1 Joint Probability Distributions

The **joint probability mass function** of discrete random variables X and Y, denoted as $f_{XY}(x, y)$ satisfies

1. $f_{XY}(x, y) \geq 0$
2. $\sum_X \sum_Y f_{XY}(x, y) = 1$
3. $f_{XY} = P(X = x, Y = y)$

The **joint probability density function** for the continuous random variables X and Y, denoted as $f_{XY}(x, y)$, satisfies the following properties:

1. $f_{XY}(x, y) \geq 0$ for all x, y
2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x, y)\, dx\, dy = 1$
3. For any region R of a two-dimensional space,

$$P((X, Y) \in R) = \iint_R f_{XY}(x, y) dx\, dy \tag{1}$$

Example 5.2 Joint p.d.f of (X, Y) is

$$f(x, y) = \begin{cases} 6(10^{-6}) \exp -0.001x - 0.002y & 0 < x < y < \infty \\ 0 & \text{Else} \end{cases}$$

Find $P(X < 1000, Y < 2000)$

Ans.

$$6(10^{-6}) \int_{x=0}^{1000} \left(\int_{y=x}^{2000} e^{-0.002y}\, dy \right) e^{-0.001x} dx = 0.9155$$