple two period model of the government's debt valuation equation:

$$
\begin{aligned}
\frac{B_{-1}}{P_0} &= s_0 + \beta \mathbb{E}_0[s_1] && (1)\\
&= s_0 + \frac{1}{1+i_0}\frac{B_0}{P_0} && (2)\\
&= s_0 + \beta_0 \mathbb{E}\left[\frac{P_0}{P_1}\right]\frac{B_0}{P_0} && (3)
\end{aligned}
$$

If the central bank raises interest rates at $t_0$, then $i_0$ will increase in equation 2. Since $P_0$ is already determined at $t_0$, it must be that $P_1$ — next period's inflation — will increase. Therefore, an increase in interest rates today leads to inflation tomorrow. This is the natural frictionless benchmark; absent any frictions, inflation is increasing in nominal interest rates.[7]

Introducing long-term debt, deals with this issue by enabling bond prices to absorb the impact of unexpectedly higher interest rates. As interest rates rise, long-term bond prices fall, which temporarily pushes inflation down. However, this is only temporary, since in a fiscal-dominant regime monetary policy can only manipulate the path of the price level, but it can't determine the overall price level in the long-run.

### *3.2 Fiscal Theory's Core Ingredients*

For the core ingredients of the fiscal theory mechanism, I follow Cochrane (2022b) and the notation employed in that paper. Let's turn to the government's budget constraint,

[6] I emphasize that these dynamics are unrealistic "for the U.S." because the U.S. government issues much long-term debt, in addition to short-term debt. However, there *are* countries that issue primarily, or even overwhelmingly, short-term debt in their own currency. For these economies, the dynamics of short-term debt models might well be realistic, especially in the case of a rollover crisis

[7] This is true anyway, though. As discussed on page 11 of Cochrane (2023), higher interest rates lead to inflation in the *long-run*. It's only due to frictions that we get the phenomenon of higher interest rate → lower inflation in the short run.

which is the following:

$$\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)} + M_{t-1} = P_t s_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)} + M_t \tag{4}$$

where $M_t$ is money, which does not earn interest.[8] $B_{t-1}^{t+j}$ is zero-coupon nominal debt outstanding at $t-1$ that is due at the beginning of period $t+j$. $Q_t^{t+j}$ is the price of the bond at time $t$. $s_t$ is the real primary surplus, which, if negative, becomes a real primary deficit. $P_t$ is the price level. Equation 4 says that the government's primary surplus is equal to the change in debt between period $t-1$ an period $t$. Equivalently, it says that the previous period's debt is financed by today's surpluses, plus whatever borrowing is needed to finance the residual.

Therefore, the end-of-period market value of debt is:

$$V_t = M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)}$$

We can specify the return on the entire government bond portfolio as follows:

$$R_{t+1}^n = \frac{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}}{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}} \tag{5}$$

This equation tells us that the return on government debt is determined by how bond prices change from one day to another. After a series of substitutions and log-linearizing, we end up with the debt accumulation equation:

$$\rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - s_{t+1} \tag{6}$$

Where lowercase variables represent deviations from the steady state and $\pi$ is inflation. The other equation of relevance to fiscal theory is the surplus process. As we will see,

[8]This is considered a form of short-term debt.

the surplus process serves much like the Taylor rule in that it eliminates equilibria. When fiscal policy is active and monetary policy passive, the surplus process, in tandem with a passive Taylor rule, ultimately selects a unique solution. In particular, “expected inflation” is determined by the interest rate rule, whereas “unexpected inflation” is pinned down by the surplus process. This mechanism is called a *fiscal theory of monetary policy*, which implies that the central bank, through interest rate manipulation, can alter the time path of inflation, but not the long-run level.

To get a realistic surplus process, we want something that is s-shaped — which means that a deficit shock (negative surplus) must eventually be followed by (positive) surpluses. To that end, the surplus process is written as a moving average:

$$s_t = a(L)\varepsilon_t = \sum_{j=0}^{\infty} a_j \varepsilon_{t-j} \tag{7}$$

When paired with the following conditions, we get a clearer picture of how the surplus process will be specified and its parameters calibrated to make the model consistent with empirical facts:[9]

$$\Delta \mathbb{E}_{t+1}\pi_{t+1} = -\Delta \mathbb{E}_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j} \tag{8}$$

$$v_t = \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j s_{t+1+j} \tag{9}$$

Combining equation 7 with 8 and 9, we have:

$$\Delta \mathbb{E}_{t+1}\pi_{t+1} = -\Delta \mathbb{E}_{t+1} \sum_{j=0}^{\infty} \rho^j a_j \varepsilon_{t+1} \tag{10}$$

$$v_t = \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j a_j \varepsilon_{t+1} \tag{11}$$

[9]The derivations for equaitons 6-8 can be found in Cochrane (2022a,b). The formula that gives us unexpected inflation in terms of surpluses comes form Hansen et al. (2019).

Therefore, the MA coefficients of $a(\rho)$ are what ultimately determine the extent to which inflation devalues debt following shocks to the surplus process. There are four possibilities: **i)** $a(\rho) \gg 1$ **ii)** $a(\rho) = 1$ **iii)** $a(\rho) \ll 1$ and **iv)** $a(\rho) = 0$.

Case i implies that an unexpectedly high surplus will be followed by high surpluses in the future, which will raise the value of debt, or, equivalently, that deficit shocks will lower the value of debt, both of which are counterfactual. Case ii means that there will be no change in the value of debt after a deficit shock. The intuition is that inflation will devalue the new debt by exactly the amount of the deficit, leaving total debt unchanged. In both cases i and ii, the government finances deficits by inflating them away. However, in case i, inflation reacts *more* than one-for-one to the deficit shock, since agents believe that current deficits will be followed by future deficits. Case iv means that a deficit shock has no impact on inflation. This is because all deficits are anticipated to be repaid in full.

Case iii is ultimately the one that is worked with in the literature, and therefore, the one we will be assuming here. When $a(\rho) \ll 1$, a deficit shock raises the value of debt as the government commits to covering at least a portion of its deficits by borrowing and repaying new debts with future surpluses. Clearly, this is the most realistic case when applied to the U.S. and Western Europe.[10]

A plausible specification for the surplus process is $s_{t+1} = \alpha v_t + u^s_{t+1}$. Governments spend money in excess of tax receipts, generating deficits that they repay a fraction $\alpha$ of. So far, we've specified the *shape* of the surplus process through a moving average representation. However, in reality, the data shows that surpluses move with the business cycle. Taxes revenues rise during expansions, and fall during contractions. It would also make sense that inflation affects surpluses to some extent since tax brackets adjust slowly (or not at all) to inflation, generating more government revenue, while government continues its spending without indexing to inflation, thereby potentially lowering real spending. Although the data isn't clear on this, we will follow the literature in assuming a moderately positive response

[10]See data in Cochrane (2022a)

in surpluses to inflation.

Hence, our final specification for the surplus process turns out to be:

$$s_{t+1} = \theta_{s\pi}\pi_{t+1} + \theta_{sx}x_{t+1} + \alpha v_t + u^s_{t+1} \tag{12}$$

Where $\theta_{s\pi} = 0.25$, $\theta_{sx} = 1$, and $\alpha = 0.2$.[11] This is what will go into our final model and will be the fiscal theory analog of the Taylor rule. The surplus process, alongside the Taylor rule, will determine inflation uniquely.

Equations 6 and 12, then, are the key elements of fiscal theory. Following the the standard recipe, we will include a Taylor rule, along with a New-Keynsian Phillip's Curve (NKPC) and IS equation. We will also specify the evolution of the return on the government debt portfolio as $r^n_{t+1} = \omega q_{t+1} - q_t$, where $q_t$ is the log-linearized price of the government debt portfolio at time $t$. The contribution of this paper is to connect $q_t$ to the financial sector and the balance sheets of intermediaries. Therefore, movements in bank net worth, leverage, SDFs, and other factors will pin down $q_t$, which ultimately determines $r^n_t$, the return on the government debt portfolio, which directly influences debt.

Up to now, I've given a lay of the land as it stands in the existing FTPL literature. At this point we turn to the household block, and then the financial block, which is the contribution of this paper. I import the core fiscal ingredients discussed in this previous section, which are the return equation in the previous paragraph, along with equations 6 and 12. The remainder of the model's equilibrium conditions will be adjusted in accordance with the introduction of financial frictions.

[11] Source: ibid

### 3.3 Households

There is a representative household that maximizes utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ U(C_t, N_{h,t}) + V\left(\frac{D_t}{P_t}\right)\right]$$

$U(\cdot)$ and $V(\cdot)$ are increasing and concave in their arguments. $C_t$ is consumption at period $t$, $N_{h,t}$ is the household's labor supply at time $t$. The household also receive liquidity services from holding deposits $D_t$. Households respect a budget constraint:

$$P_t C_t + B_{h,t} + D_t \leq W_t N_{h,t} + \Pi_{f,t} + \Psi_{b,t} + (1 + i_{t-1})(B_{h,t-1} + D_{t-1})$$

Where $P_t$ is the price of the consumption good. The household's labor earns it a wage rate $W_t$. The household places its savings into either *short-term* government debt $B_{h,t}$ or deposits that are both remunerated at the rate $i$. The household also receives the profits of the firm and bank — $\Pi_{f,t}$ and $\Psi_{b,t}$.

Household holdings of short-term government debt and deposits are remunerated at the same interest rate since both are backed by guarantees of repayment (at least partially in the case of deposits). The central bank effectively backs the government debt, and the banks partially back deposits with reserves. This is the intuition for remunerating both at the same rate. I don't include reserves on the intermediaries balance sheet. This would be an interesting extension of financial frictions in the FTPL setting, especially for looking at how the central bank's adjustments to the required reserve ratio might affect the dynamics of the model.

The Euler equation yields us the following specification for the stochastic discount factor (SDF):

$$\mathbb{E}_t[\Lambda_{t,t+1}] = \frac{1}{1 + i_t} \tag{13}$$

Thus, the expected value of the SDF is equal to the inverse of the gross nominal rate of interest. The SDF is specified as:

$$\Lambda_{t,t+1} = \beta \frac{U_c(C_{t+1})}{U_c(C_t)} \frac{P_t}{P_{t+1}}$$

Finally, linearizing the household Euler equation gives us the IS equation:

$$y_t = E_t y_{t+1} - \gamma_x (i_t - E_t \pi_{t+1}) \tag{14}$$

Where $y$ is output.

### *3.4 Firms*

An important element of the financial frictions is the fact that firms will ultimately borrow from the bank in order to finance their inputs, which consists of wages paid to their workers. Hence, the firm borrows to finance it's payroll:

$$W_t N_{h,t} = L_t$$

Where $W_t$ is the wage and $N_{h,t}$ is labor. Firms take out a loan $L_t$ which is remunerated at the interest rate $i_t^L$, with endogenous probability of default $\phi_{d,t+1}$. Therefore, the expected value of loan repayment is:

$$(1 + i_t^L)(1 - \phi_{d,t+1})L_t$$

The monopolistic competitor i's profit function, therefore, is given by:

$$\Psi_t(i) = P_t(i)Y_t(i) - (1 + i_t^L)(1 - \phi_{d,t+1}(i))W_t N_{h,t}(i)$$

with real marginal cost equal to:

$$MC_t(i) = (1 + i_t^L)(1 - \phi_{d,t+1}(i))\frac{W_t}{P_t}$$

These ingredients ultimately give us a representation of the NKPC that includes the firm's marginal cost in its determination. The details of its derivation are given in the appendix. The NKPC becomes:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t \tag{15}$$

### *3.5 Financial Frictions*

The financial frictions incorporated into this model are borrowed from Gertler and Kiyotaki (2010).[12] I incorporate the bank's balance sheet setup from GK, with the addition of bank holdings of long-term government debt. In order to issue loans and purchase government debt, banks must draw from their assets, which consists of their deposits and net worth. Therefore, shocks to the value of government debt shock the bank's net worth.

At date $t$ the bank chooses loans $L_t$ with ex post real gross return $R_{t+1}^L$, and deposits $D_t$ that must be repaid at the ex post real gross deposit rate $R_{t+1}^D$.

Net worth at $t$ is $N_t$. Let the household stochastic discount factor be $\Lambda_{t,t+1}$, and let $\Omega_{t+1}$ denote the shadow value of one unit of bank net worth next period.

The bank's balance sheet is defined as:

$$A_t \equiv L_t + Q_t B_{b,t} \;=\; N_t + D_t \tag{16}$$

Here, $Q_t B_{b,t}$ is the nominal market value of the bank's portfolio of government debt. Asset-market equilibrium entails that all bonds issued by the government are held by the banks and households: $B_{b,t} + B_{h,t} = B_t$. All elements of the bank's balance sheet are in

[12] Henceforth "GK."

nominal terms. The reason only $B_{b,t}$ needs a "price" is because all other instruments are one-period nominal claims whose market value equals their face value. Long-term government debt, on the other hand, is a multi-period asset whose current value depends on the present value of future coupon payments. For this reason, what matters for banks is $Q_t B_{b,t}$ rather than the face value $B_{b,t}$ so that their balance sheet reflects the *market value* of the long-term bond portfolio.

Following Cochrane (2022), the market value of the bank's government debt portfolio is defined as:

$$Q_t B_{b,t} \equiv \sum_{j=1}^{\infty} B_{b,t}^{t+j} Q_t^{t+j} = B_{b,t} \sum_{j=1}^{\infty} \omega^{j-1} Q_t^{t+j}$$

As $B_{b,t}^{(t+j)} = B_{b,t}\omega^{j-1}$ and $Q_t = \sum_{j=1}^{\infty} \omega^{j-1} Q_t^{t+j}$

Gross returns on assets pay out next period. Net worth at $t+1$ is

$$N_{t+1} = R_{t+1}^L L_t + R_{t+1}^n Q_t B_{b,t} \; - \; R_{t+1}^D D_t. \tag{17}$$

Where $R_{t+1}^n$ is the return on the government debt portfolio.

I use 1 to eliminate $D_t$ and organize everything into a risk-free part plus excess returns. First add and subtract $R_{t+1}^D L_t$, $R_{t+1}^D Q_t B_{b,t}$ and $R_{t+1}^D R_t$ inside 16: Which gives us:

$$N_{t+1} = R_{t+1}^D N_t + (R_{t+1}^L - R_{t+1}^D) L_t + (R_{t+1}^n - R_{t+1}^D) Q_t B_{b,t} \tag{18}$$

It's important to note here that $E_t R_{t+1}^L = (1 + i_t^L)(1 - \phi_{d,t+1})/\Pi_{t+1}$. The expected return on loans accounts for the endogenous probability of default, $\phi_{d,t+1}$. The objective of the bank is to maximize its expected wealth over time:

$$\mathscr{V}_t = \mathbb{E}_t \sum_{i=1}^{\infty} (1 - \sigma_B) \sigma_B^{i-1} \Lambda_{t,t+i} N_{t+i} \tag{19}$$

Here, we introduce key elements from the GK framework. The parameter $\sigma_B$ represents the fact that the banker exits at the end of period $t$ with probability $(1-\sigma_B)$, in which case he simply consumes $N_t$, and survives with probability $\sigma_B$, in which case he continues operating the bank and earns continuation value $\mathscr{V}_t(n_t)$. What's the purpose of this? If bankers did not exit and become consumers with probability $(1-\sigma_B)$, then, as banks earn positive profits each period, $N_t$ would accumulate to a point where bankers would be able to self-finance, no longer needing deposits, eliminating loan-deposit spreads, and with it, the financial frictions.

We guess the following form for the value function of the bank:

$$\mathscr{V}_t \;=\; \mathbb{E}_t[\Lambda_{t,t+1}\,\Omega_{t+1}\,N_{t+1}]\,. \tag{20}$$

Where $\Omega_{t+1}$ is the bank's shadow value of a unit of net worth. From the banker's Bellman equation,

$$\mathscr{V}_{t-1}(N_t) = E_{t-1}\Big[\Lambda_{t-1,t}\big((1-\sigma_B)N_t + \sigma_B \max_{L_t,B_{b,t}} \mathscr{V}_t(N_{t+1})\big)\Big] \tag{21}$$

Which means that we maximize

$$\mathscr{V}_t = E_t\big\{\Lambda_{t,t+1}\Omega_{t+1}[R^D_{t+1}N_t + (R^L_{t+1} - R^D_{t+1})L_t + (R^n_{t+1} - R^D_{t+1})Q_t B_{b,t}]\big\}. \tag{22}$$

subject to:

$$\mathscr{V}_t \geq \Theta_B(L_t + Q_t B_{b,t}) \tag{23}$$

This constraint is a limit on the divertability of funds by bankers. When banks obtain funds from households, the bank's manager may divert a fraction of the assets it receives to their own family. This constraint in equation 23 says that a fraction $(1-\Theta_B)$ of funds can be reclaimed by creditors. Therefore, the bank's franchise value $\mathscr{V}_t$ must be at least as great as its gain from diverting funds.

In equilibrium the constraint binds. Divide 23 by $N_t$ and use $\phi_t \equiv \frac{L_t + Q_t B_{b,t}}{N_t}$,

$$\frac{\mathscr{V}_t}{N_t} = \Theta_B \phi_t. \tag{24}$$

I call $\phi_t$ the bank's leverage. Solving for the bond portfolio price, we end up with:

$$Q_t = \frac{E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(1+\omega Q_{t+1})\big]}{E_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big] - \mu_t\Theta_B} \tag{25}$$

Where the Lagrange multiplier $\mu_t$ comes out to:

$$\mu_t = \frac{E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]}{\Theta_B - E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]} \tag{26}$$

Where $R^L_{t+1}$ and $R^D_{t+1}$ are the returns on loans and deposits, respectively. The price of the government debt portfolio now depends upon credit spreads, the SDF, and the bank's shadow value of net worth. We see that today's price of the debt portfolio is increasing in its continuation value, $Q_{t+1}$. As bonds are expected to appreciate in the future, people demand more of them today, bidding up the price.

When the spread $R^L - R^D$ increases, the bank would like to issue more loans. However, they are constrained by the incentive constraint. Although both loans and government bonds appear in the incentive constraint 23, loans tighten the constraint because they generate non-pledgeable and state-contingent returns. Government bonds, on the other hand, raise the bank's net worth without worsening diversion incentives. Hence, an increase in the loan–deposit spread tightens the constraint, raising the value of assets like government bonds. Therefore, an increase in $R^L - R^D$ increases the incentive for the bank to expand lending. As the bank approaches the IC constraint in 23, this raises $\mu_t$, which pushes bond prices up.

However, the effects of $R^L - R^D$ are counterbalanced by the effects of the intermediary pricing kernel $\Lambda\Omega$. The price of the debt portfolio is *decreasing* in the bank's pricing kernel. When this object rises, it either means the economy is in a bad state, or the bank, for whatever reason, is experiencing balance-sheet tightness, pushing up its shadow value of net worth. As a result, banks discount future bond payoffs more heavily, which lowers their present value. The rise in bank discounting of future cash flows pushes down the price of the debt portfolio. This is analogous to the baseline responses of bond prices in FTPL models Cochrane (2022a,b) with long-term debt, where rises in discount rates lower $Q_t$.

Recall that $\Omega$ is the bank's shadow value of net worth. It has the following specification:

$$\Omega_t = (1 - \sigma_B) \cdot 1 + \sigma_B \cdot \Theta_B \phi_t \tag{27}$$

which is an increasing function of leverage $\phi_t$, which itself is a function of net worth and bank assets. Therefore, bank leverage and net worth are key elements that determine the intermediary pricing kernel, which in turn is a major of the price of government debt. Also recall that, log-linearizing equation 5, we end up with:[13]

$$r^n_{t+1} = \omega q_{t+1} - q_t \tag{28}$$

Alas, we have our link between the valuation equation of government debt and financial intermediaries. Bank leverage in equation 27 determines the shadow value of bank net worth $\Omega$, which, along with the SDF, determines the intermediary's pricing kernel and, furthermore, bond prices. As bank net worth falls, leverage (and therefore $\Omega$) rise, which causes $Q$ to fall — transmitting directly into 27. Therefore, since we know there is a strong relationship between discount rates and inflation, and since we now see the link between the financial sector and discount rate determination, we can now more fully understand how financial frictions will operate in the FTPL environment as we turn to the IRFs.

[13] See appendix of Cochrane (2022a) for derivation.

# 4 Impulse Responses

At this point we turn to the analysis of the impulse response functions. It seemed appropriate to me to provide a benchmark against which one could compare the model with financial frictions. This will clarify the role of these frictions and how they alter the dynamics of the model. Since the virtually the entirety of the FTPL block in the model is imported from Cochrane (2022b), this will be our baseline. Cochrane's "Fiscal Theory of Monetary Policy" contains frictions in the form of price stickiness, which I include in my paper as well. However, the NKPC now includes firms' marginal cost, as opposed to output.

The key differentiating factor between the fiscal block in the baseline model and mine is the determination of bond prices, which is ultimately an AR(1) process that also responds to output and inflation in the baseline setup. At the end of the previous section I discussed what drives bond prices in my model, which are now intimately linked to the financial sector.

We will conduct four experiments that will enable us to compare the results of our model against the baseline (no financial frictions) model: 1) A monetary policy shock in an environment with no policy rules (fiscal and monetary policy rules are shut off). 2) A monetary policy shock with policy rules. 3) A fiscal policy shock with no policy rules. 4) A fiscal policy shock with policy rules.

Denote $\vec{\theta} = [\theta_{ix} \;\; \theta_{i\pi} \;\; \theta_{sx} \;\; \theta_{s\pi}]$ as the vector (or set) of policy rule parameters. To say that we are shutting off the policy rules in two of these experiments, it simply means that $\vec{\theta} = \mathbf{0}$. This exercise gives us a look into how FTPL alone can produce realistic dynamics of inflation and output, with the aid of policy tools — the impulse responses with policy rules turned off can be found in the appendix. Of particular interest are the simulations with policy rules, which is what we turn our attention to here.

Figure 2 compares the impulse responses of the baseline model to the financial frictions (FF) model in the context of a monetary policy shock with policy rules.

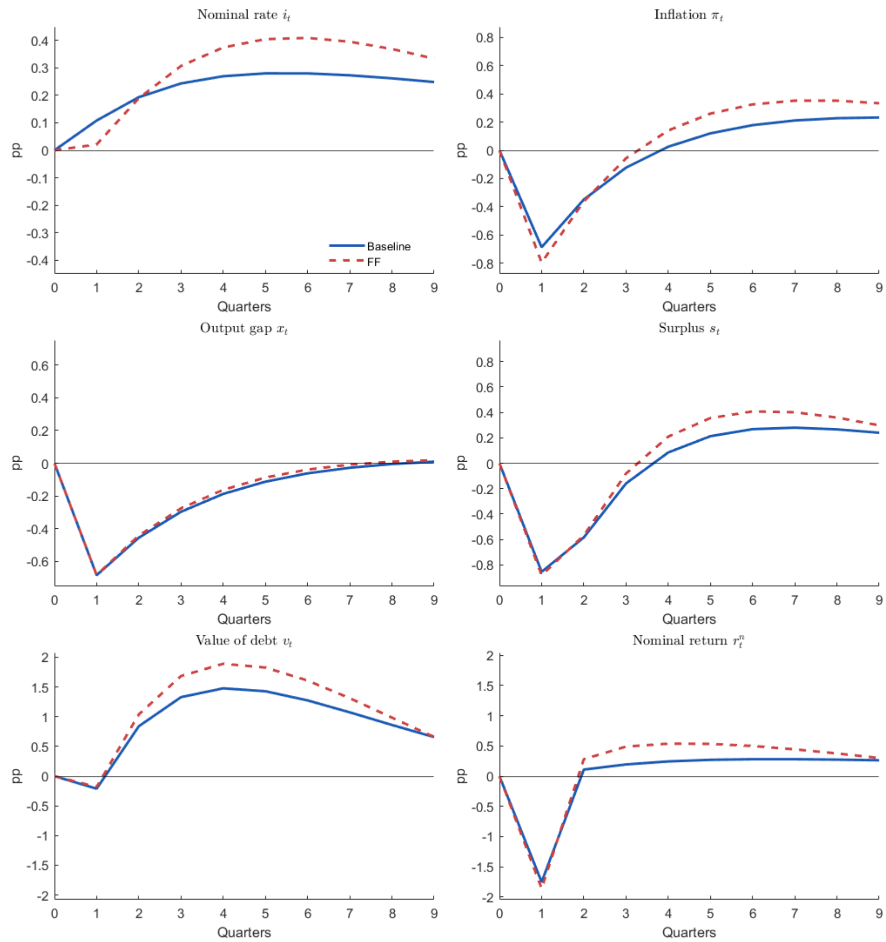


Figure 2: MP Shock - With Rules

Impulse responses to a monetary policy shock where rules are on $\vec{\theta} \gg \mathbf{0}$. The solid blue lines represent the baseline model from Cochrane (2022b). The orange dashed lines represent the model with financial frictions (FF).

As we'd hope, a monetary policy shock that *increases* interest rates pushes inflation and output down at near-term horizons. The relatively flat response of $i_t$ is owing to the policy rule's response to the immediate drop in output and inflation, which leads $i_t$ to rise by substantially less than its shock. In the baseline specification we get immediate liftoff, but under FF, the interest rate doesn't lift off until after period 1. After period two, we see both higher amplitude and persistence in the interest rate's dynamics under FF than the baseline.

The reason for the one period lag in the interest rate rise under FF is the fact that inflation falls by about an additional 0.11 percentage points compared to the baseline. When then see larger inflation over the medium and long term in the FF model.

There are several forces behind the larger and more persistent inflation response relative to the baseline. First, the Phillips curve now responds to firms' marginal costs, which in this model depend not only on output but also on bank credit conditions, including lending volumes, lending rates, and default risk. As the policy rate rises, bank lending contracts sharply.[14] Since, in this model, marginal cost effectively depends on the gap between output and credit, a fall in lending that is larger and more persistent than the fall in output raises marginal cost. On impact, both output and lending drop, but lending drops by more, and over the medium term the continued shortfall of credit relative to activity keeps marginal costs elevated. This additional and persistent pressure on marginal cost feeds into the Phillips curve and generates stronger and more drawn-out inflation dynamics than in the baseline model without financial frictions.

| Model | $\Delta E_1 \pi_1$ | $-\Delta E_1(r_1^n)$ | $=$ | $-\sum_{j=0}^{\infty} \Delta E_1 s_{1+j}$ | $+\sum_{j=1}^{\infty} \Delta E_1 \left(r_{1+j}^n - \pi_{1+j}\right)$ |
|---|---|---|---|---|---|
| Baseline | $(-0.69)$ | $-(-1.75)$ | $=$ | $-(0.28)$ | $+(1.34)$ |
| FF | $(-0.80)$ | $-(-1.86)$ | $=$ | $-(-1.57)$ | $+(-0.51)$ |

Table 1: Decomposition for a monetary shock with policy responses.

Table 1 follows Cochrane (2022a,b) in analyzing the decomposition of inflation.[15] The 1% contractionary monetary policy shock leads to and immediate fall in inflation of 0.69% in the baseline model, as opposed to 0.80% under FF — a 16% greater fall in inflation over the immediate term. Although lending rates rise in wake of the policy rate shock, lending volume falls by *less* than economic activity (output), which means that marginal costs actually create additional downward pressure on inflation over the immediate term. Of course, as explained above, this trend reverses over longer horizons. Although marginal cost isn't included in the decomposition of Table 1, the table provides us with some interesting contrasts in variables that *are* influenced by marginal cost.

[14]See appendix figures for IRFs and other details.

[15]See online appendix (section 6.1) of Cochrane (2022a) for derivation of this identity.

In the baseline model we see that real interest rates rise, which entails a higher real returns on government debt. This contributes positively to inflation and is offset by the ensuing rise in expected future surpluses raised in response to these higher returns.

Under FF, the signs are reversed. Real returns on debt fall over time, which create deflationary pressure. However, this is partially offset by the fall in future surpluses that follow as a result of the decrease in the expected rate of return on the debt, which results in inflationary pressure. However, the magnitude of the fall in government debt returns dominates, which leads generates, overall, disinflation.

Consequently, FFs lead to the same directional change in inflation as the baseline model (a fall in inflation), but for quite different reasons. The amplification of interest rates over the medium term leads to higher inflation over the same horizon. This is an artifact of fiscal dominance. Over the short-term, in both models, yields on long-term bonds rise, which means that bond returns $r^n$ fall immediately. However, as interest rates continue to rise, so do expected bond returns, and with it, debt.

The movement of $r^n$ is tied to the behavior of the bank's balance sheet. Recall what equation 29 tells us about the determination of bond returns: they are determined by the evolution of bond prices. As stated above, as bank balance sheets deteriorate, bond prices rise today (at time $t$), which causes $r^n_{t+1}$ to fall. This story is precisely what we see in the early horizons of $r^n$ in figure two. There is very little change in the initial fall of $r^n$. However, over medium and longer term horizons, the nominal return on government debt exceeds the baseline. Why is this happening? The answer is simple: in the FF model, after the immediate rate hike, bank net worth falls faster than assets, which means leverage increases. Once leverage spikes, the no-diversion constraint becomes more binding. As a result, $\mu_t$ increases, which means banks value government debt in so far as it relaxes this constraint. This immediately raises the bond price $Q_t$, leading to lower returns on nominal debt $r^n_{t+1}$, which the bank accepts due to the fact that these assets provide a kind of "collateral premium." After a few quarters have elapsed, lending has dropped of significantly, which leads $\mu_t$ to decrease. The SDF is a decreasing function of interest rates.

Therefore, nominal interest rates operate through the financial sector in a manner that amplifies virtually every ingredient of interest in wake of a monetary policy shock. This is due to the fact that surpluses only cover a portion of the debt accumulated, which means inflationary pressure arises over the medium term. Therefore, we have marginal costs amplifying interest rates over the medium term, which pushes the rate of return on government debt up, and therefore the value of debt, which generates additional inflation due to the inability of surpluses to fully cover the increase in the value of debt.

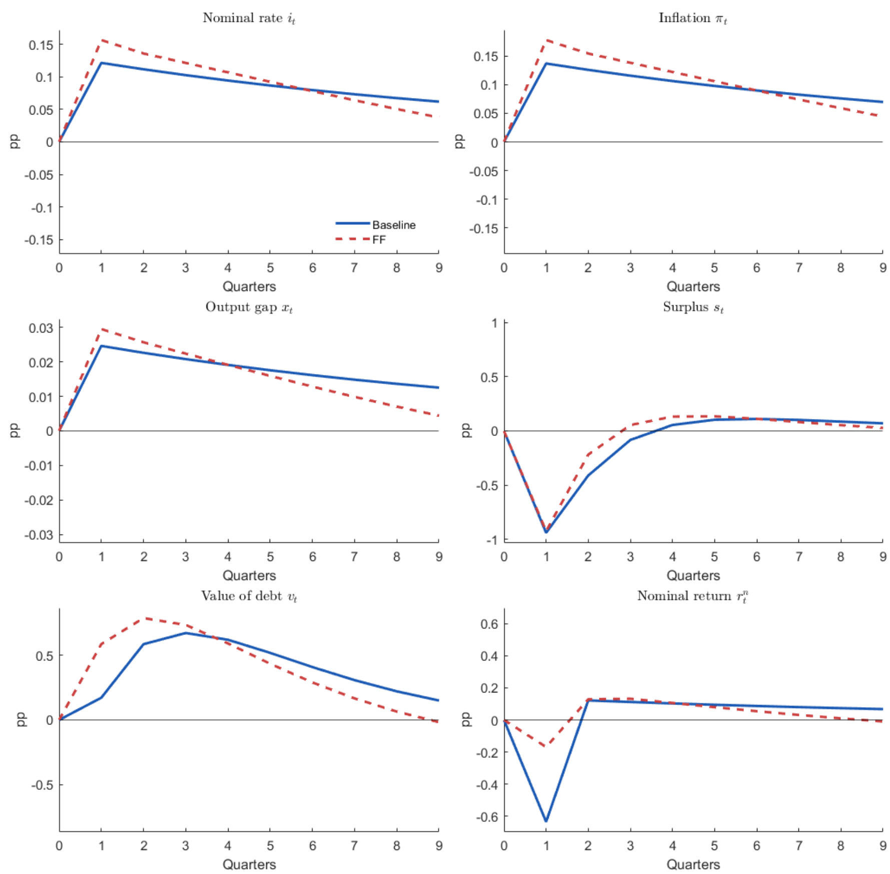


Figure 3: FP Shock - With Rules

We move on to a fiscal policy shock in an environment of policy rules. A "fiscal shock" entails a sudden 1% decrease in expected government surpluses.

The initial shock to surpluses is -1%. However, the sum of the exogenous shock's con-

| Model | $\Delta E_1 \pi_1$ | $-\Delta E_1(r_1^n)$ | $=$ | $-\sum_{j=0}^{\infty} \Delta E_1 s_{1+j}$ | $+\sum_{j=1}^{\infty} \Delta E_1 \left(r_{1+j}^n - \pi_{1+j}\right)$ |
|---|---|---|---|---|---|
| Baseline | (0.14) | $-(-0.63)$ | $=$ | $-(-0.82)$ | $+(-0.04)$ |
| FF | (0.18) | $-(-0.17)$ | $=$ | $-(-1.16)$ | $+(-0.81)$ |

| Model | $\sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j}$ | $=$ | $-\sum_{j=0}^{\infty} \rho^j \Delta E_1 s_{1+j}$ | $+\sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_1 r_{1+j}$ |
|---|---|---|---|---|
| Baseline | (0.79) | $=$ | $-(-0.82)$ | $+(-0.02)$ |
| FF | (0.66) | $=$ | $-(-1.16)$ | $+(-0.50)$ |

Table 2: Decomposition for a fiscal shock with policy responses.

Derivations for these decompositions are found in Cochrane (2022b).

tribution to changes in surpluses is -2%. In the baseline model, the shock results in a -0.82% change in expected future surpluses. The instantaneous change in inflation is 0.14 in the baseline model and 0.18 in the model with financial frictions. However, total weighted inflation goes up by 0.79% in the baseline, and by 0.66% under FF.[16]

Additionally, the weighted sum of surpluses is even more negative under FF than the baseline, yet we get less cumulative inflation. How is this possible, given fiscal theory's predictions that the price level should adjust so that the real value of nominal government debt equals the present value of future primary surpluses? Surpluses fall by an additional 0.34% *more* under FF than under the baseline, yet we get 0.13% less inflation under FF.

As can be seen in Table 2, clearly the discount rate variation accounts for the fact that we have a larger unfunded deficit under FF with less inflation. Note that $r_{t+1} = r_{t+1}^n - \pi_{t+1}$ is *real* return on government debt. Look at the bottom right panel of Figure 3. The fall in the nominal returns on government debt in the immediate aftermath of the shock is less than under the baseline. However, the peak of the recovery at horizon 2 is just as large as the baseline, but we see a much quicker reversion to trend under FF. Therefore, we have a more negative cumulative change in nominal and real discount rates. This raises the value

[16]The "weight" $\omega^j$ is the geometric maturity structure of long-term debt.

of debt, and acts a deflationary countermeasure to the inflationary pressure brought on by the fall in future surpluses. So what's behind the change in discount rates? To answer this, we need to see what's happening on the financial side.

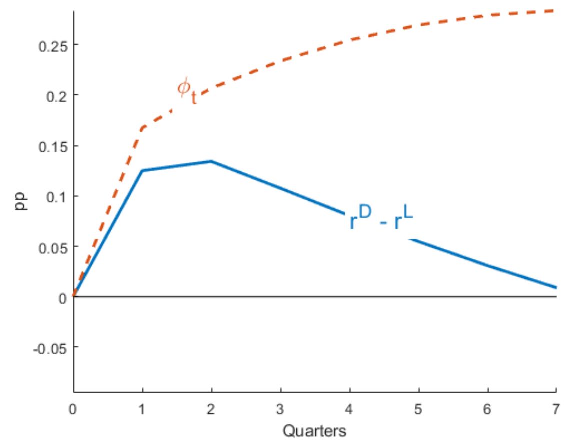


Figure 4: Impulse Responses of Leverage and Interest Rate Spreads

The orange dashed line $\phi_t$ is bank leverage from equation 24. The blue line is the spread between real deposit and loan rates.

Figure 4 plots the bank leverage $\phi_t$ and the spread between real deposit and loan rates. The rise in leverage causes the bank's shadow value of net worth $\Omega$ to increase, which puts upward pressure on $Q_t$, which pushes $r^n$ down. The interest rate on deposits responds more strongly to the fiscal shock than loan rates. Both nominal rates increase, but deposit rates increase by more. Bond prices are decreasing in this spread, which puts upward pressure on $r^n$, counterbalancing the downward pressure created by $\Omega$. Since $r^n$ falls in wake of the shock, this shows that the effects of the leverage (net worth) channel dominate overall. This explains the smaller response of $r^n$ in the immediate aftermath of the shock — $Q_t$ falls by less under FF than in the baseline.

It should be noted that this explains the front-loaded accumulation of debt under FF, relative to the baseline. The debt accumulation equation is increasing in bond returns. Since the initial fall in nominal returns is muted under FF relative to the baseline, the accumulation of debt at early horizons is amplified. However, after a few quarters, the additional inflation pressure and stronger recovery of surpluses bring the accumulation of debt below the rate of increase displayed in the baseline model.

So how do discount rates experience a larger cumulative fall under FF than in the baseline? What is obscured from Figure 3 is that, after quarter 10, nominal returns fall slightly below zero for a few quarters before returning to trend, along with inflation. This mimics the s-shaped pattern of surpluses over longer horizons. Surpluses, by construction, follow an s-shaped pattern: when we have a fiscal shock that leads to a fall in surpluses and accumulation of debt, the government must raise future surpluses to at least partially repay the debt.

The surplus process feeds in equation 12 feeds into the debt accumulation equation 6. This means the surplus equation affects the price of the debt portfolio and, therefore, nominal returns $r^n$. In the baseline model, bond price variation follows the future path of the policy rate, which leads to largely smooth dynamics that result in $r^n$ settling at zero after recovery without any oscillations. However, in my model, financial sector dynamics feed directly into the bond pricing equation. Moreover, bond pricing feed into the bank's balance sheet and leverage. This generates a feedback mechanism where changes in bond prices $Q_t$ change the banks asset holdings, which directly affects leverage $\phi_t$ that then affects the bank's shadow value of net worth $\Omega_{t+1}$, which feeds back into $Q_t$.

Therefore, the s-shaped surplus process, along with the bond-price feedback mechanism on the financial side, help contribute to the s-shaped responses in the returns on government debt and inflation. However, the main driver of the s-shaped dynamics in $r^n$ and inflation are the differing degrees of persistence among key variables on the financial side. Referring again to Figure 4, we see that there is more persistence in leverage than in the interest rate spread. Recall that increasing leverage results in decreasing $r^n$. After the initial rise of $r^D - r^L$, which pushes $r^n$ up, it corrects and begins to revert back to trend faster than leverage. Since leverage remains elevated, this puts upward pressure on $Q_t$, pushing $r^n$ down. Hence the oscillations in $r^n$ over time — they are owing to differential persistence in bank-level variables that pin down $Q$.

Altogether, this enables us to make sense of the fact that, although we have a larger cumulative fall in surpluses under FF, we get less cumulative inflation — discount rates absorb much of the effect. This is precisely what we wanted when adding financial frictions into the FTPL environment — a story that connects discount rates (a major driver of inflation dynamics) to a financial sector, and see how these ingredients affect the story.

The appendix includes the IRFs of specific financial variables like loans, net worth, leverage, and bond prices, which aid in visualizing the dynamics of the variables plotted in Figure 2 and Figure 3. I've also included the results of these simulations when the policy rules are shut off.

### *4.1 Extensions*

A worthy extension of this kind of model would naturally be to extend this into a continuous time setting à la Brunnermeier and Sannikov (2014) or He and Krishnamurthy (2013). Fiscal and monetary shocks, by their nature, generate financial market volatility that transmits to asset prices. See how changes in risk premia transmit to bond prices would be of extreme interest. The latter paper would especially be interesting for understanding how pricing kernels and discount rates interact, and in turn, inflation.

In the discrete-time setting laid out in this paper, it would also be interesting to incorporate reserve balances into the bank balance sheet that are remunerated at the policy rate. Then one could incorporate a reserve requirement that could be manipulated by the monetary authority. Under active fiscal policy, manipulating reserve ratios would, again, only alter the path of inflation, not the cumulative change. Nonetheless, it would be interesting to see how it affects the dynamics.

# 5 Conclusion

Incorporating financial frictions into a fiscal theory framework doesn't fundamentally alter the logic of FTPL. However, it does add some color to the story. The story is that, by adding financial frictions, the financial sector amplifies the responses of macro-aggregates across all horizons in wake of a monetary policy shock, and across near-term horizons following a fiscal shock. A major driver of the monetary shock dynamics are, obviously, the policy rate, but also credit as it responds to the policy rate. This affects default risk and lending rates, all of which affect firms' marginal costs, leading to more drawn-out inflation via the Phillips curve.

On the fiscal side, a shock to surpluses leads to immediate amplification of virtually all macro-aggregates over and above what is observed in the baseline model, with the exception of discount rates. Although surpluses fall more over time, we get less inflation under FF as discount rates do the work of acting as a shock absorber. This is owing to the dynamics of bank leverage and interest rate spreads that help pin down bond prices. Higher and more persistent bank leverage relative to the deposit-loan spread creates oscillations in discount rates over the long term that absorb much cumulative inflation. However, the immediate impact of inflation is more front loaded under financial frictions for similar reasons as what happens under monetary policy shocks — namely, loan and default rates play a major role through marginal costs.

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# A Appendix

*A.1 Banks*

At date $t$ the bank chooses loans $L_t$ with ex post real gross return $R^L_{t+1}$, and deposits $D_t$ that must be repaid at the ex post real gross deposit rate $R^D_{t+1}$.

Net worth at $t$ is $N_t$. Let the household stochastic discount factor be $\Lambda_{t,t+1}$, and let $\Omega_{t+1}$ denote the shadow value of one unit of bank net worth next period. Let $(1 + g_{t+1})$ be the growth factor between $t$ and $t + 1$.

The bank's balance sheet is defined as:

$$A_t \equiv L_t + Q_t B_{b,t} \; = \; N_t + D_t \tag{A.1}$$

Here, $Q_t B_{b,t}$ is the nominal market value of the bank's portfolio of government debt. All elements of the bank's balance sheet are in nominal terms. The reason only $B_{b,t}$ needs a "price" is because all other instruments are one-period nominal claims whose market value equals their face value. Long-term government debt, on the other hand, is a multi-period asset whose current value depends on the present value of future coupon payments. For this reason, what matters for banks is $Q_t B_{b,t}$ rather than the face value $B_{b,t}$ so that their balance sheet reflects the *market value* of the long-term bond portfolio.

Following Cochrane (2022), the market value of the bank's government debt portfolio is defined as:

$$Q_t B_{b,t} \equiv \sum_{j=1}^{\infty} B^{t+j}_{b,t} Q^{t+j}_t = B_{b,t} \sum_{j=1}^{\infty} \omega^{j-1} Q^{t+j}_t$$

As $B^{(t+j)}_{b,t} = B_{b,t}\omega^{j-1}$ and $Q_t = \sum_{j=1}^{\infty} \omega^{j-1} Q^{t+j}_t$

Gross returns on assets pay out next period. Net worth at $t+1$ is

$$N_{t+1} = R^L_{t+1} L_t + R^n_{t+1} Q_t B_{b,t} \; - \; R^D_{t+1} D_t. \tag{A.2}$$

Where $R^n_{t+1}$ is the return on the government debt portfolio.

I use 1 to eliminate $D_t$ and organize everything into a risk-free part plus excess returns. First add and subtract $R^D_{t+1}L_t$, $R^D_{t+1}Q_tB_{b,t}$ and $R^D_{t+1}R_t$ inside 2:

$$N_{t+1} = R^D_{t+1}L_t + R^D_{t+1}Q_tB_{b,t} + -R^D_{t+1}D_t + (R^L_{t+1} - R^D_{t+1})L_t \\ +(R^n_{t+1} - R^D_{t+1})Q_tB_{b,t}$$

Which gives us:

$$N_{t+1} = R^D_{t+1}N_t + (R^L_{t+1} - R^D_{t+1})L_t + (R^n_{t+1} - R^D_{t+1})Q_tB_{b,t} \tag{A.3}$$

This leads us to the following value function:

$$V_t \;=\; \mathbb{E}_t[\Lambda_{t,t+1}\,\Omega_{t+1}\,N_{t+1}]\,. \tag{A.4}$$

Substitute 3 into 4. Since $N_t, L_t, B_{b,t}$ are chosen at $t$, they are outside the expectation:

$$\frac{V_t}{N_t} = \mathbb{E}_t\Big[\Lambda_{t,t+1}\,\Omega_{t+1}\,R^D_{t+1}\Big] + \mathbb{E}_t\Big[\Lambda_{t,t+1}\,\Omega_{t+1}\,(R^L_{t+1} - R^D_{t+1})\Big]\,\frac{L_t}{N_t} + \mathbb{E}_t\Big[\Lambda_{t,t+1}\,\Omega_{t+1}\,(R^n_{t+1} - R^D_{t+1})\Big]\,\frac{Q_tB_{b,t}}{N_t} \tag{A.5}$$

Following Gertler, Kiyotaki (2010), the banker can divert a fraction of assets; households require the franchise value to exceed the diversion payoff. With $\Theta_B \in (0,1)$ and total assets $A_t$,

$$V_t \;\geq\; \Theta_B\,(L_t + Q_tB_{b,t}). \tag{A.6}$$

In equilibrium the constraint binds. Divide 6 by $N_t$ and use $\phi_t \equiv \frac{L_t + Q_tB_{b,t}}{N_t}$,

$$\frac{V_t}{N_t} \;=\; \Theta_B\phi_t. \tag{A.7}$$

We equate 5 and 7 and solve for $L_t$. We end up with following loan identity:

$$L_t = \frac{\mathbb{E}_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big]\,N_t - \big\{\Theta_B - E_t\left[\Lambda_{t,t+1}\Omega_{t,t+1}(R^n_{t+1} - R^D_{t+1})\right]\big\}\,Q_t B_{b,t}}{\Theta_B - \mathbb{E}_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]} \tag{A.8}$$

I let banks hold the long-term government debt portfolio with geometric maturity parameter $\omega$. Following Cochrane (2022), the gross return on this portfolio is

$$R^n_{t+1} = \frac{1 + \omega Q_{t+1}}{Q_t}. \tag{A.9}$$

For the derivation of $\Omega_{t+1}$ we do the following:

From the banker's Bellman equation,

$$V_{t-1}(N_t) = E_{t-1}\Big[\Lambda_{t-1,t}\big((1-\sigma_B)N_t + \sigma_B \max_{L_t, B_{b,t}} V_t(N_{t+1})\big)\Big] \tag{A.10}$$

the banker exits at the end of period $t$ with probability $(1-\sigma_B)$, in which case he simply consumes $n_t$, and survives with probability $\sigma_B$, in which case he continues operating the bank and earns continuation value $V_t(n_t)$.

The incentive constraint binds in equilibrium,

$$V_t = \Theta_B(L_t + Q_t B_{b,t}) \tag{A.11}$$

which, dividing and multiplying by $N_t$, gives

$$V_t = \Theta_B \frac{L_t + Q_t B_{b,t}}{N_t}\,N_t = \Theta_B \phi_t\,N_t \tag{A.12}$$

where $\phi_t \equiv (L_t + Q_t B_{b,t})/N_t$ is the banker's leverage ratio.

Substituting 14 into 12 yields

$$V_{t-1}(N_t) = E_{t-1}\Big[\Lambda_{t-1,t}\big((1-\sigma_B)N_t + \sigma_B\Theta_B\phi_t N_t\big)\Big] \tag{A.13}$$

$$= E_{t-1}\Big[\Lambda_{t-1,t}\big((1-\sigma_B)+\sigma_B\Theta_B\phi_t\big)N_t\Big] \tag{A.14}$$

Assume, as conjectured, that the value function is homogeneous of degree one in net worth:

$$V_{t-1}(N_t) = E_{t-1}\big[\Lambda_{t-1,t}\,\Omega_t\,N_t\big] \tag{A.15}$$

Comparing 16 and 17, the coefficients on $n_t$ inside the expectation must coincide, implying

$$\Omega_t = (1-\sigma_B)+\sigma_B\Theta_B\phi_t \tag{A.16}$$

A unit of net worth at $t$ yields a payout of 1 if the banker exits (probability $1-\sigma_B$), and a franchise value $\Theta_B\phi_t$ if she survives (probability $\sigma_B$). Thus the expected value per unit of net worth is

$$\Omega_t = (1-\sigma_B)\cdot 1+\sigma_B\cdot\Theta_B\phi_t.$$

To derive an equation for $B_{b,t}$, take (8) and plug it into for $L_t$ in the bank's budget constraint (1) and solve for $B_{b,t}$. This gets us an equation for the bank's holdings of long-term government debt:

$$Q_tB_{b,t} = \frac{\Theta_B - E_t[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1}-R^D_{t+1})]D_t + (\Theta_B - E_t[\Lambda_{t,t+1}\Omega_{t+1}R^L_{t+1}])N_t}{E_t[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1}-R^L_{t+1})]} \tag{A.17}$$

In equilibrium, the constraint binds, so

$$\frac{V_t}{N_t} = \Theta_B\varphi_t. \tag{A.18}$$

Given $N_t$ and the continuation value the bank chooses $(\frac{L_t}{N_t}, \frac{Q_tB_{b,t}}{N_t})$ to maximize the value per unit of net worth subject to the incentive constraint:

$$\max \quad E_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big] + E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1}-R^D_{t+1})\big]\frac{L_t}{N_t} + E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1}-R^D_{t+1})\big]\frac{Q_tB_{b,t}}{N_t} \tag{A.19}$$

$$\text{s.t.} \quad E_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big] + E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]\frac{L_t}{N_t} + E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1})\big]\frac{Q_t B_{b,t}}{N_t} \;\geq\; \Theta_B\left(\frac{L_t}{N_t} + \frac{Q_t B_{b,t}}{N_t}\right). \tag{A.20}$$

Introduce a Lagrange multiplier $\mu_t \geq 0$ on the constraint 32. The Lagrangian for the per-unit problem is

$$\begin{aligned}\mathcal{L}_t = {} & E_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big] + E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]\frac{L_t}{N_t} + E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1})\big]\frac{Q_t B_{b,t}}{N_t} \\ & + \mu_t\big(E_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big] + E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]\frac{L_t}{N_t} + E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1})\big]\frac{Q_t B_{b,t}}{N_t} - \Theta_B(\frac{L_t}{N_t} + \frac{Q_t B_{b,t}}{N_t})\big).\end{aligned} \tag{A.21}$$

Assuming an interior solution in loans, the first-order condition with respect to $\frac{L_t}{N_t}$ is

$$\frac{\partial \mathcal{L}_t}{\partial x_t} = E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big] + \mu_t(E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big] - \Theta_B) = 0. \tag{A.22}$$

Rearrange 34:

$$E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big] + \mu_t E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big] - \mu_t\Theta_B = 0, \tag{A.23}$$

$$E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big](1 + \mu_t) = \mu_t\Theta_B. \tag{A.24}$$

Solve 36 for $\mu_t$:

$$E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big](1 + \mu_t) = \mu_t\Theta_B \quad \Rightarrow \quad E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big] = \mu_t(\Theta_B - E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]) \tag{A.25}$$

$$\Rightarrow \quad \mu_t = \frac{E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]}{\Theta_B - E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]}. \tag{A.26}$$

Recalling the definition of $B_t$, we obtain

$$\mu_t = \frac{E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]}{\Theta_B - E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]} \tag{A.27}$$

If there is also an interior solution in bond holdings $y_t$, we can analogously use the FOC with respect to $y_t$:

$$\frac{\partial \mathcal{L}_t}{\partial y_t} = E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1}) + \mu_t(E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1}) - \Theta_B) = 0, \tag{A.28}$$

which implies

$$E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1})(1 + \mu_t) = \mu_t\Theta_B \quad \Rightarrow \quad \mu_t = \frac{E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1})}{\Theta_B - E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1})} = \frac{E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1})\big]}{\Theta_B - E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1})\big]}. \tag{A.29}$$

If both $L_t/N_t$ and $Q_tB_{b,t}/N_t$ in equilibrium, the two expressions for $\mu_t$ must coincide, imposing a relationship between the loan and bond excess returns under the bank's SDF.

The bank's net worth evolves according to

$$N_{t+1} = R^D_{t+1}N_t + (R^L_{t+1} - R^D_{t+1})L_t + (R^n_{t+1} - R^D_{t+1})Q_tB_{b,t}, \tag{A.30}$$

and the banker values net worth using the augmented SDF

$$V_t = E_t\big[\Lambda_{t,t+1}\Omega_{t+1}N_{t+1}\big]. \tag{A.31}$$

The bank faces the incentive (no-diversion) constraint

$$V_t \;\geq\; \Theta_B\big(L_t + Q_tB_{b,t}\big), \tag{A.32}$$

with Lagrange multiplier $\mu_t \geq 0$.

Form the Lagrangian (suppressing terms that do not depend on $B_{b,t}$),

$$\mathcal{L}_t = E_t\big[\Lambda_{t,t+1}\Omega_{t+1}N_{t+1}\big] + \mu_t\Big(\Theta_B(L_t + Q_tB_{b,t}) - V_t\Big). \tag{A.33}$$

Using 42, the marginal effect of $B_{b,t}$ on $N_{t+1}$ is

$$\frac{\partial N_{t+1}}{\partial B_{b,t}} = (R^n_{t+1} - R^D_{t+1})Q_t. \tag{A.34}$$

The first-order condition with respect to $B_{b,t}$ is then

$$0 = \frac{\partial \mathcal{L}_t}{\partial B_{b,t}} = E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1})Q_t\big] + \mu_t \Theta_B Q_t. \tag{A.35}$$

Divide by $Q_t$ to obtain

$$E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^D_{t+1})\big] = -\mu_t \Theta_B. \tag{A.36}$$

Rearrange 48 as

$$E_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^n_{t+1}\big] = E_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big] - \mu_t \Theta_B. \tag{A.37}$$

The long bond has return

$$R^n_{t+1} = \frac{1 + \omega Q_{t+1}}{Q_t}, \tag{A.38}$$

where $\omega \in (0,1)$ parameterizes the geometric maturity structure.

Substitute 50 into the left-hand side of 49:

$$E_t\left[\Lambda_{t,t+1}\Omega_{t+1}\frac{1 + \omega Q_{t+1}}{Q_t}\right] = E_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big] - \mu_t \Theta_B. \tag{A.39}$$

Since $Q_t$ is known at time $t$, pull it outside the expectation:

$$\frac{1}{Q_t}E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(1 + \omega Q_{t+1})\big] = E_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big] - \mu_t \Theta_B. \tag{A.40}$$

Multiply both sides of 52 by $Q_t$:

$$E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(1 + \omega Q_{t+1})\big] = Q_t\Big(E_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big] - \mu_t \Theta_B\Big). \tag{A.41}$$

Finally, solve 53 for $Q_t$ to obtain the new bond pricing equation:

$$Q_t = \frac{E_t\left[\Lambda_{t,t+1}\Omega_{t+1}(1+\omega Q_{t+1})\right]}{E_t\left[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\right] - \mu_t\Theta_B} \tag{A.42}$$

Equation 54 shows that the long-bond price $Q_t$ depends on the bank's augmented stochastic discount factor $\Lambda_{t,t+1}\Omega_{t+1}$ and on the tightness of the incentive constraint through the multiplier $\mu_t$.

### *A.2 Households*

The representative household maximizes lifetime utility:

$$\max_{\{C_t, N_{h,t}, B_{h,t}, D_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \theta \frac{N_{h,t}^{1+\eta}}{1+\eta} + \varphi_d \ln\left(\frac{D_t}{P_t}\right)\right) \tag{A.43}$$

subject to the budget constraint:

$$P_t C_t + B_{h,t} + D_t \le W_t N_{h,t} + \Pi_{f,t} + \Psi_{b,t} + (1+i_{t-1})(B_{h,t-1} + D_{t-1}) \tag{A.44}$$

The household consumes $C_t$, supplies labor $N_{h,t}$ at wage rate $W_t$, holds short-term public debt $B_{h,t}$, which is remunerated at rate $i_t$; holds deposits $D_t$, which are remunerated at rate $i_t^D$. $\Pi_{f,t}$ and $\Psi_{b,t}$ are firm and bank profits, respectively.

The consumer only holds debt of near-term maturity, where as banks hold all of the long-term debt. Taken together then, households and banks hold all of the government's debt, so:

$$B_{b,t} + B_{h,t} = B_t$$

Taking FOCs gets us the Euler equation through the following steps:

$$\mathcal{L}'(C_t) \to C_t^{-\sigma} = \lambda_t P_t \tag{A.45}$$

$$\mathcal{L}'(B_{h,t+1}) = \lambda_t = \beta E_t[\lambda_{t+1}(1+i_t)] \tag{A.46}$$

$$\Rightarrow 1 = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)(1+i_t)\right] \tag{A.47}$$

which is our Euler equation.

This gives us our SDF:

$$1 = E_t[\Lambda_{t,t+1}(1+i_t)] \tag{A.48}$$

Linearizing (24), we get:

$$E_t[c_{t+1} - c_t] = \gamma_x\omega + \gamma_x(i_t - E_t\pi_{t+1}) \tag{A.49}$$

where $\gamma_x = \frac{1}{\sigma}$ and $\omega = -\log\beta$. Setting consumption equal to output $c_t = y_t$, we get the IS equation:

$$y_t = E_t y_{t+1} - \gamma_x(i_t - E_t\pi_{t+1}) \tag{A.50}$$

From the Household FOCs we also have:

$$\lambda w_t = \theta N_{h,t}^{\eta} \tag{A.51}$$

with $y_t = \frac{C_t}{P_t}$, we get:

$$y_t^{-\sigma} w_t = \theta N_{h,t}^{\eta} \tag{A.52}$$

In the case of firms, we will see that: $L_t = w_t N_{h,t}$. Therefore, after some rearranging:

$$w_t = (\theta y_t^{\sigma} L_t^{\eta})^{\frac{1}{1+\eta}} \tag{A.53}$$

Taking the FOC of the household's equation with respect to $D_t$, rearranging terms, and substituting definitions of $\lambda_t$ from the household block and $D_t$ from the banking block, we end up with:

$$(1 + i_t^D) = \frac{1}{\beta}\left(\Pi_{t+1}\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} - \phi_d C_{t+1}^{\sigma} P_{t+1}\frac{\phi_d}{D_t}\right)$$

Where:

- $y_t = \frac{C_t}{P_t}$
- $l_t = \frac{L_t}{P_t}$
- $n_{h,t} = \frac{N_{h,t}}{P_t}$
- $R_{t+1}^D = \frac{1+i_t^D}{\Pi_{t+1}}$

Which gives us:

$$D_t = \frac{\varphi_d y_t^{\sigma}}{1 - E_t \Lambda_{t,t+1}(1 + i_t^D)} = \frac{\varphi_d y_t^{\sigma}}{1 - \frac{1+i_t^D}{1+i_t^B}}$$

### *A.3 Firms*

Demand for the individual monopolist $i \in [0, 1]$ is:

$$Q_i = \left(\frac{P_i}{P}\right)^{-\eta} Y \tag{A.54}$$

The firm uses labor $N_h$ to produce output $Y$.

Firms take out loans from the banks to finance input (labor) costs:

$$W_t N_{h,t} = L_t \tag{A.55}$$

Where $W_t$ is the wage and $N_{h,t}$ is labor. Firms take out a loan $L_t$ which is remunerated at the interest rate $i_t^L$, with probability of default $\phi_{d,t+1}$. Therefore, the expected value of loan repayment is:

$$(1 + i_t^L)(1 - \phi_{d,t+1})L_t \tag{A.56}$$

The monopolistic competitor's profit function, therefore, is given by:

$$\Psi_t(i) = P_t(i)Y_t(i) - (1 + i_t^L)(1 - \phi_{d,t+1}(i))W_t N_{h,t}(i) \tag{A.57}$$

with real marginal cost equal to:

$$MC_t(i) = (1 + i_t^L)(1 - \phi_{d,t+1}(i))\frac{W_t}{P_t} \tag{A.58}$$

The optimal price is mark-up over the marginal cost:

$$\frac{P_i^*}{P} = \frac{\eta}{\eta - 1} MC(i) \tag{A.59}$$

This gives us our optimal price formula:

$$p_t^* = p_t + mc_t \tag{A.60}$$

Where $\log\left(\frac{\eta}{\eta-1}\right)$ drops out when examining deviations from steady state.

From here, we can consider the profit function $\Psi_t(x_t)$ for the representative monopolistically competitive firm with current price $x_t$. We write the quadratic loss function of the firm as:

$$L_t = \frac{K}{2}(p_t^* - x_t)^2 \tag{A.61}$$

where K is the absolute value of the second derivative of the profit function.

The representative firm gets to reset its price at any period with probability $(1 - \theta)$. the objective of the the firm resetting its price at period $t$ is:

$$\min_{x_t} = \sum_{j=0}^{\infty}(1-\theta)\theta^j\beta^j\frac{K}{2}(E_t p^*_{t+j} - x_t)^2 \tag{A.62}$$

where $\beta$ is the discount factor and $(1-\theta)\theta^j$ is the probability that the firm will not reset its price until period $t+j+1$.

Optimizing this objective function and doing some algebra yields us the classic New Keynesian Phillips Curve:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t \tag{A.63}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$

Recall that the firm's MC as defined in (35). Combining equations (33) and (34), we arrive at the condition for default:

$$\phi_{d,t+1} = 1 - \frac{Y_t}{(1+i^L_t)L_t/P_t} \tag{A.64}$$

Plugging this into the firm's MC equation we get

$$mc_t = \frac{Y_t}{L_t}w_t \tag{A.65}$$

Using the FOC for labor

$$\lambda_t w_t = \theta N^{\eta}_{h,t} \quad \text{with} \quad \lambda_t = \frac{C_t^{-\sigma}}{P_t} \quad \text{and} \quad C_t = Y_t \tag{A.66}$$

Along with $W_t = \frac{L_t}{N_{h,t}}$ from (32), we get:

$$w_t = (\theta Y_t^{\sigma} L_t^{\eta})^{\frac{1}{1+\eta}} \tag{A.67}$$

$$mc_t = \frac{Y_t}{L_t}\left(\theta^{\frac{1}{1+\eta}} Y_t^{\frac{\sigma}{1+\eta}} L_t^{\frac{\eta}{1+\eta}}\right)$$

n

$$mc_t = \left(\theta^{\frac{1}{1+\eta}} Y_t^{\frac{1+\sigma+\eta}{1+\eta}} L_t^{\frac{-1}{1+\eta}}\right) \tag{A.68}$$

$$mc_t = \left(\frac{\theta}{L_t}\right)^{\frac{1}{1+\eta}} Y_t^{\frac{1+\sigma+\eta}{1+\eta}} \tag{A.69}$$

To get $R^L_{t+1}$, observe that $E_t R^L_{t+1} = \frac{(1+i^L_t)(1-\phi_{d,t+1})}{\Pi_{t+1}}$. But since we have already derived $mc_t = (1+i^L_t)(1-\phi_{d,t+1})w_t$, we simply have

$$R^L_{t+1} = \frac{mc_t/w_t}{\Pi_{t+1}}$$

### *A.4 Cochrane Core Eq. Conditions*

$$y_t = E_t y_{t+1} - \gamma_x\big(i_t - E_t \pi_{t+1}\big) \tag{A.70}$$

$$\pi_t = \beta\, E_t \pi_{t+1} + \lambda\, mc_t \tag{A.71}$$

These two equations above are slightly different from Cochrane's.

$$i_t = \theta_{i\pi}\, \pi_t + \theta_{ix}\, y_t + u^i_t \tag{A.72}$$

$$s_t = \theta_{s\pi}\, \pi_t + \theta_{sx}\, y_t + \alpha\, v^*_t + u^s_t \tag{A.73}$$

$$\rho v^*_{t+1} = v^*_t + r^n_{t+1} - E_t \pi^*_{t+1} - s_{t+1} \tag{A.74}$$

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - s_{t+1} \tag{A.75}$$

$$r^n_{t+1} = \omega\, q_{t+1} - q_t \tag{A.76}$$

$$u^i_{t+1} = \rho_i\, u^i_t + \varepsilon^i_{t+1}, \tag{A.77}$$

$$u^s_{t+1} = \rho_s\, u^s_t + \varepsilon^s_{t+1} \tag{A.78}$$

*A.5 New Equilibrium Conditions*

$$Y_t = C_t \tag{A.79}$$

$$w_t = (\theta y_t^{\sigma} L_t^{\eta})^{\frac{1}{1+\eta}} \tag{A.80}$$

$$mc_t = \left(\frac{\theta}{L_t}\right)^{\frac{1}{1+\eta}} Y_t^{\frac{1+\sigma+\eta}{1+\eta}} \tag{A.81}$$

$$D_t = Q_t B_{b,t} + L_t - N_t \tag{A.82}$$

$$R^D_{t+1} = \frac{1+i_t}{\Pi_{t+1}} \tag{A.83}$$

$$R^L_{t+1} = \frac{mc_t/w_t}{\Pi_{t+1}} \tag{A.84}$$

$$R^n_{t+1} = \frac{1+\omega Q_{t+1}}{Q_t} \tag{A.85}$$

$$E_t \Lambda_{t,t+1} = \frac{1}{1+i_t} \tag{A.86}$$

$$\Omega_t = (1-\sigma_B) + \sigma_B \Theta_B \phi_t \tag{A.87}$$

$$\phi_t = \frac{L_t + Q_t B_{b,t}}{N_t} \tag{A.88}$$

$$N_{t+1} = R^D_{t+1} N_t + (R^L_{t+1} - R^D_{t+1}) L_t + (R^n_{t+1} - R^D_{t+1}) Q_t B_{b,t} \tag{A.89}$$

$$L_t = \frac{\mathbb{E}_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big] N_t - \big\{\Theta_B - E_t\left[\Lambda_{t,t+1}\Omega_{t,t+1}(R^n_{t+1} - R^D_{t+1})\right]\big\} Q_t B_{b,t}}{\Theta_B - \mathbb{E}_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]} \tag{A.90}$$

$$Q_t B_{b,t} = \frac{\Theta_B - E_t[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})] D_t + (\Theta_B - E_t[\Lambda_{t,t+1}\Omega_{t+1}R^L_{t+1}]) N_t}{E_t[\Lambda_{t,t+1}\Omega_{t+1}(R^n_{t+1} - R^L_{t+1})]} \tag{A.91}$$

$$Q_t = \frac{E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(1+\omega Q_{t+1})\big]}{E_t\big[\Lambda_{t,t+1}\Omega_{t+1}R^D_{t+1}\big] - \mu_t \Theta_B} \tag{A.92}$$

$$\mu_t = \frac{E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]}{\Theta_B - E_t\big[\Lambda_{t,t+1}\Omega_{t+1}(R^L_{t+1} - R^D_{t+1})\big]} \tag{A.93}$$

$$\Theta_B(Q_t B_{b,t} + L_t) = E_t[\Lambda_{t,t+1}\Omega_{t,t+1}N_{t+1}] \tag{A.94}$$

*A.6 Steady State:*

In steady state, first note from equation 24 that we have:

$$1 + i = \frac{\Pi}{\beta}$$

Which, from 63, means that we also have:

$$\Lambda = \frac{\beta}{\Pi}$$

The steady state of $Y, w, mc, D, R^D, R^L, R^n, \Omega,$ and $\phi$ are as follows:

$$\begin{aligned} Y &= C \\ w &= (\theta Y^{\sigma} L^{\eta})^{\frac{1}{1+\eta}} \\ mc &= \left(\frac{\theta}{L}\right)^{\frac{1}{1+\eta}} Y^{\frac{1+\sigma+\eta}{1+\eta}} \end{aligned}$$

Since we have $R^L = (mc/w)/\Pi$, combining the steady state conditions of $mc$ and $w$, assuming $Y = 1$, gives us:

$$R^L = \frac{1}{L}$$

We already have $R^D$ pinned down above, but here I assume a spread between $R^L$ and $R^D$ that gives us $R^L$, and thus, $L$. I also normalize $N = 1$. At this point, the following conditions enable us to solve for steady state:

$$
\begin{aligned}
D &= QB_b + L - N \\
R^D &= \frac{1+i^D}{\Pi} \\
R^L &= \frac{mc/w}{\Pi} \\
R^n &= \frac{1+\omega Q}{Q} \\
\Omega &= (1-\sigma_B) + \sigma_B \Theta_B \phi \\
\phi &= \frac{L + QB_b}{N}
\end{aligned}
$$

Equation 104 can be rearranged to find the steady state price of the government debt portfolio, which is:

$$
Q = \frac{\Lambda\Omega}{\Lambda\Omega(R^D - \omega) - \mu\Theta_B}
$$

Then we have our multiplier $\mu_t$:

$$
\mu = \frac{\Lambda\Omega(R^L - R^D)}{\Theta_B - \Lambda\Omega(R^L - R^D)}
$$

And finally, from equations 100 and 106, we get:

$$
\Theta_B \phi = \Lambda\Omega
$$

Plugging in the definition of $\Omega$, we get:

$$\Theta_B\phi = \Lambda((1-\sigma_B) + \sigma_B\Theta_B\phi)$$

Which enables us, from our parameter calibrations to recover steady state leverage $\phi$, which tells us steady state $\Omega$, then $\mu$, then $R^n$ and so on.

### *A.7 Calibration*

| **Parameter / Steady State** | **Value** |
|---|---|
| Discount factor $\beta$ | 0.99 |
| Risk aversion $\sigma$ | 2 |
| Labor elasticity $\eta$ | 1 |
| Calvo stickiness $\theta$ | 0.5 |
| NKPC slope $\lambda$ | 0.50 |
| Maturity structure $\omega$ | 0.90 |
| Bank survival probability $\sigma_B$ | 0.975 |
| Stochastic Discount Factor $\Lambda$ | 0.99 |
| Fraction of assets divertable $\Theta_B$ | 0.383 |
| $\theta_{i\pi}$ | 0.80 |
| $\theta_{ix}$ | 0.50 |
| Steady-state leverage $\phi$ | 1.8596 |
| Steady-state nominal rate $1+i$ | 1.01 |
| Steady-state inflation $\Pi$ | 1 |
| Steady-state output $Y$ | 1 |
| Steady-state shadow value of net worth $\Omega$ | 0.7194 |

| **Parameter / Steady State** | **Value** |
|---|---|
| Steady-state multiplier $\mu$ | 0.0151 |
| Steady-state bond price $Q$ | 9.82 |
| Steady-state bond return $R^n$ | 1.0019 |
| Steady-state loan return $R^L$ | 1.018 |
| Steady-state deposit return $R^D$ | 1.01 |
| Steady-state loans $L$ | 0.9823 |
| Steady-state deposits $D$ | 0.85628 |
| Steady-state bonds $B_b$ | 0.089 |
| Steady-state equity $N$ | 1 |

*A.8 Linearized Conditions*

$$y_t = E_t y_{t+1} - \gamma_x \big(i_t - E_t \pi_{t+1}\big) \tag{A.95}$$

$$\pi_t = \beta\, E_t \pi_{t+1} + \lambda\, mc_t \tag{A.96}$$

$$i_t = \theta_{i\pi}\, \pi_t + \theta_{ix}\, y_t + u^i_t \tag{A.97}$$

$$s_t = \theta_{s\pi}\, \pi_t + \theta_{sx}\, y_t + \alpha\, v^*_t + u^s_t \tag{A.98}$$

$$\rho v^*_{t+1} = v^*_t + r^n_{t+1} - E_t \pi^*_{t+1} - s_{t+1} \tag{A.99}$$

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - s_{t+1} \tag{A.100}$$

$$r^n_{t+1} = \omega\, q_{t+1} - q_t \tag{A.101}$$

$$u^i_{t+1} = \rho_i\, u^i_t + \varepsilon^i_{t+1}, \tag{A.102}$$

$$u^s_{t+1} = \rho_s\, u^s_t + \varepsilon^s_{t+1} \tag{A.103}$$

$$w_t = \omega_x y_t + \mu_\ell \ell_t \tag{A.104}$$

$$mc_t = \gamma y_t - \delta_\ell \ell_t \tag{A.105}$$

$$d_t = \phi_{qb}(q_t + b_{b,t}) + \phi_l \ell_t - \phi_n n_t \tag{A.106}$$

$$r^L_{t+1} = mc_t - w_t - \pi_{t+1} \tag{A.107}$$

$$r^D_{t+1} = i_t - \pi_{t+1} \tag{A.108}$$

$$i_t + \tilde{\lambda}_{t,t+1} = 0 \tag{A.109}$$

$$\tilde{\omega}_{t,t+1} = \#\phi_t \tag{A.110}$$

$$\gamma_L \ell_t + \xi_{qb}(q_t + b_{b,t}) = \tilde{\lambda}_{t,t+1} + \tilde{\omega}_{t,t+1} + n_t \tag{A.111}$$

$$\tilde{\phi}_t = \gamma_l \ell_t + \xi_{qb}(q_t + b_{b,t}) - n_t \tag{A.112}$$

$$n_{t+1} = \gamma_n n_t + \alpha_{r\ell} r^L_{t+1} + \alpha_\ell \ell_t + \alpha_{rn} r^n_{t+1} - \alpha_{rd} r^D_{t+1} - \alpha_{qb}(q_t + b_{b,t}) \tag{A.113}$$

$$\ell_t = \psi_{lo}(\tilde{\lambda}_{t+1} + \tilde{\omega}_{t+1}) + \sigma_n n_t + \sigma_{rn} r^n_{t+1} + \sigma_{r\ell} r^L_{t+1} - \sigma_{rd} r^D_{t+1} - \sigma_{qb}(q_t + b_{b,t}) \tag{A.114}$$

$$q_t + b_{b,t} = \beta_{rn} r^n_{t+1} + \beta_{rd} r^D_{t+1} - \beta_{lo}(\tilde{\lambda}_{t+1} + \tilde{\omega}_{t+1}) - \beta_{r\ell} r^L_{t+1} - \beta_d d_t - \beta_n n_t \tag{A.115}$$

$$q_t = \delta_q q_{t+1} + \delta_\mu \tilde{\mu}_t - \delta_{lo}(\tilde{\lambda}_{t+1} + \tilde{\omega}_{t+1}) - \delta_{rd} r^D_{t+1} \tag{A.116}$$

$$\tilde{\mu}_t = \kappa_{lo}(\tilde{\lambda}_{t+1} + \tilde{\omega}_{t+1}) + \kappa_{r\ell} r^L_{t+1} - \kappa_{rd} r^D_{t+1} \tag{A.117}$$

$$\begin{aligned}
\ell_t = \psi_{lo}\{(-\theta_{i\pi}\pi_t - \theta_{ix} y_t - u^i_t + \pi_{t+1}) + \#[\gamma_\ell \ell_t + \xi_{qb}(q_t + b_{b,t}) - n_t]\}+ \\
\sigma_n\{\gamma_n n_{t-1} + \alpha_{r\ell}(y_{t-1} - \ell_{t-1} - \pi_t) + \alpha_\ell \ell_{t-1} + \alpha_{rn}(\omega q_t - q_{t-1})- \\
\alpha_{rd}[\theta_{i\pi}\pi_{t-1} + \theta_{ix} y_{t-1} + u^i_{t-1} - \pi_t] - \alpha_{qb}(q_{t-1} + b_{b,t-1})\}+ \\
\sigma_{rn}(\omega q_{t+1} - q_t) - \sigma_{qb}(q_t + b_{b,t}) + \sigma_{r\ell}(y_t - \ell_t - \pi_{t+1})- \\
\sigma_{rd}(\theta_{i\pi}\pi_t + \theta_{ix} y_t + u^i_t - \pi_{t+1})
\end{aligned}$$

$$
\begin{aligned}
(1-\psi_{lo}\,\#\,\gamma_L+\sigma_{r\ell})\,\ell_t = &\Big[\sigma_n(\alpha_{rd}-\alpha_{r\ell})-\psi_{lo}\theta_{i\pi}-\sigma_{rd}\theta_{i\pi}\Big]\pi_t\\
&+\ (\psi_{lo}-\sigma_{r\ell}+\sigma_{rd})\,\pi_{t+1}\ -\ \sigma_n\alpha_{rd}\theta_{i\pi}\,\pi_{t-1}\\
&+\ (\sigma_{r\ell}-\psi_{lo}\theta_{ix}-\sigma_{rd}\theta_{ix})\,y_t\ +\ \sigma_n\big(\alpha_{r\ell}-\alpha_{rd}\theta_{ix}\big)\,y_{t-1}\\
&+\ \big(\psi_{lo}\,\#\,\xi_{qb}+\sigma_n\alpha_{rn}\omega-\sigma_{rn}-\sigma_{qb}\big)q_t\ +\ \sigma_{rn}\omega\,q_{t+1}\\
&-\ \sigma_n(\alpha_{rn}+\alpha_{qb})\,q_{t-1}\\
&+\ (\psi_{lo}\,\#\,\xi_{qb}-\sigma_{qb})\,bb_t\ -\ \sigma_n\alpha_{qb}\,bb_{t-1}\\
&-\ \psi_{lo}\,\#\,n_t\ +\ \sigma_n\gamma_n\,n_{t-1}\\
&+\ \sigma_n(\alpha_\ell-\alpha_{r\ell})\,\ell_{t-1}\\
&-\ (\psi_{lo}+\sigma_{rd})\,u_{i,t}\ -\ \sigma_n\alpha_{rd}\,u_{i,t-1}.
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_\ell\ell_t = \Gamma_\pi\pi_{t+1}-\Phi_\pi\pi_t-A_\pi\pi_{t-1}-\Gamma_y y_t+\Phi_y y_{t-1}+\Gamma_q q_{t+1}+\Phi_q q_t-A_q q_{t-1}+\Gamma_b b_{b,t}-\Phi_b b_{b,t-1}-\\
\Gamma_n n_t+\Phi_n n_{t-1}-\Phi_\ell\ell_{t-1}+\Gamma_u u^i_t-\Phi_u u^i_{t-1}
\end{aligned}
\tag{A.118}
$$

$$
\begin{aligned}
\delta_q q_{t+1} = &\Big[1+(\delta_{lo}-\delta_\mu\kappa_{lo})\xi_{qb}\Big]q_t+\Big[(\delta_{lo}-\delta_\mu\kappa_{lo})\gamma_L+\delta_\mu\kappa_{r\ell}\Big]\ell_t\\
&+(\delta_{lo}-\delta_\mu\kappa_{lo})\xi_{qb}\,bb_t-(\delta_{lo}-\delta_\mu\kappa_{lo})n_t+\Big[-\ \delta_\mu\kappa_{r\ell}+(\delta_\mu\kappa_{rd}+\delta_{rd})\theta_{ix}\Big]y_t\\
&+(\delta_\mu\kappa_{rd}+\delta_{rd})\theta_{i\pi}\,\pi_t+\Big[\delta_\mu\kappa_{r\ell}-(\delta_\mu\kappa_{rd}+\delta_{rd})\Big]\pi_{t+1}+(\delta_\mu\kappa_{rd}+\delta_{rd})u^i_t.
\end{aligned}
$$

Parameters consolidated and computed gives:

$$q_{t+1} = \Psi_q \, q_t + \Psi_\ell \, \ell_t - \Psi_b \, b_{b,t} + \Psi_n \, n_t - \Psi_y \, y_t + \Lambda_\pi \, \pi_t - \Psi_\pi \, \pi_{t+1} + \Psi_u \, u_t^i. \tag{A.119}$$

Rearranging the NKPC, we have:

$$\pi_{t+1} = \frac{1}{\beta}\pi_t - \frac{\lambda}{\beta}(\gamma y_t - \delta_l \ell_t) \tag{A.120}$$

Forwarding equation 130 by one period:

$$\begin{aligned} \Gamma_\ell \, \ell_{t+1} = \Gamma_\pi \, \pi_{t+2} - \Phi_\pi \, \pi_{t+1} - A_\pi \, \pi_t - \Gamma_y \, y_{t+1} + \Phi_y \, y_t + \Gamma_q \, q_{t+2} + \Phi_q \, q_{t+1} - A_q \, q_t + \\ \Gamma_b \, b_{b,t+1} - \Phi_b \, b_{b,t} - \Gamma_n \, n_{t+1} + \Phi_n \, n_t - \Phi_\ell \, \ell_t + \Gamma_u \, u_{t+1}^i - \Phi_u \, u_t^i \end{aligned} \tag{130'}$$

Plugging 131 and 132 into 130', we get:

$$\begin{aligned} -\Omega_\ell \, \ell_{t+1} = \Pi_\pi \, \pi_{t+1} - \Upsilon_y \, y_{t+1} + \Theta_q \, q_{t+1} + B_b \, b_{b,t+1} - N_n \, n_{t+1} + \Psi_\mu \, u_{t+1}^i \\ - \Phi_\pi \, \pi_t + \Gamma_y \, y_t - R_q \, q_t - X_b \, b_{b,t} + P_n \, n_t - \Lambda_\ell \, \ell_t - \S_u \, u_t^i. \end{aligned} \tag{A.121}$$

Eliminating redundant equations in 127:

$$\begin{aligned} \beta_{lo} \, \# \, \xi_{qb} \, b_{b,t+1} = {} & (\beta_{rn}\omega - \beta_{lo} \, \# \, \xi_{qb}) \, q_{t+1} - (1 + \beta_{rn} + \beta_d \phi_{qb}) \, q_t - (1 + \beta_d \phi_{qb}) \, b_{b,t} \\ & - \beta_{lo} \, \# \, \gamma_\ell \, \ell_{t+1} + (\beta_{r\ell} - \beta_d \phi_\ell) \, \ell_t + (\beta_{rd}\theta_{ix} + \beta_{lo}\theta_{ix} - \beta_{r\ell}) \, y_t \\ & + \beta_{lo} \, \# \, n_{t+1} + (\beta_d \phi_n - \beta_n) \, n_t + (\beta_{rd}\theta_{i\pi} + \beta_{lo}\theta_{i\pi}) \, \pi_t + (\beta_{r\ell} - \beta_{rd}) \, \pi_{t+1} \\ & + (\beta_{rd} + \beta_{lo}) \, u_t^i. \end{aligned}$$

Then we have:

$$\Upsilon_{\rm b}\, b_{b,t+1} = \Gamma_{\rm qf}\, q_{t+1} - \Gamma_{\rm qp}\, q_t - \Upsilon_{\rm bp}\, b_{b,t} - \Psi_{\rm ellf}\, \ell_{t+1} + \Psi_{\rm ellp}\, \ell_t$$
$$- \Lambda_{\rm y}\, y_t + \Xi_{\rm nf}\, n_{t+1} - \Xi_{\rm np}\, n_t + \Theta_\pi\, \pi_t + \Theta_{\pi f}\, \pi_{t+1} + \Omega_{\rm u}\, u_t^i + \delta_{b,t+1}.$$

Eliminating redundant equations in 125:

$$n_{t+1} = \gamma_n\, n_t + (\alpha_{r\ell} - \alpha_{rd}\theta_{ix})\, y_t + (\alpha_\ell - \alpha_{r\ell})\, \ell_t - \alpha_{rd}\theta_{i\pi}\, \pi_t + (\alpha_{rd} - \alpha_{r\ell})\, \pi_{t+1}$$
$$+ \alpha_{rn}\omega\, q_{t+1} - (\alpha_{rn} + \alpha_{qb})\, q_t - \alpha_{qb}\, b_{b,t} - \alpha_{rd}\, u_t^i.$$

Then we have:

$$n_{t+1} = \Upsilon_n\, n_t + P_y\, y_t - \Upsilon_\ell\, \ell_t - \Xi_\pi\, \pi_t - P_{\pi f}\, \pi_{t+1}$$
$$+ \Omega_{qf}\, q_{t+1} - \Omega_{qp}\, q_t - \Xi_b\, b_{b,t} - \Lambda_u\, u_t^i.$$

### *A.9 MODEL*

$$\boxed{q_{t+1} + \Psi_\pi\, \pi_{t+1} = \Psi_q\, q_t + \Psi_\ell\, \ell_t - \Psi_b\, b_{b,t} + \Psi_n\, n_t - \Psi_y\, y_t + \Lambda_\pi\, \pi_t + \Psi_u\, u_t^i + \delta_{rn,t+1}}$$

$$\boxed{-\Omega_\ell\, \ell_{t+1} - \Pi_\pi\, \pi_{t+1} + \Upsilon_y\, y_{t+1} - \Theta_q\, q_{t+1} - B_b\, b_{b,t+1} + N_n\, n_{t+1} - \Psi_\mu\, u_{t+1}^i = -\Phi_\pi\, \pi_t + \Gamma_y\, y_t - R_q\, q_t - X_b\, b_{b,t} + P_n\, n_t - \Lambda_\ell\, \ell_t - \S_u\, u_t^i}$$

$$\boxed{\Upsilon_{\rm b}\, b_{b,t+1} + \Psi_{\rm ellf}\, \ell_{t+1} - \Gamma_{\rm qf}\, q_{t+1} - \Xi_{\rm nf}\, n_{t+1} - \Theta_{\pi f}\, \pi_{t+1} = -\Gamma_{\rm qp}\, q_t - \Upsilon_{\rm bp}\, b_{b,t} + \Psi_{\rm ellp}\, \ell_t - \Lambda_{\rm y}\, y_t - \Xi_{\rm np}\, n_t + \Theta_\pi\, \pi_t + \Omega_{\rm u}\, u_t^i + \delta_{b,t+1}}$$

$$\boxed{n_{t+1} + P_{\pi f}\, \pi_{t+1} - \Omega_{qf}\, q_{t+1} = \Upsilon_n\, n_t + P_y\, y_t - \Upsilon_\ell\, \ell_t - \Xi_\pi\, \pi_t - \Omega_{qp}\, q_t - \Xi_b\, b_{b,t} - \Lambda_u\, u_t^i}$$

$$\boxed{y_{t+1} = \left(1 + \sigma\theta_{ix} + \frac{\sigma\lambda}{\beta}\right) y_t + \sigma\left(\theta_{i\pi} - \frac{1}{\beta}\right) \pi_t + \sigma u_{i,t} + \delta_{x,t+1}.}$$

$$\boxed{\pi_{t+1} = \frac{1}{\beta}\, \pi_t - \frac{\lambda\gamma}{\beta}\, y_t + \frac{\lambda\delta}{\beta}\, \ell_t - \beta_i\, \varepsilon_{i,t+1} - \beta_s\, \varepsilon_{s,t+1}}$$

$$\boxed{u_{i,t+1} = \rho_i u_{i,t} + \varepsilon_{i,t+1}.}$$

$$\boxed{u_{s,t+1} = \rho_s u_{s,t} + \varepsilon_{s,t+1}.}$$

$$\mathbf{z}_t \equiv \begin{bmatrix} y_t \\ \pi_t \\ q_t \\ u_t^i \\ u_t^s \\ \ell_t \\ b_{b,t} \\ n_t \end{bmatrix}, \qquad \boldsymbol{\varepsilon}_{t+1} \equiv \begin{bmatrix} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix}, \qquad \boldsymbol{\eta}_{t+1} \equiv \begin{bmatrix} \delta_{x,t+1} \\ \delta_{rn,t+1} \\ \delta_{b,t+1} \end{bmatrix}.$$

The model is written as

$$\underbrace{\mathbf{A}}_{8\times 8} \mathbf{z}_{t+1} = \underbrace{\mathbf{B}}_{8\times 8} \mathbf{z}_t + \underbrace{\mathbf{C}}_{8\times 2} \boldsymbol{\varepsilon}_{t+1} + \underbrace{\mathbf{D}}_{8\times 3} \boldsymbol{\eta}_{t+1}.$$

**Matrices.** With the following ordering $[x, \pi, q, u^i, u^s, \ell, b_b, n]$,

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Psi_\pi & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \Upsilon_y & -\Pi_\pi & -\Theta_q & -\Psi_\mu & 0 & -\Omega_\ell & -B_b & N_n \\ 0 & -\Theta_{\pi f} & -\Gamma_{\text{qf}} & 0 & 0 & \Psi_{\text{ellf}} & \Upsilon_b & -\Xi_{\text{nf}} \\ 0 & P_{\pi f} & -\Omega_{qf} & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$
\mathbf{B} \;=\; \begin{pmatrix}
1+\sigma t_{ix}+\frac{\sigma\lambda}{\beta} & \sigma t_{i\pi}-\frac{\sigma}{\beta} & 0 & \sigma & 0 & 0 & 0 & 0 \\
-\frac{\lambda\gamma}{\beta} & \frac{1}{\beta} & 0 & 0 & 0 & \frac{\lambda\,\delta_\ell}{\beta} & 0 & 0 \\
-\Psi_y & \Lambda_\pi & \Psi_q & \Psi_u & 0 & \Psi_\ell & -\Psi_b & \Psi_n \\
0 & 0 & 0 & \rho_i & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \rho_s & 0 & 0 & 0 \\
\Gamma_y & -\Phi_\pi & -R_q & -\S_u & 0 & -\Lambda_\ell & -X_b & P_n \\
-\Lambda_y & \Theta_\pi & -\Gamma_{\mathrm{qp}} & \Omega_u & 0 & \Psi_{\mathrm{ellp}} & -\Upsilon_{\mathrm{bp}} & -\Xi_{\mathrm{np}} \\
P_y & -\Xi_\pi & -\Omega_{qp} & -\Lambda_u & 0 & -\Upsilon_\ell & -\Xi_b & \Upsilon_n
\end{pmatrix}.
$$

$$
\mathbf{C} \;=\; \begin{pmatrix}
0 & 0 \\
-\,\beta_i & -\,\beta_s \\
0 & 0 \\
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{pmatrix}, \qquad
\mathbf{D} \;=\; \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{pmatrix}.
$$

**Solution** Define

$$
\mathbf{F} \equiv \mathbf{A}^{-1}\mathbf{B}, \qquad \mathbf{F} = \mathbf{Q}\,\boldsymbol{\Lambda}\,\mathbf{Q}^{-1} \quad \text{(eigenvalue decomposition).}
$$

Let the index set of unstable roots be $\mathcal{F} \equiv \{j:\ |\lambda_j| \geq 1\}$, and stable roots $\mathcal{B} \equiv \{j:\ |\lambda_j| < 1\}$. Form the selector matrices

$$
\mathbf{E}_f \in \mathbb{R}^{|\mathcal{F}|\times 8}, \quad (\mathbf{E}_f)_{k,\ j_k} = 1 \text{ for } j_k \in \mathcal{F}; \qquad \mathbf{E}_b \in \mathbb{R}^{|\mathcal{B}|\times 8}, \quad (\mathbf{E}_b)_{k,\ j_k} = 1 \text{ for } j_k \in \mathcal{B}.
$$

The mapping from shocks to the stable eigenspace is

$$\boldsymbol{\zeta} \;=\; \mathbf{E}_b\,\mathbf{Q}^{-1}\,\mathbf{A}^{-1}\Big(\mathbf{C} \;-\; \mathbf{D}\left[\mathbf{E}_f\,\mathbf{Q}^{-1}\,\mathbf{A}^{-1}\mathbf{D}\right]^{-1}\mathbf{E}_f\,\mathbf{Q}^{-1}\,\mathbf{A}^{-1}\mathbf{C}\Big).$$

## *A.10 Additional figures*

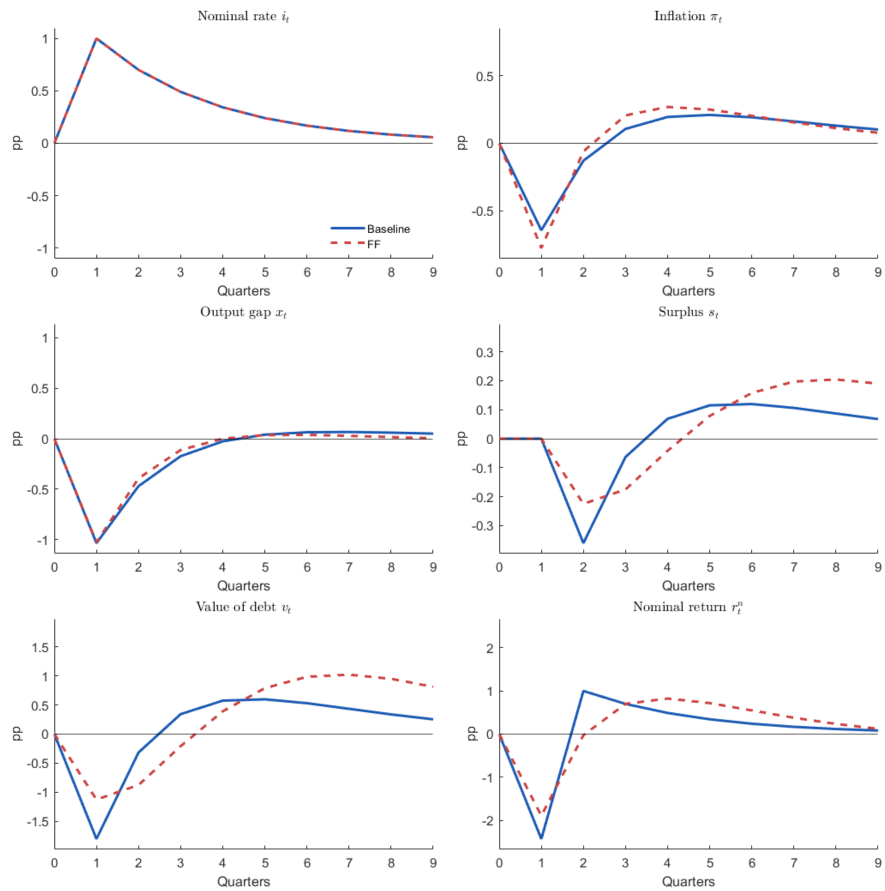



Figure 5: MP Shock - No Rules

Figure 5 shows the responses of the variables to a monetary policy shock with policy rules shut off, meaning $\vec{\theta} = \mathbf{0}$. Note that this includes the policy responses in the Taylor rule as well as the surplus equation. Therefore, surpluses purely respond to changes in debt (partially). The intuition for the dynamics of the variables in this model resembles that of Figure 2 in section 4.

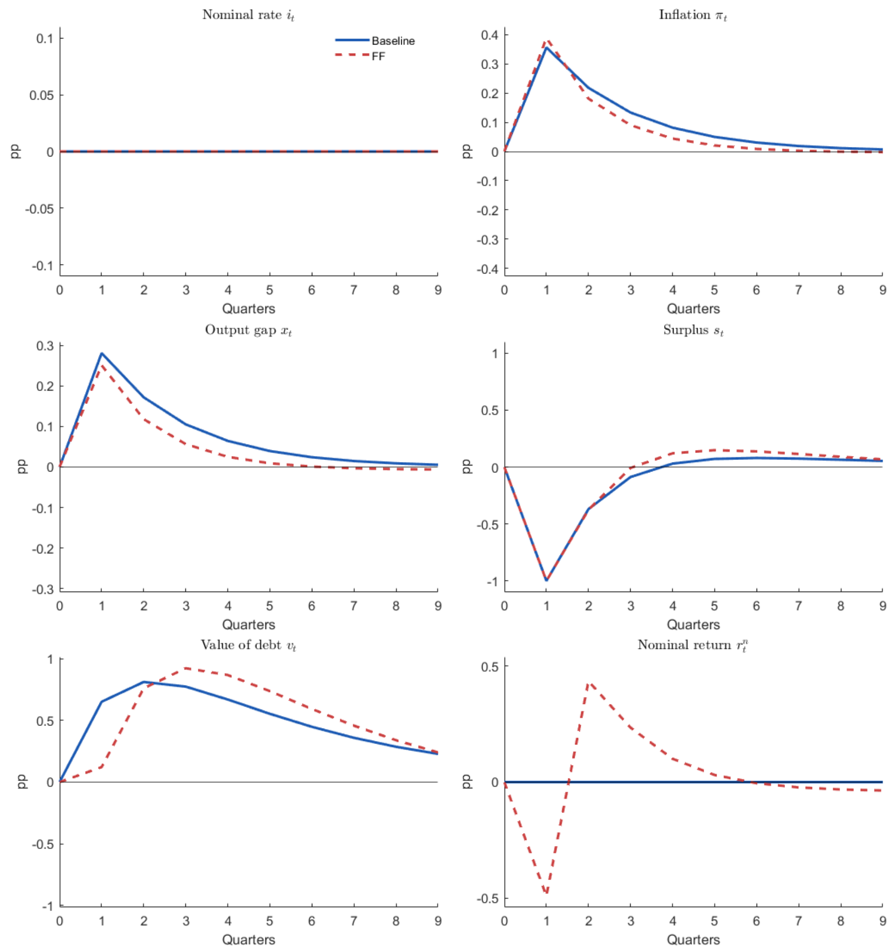
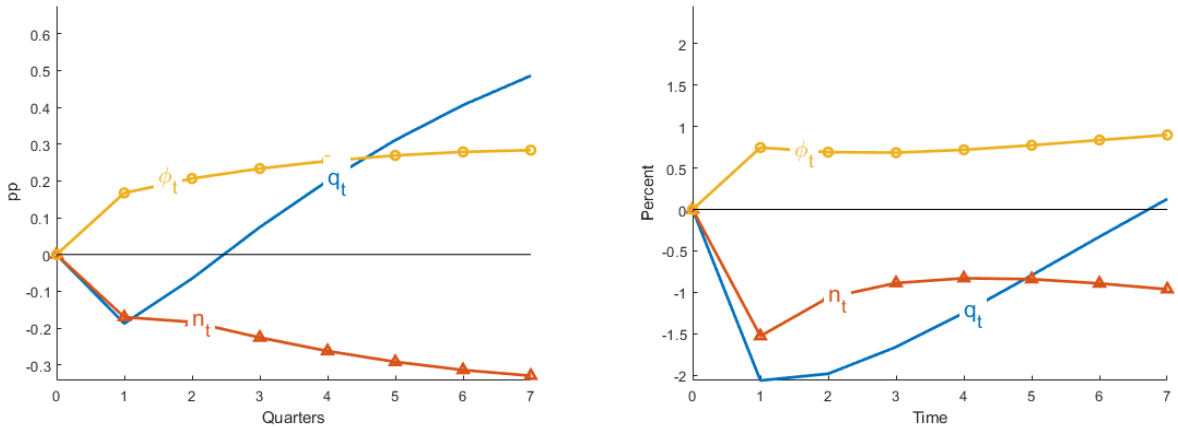



Figure 6: FP Shock - No Rules

Figure 6 is the response of the core variables to a fiscal shock (negative shock to surpluses) where policy rules are shut off. The fundamental intuition of the difference in dynamics is similar to what we encounter in Figure 3 in section 4. However, there are a few important differences. For one, the response of debt is pushed forward relative to the baseline model. This is entirely driven by the fall in $r^n$ at early horizons - whereas in the baseline, $r^n$ doesn't respond to the surplus shock. This is owing to the fact that, in the baseline, discount rates are largely driven by the policy rate. Under FF, the policy rate is an element, but financial frictions are also relevant.

This plot makes it plain that bank leverage and net worth are relevant to discount rate determination, since the policy rate doesn't even move. Going from the baseline to FF, the financial frictions are entirely responsible for the determination of the discount rate in this scenario.




**FP Shock - Rules** **MP Shock - Rules**

Figure 7: Response of Financial Variables to Shocks with Rules

Figure 7 plots the responses of key financial variables to the shocks discussed in section four (shocks with policy rules turned on). The left panel shows the responses of bond prices $q_t$, bank leverage $\phi_t$, and bank net worth $n_t$ to a fiscal policy shock (negative surplus shock). The right panel shows the same variables as they respond to a contractionary monetary policy shock.