24.1 Electric Flux

The product of the magnitude of the electric field and surface area perpendicular to the field is called **electric flux** Φ_E [N $\cdot$m^2/C]

We notice that since we only care about the field lines perpendicular to the surface, we can find the angle between the field line and a normal vector to the surface.

$$\Phi_E = EA_\perp = EA\cos\theta$$

For maximum value of flux, we would like $\theta = 0°$, and for minimum we would like $\theta = 90°$
Let there be $\vec{\boldsymbol{A}}$ which has magnitude of surface area and has a vector normal to the surface. Now, let us make this area $\vec{\boldsymbol{A}}$ really small to $\Delta\vec{\boldsymbol{A}}$. And let us add each of these small electric fluxes at every area.

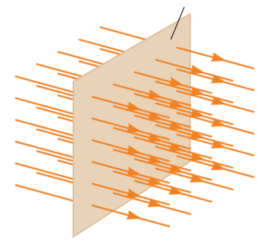

Figure 24.1 Field lines representing a uniform electric field penetrating a plane of area perpendicular to the field.

$$\Phi_E \approx \sum \vec{\boldsymbol{E}}_{\boldsymbol{i}} \cdot \Delta\vec{\boldsymbol{A}}_{\boldsymbol{i}}$$

Electric Flux

$$\Phi_E = \int_{\text{Surface}} \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} \tag{1}$$

The net flux is proportional to the number of field lines entering and leaving the surface.

$$‘\Phi_E \propto l_{leaving} - l_{entering}$$

An Integral over a closed surface becomes,

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}$$

If there exists no charge inside, and the surface is closed, the electric flux is always zero. As the number of field lines entering is the same number of field lines leaving.

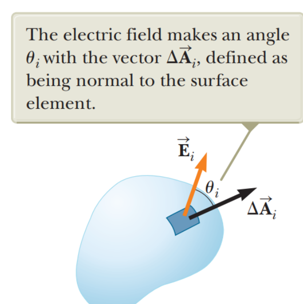

Figure 24.3 A small element of surface area ΔA_i in an electric field.

24.2 Gauss Law

Assume a point charge at $+q$ inside a closed surface (sphere)
of radius r. Since the field lines from the positive charge are going to be perpendicular to the surface, and therefore are also going to be parallel to the surface of our sphere. (Parallel to $\vec{\boldsymbol{A}}$).
We can rewrite the equation more simply as,

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \oint E dA = E \oint dA$$

The surface area of a sphere is given as,

$$A = 4\pi r^2$$

Plugging that in for our A when we integrate,

$$\Phi_E = E(4\pi r^2) = k_e \frac{q}{r^2}(4\pi r^2) = 4\pi k_e q$$

We however know from previous chapters that $k_e = 1/4\pi\epsilon_0$

$$\Phi_E = \frac{1}{4\pi\epsilon_0} 4\pi q$$

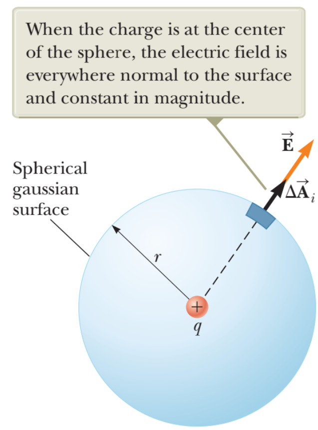

Figure 24.6 A spherical gaussian surface of radius r surrounding a positive point charge q.

Simplifying, we the get,

Gauss' Law

$$\Phi_E = \frac{q}{\epsilon_0} \tag{2}$$

The net flux for any Gaussian Surface (closed surface) surrounding a point charge q is given by q/ϵ_0 and in independent of the shape of that surface

But note that,

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \oint (\vec{\boldsymbol{E_1}} + \vec{\boldsymbol{E_2}} + ...) \cdot d\vec{\boldsymbol{A}}$$

So instead, we can generalize this to all charges inside the closed surface,

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \frac{q_{inside}}{\epsilon_0}$$

Where $q_{inside} = q_1 + q_2 + ... + q_n$ for all n charges inside the closed surface

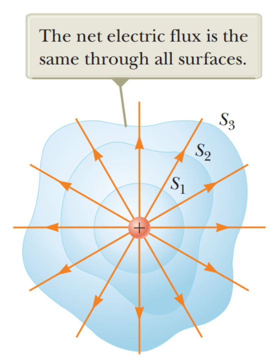

Figure 24.7 Closed surfaces of various shapes surrounding a positive charge.

24.3 Applications of Gauss' Law

$$\Phi_E = \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}$$

1. The value of the electric field can be argued by symmetry to be constant over the portion of the surface
2. The dot product can be simplified to simple multiplication of EdA if the vectors are parallel
3. The The dot product is zero if the vectors are perpendicular such that $\theta = 90°$
4. The electric field is zero over the portion of the surface

The goal in this unit is to find the simplest closed surface to enclose your object inside. Such that the field vectors would be parallel to E and everything else would cancel.

Surface Examples include, how to find the electric field, given surfaces such as,

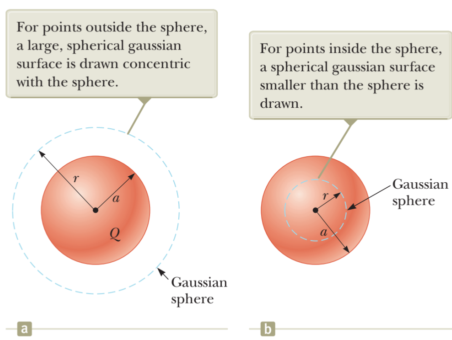

Figure 24.10 (Example 24.3) A uniformly charged insulating sphere of radius a and total charge Q. In diagrams such as this one, the dotted line represents the intersection of the gaussian surface with the plane of the page.

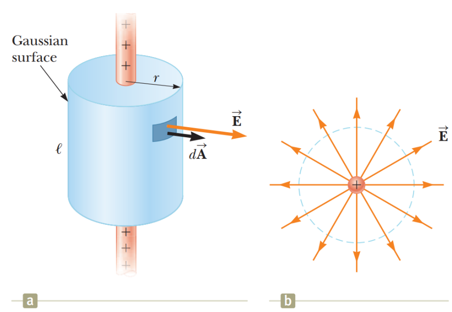

Figure 24.12 (Example 24.4) (a) An infinite line of charge surrounded by a cylindrical gaussian surface concentric with the line. (b) An end view shows that the electric field at the cylindrical surface is constant in magnitude and perpendicular to the surface.

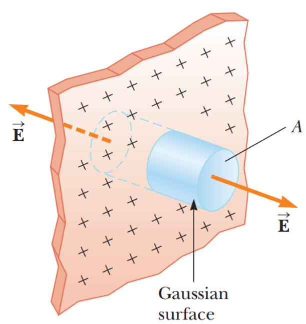

Figure 24.13 (Example 24.5) A cylindrical gaussian surface penetrating an infinite plane of charge. The flux is EA through each end of the gaussian surface and zero through its curved surface.

24.4 Conductors in Electrostatic Equilibrium

When there is no net movement of charge within a conductor, the conductor is said to be in **electrostatic equilibrium**. A conductor in electrostatic equilibrium has the following properties:

1. The electric field is zero everywhere inside the conductor, whether the conductor is solid or hollow.
2. If the conductor is isolated and carries a charge, the charge resides on the surface.
3. The electric field at a point on the outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude of σ/ϵ_0, wher'e σ is the suface charge density at that point.
4. On an irregularly shaped object, the surface charge density is greatest at locations where the curvature is the smallest.

Notice in this figure, when an external field is applied, electrons accelerate against the field moving towards the left of the slab. Positively charged particles move in the direction of the field and therefore gathering at the right side of the slab. This causes there to be an electric field inside the conductor which opposes the external field. Given enough time, the inner field will get strong enough such that the net field is zero.

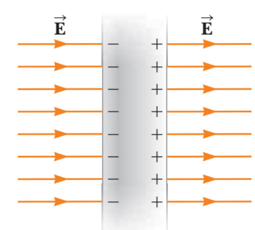

Figure 24.16 A conducting slab in an external electric field $\vec{\mathbf{E}}$. The charges induced on the two surfaces of the slab produce an electric field that opposes the external field, giving a resultant field of zero inside the slab.

Since there is no net field inside a charged conductor, any net charges must reside on the surface of the conductor.

Since the conductor is electrostatic equilibrium, the $\vec{\boldsymbol{E}}$ has to be perpendicular to the surface as to not move charges inside the conductor. Therefore, we can simply use the algebraic version referring to 24.3 point 2.

$$\Phi_E = \oint EdA = EA = \frac{q_{inside}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

Therefore, the electric field at the surface of a conductor is,

$$E = \frac{\sigma}{\epsilon_0}$$

When we took the surface integral on the previous page we found only the net charge due to the charges on the surface

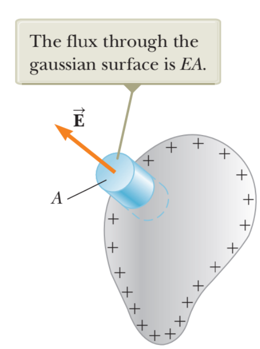

Figure 24.18 A gaussian surface in the shape of a small cylinder is used to calculate the electric field immediately outside a charged conductor.