8 Continuous Random Variable

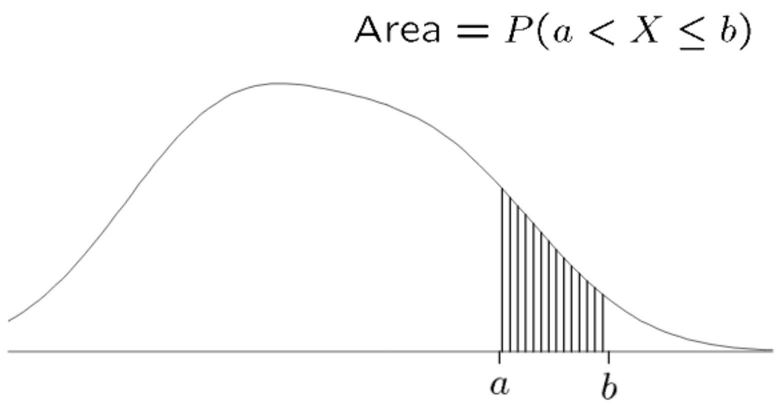
Continuous random variables are variables that arise from measurements that can take any value within intervals.

Recall that p.m.f $f(x)$ can be represented by masses at discrete points on a real line. For continuous random variables, these masses are smeared continuously between discrete points. Thus giving rise to a density function as we try to describe the mass per unit length on the x-axis, we call this distribution a **probability density function(p.d.f)** $f(x)$. The p.d.f describes probability of a continuous random variable X.

For a continuous random variable X, a probability density function is a function such that

1. $f(x) \geq 0$: The probability density function is always greater than or equal to zero.
2. $\int_{-\infty}^{+\infty} f(x)dx = 1$: The total area under the graph of $f(x) = 1$

   The area under $f(x)$ on any interval equals the true probability that a measurements falls in the interval.
3. $P(a \leq X \leq b) = \int_a^b f(x)dx =$ area under $f(x)$ from a to b for any a, b
4. $P(X = x) = 0$: Since there is no area defined, no measurement falls in an infinitely small range



Example 4.2 Hole diameter X has p.d.f

$$f(x) = \begin{cases} 20e^{-20(x-12.5)} & x \geq 12.5 \\ 0 & \text{else} \end{cases}$$

Find $P(X > 12.60)$ and $P(12.5 < X < 12.6)$

Ans.

$$P(X > 12.60) = \int_{12.60}^{\infty} f(x)dx = \int_{12.60}^{\infty} 20e^{-20(x-12.5)}dx = -e^{-20(x-12.5)}|_{12.6}^{\infty} = 0.135$$

$$P(12.5 < X < 12.6) = \int_{12.50}^{12.60} f(x)dx = \int_{12.50}^{12.60} 20e^{-20(x-12.5)}dx = -e^{-20(x-12.5)}|_{12.5}^{12.6} = 0.865$$