MEASUREMENT UNCERTAINTY IN NON-LINEAR BEHAVIOURAL MODELS OF MICROWAVE AND MILLIMETRE-WAVE AMPLIFIERS

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A thesis submitted in partial fulfillment for the degree of Doctor of Philosophy

in the
Advanced Technology Institute and Department of Electronic Engineering
Faculty of Engineering and Physical Sciences
University of Surrey

September 2017

Declaration of Authorship

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Laurence Stant (Author)	Date

"What error drives our eyes and ears amiss? Until I know this sure uncertainty I'll entertain the offered fallacy."

William Shakespeare, The Comedy of Errors

"That's right!" shouted Vroomfondel, "we demand rigidly defined areas of doubt and uncertainty!"

Douglas Adams, The Hitchikers Guide to the Galaxy

Abstract

Abstract goes here

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Acknowledgements

I want to thank...

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1 Introduction

1.1 Wireless Communications

1.2 Amplifier Measurement and Modelling

1.3 The Role of Uncertainty in Measurement

1.4 Thesis Structure

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2 Radio Frequency and Microwave Measurements

2.1 Introduction

To characterise nonlinear behavioural models, the radio frequency (RF) response of a device to electromagnetic wave stimuli must be measured. When compared with DC (and low frequency) measurements, RF and microwave measurements present significant additional challenges. For DC systems, it is desirable to propagate voltages through a circuit with minimal loss in amplitude. To achieve this effectively, components are typically designed with high input impedance and low output impedance. With RF systems, circuit components and interconnects can be of the order of a quarter-wavelength in length, and therefore signals must be treated as electromagnetic waves to account for different behaviour at these frequencies. When a travelling wave encounters a discontinuity in impedance, such as a cable connector or on-wafer structure, some of the power in the wave is reflected. The amount of reflected power is proportional to the size of the impedance mismatch between each side of the discontinuity. Hence, for RF systems, the transmission of power is the focus of the circuit designer. The measurement of power flowing through a transmission line is complicated by three key factors. Firstly, because the waves are travelling, the instantaneous voltage at any point on the transmission line will vary between the peak-to-peak values of the wave. Secondly, there are waves travelling in both directions along the transmission line which must be measured separately. Finally, the power of the wave is a complex quantity which consists of both magnitude and phase. To perform these measurements, a specialist instrument called a vector network analyser (VNA) can be used. In this chapter, the concepts and measurements associated with this instrument are introduced, which will be used later in the thesis to understand the uncertainty contributions from measurements to nonlinear behavioural models.

2.2 Electromagnetic Wave Parameters

2.2.1 Wave Definitions

To describe the power of electromagnetic waves propagated through a transmission line, several definitions are in use in industry and academia for either accuracy or convenience. To avoid confusion in this document, these will now be defined. Information presented in this section has been obtained from [1-5].

Travelling Waves

Travelling waves represent a solution to Maxwell's equations along a transmission line. They are physical and measurable via slotted line experiments or thru-reflect-line calibrations [6] (see Chapter X). Travelling waves are defined by the total transverse electric and magnetic fields $\boldsymbol{E}_\mathrm{t}$ and $\boldsymbol{H}_\mathrm{t}$ of a single propagating mode at each frequency:

$$\boldsymbol{E}_\mathrm{t} = c^{+}e^{-\gamma z}\boldsymbol{e}_\mathrm{t} + c^{-}e^{+\gamma z}\boldsymbol{e}_\mathrm{t}, \quad \boldsymbol{H}_\mathrm{t} = c^{+}e^{-\gamma z}\boldsymbol{h}_\mathrm{t} - c^{-}e^{+\gamma z}\boldsymbol{h}_\mathrm{t} \tag{2.1}$$

where, following the notation of [5], $\boldsymbol{e}_\mathrm{t}$ and $\boldsymbol{h}_\mathrm{t}$ are the un-normalized electric and magnetic fields of the modal solution of Maxwells equations in transmission line, $\gamma = a + ib$ is the complex propagation constant of the mode, z is the direction of propagation, and c^{+} and c^{-} are complex quantities representing the un-normalized forward and backward amplitude of the mode, respectively.

Equivalent-Circuit Voltage and Current

To represent travelling waves as equivalent low frequency circuit parameters such as voltage and current, a normalisation is chosen to derive a characteristic impedance for the transmission line. This normalisation takes the form

$$\boldsymbol{E}_\mathrm{t}(z) = \frac{v(z)}{v_0}\boldsymbol{e}_\mathrm{t}, \quad \boldsymbol{H}_\mathrm{t}(z) = \frac{i(z)}{i_0}\boldsymbol{h}_\mathrm{t}, \tag{2.2}$$

where v_0 and i_0 are normalisation constants that allow v and i to take units of root-mean-square voltage and current, respectively [5].

Pseudowaves

Equivalent voltages and currents cannot be used in lossy transmission lines where the electric and magnetic fields are out of phase. To account for this and provide a solution which can be used with conventional circuit design methodologies (e.g. Smith chart techniques [7]) and simulators, pseudowaves can be used. This representation is defined with a reference impedance, Z_{ref}, which can be chosen by the user, but is typically 50-Ω in conventional measurements. The forward and backward pseudowaves a and b can be written as:

$$a(Z_{\text{ref}}) = \left[\frac{|v_0|}{v_0}\frac{\sqrt{\Re(Z_{\text{ref}})}}{2|Z_{\text{ref}}|}\right](v + iZ_{\text{ref}}), \quad b(Z_{\text{ref}}) = \left[\frac{|v_0|}{v_0}\frac{\sqrt{\Re(Z_{\text{ref}})}}{2|Z_{\text{ref}}|}\right](v - iZ_{\text{ref}}) \tag{2.3}$$

Power Waves

Finally, power waves are defined so that the relationship $P = |a|^2 - |b|^2$ is true for any reference impedance, where P is the power transmitted through the transmission line and a and b are the forward and backward power waves, respectively. They are defined as:

$$a(Z_{\text{ref}}) = \frac{v + iZ}{2\sqrt{\Re(Z_{\text{ref}})}}, \quad b(Z_{\text{ref}}) = \frac{v - iZ}{2\sqrt{\Re(Z_{\text{ref}})}}. \tag{2.4}$$

Data taken from Keysight instruments used later in this work is presented in power wave format, with units of square-root Watts. To convert these values into decibels referenced to 1 milliwatt, the following formula is used:

$$P(\text{dBm}) = 10\log_{10}(P(\sqrt{W})^2) + 30 \tag{2.5}$$

2.2.2 Derived Metrics and Figures of Merit

The behaviour of a linear microwave device can be completely defined by the complex ratio of electromagnetic waves which are scattered at each port to those which are incident at each port. The combination of these ratios constitutes the scattering parameters (s-parameters) of a microwave device and are used extensively in the design and measurement of microwave systems[ref]. The formal definitions of the s-parameters for a two-port device are

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}, \quad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}, \quad S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}, \quad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}, \tag{2.6}$$

where both a and b can be expressed in either pseudowave or power wave representation. The term scattered can be interchanged with transmitted and reflected depending on if the scattered wave is output on a different port, or the same port, to the incident wave, respectively. A signal

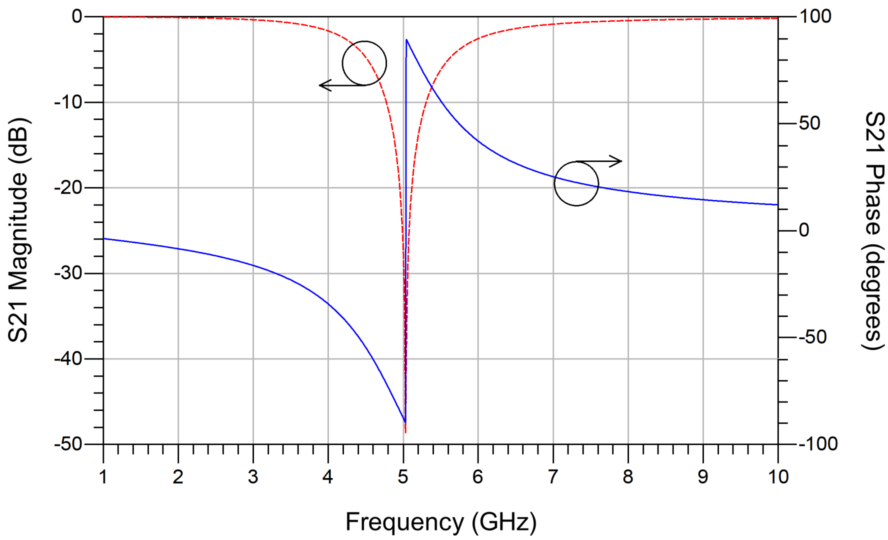

Figure 2.1: The frequency dependence of the magnitude (red dotted trace) and phase (blue solid trace) of S_{21} for a bandstop filter.

flow diagram is provided in Fig. 1 showing the relationship between equivalent-circuit voltage and current, pseudowaves/power waves and s-parameters for a two-port device. The s-parameters of all microwave devices will exhibit some degree of frequency dependence. This effect originates from physical processes occurring in the device and can either be a benefit or hinderance to a design. Most passive components (including cables) will have a usable bandwidth which is an unwanted limitation, whereas microwave filters are a ubiquitous component where the same fixed bandwidth is the main purpose of the device. To capture this frequency dependence, s-parameters are measured across a frequency range and stored in a table, usually in Touchstone format (see Fig.2). An example of the frequency dependence of a filter is shown in Fig. 3. For a device operating in the linear regime, if multiple stimuli at different frequencies are incident on the device, they will not interact with each other. The scattered waves will have the same frequency components as if the stimulus at each frequency was applied separately. This is called the frequency superposition principle and does not apply to nonlinear operating regimes, which will be discussed later in this chapter.

Scattering parameters are often expressed in matrix form, where the column index is the scattered port, and the row index is the incident port. For a two-port device, the s-parameter

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6.00000000000000E+9 1.53465270996094E-2 -6.19812011718750E-2 1.51917338371277E-5 5.5294483900070
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Figure 2.2: An short example Touchstone file showing a two-port measurement at two frequencies. The rows continue to the right of the figure.

matrix would be

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{2.7}$$

The most interesting characteristic of a two-port microwave device is often the effect which it has on a transmitted wave in the forward direction (S_{21}). If the device increases the magnitude of the incident signal this metric is called gain, otherwise it is called insertion loss. Typically gain is associated with active devices (those which are powered from an external source separate to the incident microwave signals) such as amplifiers, and insertion loss is associated with passive devices (those with no external power source) such as attenuators, splitters and mixers. The power gain (operating gain) and insertion loss relating to S_{21} can be calculated using

$$\text{Power Gain} = 10 \log_{10} |S_{21}|^2 \text{ dB}, \tag{2.8}$$

and

$$\text{Insertion Loss} = -10 \log_{10} |S_{21}|^2 \text{ dB}, \tag{2.9}$$

respectively.

Optimal transmission in microwave systems requires impedance matching between components, and it is inevitable that this matching will not be perfect and so some power will be reflected in a two-port device. Therefore, the match of a device is another important measurement, which is dependent on the voltage reflection coefficient (Γ) of the device and can be related to the impedance of a source and load by

$$\Gamma_{xx} = S_{xx} = \frac{Z_\text{L} - Z_\text{S}}{Z_\text{L} + Z_\text{S}}, \tag{2.10}$$

where x is a port index. A more thorough definition of voltage reflection coefficient for a two-port device includes any effect from the impedance seen at the other port, and for the case of input match is calculated as

$$\Gamma_{11} = S_{11} + \frac{S_{12}S_{21}\Gamma_\text{L}}{1 - S_{22}\Gamma_\text{L}}, \tag{2.11}$$

where Γ_L is the voltage reflection coefficient of the load connected to the device. For amplifiers, the amount of isolation (reduction of S_{12}) is an important characteristic of the device, whereby a fully isolated amplifier ($S_{12} = 0$) is said to be unilateral and equations 2.10 and 2.11 are equivalent.

For active devices, such as amplifiers, it can also be useful to consider the power reflected at the input when calculating the power gain of the device. The transducer gain of a device accounts for this potential loss of power at the input and provides a more portable metric which is not dependent on the impedance of the measurement setup. It is defined as

$$G_T = \frac{1 - |\Gamma_S|^2}{1 - |\Gamma_{in}\Gamma_S|^2} |S_{21}|^2 \frac{1 - |\Gamma_L^2|^2}{1 - |S_{22}\Gamma_L|^2}, \tag{2.12}$$

where Γ_{in} is the input match of the device.

For all devices operating in the linear regime, any reflected or transmitted wave will have a frequency equivalent to that incident to the device. In addition, the stimulus power that was used to measure the s-parameters is not important as the ratio of scattered to incident wave magnitude is not dependent on this quantity. However, when microwave devices operate in the nonlinear regime, these conditions no longer apply, and s-parameters cannot be used to capture the full behaviour of the device.

2.3 Measurements of Nonlinear Devices

Microwave devices operating in the nonlinear regime exhibit three differences from their linear counterparts which are significant to the designer:

1. The amplitude of electromagnetic waves scattered from the device are not linearly dependent on the amplitude of waves incident. This is the cause of features such as gain compression and gain expansion in amplifiers. Some of these effects are solely due to the nonlinear sources inside the device, while others are a symptom of the combined response of the nonlinearity and the power supply. A typical gain compression curve is shown in Fig. 4.

2. The frequency superposition principle does not apply, and instead the frequency spectrum of scattered waves contains components at frequencies other than those incident upon it. Rather than the incident signals purely summing inside the device, they are also multiplied

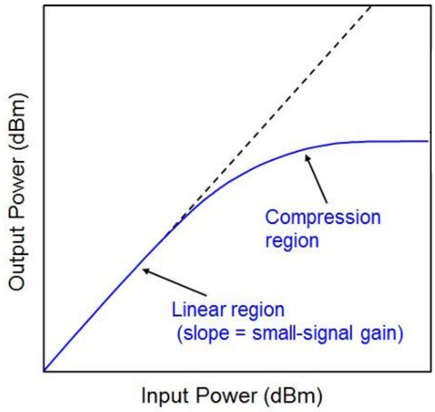

Figure 2.3: Gain compression occurs when an amplifier is driven into a nonlinear operating regime.

with each other (frequency mixing), as shown by

$$b = c_0 + c_1 a + c_2 a^2 + c_3 a^3 + \cdots , \tag{2.13}$$

$$\alpha = \beta = 2\pi\omega t, \tag{2.14}$$

$$a(t) = A\cos(\alpha), \tag{2.15}$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}(\cos(\alpha+\beta) + cos(\alpha-\beta)), \tag{2.16}$$

$$a^2(t) = \frac{1}{2}A^2[\cos(2\pi(2\omega)t) + 1], \tag{2.17}$$

$$a^3(t) = \frac{1}{4}A^3[\cos(2\pi(3\omega)t) + 3\cos(2\pi\omega t)]. \tag{2.18}$$

Here, a and b are the incident and scattered waves for the device, c_i are coefficients of the device's nonlinear transfer function, and $a(t)$ is a wave in the time domain with amplitude A and frequency ω. For stimuli with a single frequency (α=β, as above), integer multiples of that frequency will be scattered from the device (harmonics). For stimuli with multiple tones ($\alpha \neq \beta$), additional products from combinations of the incident tone frequencies will be scattered (intermodulation). If the nonlinear device is incident with a fixed bandwidth of frequencies, such as the case for communications signals, then sidebands will be produced around the harmonics of the oscillator frequencies. This effect can be troublesome in practical designs where the unwanted sidebands overlap with the useful microwave bandwidth, distorting the signal. For this reason, it is important for designers to be able to accurately measure and characterise this nonlinear effect. Fig. 5 shows example spectra of these effects.

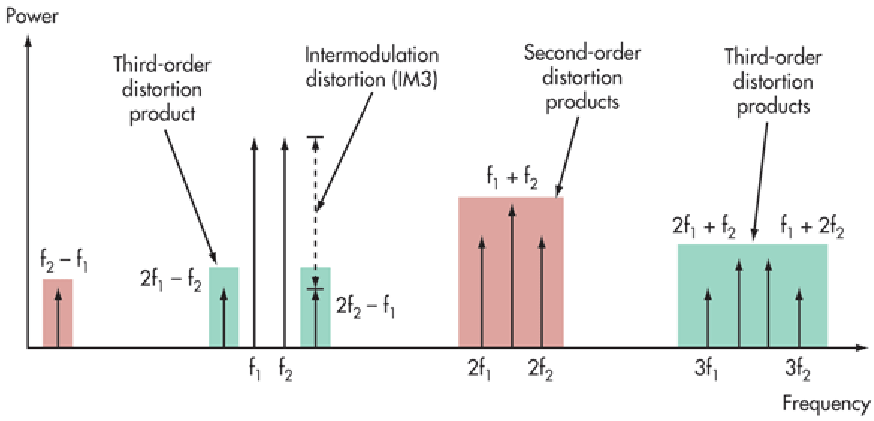

Figure 2.4: Intermodulation products from two tones within the cellular channel bandwidth f_1 and f_2. The second order products, and upper third order products, can be easily filtered out. However, the lower third order products $2f_1 - f_2$ and $2f_2 - f_1$ are located within the channel bandwidth and interact with the useful data, increasing EVM and BER **Hall2013**.

3. The amplitude of scattered waves with multiple incident waves is dependent (nonlinearly) on the phase of the incident waves. In the linear regime the superposition principle prevents this, but now there is a nonlinear dependence which can have significant effects on the amplitude of scattered waves. Designers must consider this when building efficient nonlinear amplifiers, which leads to the practice of accurately terminating scattered harmonic frequencies at an optimum phase. This will be covered in more detail in chapter X when we discuss nonlinear device models.

The result of these differences is that the measurement requirements for nonlinear devices are considerably larger than for linear devices. The nonlinear dependencies on stimulus power and phase means that ratioed measurements no longer fully capture the device response, and absolute measurements of the magnitude and phase of both the incident and scattered waves is required. The production of scattered waves at frequencies different to those in the stimulus demands an additional dimension of measurements. In contrast to These complications must be met with changes to both the measurement system and the method of storing the results.

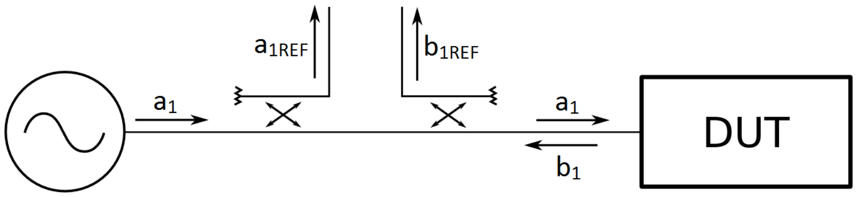

Figure 2.5: A one-port simple reflectometer. a_1 is the incident wave generated by the source, which is admitted to the DUT while also being sampled by the directional coupler and sent to the reference receiver via a_{1REF}. The reflected wave, b_1, is also sampled by another directional coupler and sent to the test receiver as $b_1 REF$, with the remaining power dissipated at the matched source.

2.4 Vector Network Analysers

To measure the incident and scattered waves for a DUT and calculate the s-parameters as in (5), a vector network analyser (VNA) is typically used. The VNA is a quintessential piece of RF and microwave instrumentation and is found in most if not all such laboratories. Due to the challenging nature of measurements at these frequencies, it is a complicated instrument with many internal parts. This section explains how the VNA functions and the procedures behind its calibration. For a good history of VNA architecture and product development please refer to [Teppati Camb, Dunsmore Wiley].

2.4.1 Architecture

The origin of the VNA lies in an early instrument called a reflectometer. Designed in 1947 by Parzen and Yalow [x], it became an invaluable tool for characterising transmission lines used in telecommunication systems. Shown in Figure x, the incident signal is generated by a swept signal source and passes through the directional coupler before arriving at the DUT. The voltage reflection coefficient of the DUT will cause an amount of incident power to be reflected, which passes back through the coupler before being absorbed by the source (which has very low reflection). The directional couplers allow the waves travelling between the source and the DUT to be sampled by complex receivers, filtering the two waves by their direction of travel thus allowing the incident and scattered waves to be separated for measurement.

The limitation of a single reflectometer is that it can only measure waves at one port of a

DUT, therefore preventing transmission measurements. By adding a second reflectometer and synchronising the stimuli and measurements, it is possible to measure all s-parameters of a two-port device. This is the fundamental structure of a VNA, and most variations consist of changing the number of sources or receivers to optimise the instrument for cost or performance. Many older designs use an economical single source which is switched between both ports, whereas now the price of sources has falled, there are instruments available with two independent sources, which allows two-tone and some types of nonlinear measurements. These more versatile units often also expose more connections between internal components (e.g. the couplers and receivers) to allow the user to perform non-standard measurements or to add attenuation or preamplification for extreme stimulus powers. Modern VNAs also offer the option of measuring more than two ports, which are referred to as multi-port measurements. Several manufacturers offer four-port instruments which include four reflectometers (with usually two sources), although with external switching networks it is possible to expand this up to 48 ports [http://www.microwavejournal.com/articles/21785-vector-network-analysis-with-up-to-48-ports]. The basic block diagram of a modern two-port double-reflectometer VNA is shown in Fig. 9. To measure both stimulus conditions for the two-port S-parameter equations in (5), the sources alternate between delivering power and acting as a load for each measurement. As the source is swept the a and b waves for all ports are measured against frequency, from which the VNA software calculates the S-parameters. The receivers sampling the incident waves are known as the reference receivers and those sampling the scattered waves are called measurement or test receivers.

To perform S-parameter measurements using a VNA, the user must set both the frequency span and number of frequency points. They may also change settings of intermediate frequency bandwidth (IFBW) and numerical averaging, both of which reduce measurement noise by applying digital filtering but can consequently increase acquisition time. The user will then perform a calibration, which corrects for any response present in the measurement setup that is not caused by the DUT. When the system is calibrated physical measurement planes are defined, where only effects of the signal path on the DUT side of the planes are incorporated in the measurement results. This is illustrated in Fig. 10. Once this step is complete, the VNA is ready for use. However, it is good practice to first check that calibration was successful by measuring some known devices (verification), or to use techniques such as ripple extraction (discussed in Chapter 4) to measure the residual uncertainty. This process characterises remaining error which the calibration failed to correct.

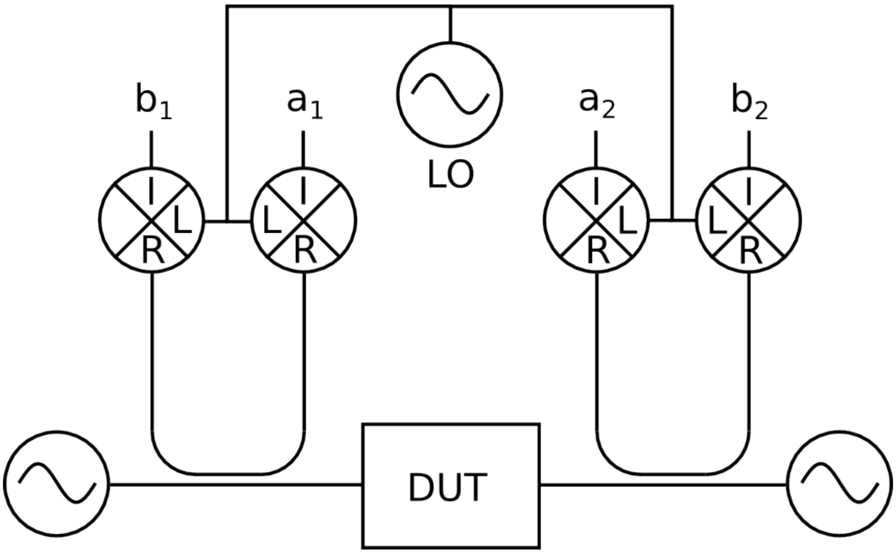

Figure 2.6: A modern two-source mixer-based VNA, which employs heterodyning to allow measurements at microwave frequencies. Two directional couplers are located between each source and the DUT and are connected back to back. These sample waves travelling in both directions and are connected to mixers which downconvert the microwave frequencies (R) into intermediate frequencies (I) which can be sampled by the complex receivers. The shared local oscillator (LO) feeding the mixers preserves phase coherence between the receivers. This configuration is known as a two-port double-reflectometer VNA. Figure adapted by author from **Root2013**.

2.4.2 Error Models

To remove the effect of the measurement setup from the measured device response, an error model is formed to capture the response of the measurement setup during calibration. These error models are stored in the memory of the VNA and are typically de-embedded from the measured device response before the results are presented to the user (although the raw measurements can still be obtained for separate post-processing). Because the measurement setup response is frequency dependent, the error model coefficients are characterised across the measurement bandwidth and are either applied at each measurement frequency or linearly interpolated.

One-Port Model

The classic one-port error model can be obtained through analysis of the signal flow diagram of a one-port VNA shown in Fig. 6. One can write the relationship between the measured (Γ_{M}) and absolute (Γ_{A}) reflection coefficients as

$$\Gamma_{\mathrm{M}} = D + \frac{T\Gamma_{\mathrm{A}}}{1 - M\Gamma_{\mathrm{A}}}, \tag{2.19}$$

where D, M and T are error coefficients which capture the unwanted response of the measurement setup. For this model, the three coefficients each have a physical meaning as they are caused by separate physical effects (illustrated in Fig. 7):

- **Directivity** (D) is caused by the nonideal operation of the directional couplers used to separate the incident and reflected waves inside the VNA. In practice, some amount of incident wave will travel into the test receiver port (vica versa??), reducing the measured gain of the device under test.

- **Test port match** (M) results from the impedance of the VNA test port (either the original test port or the extended measurement plane including any cables or other components in the setup) being different from the characteristic impedance of the measurement, which is typically 50-Ω. This effect will cause some of the incident wave to be reflected at the test port which is not due to the device response.

- **Reflection tracking** (T) characterises the insertion loss of the couplers and other measurement components between the reference receiver and the test receiver.

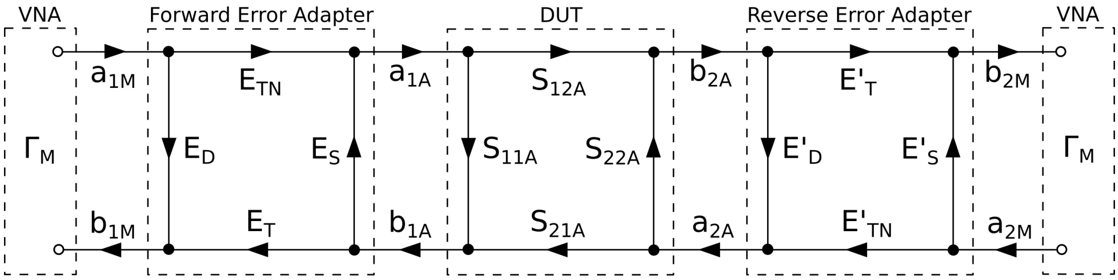

Figure 2.7: The 8-term error model for a two-port measurement. E_D, E_S, and E_T are the same as for the one-port model, except there are now sets of each for both ports. These extra terms account for different error values when the incident signal is sourced from each port. Additionally for the two-port case, a transmission term E_{TN} has been added for each direction.

8-Term Model

Devices with two or more ports require transmission measurements in addition to the reflection measurements which can be corrected using the one-port model. A popular two-port error model, the 8-term model, adds two transmission terms to the method used for the one-port model. This is shown in Fig. blah.

2.4.3 Calibration

2.5 Large Signal Network Analysers

2.5.1 Absolute 8-Term Error Model

2.5.2 Power Meter Calibration

2.5.3 Phase References

2.6 Conclusions

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3 Measurement Uncertainty

3.1 Introduction

A measurement is an observation of a physical effect or quantity which provides useful information. This information, through the ages, has been used to facilitate advancement of both scientific knowledge and industrial development - from the production of standardised stone blocks to build the pyramids of ancient Egypt, to the production of standardised car parts to build Henry Ford's Model T. In the scientific realm, advanced measurement techniques at laboratories such as CERN are used to convince the world that new subatomic particles exist.

To communicate information about a measurement, the recipient needs to be able to either make or imagine a similar observation to that of the original measurer (or metrologist). The simplest way of doing this is to provide the recipient with the same physical effect or quantity for which to make their own observation (if you require a new nut for a bolt from a hardware shop, you might intuitively take the bolt with you), however, this can be inconvenient or impractical with larger objects, or if the recipient is located far away. Instead, you might substitute the physical effect or quantity with a more portable representation. For example, if you were to measure the size of a doorway to see if a new piece of furniture may fit through it, you might cut a piece of string to the same length and use this as the representation of the width of the item. However, this approach is very wasteful and also impractical for many physical effects (temperature, flow, pressure).

A solution widely thought to have been first established in the 3rd or 4th Millennium BC (see Figure 3.1), is a system of units. In such a system, a discretised value of a quantity is standardised and knowledge of its value is disseminated to all people who wish to use it. Typically, a range of discrete values are chosen, such that the system of units can be conveniently used to represent all measurements. Knowledge of the discretised values is obtained from a primary standard which

Figure 3.1: Egyptian royal cubit rod of Maya (treasurer of King Tutankhamun) 1336–1327 BC. The cubit is thought to be the earliest attested standard measure of length, first used in the 3rd or 4th Millennium BC.

becomes the definition of the unit and is used to create copies of the standard which can be given to users of the unit system to perform measurements with. The most common method of performing measurements with a unit system is to use a standard to calibrate a measuring instrument, which can then be used to measure an arbitrary value of a quantity in the units defined by the standard.

The introduction of a regulated system of units enables commerce, as traded goods can be reliably valued between merchants across cities. This application is encountered by all citizens, and so there is a high demand for standards to be produced from the primary standard and physically distributed. It becomes impractical to create all standards by copying the primary standard directly (in some cases because the value of the primary standard is perturbed each time it is measured), and so a tiered organisational structure of standards is used. In this structure, there is a tier consisting of a small number of standards which are created directly from measurements of the primary standard, followed by subsequent tiers of larger numbers of standards which are derived from measurements of those in the previous tier. For any standard produced, it should be possible to trace the lineage back to a measurement of the primary standard. This is referred to as a traceability chain (see Figure 3.2) and it is a fundamental tenet of metrology. Measurements with a shorter traceability chain are considered more traceable than those with longer chains.

Today, the primary standards are maintained in most countries by a National Measurement Institute (NMI) and co-ordinated by the Bureau of International Weights and Measures (BIPM). To accommodate international trade and compatibility, a routine process of inter-comparisons is undertaken to ensure that the values of the primary standards between countries are in agreement.

Secondary standards are also kept by the NMIs and are used to reduce excessive wear to the primary standard caused by frequent measurements (and also to reduce bottlenecks caused by having a single standard). They are calibrated against the primary standard as infrequently as possible, again to reduce wear. Secondary standards are used by the NMI to characterise working standards which are sent to them by manufacturers and research institutes. Another important task of each NMI is to perform investigations to discover new and improved methods of measurement, which make use of secondary standards to better compare the accuracy of different methods.

Working standards are used, for example, by instrumentation manufacturers who may use them to calibrate their products before shipping to the customer, and more generally the standards can be used to calibrate test equipment to identify faulty products. Larger research institutes typically use working standards to recalibrate instrumentation prior to performing very sensitive measurements. To ensure that product specifications and scientific measurements are traceable and of high quality, accreditation services such as the United Kingdom Accreditation Service (UKAS) exist to certify manufacturers and laboratories that demonstrate good measurement practice and use traceable measurements [2].

The selection of quantities for which primary standards are kept is only a subset of those for which recognised units exist. This is because many units are derived quantities, where their value can be obtained by calculation using definitions of other units. For example, the definition of the unit of resistance (R, ohms) can be derived from that of voltage (V, volts) and current (I, amperes), because R=V/I. The eight fundamental "base" units which make up the International System of Units (SI), are the metre, kilogram, second, ampere, kelvin, candela and mole. From these unit definitions, it is possible to define any other derived unit in use. NMIs will usually keep secondary standards of most derived quantities that users may wish to calibrate against, which are traceable to one or more primary standards of the base units. Although traditionally all primary standards were defined by physical artefacts (e.g. metallic weights, burning candles), these are being gradually replaced by definitions involving physical constants (e.g. Plank, Boltzmann), which do not degrade over time or use. The "Ninth SI Units" [3], a proposition recently accepted by the BIPM, covers the redefinition of four of the SI units (the ampere, the kilogram, the kelvin and the mole) which will come into effect by May 2019.

The crucial effect of traceability on measurements is the confidence in their results. Measurements with poor traceability (longer chains) will produce results which are likely to be less accurate than those with better traceability (shorter chains). The reason for this is measurement uncertainty, which will now be explained.

It is impossible to know the true value of a quantity being measured as many undesirable physical effects typically occur during the measurement process. These effects contribute error (an unwanted perturbation) to the measured value, causing a reduction in accuracy (the deviation of the measured value from the true value). Typical sources of error in measurement include thermal noise, imperfect calibration and drift of environmental conditions from those at which a measuring instrument was calibrated. In some cases, it is possible to quantify and correct for these errors, but there are often many sources (some of which contribute very small errors) which cannot be corrected for. This is because either the error cannot be quantified or the value of the error will change over the duration of the measurement process (random errors). Any source of error which cannot be removed from a measurement becomes a source of uncertainty, because the deviation of the measured value from the true value due to this source of error is uncertain. If it is possible to quantify the amount of uncertainty in a measurement, then a degree of confidence can be formed about its value. If every measurement has an associated uncertainty in its value, then any measurement involving the results of previous measurements will include uncertainty contributions from both measurements. Measurements with good traceability involve fewer sources of uncertainty than those with poor traceability, leading to a higher degree of confidence in the former. It is because of this fact that NMIs strive to reduce the uncertainties in their primary standard definitions, which in turn reduces the uncertainty in all traceable measurements.

Because it is impossible to know the amount of error in a source of uncertainty, probability and statistical theories are used to instead describe the amount of uncertainty associated with it. By the nature of these theories there are often several methods which can be used to obtain a result, which sometimes provide different values. To ensure consistency and portability of uncertainty definitions, measurement guides were created in each industry and area of science, which specialised in processing the results of typical measurements. In addition, different guides were produced depending on the level of accuracy required - as more accurate measurements often require more effort to complete. Although this practice allowed suitable measurement comparisons within each field (e.g. chemistry, mechanical engineering), ambiguities still existed in uncertainty definitions between fields. To address this, a landmark document was published in 1993 by the International Organisation for Standardisation (ISO), the Guide to the Expression of Uncertainty in Measurement (GUM) [4]. This document was the work of representatives from seven international organisations: the BIPM, the International Organisation of Legal Metrology (OIML), the International Electrotechnical Commission (IEC), the ISO, the International Federation of Clinical Chemistry and Laboratory Medicine (IFCC), the International Union of Pure and Applied Chemistry (IUPAC), and the International Union of Pure and Applied Physics (IUPAP). The GUM, updated in 2008 [5], is still used today as a reference for the evaluation of measurement uncertainty in many laboratories and industries across the world. The seven original organisations which wrote the GUM, together with the International Laboratory Accreditation Cooperation (ILAC, of which UKAS is a member), form the Joint Committee for Guides in Metrology (JCGM), who maintain the GUM and subsequent additional documents. These additional documents consist of the International Vocabulary of Metrology (VIM) [6] and two supplements to the GUM [7,8]: Supplement 1 covers the use of a Monte Carlo method [9] in uncertainty evaluation; Supplement 2 is used where more than one quantity is measured at the same time (multivariate). Throughout this dissertation, the methodologies presented in the GUM will be used. The international authority of the guide, developed by seven international organisations (including the two global standardisation bodies IEC and ISO), gives strong motivation to use it as a basis for a framework to evaluate uncertainty in measurement. This Chapter describes the evaluation of uncertainty prescribed in the GUM and highlights an inconsistency in the current version of the GUM and associated documents (which can have a profound effect on electromagnetic measurements).

3.2 The Measurement Process

In contrast to basic evaluations of uncertainty, where only repeat measurements of the quantity of interest are analysed, the GUM prescribes a more rigorous approach, which defines a mathematical model of the measurement process (measurement model) and propagates uncertainty through that model to the result (measurands). This allows any uncertainties from previous measurements, including those involving standards in the traceability chain, to be included in the result. The measurement model can be simple, such as measuring resistance using input quantities of voltage and current, or complicated and multivariate, requiring many input quantities and producing many output quantities. In some cases, the measurement model may not be known and can be defined as a black box, but this has certain limitations discussed later with Monte Carlo methods.

The GUM defines a process that is to be followed when evaluating uncertainty in measurement. It consists of the following steps:

1. Modelling the measurement.

2. Evaluating standard uncertainty of input quantities.

3. Determining combined standard uncertainty of the measurands.

4. Determining expanded uncertainty of the measurands.

where standard uncertainty is an uncertainty expressed as a standard deviation and expanded uncertainty defines an interval encompassing a large fraction of the distribution of values that could reasonably be attributed to the measurand.

3.2.1 Modelling the Measurement

3.2.2 Evaluating Standard Uncertainty of Input Quantities

Category A Evaluation

Category B Evaluation

3.2.3 Evaluating Combined Standard Uncertainty

Monte Carlo Methods

Law of Propagation of Uncertainty

Finite Difference Methods

3.2.4 Expanded Uncertainty and Coverage Intervals

3.3 Sensitivity Analysis

3.4 Conclusions

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4 Evaluating Uncertainty in Vector Network Analyser Measurements

4.1 Introduction

4.2 Evaluating Residual Error in VNA Calibrations

4.2.1 Imperfect Calibration Standards

4.2.2 The Ripple Technique

4.2.3 Application of The Ripple Technique at Sub-Millimetre Wavelength in Rectangular Metallic Waveguide

4.3 Sources of VNA Measurement Uncertainty

4.3.1 Calibration Standards

4.3.2 Sources of Random Error

4.3.3 Additional Sources

4.4 Software Frameworks for VNA Uncertainty Evaluation

4.4.1 Standalone Vendor Tools

4.4.2 Keysight PNA-X Dynamic S-Parameter Uncertainty Option

4.4.3 METAS VNA Tools II

4.4.4 NIST Microwave Uncertainty Framework

4.5 Sources of MHVNA Measurement Uncertainty

4.5.1 Power Calibration

4.5.2 Phase References

5 Propagating Measurement Uncertainty into Nonlinear Behavioural Models

5.1 Introduction

5.2 The X-Parameter Model

5.2.1 Model Definition

5.2.2 Extraction Procedure

5.2.3 Applications of X-Parameters

5.3 Design and Simulation using Nonlinear Behavioural Models Incorporating Measurement Uncertainty

5.4 Conclusions

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6 Applications of Nonlinear Behavioural Models Incorporating Measurement Uncertainty

6.1 Introduction

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6.2 Prediction of Optimum Load Match and Delivered Power using X-Parameters Incorporating Measurement Uncertainty

6.3 Conclusions

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7 Conclusions

7.1 Further Work

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