5 Independence

When two probabilities are **independent** it means that knowledge that other occurred, does not change the probability of the other. Mathematically speaking, we arrive at the conclusion below

$$P(B|A) = P(B) \tag{1}$$

From this equation we can see that the occurrence of event A does not change $P(B)$. Since the probabilities do not affect each other, we call A and B independent events. We can recall from the earlier section

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \tag{2}$$

In order to obtain equation (1) from equation (2), the following has to be true

$$P(A \cap B) = P(A)P(B) \tag{3}$$

This answer follows symmetry as $P(A \cap B) \equiv P(B \cap A)$.

The following are of independent events: A', B or A', B' or A, B'. For each pair we see that the probability of the other does not get affected. For example, $P(A'|B)$ since we know that $B \subset A'$ so it does not change the probability of A' occurring.

Events $A_1, A_2, ..., A_n$ are independent if

$$P(\cap_{j=1}^{k} A_{i_j}) = \prod_{j=1}^{k} P(A_{i_j}) \tag{4}$$

for any $k \geq 2$ and any subset of events $\{ A_{i_1}, ..., A_{i_k} \}$

Equation (4) simply states that any event A_n is independent if the intersection of any subset**s** A_{i_k}, where $k \geq 2$ would simply yield the product of the probability of the events.

Example 30% of a company's washing machines require services under warranty, 10% of its dryers needs service as well. If someone pays for a washer and dryer, what is the probability that both machines require service? Assume independence

Ans. Let A be the event that the washer needs service, and let B be the event that the dryer needs service.

$$P(A \cap B) = P(A)P(B) = (0.1)(0.3) = 0.03$$

5.1 Reliability

The reliability of a device or system is the probability that it operates for a specified duration.

Let A and B be components in a circuit

For parallel circuits, the reliability would be the probability of either component working (this is because if one or the other works, the circuit will work).

$$P(\text{Reliability}) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For series circuits, the reliability would be the probability of both components working (this is because if one or the other fails, the circuit will not work).

$$P(\text{Reliability}) = P(A \cap B) = P(A)P(B)$$

If given probability of failure and asked for the total failure probability, then parallel would be the probability of both failing, and series would be the probability of either failing.