29.1 Magnetic Field

Magnetic Field and Force

The region of space surrounding a charge contains a **magnetic field $\vec{B}$**

The force produced by the field is,

$$\vec{F}_B = q\,\vec{v} \times \vec{B} \tag{1}$$

Always use the right hand rule, from the cross product we can see that $\vec{F}_B$ is always going to be perpendicular to the plane created by the $\vec{B}$ and *vecv* vectors.

To use the right hand rule, point your fingers in the direction of $\vec{v}$, then curl your hand towards the $\vec{B}$ vector. The direction your thumb points is the direction that $\vec{F}_B$ acts in.

Scalar Magnetic Force

Using the definition of a scalar cross product,

$$F_B = |q|vB\sin\theta \tag{2}$$

where θ is the angle between $\vec{v}$ and $\vec{B}$
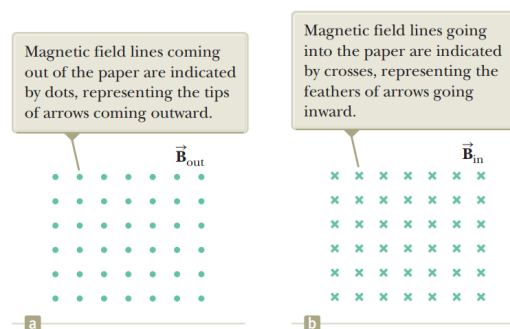
For maximum force, we would like the velocity and field of the magnetic field to point perpendicular to each other as $\sin 90° = 1$

$$[B] = 1\text{T} = \text{N}/(\text{Cm}/\text{s})$$

29.2 Motion of a Charged Particle in a Uniform Magnetic Field

On the left the two figure shows 2 orientations for the magnetic field. The notation for inside and outside the page magnetic field is given as $\vec{B}_{in}$ and $\vec{B}_{out}$ respectively.

Notice that if a particle was moving perpendicular to the field, it would always have a centripetal force $\vec{F}_B$ acting on the charged particle.

Magnetic field lines coming out of the paper are indicated by dots, representing the tips of arrows coming outward.

$\vec{B}_{out}$

Magnetic field lines going into the paper are indicated by crosses, representing the feathers of arrows going inward.

$\vec{B}_{in}$

Due to this nature, we can then determine how the particle would behave in such a field.

$$\sum F = F_B = ma$$

$$F_B = qvb\sin(90°)$$
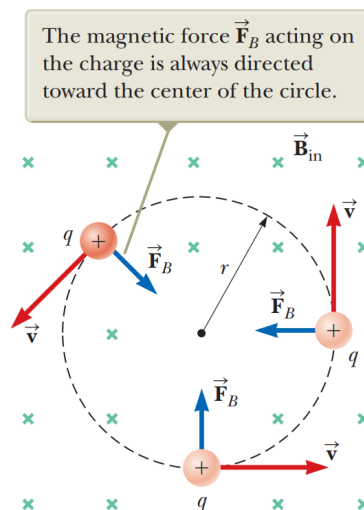
But since $\vec{F}_B$ is a centripetal force,

$$qvb = \frac{mv^2}{r}$$

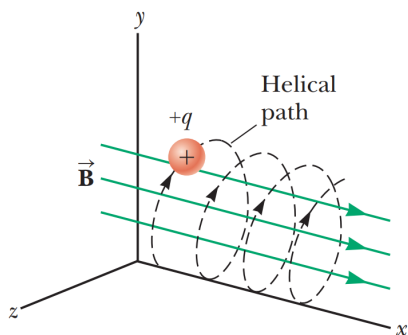Solving for r, we can see the radius of our motion,

$$r = \frac{mv}{qB}$$

Similarly, since we know $\omega = \frac{v}{r}$, we can say,

$$\omega = \frac{qB}{m}$$

The magnetic force $\vec{F}_B$ acting on the charge is always directed toward the center of the circle.

Therefore, using the idea of centripetal motion, we can find the period.

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

The above is how the charged particle would move in the field assuming it also had a constant speed in an axis not affected by the magnetic field

29.3 Applications Involving Charged Particles Moving in A Magnetic Field

If there was a magnetic field with a electric field, we would have the equation,

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

If there was a magnetic field into the page, the positive charge is moving upwards, and there was an electric field to the right. The equation would simplify to,

$$v = \frac{E}{B}$$

For mass spectrometry, we get the equation,

$$ma_c = qvB$$
$$\frac{m}{q} = \frac{qB_o}{v}$$

Side note: The cyclotron was a device that could accelerate charges to very high speeds.

We can find from the previous equations that kinetic energy of a particle in a cyclotron as,

$$K = \frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$

29.4 Magnetic Force Acting on a Current Carrying Conductor

Since we know that for one charge,

$$F_B = q\vec{v} \times \vec{B}$$

So, inside a conductor we know that,

$$I = nqv_d A$$

We can find $\vec{F_B}$ if we multiplied it by the number of charges in the wire (multiply by the volume· charge density).

$$\vec{F_B} = (q\vec{v_d} \times \vec{B})qAL$$

We can rewrite this,

$$\vec{F_B} = I\vec{L} \times \vec{B}$$

where I is the current, $\vec{L}$ being a vector in the direction of the current with magnitude as the length of wire, and $\vec{F_B}$ is the magnetic field.
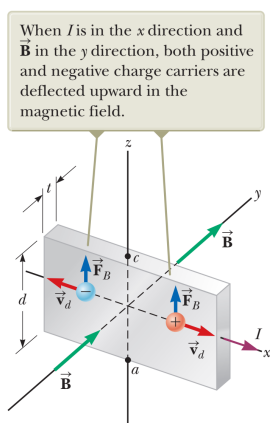
If we were to consider an extremely small segment $d\vec{s}$, we can return the total force on an arbitrarily shaped object as,

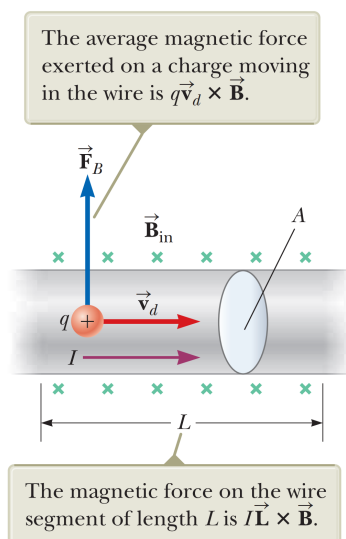$$\vec{F_B} = I \int_a^b d\vec{s} \times \vec{B}$$

where a and b are the end points of a strip of conductive material in a magnetic field.

29.6 The Hall Effect

When a current carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to the current and magnetic field. This is called the **Hall Effect**.

The average magnetic force exerted on a charge moving in the wire is $q\vec{v}_d \times \vec{B}$.

The magnetic force on the wire segment of length L is $I\vec{L} \times \vec{B}$.

Notice in the diagram below, since there is a magnetic field in the negative y direction. This causes a force $\vec{F_B} = q\vec{vd} \times \vec{B}$ upwards on negative charge carriers travelling towards the negative x inside the conductor.

This causes most of the negative charges to accumulate at the top of the conductor and most positive charges towards the bottom of the conductor. This is the potential difference, and eventually all the charges inside the conductor reach equilibrium. $\vec{F_{net}} = 0$

When I is in the x direction and $\vec{B}$ in the y direction, both positive and negative charge carriers are deflected upward in the magnetic field.

Figure 29.26 To observe the Hall effect, a magnetic field is applied to a current-carrying conductor. The Hall voltage is measured between points a and c.

The potential difference is named **Hall Voltage** ΔV_H Notice, we can use the particle in equilibrium model to show,

$$0 = F_B - F_E = qv_dB - qE_H$$

Solving for Hall's Voltage,

$$E_H = v_dB$$

Since we also know that,

$$v_d = \frac{1}{nqA} \qquad\qquad \Delta V_H = E_Hd$$

We can rewrite the previous equation as,

Hall Voltage

$$\Delta V_H = v_d B d = \frac{IBd}{nqA} = \frac{R_H IB}{t} \tag{3}$$

Where, R_H is called the **Hall Coefficient**, $R_H = 1/nq$. Note, we expressed the area A as $A = dt$, where d is the length and t is the thickness.

$$n = \frac{N_A}{V} = \frac{N_A \rho}{M} \tag{4}$$

This is a useful note as, n is the electron density, N_A is Avagradro's Number, $N_A = 6.022 \times 10^{23}$ atoms/mole

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