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Secular Trends in Real Rates

by

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Abstract

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In this paper, we investigate the joint dynamics of the long-run components of interest rates and inflation for eight advanced economies. The approach is broadly based on Del Negro et al. [2018]. The Kalman filter is used to operate the trend-cycle decompositions. Our findings are as follows. First, the co-movement of real interest rates among advanced economies can be explained by the existence of a world real interest rate. Second, the trends in real interest rates of advanced economies have been converging to the world trend since the 1980s. Third, the trend in the world real interest rate decreased by 150 basis points since the 1980s. Fourth, further analysis on these results suggest that low-frequency movements of the world real interest rate can be explained by global changes in technological progress and growth expectations, as well as shifts in the saving and investment schedules. We estimate that global demographics development and changes in the level of public investment in advanced economies can account for 85 basis points of the fall in the world real interest rate of the past decades.

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Introduction

A simple research on Google trends shows that the interest for secular stagnation took off in major advanced economies since precisely November 2013 and never returned to its previous level. This particular date comes as no surprise as it corresponds to the day of famous economist Larry Summers' presentation at the International Monetary Fund (IMF) Research Conference on secular stagnation. A matter which he considered to be, at the time, the most pressing economic issue of our days.

The dread of secular stagnation is not recent, though it has certainly bloomed over the past decades. 80 years ago, Hansen [1939] feared that following the Great Depression, the future of the United States' economy would prove as gloom as the last decade. His research on the reasons behind a potential long-term stagnation drove him to investigate low-frequency movements in the structural drivers of economic growth. He found that a downward tendency of both population and productivity growth as well as a shift in investment demand would eventually lead to a long-term fall in interest rates. Though Hansen's predictions did not come to happen, as the U.S. experienced exploding rates in both birth and technological progress following World War II, his theory would be discussed for decades to come during times of recession. And today, when real interest rates seem to reach all-time lows among all advanced economies, might be an appropriate time to bring back the issue.

It has been usual policy until now for governments to lower short-term interest rates to boost economic growth during times of recession. However, from the never ending fall in interest rates over the world since 1990, it seems that most major economies will soon be facing the dreaded zero lower bound, if they have not already. From the rapid and identical movements of real interest rates in many advanced economies, one might wonder if macroeconomic policy is the sole reason or if other fundamental variables are to blame. Establishing what drives real interest rates over the world, whether they are driven by global or country-specific determinants is crucial in this time of extreme uncertainty and prolonged recession. Particularly, it will prove useful to evaluate the extent of the role of monetary policy in the midst of this long-term decline, especially with many central banks experimenting negative interest rate policies.

We present in this paper an analysis on real interest rates in two parts. The first part aims to uncover the existence of a world real interest rate which drives the movements of real interest rates in all advanced economies, and to estimate its low-frequency movements. We also intent to identify the structure of deviations of advanced economies from said world trend : whether they are country-specific trends, and so driven by secular changes, or cyclical shocks. To do so, we use data on short-term and long-term nominal interest rates as well as inflation for 8 OECD countries.[1]. We perform on the collected data a recursive trend-cycle decomposition using a Kalman filter and a fixed-interval smoother. Then, in a second part, the paper explores the long-term drivers of the previously estimated trends through multiple econometric regressions and specifications. To make the most of the first-part results, the regressions are separated by world and idiosyncratic trends. A multivariate regression using global explanatory variables or weighted averages presents the secular drivers of the global real interest rate which affects all advanced economies. Finally, a set of panel regressions on all the estimated country-specific trends shows how the movements of real interest rates in a particular economy can be explained through own-country determinants.

In the first part, our results testify of strong long-term co-movements of real interest rates, inflation and term spreads across all sampled economies. The common trend we have identified as the world interest rate denotes of important global movements throughout history, such as low real interest rates after World War I and II and throughout the Great Depression, or the historical high during the 1980s. It also captures the notorious subsequent fall in real interest rates over the world. Our estimations show that the trend in the world interest rate has decreased by 150 basis points since 1990. The real interest rate in 2016 is the lowest registered in over 60 years. In the second part, we find that the world interest rate is driven at significant level by global changes in educational attainment, level of public investment, technological progress, age demographics, catch up growth and expectations of growth. Shifts in the investment and saving schedules driven by demographics development and changes in the level of public investment account for the most part of the decrease in real interest rates of the past decades. Finally, the panel regression results indicate that only changes in inequality level have had a negative and significant effect on country-specific trends. None of the other variables has proved reliable in explaining the low-frequency movements of the idiosyncratic components of real

[1] Canada, France, Germany, Italy, Japan, Switzerland, the U.K. and the U.S.A

interest rates, implying other determinants are at stake.

As such, this paper relates to two major strands of research : estimation of the world interest rate and investigation of the secular drivers of real interest rates across the world. While theory advocates that capital and goods mobilities in the market ensure that real interest rates are equal in monetary value across all economies, empirical evidence strongly suggests that they are, in fact, not (see Edison and Pauls [1993]; Fujihara and Mougoué [1996]; Kugler and Neusser [1993]). Sources of deviation from the uncovered interest rate parity condition have been highly researched among neoclassical economists and so have the co-movements of advanced economies' real interest rates. Early on, Barro and Sala-i Martin [1990] showed interest in the movements of real interest rates in ten OECD countries and why they seemed to have all sporadically dropped throughout the 1980s. They find that real interest rates in advanced economies are driven primary by global factors rather than domestically. Barro and Sala-i Martin [1990] measured the world real interest rate as the GDP-weighted average of 9 of their countries of interest, whereas we impose no arbitrary weight on the structure of the world real interest rate. Moreover we aim to extract secular trends, contrary to their focus on high-frequency fluctuations. The difference in time horizons between both models illustrate this important distinction : their sampled data comes from the period 1959-89, while our data covers the period 1870-2016. Gagnon and Unferth [1995] explored the idea of a world real interest rate using panel estimations on 9 OECD countries over a 16-year time span (1977-1993). We offer a model with a much longer time horizon which is additionally able to extract low-frequency co-movements in real interest rates over the world, notably by using structures of both short-term and long-term interest rates. King and Low [2014] estimated the long term world real interest rate as unweighted and weighted averages of the long term real interest rates on 6 of the G-7 member states. While they had to exclude Italy since its long term interest rate demonstrated strong fluctuations of the risk premium, our model is able to directly accommodate such high-frequency movements through its trend-cycle decomposition properties. Similarly to Barro and Sala-i Martin [1990], they present results using a predetermined structure of the world real interest rate, either weighting all economies equally or by share of total GDP. We also differ in purpose with King and Low [2014], as we are more interested in extracting and explaining low-frequency fluctuations of the world real interest rates rather than overall movements. Desroches and Francis

[2006] published a paper exploring the reason behind the fall of G7 countries' long term interest rates to levels "not seen since the 1960s". They notice that they all seemed to be moving together and used a Kalman filter to extract a world real interest rate, in the same way that we have in our paper. They show similar results to our unconstrained analysis, pointing to an estimation of higher-frequency movements of the world real interest rate. More recently, Del Negro et al. [2018] investigated the historically low interest rates in the world economy through the same sampled countries as Desroches and Francis [2006] using Bayesian estimations. Our research is mostly based on their theory and results, although we use a different approach to perform the model decomposition. We differentiate from both Desroches and Francis [2006] and Del Negro et al. [2018] by adding Switzerland to the sample of countries and pursuing the analysis on the results, notably using multivariate regressions on both world and country-specific trends.

Many other researchers, while not directly incorporating estimations of global real interest rates, have aimed to identify the determinants of fluctuations in the real interest rates of advanced economies. Blanchard and Summers [1984] were discussing the subject in the midst of the high peak of real interest rates in the early 1980s. Even this early, they believed that interest rates of the OECD countries were driven not by own-country determinants but rather by world factors. They also conclude that the exceptionally high rates of the time were not solely due to monetary and fiscal policies. Later, focusing on the decrease in real interest rates following the 1980s, Bernanke [2005] explored a global saving glut, he defined as a "significant increase in the global supply of saving", as the main reason behind low interest rates around the world. Desroches and Francis [2006] in their analysis of the world interest rate mention investment and desired savings, notably through labour force growth and age structure respectively, as significant and long-term drivers. Most of these studies, we differ first by extracting first our own estimations of the trends in real interest rates, and second by conducting separate analysis on global and country-specific components of said trends. The aim is to identify how specifically the overall trends are influenced by world factors movements and change in their own-country determinants.

The rest of the paper proceeds as follows. The first section introduces our theoretical framework in a first part, goes on to discuss the long-term implications on which we are basing our analysis and lastly, presents the econometrical tools and assumptions behind our results. The second section briefly presents the data, the restrictions and the initialization regarding the trend-cycle decomposition and subsequently presents and discusses the results of all estimated trends. Section 3 opens the discussion on the secular drivers of trends in real interest rates and pursue further analysis on the previously estimated variables. Section 4 concludes.

1 Model

This section will present in three parts the theoretical model, the long-run assumptions and the econometric framework with which we are working to produce estimations of trends in real interest rates.

1.1 Theoretical framework

We develop here the theoretical concepts which are at the foundation of this paper. The theory, directly reproduced from Del Negro et al. [2018], is built on consumption-based asset pricing properties and relate to the recent literature about exchange rates and deviations from UIP (Convenience yield in Valchev [2020], Jiang et al. [2018]; liquidity premium in Nagel [2014], Engel [2015], Itskhoki and Mukhin [2017]; currency risk premium in Gabaix and Maggiori [2014]).

The building block of the model developed in Del Negro et al. [2018] is the 3-month U.S. Treasury bill pricing Euler equation,

$$\mathbb{E}_t\left[M_{t+1}^{US}(1+CY_{t+1}^{US})(1+CY_{t+1}^{\$})(1+R_t^{\$})\frac{P_t^{\$}}{P_{t+1}^{\$}}\right]=1, \tag{1.1}$$

where $R_t^{\$}$ is the net return of the bond in U.S. dollars and M_{t+1}^{US} the stochastic discount factor of U.S. investors. Equation (1.1) is the augmented Euler equation in which $(1+CY_{t+1}^{US})(1+CY_{t+1}^{\$})$ represents a *convenience yield for safety and liquidity* associated with both the asset and investors. This convenience yield reflects the value that investors attribute to the safety and liquidity characteristics of an asset. If and how much the investors are willing to pay for such conveniences could result in a return spread between assets with similar payoffs and maturities. In an international setting, U.S. investors participate in the equilibrium pricing of foreign government bonds, which can be represented through similar Euler equations.[2] As such, the pricing equation of a short-term U.K.

[2]For illustration purposes, we arbitrarily choose to use the United Kingdom as representation of a foreign governement and thus the British pound sterling as the foreign currency, but the same demonstration could be done with any advanced economy and its currency.

bond issued in British £ from the side of U.S. investors would be,

$$\mathbb{E}_t\left[M_{t+1}^{US}(1+CY_{t+1}^{US})(1+CY_{t+1}^{£})(1+R_t^{£})\frac{S_{t+1}^{£,\$}}{S_t^{£,\$}}\frac{P_t^{\$}}{P_{t+1}^{\$}}\right]=1. \tag{1.2}$$

Equation (1.2) is the augmented international Euler equation and introduces nominal exchange rates in the setting through $S_t^{£,\$}$, which converts the price and the return of the bond from British pound sterlings to U.S. dollars. As such, a decrease in the rate $S_t^{£,\$}$ denotes an appreciation of the U.S. dollar. In both equations (1.1) and (1.2), the convenience yield is made of two terms. The first term $(1+CY_{t+1}^{US})$ is country-specific to the investors pricing the assets, in that it represents the value that U.S. investors give to safety and liquidity properties. As such, we allow the model to account for differences in the way U.S. and U.K. investors value the conveniences of a same asset. The second term, $(1+CY_{t+1}^{£})$, is asset-specific and contingent on the currency (or country) in which the bond was issued. It indicates the global value which is associated with the issuing country's government bill's convenient properties. The interaction of the two terms is what makes the overall (investor and asset specific) convenience yield.

As of now, we have only focused on the way U.S. investors price their own-country assets as well as foreign assets. Evidently, in this setting, U.K. investors participate in pricing both assets. The augmented international Euler pricing equations from the perspective of U.K. investors are,

$$\mathbb{E}_t\left[M_{t+1}^{UK}(1+CY_{t+1}^{UK})(1+CY_{t+1}^{£})(1+R_t^{£})\frac{P_t^{£}}{P_{t+1}^{£}}\right]=1 \tag{1.3}$$

$$\mathbb{E}_t\left[M_{t+1}^{UK}(1+CY_{t+1}^{UK})(1+CY_{t+1}^{\$})(1+R_t^{\$})\frac{S_t}{S_{t+1}^{£,\$}}\frac{P_t^{£}}{P_{t+1}^{£}}\right]=1, \tag{1.4}$$

Mirroring equations (1.1) and (1.2), equations (1.3) and (1.4) now introduce the SDF and convenience yields of U.K. investors. Taking first-order approximations of the pricing equations (1.1) to (1.4) lets us write,

$$\begin{aligned}
R_t^{\$} &= m_t^{US} - cy_t^{US} - cy_t^{\$} + \mathbb{E}_t[\pi_{t+1}^{\$}] \\
R_t^{£} + \mathbb{E}_t[\Delta s_{t+1}^{£,\$}] &= m_t^{US} - cy_t^{US} - cy_t^{£} + \mathbb{E}_t[\pi_{t+1}^{\$}] \\
R_t^{£} &= m_t^{UK} - cy_t^{UK} - cy_t^{£} + \mathbb{E}_t[\pi_{t+1}^{£}] \\
R_t^{\$} - \mathbb{E}_t[\Delta s_{t+1}^{£,\$}] &= m_t^{UK} - cy_t^{UK} - cy_t^{\$} + \mathbb{E}_t[\pi_{t+1}^{£}],
\end{aligned} \tag{1.5}$$

where $cy_t^i \equiv \mathbb{E}_t[\log(1 + CY_{t+1}^i)]$ is the convenience yield associated with the investors of a country i, $cy_t^{¤} \equiv \mathbb{E}_t[\log(1 + CY_{t+1}^{¤})]$ the convenience yield associated with currency ¤, $s_t^{£,\$} \equiv \log S_t^{£,\$}$ is the log nominal exchange rate and $m_t^i \equiv -\mathbb{E}_t[\log(M_{t+1}^i)]$ is the negative of the expected change in marginal utility of country i's investors. Combining either the first or the last two equations of this system is enough to yield the important condition,

$$R_t^{\$} = R_t^{£} + \mathbb{E}_t[\Delta s_{t+1}^{£,\$}] + cy_t^{£} - cy_t^{\$}. \tag{1.6}$$

While uncovered interest rate parity (UIP) theory states that under international arbitrage, the interest rates between two countries should differ only by the relative change in currency exchange rate, i.e. $R_t^{\$} = R_t^{£} + \mathbb{E}_t[\Delta s_{t+1}^{£,\$}]$, equation (1.6) says otherwise. More precisely, it implies that the standard UIP condition cannot hold in an economy where convenience yields differ between currencies. Let us then call the condition in (1.6) the modified UIP condition. Because we are interested in variables expressed in real terms, let us write the system (1.5) in the form,

$$R_t^{\$} - \mathbb{E}_t[\pi_{t+1}^{\$}] = m_t^{US} - cy_t^{US} - cy_t^{\$} \tag{1.7}$$

$$R_t^{£} - \mathbb{E}_t[\pi_{t+1}^{£}] + \mathbb{E}_t[\Delta q_{t+1}^{£,\$}] = m_t^{US} - cy_t^{US} - cy_t^{£} \tag{1.8}$$

$$R_t^{\$} - \mathbb{E}_t[\pi_{t+1}^{\$}] - \mathbb{E}_t[\Delta q_{t+1}^{£,\$}] = m_t^{UK} - cy_t^{UK} - cy_t^{\$} \tag{1.9}$$

$$R_t^{£} - \mathbb{E}_t[\pi_{t+1}^{£}] = m_t^{UK} - cy_t^{UK} - cy_t^{£}, \tag{1.10}$$

where $q_t^{£,\$} \equiv \log(S_t^{£,\$} P_t^{£} / P_t^{\$})$ is the log real exchange rate. Those results already give an interesting interpretation of real interest rates under international arbitrage. Pricing equations set by the same investors in a country i (i.e. (1.7) and (1.8) or (1.9) and (1.10)) show the structure of the real return of two assets issued in different currencies. More precisely, they show that once the real returns are expressed in equivalent consumption units (i.e. adjusted by the real exchange rate), the two assets share a joint component, $m_t^i - cy_t^i$ and are only distinguished by the idiosyncratic components $cy_t^{¤}$. This result illustrates the role of the convenience premia in explaining deviations from the real interest rate parity (RIRP) condition. Going further, there are additional conclusions that we can draw which will prove useful for our analysis. First, let us put together equations pricing a similar asset by different investors (i.e. (1.7) and (1.9) or (1.8) and (1.10)), and write the following,

$$m_t^{US} - cy_t^{US} = m_t^{UK} - cy_t^{UK} + \mathbb{E}_t[\Delta q_{t+1}^{£,\$}]. \tag{1.11}$$

Additionally, assuming risk-neutrality[3], doing the same process as (1.1) to (1.11) for assets with no convenience service yields the condition,

$$m_t^{US} = m_t^{UK} + \mathbb{E}_t[\Delta q_{t+1}^{£,\$}]. \tag{1.12}$$

Putting together equations (1.11) and (1.12) indicates,

$$cy_t^{US} = cy_t^{UK}. \tag{1.13}$$

From these last three equations of the theoretical framework, we can conclude that 1) there exists a global yield on assets with safety and liquidity properties which we will denote as cy_t^w, 2) there exists a global SDF when expressed in equivalent consumption units across all economies and 3) as such, under international arbitrage, the identity of the investors does not matter when it comes to pricing assets.

1.2 Long-run Model

As we are mainly interested in studying trends, we elaborate here on long-term relationships implied by the theoretical model. This will allow us to use economic assumptions which could not hold in a short-term setting. The long-run view of our model allows us to write expected values as trends, which, remembering that $cy_t^i \equiv \mathbb{E}_t[\log(1 + CY_{t+1}^i)]$ and $m_t^i \equiv -\mathbb{E}_t[\log(M_{t+1}^i)]$, we will denote as $\overline{m}_t^i$ and $\overline{cy}_t^i$. As such, we write equations (1.7),

[3]Risk-neutrality is implied when taking the first-order approximation of the augmented Euler equations. If that proved incorrect, additional deviations from the UIP will be reflected in the idiosyncratic term.

(1.10) and (1.12) as their long-term form,

$$\overline{R}_t^{¤} - \overline{\pi}_t^{¤} = \overline{m}_t^i - \overline{cy}_t^w - \overline{cy}_t^{¤} \tag{1.14}$$

$$\overline{m}_t^i = \overline{m}_t^j + \overline{\Delta} q_{t+1}^{j,i}. \tag{1.15}$$

Equation (1.14) states that the trend in real interest rate of a country i with currency ¤ will differ from another country only as much as their stochastic discount factor and convenience yield on currency differ. Focusing on a long-term view allows us to take an additional step with equation (1.15). Even though, when it comes to trends in real exchange rates there is a lack of consensus in the history of literature, modern literature has been pointing to evidence in favor of long-run purchasing power parity (Abuaf and Jorion [1990]; Taylor and Taylor [2004]). While we will not use this strong assumption, we will assume that there is no trend in the growth rate of the real exchange rate, i.e. $\overline{\Delta q}_t = 0$, which is implied but weaker than purchasing power parity (PPP) in the long-run[4]. Del Negro et al. [2018] conducted a brief analysis and found no trend in the growth of the real exchange rate. In Appendix B1, we present results of a trend-cycle decomposition on log real exchange rates differences consistent with the aforementioned assumption. Therefore, from (1.15) and the long-term stationarity assumption we are able to write,

$$\overline{m}_t^i = \overline{m}_t^j, \tag{1.16}$$

which implies that, as for the convenience yield, there exists a global stochastic discount factor which does not depend on nationality in the long-run. Denoting the latter $\overline{m}_t^w$, we can rewrite equation (1.14) as,

$$\overline{R}_{i,t} - \overline{\pi}_{i,t} = \overline{m}_t^w - \overline{cy}_t^w - \overline{cy}_t^i. \tag{1.17}$$

Since the identity of investors does not enter the model, the real return of two assets issued in different currencies will only differ by their return on conveniences. As we are working with government issued bonds, we can use the subscript and superscript i which denote of the country-specific characteristic. Additionally, for identification purposes we will write the trend in real rate of a country i, $\overline{r}_{i,t} \equiv \overline{R}_{i,t} - \overline{\pi}_{i,t}$, as the sum of two terms

[4]In the notation of our model, purchasing power parity would translate to $\overline{q}_t = 0$. The stationarity of the real exchange rate $\overline{\Delta q}_t = 0$ is then implied.

only : a global component $\overline{r}_t^w$ encompassing both $\overline{m}_t^w$ and $-\overline{cy}_t^w$ and an idiosyncratic component $\overline{r}_t^i$. Putting it all together with equation (1.17) allows us to write (1.14) as,

$$\overline{r}_{i,t} = \overline{r}_t^w + \overline{r}_t^i. \tag{1.18}$$

Equation (1.18) will be the basis of our model. The world component $\overline{r}_t^w$ drives the trend in real interest rates of all advanced economies, whereas, according to our theoretical framework, the country-specific component $\overline{r}_t^i$ is an indication of the existence of a convenience yield ($cy_t^{\$}$ and $cy_t^{£}$ in the previous model) as deviation from the standard RIRP. Using the wider notation $\overline{r}_t^i$ instead of $-\overline{cy}_t^i$ allows the model to account for any source of deviation from the UIP condition.[5]

In addition to equation (1.18), we mirror the trend decomposition of $\overline{r}_{i,t} = \overline{r}_t^w + \overline{r}_t^i$ using trends in inflation $\overline{\pi}_{i,t}$ and term spread $\overline{R}_{i,t}^L - \overline{R}_{i,t}$ to write,

$$\overline{\pi}_{i,t} = \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i \tag{1.19}$$

$$\overline{R}_{i,t}^L - \overline{R}_{i,t} = \overline{ts}_t^w + \overline{ts}_t^i, \tag{1.20}$$

which will allow us to identify the structure of the trends in inflation and term spread for each country i, by extracting in each overall trend (subscript i) a common component (superscript w) and a country-specific component (superscript i). The trend in the term premium is calculated as the differences of the trends in returns on long-term government bonds $\overline{R}_{i,t}^L$ and the previously discussed short-term interest rate. In the case of inflation, the global component is allowed to be weighted differently between countries through the loading λ_i^π, as there is no equivalent to international arbitrage pushing inflation rates to a common trend. Combining (1.18), (1.19) and (1.20), we can finally write our full model in trends as,

$$\begin{aligned} \overline{R}_{i,t}^L &= \overline{r}_t^w + \overline{r}_t^i + \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i + \overline{ts}_t^w + \overline{ts}_t^i \\ \overline{R}_{i,t} &= \overline{r}_t^w + \overline{r}_t^i + \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i \\ \overline{\pi}_{i,t} &= \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i. \end{aligned} \tag{1.21}$$

[5]Another potential source of deviation from UIP would be for example a political risk premium in Dooley and Isard [1980].

1.3 Econometric Framework

This final part of the model section describes the econometric tools we use to conduct our analysis and lays the underlying econometric assumptions. Because we based our theoretical model on long-term assumptions and because we are interested in studying low-frequency fluctuations, we aim to estimate all the trends listed in the system (1.21). As such, based on the collected data, we need to conduct a trend-cycle decomposition of the form,

$$\begin{aligned} R_{i,t} &= \overline{r}_t^w + \overline{r}_t^i + \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i + \tilde{R}_{i,t} \\ R_{i,t}^L &= \overline{r}_t^w + \overline{r}_t^i + \overline{ts}_t^w + \overline{ts}_t^i + \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i + \tilde{R}_{i,t}^L \\ \pi_{i,t} &= \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i + \tilde{\pi}_{i,t}, \end{aligned} \tag{1.22}$$

where $\tilde{R}_{i,t}, \tilde{R}_{i,t}^L, \tilde{\pi}_{i,t}$ are the cyclical components of the short term interest rate $R_{i,t}$, the long term interest rate $R_{i,t}^L$ and the inflation $\pi_{i,t}$ at time t.

We choose to use a recursive estimation algorithm to perform this decomposition. Using a state-space form of the system in 1.22, we construct and apply a Kalman filter followed by a fixed-interval smoother. The Kalman filter, specialized in extracting unobservable state variables using at each point of time its corresponding information set, gives us as a preliminary estimation of the decomposition for each time t. The Rauch-Tung-Striebel (fixed-interval) smoother goes over all the filtered estimations and updates each of them using the complete information set of the very last period of time of the sample, allowing to form together their best estimates of the unobservable components. More specifically to our data set and more interestingly for our purposes, we are able to perform two kinds of components extraction : while the longitudinal aspect of our data allows the Kalman filter to perform a traditional trend-cycle decomposition, the cross subsection of countries makes it possible to further separate each countries' trend in the desired global component $\overline{y}_t^w$ and its idiosyncratic counterpart $\overline{y}_t^i$ (See equations (1.18), (1.19) and (1.20) for details).

We are basing our econometric framework on the theory and nomenclature of Hamilton [1986]. The specifics behind the filter and the smoother can be found in general terms in Appendix A1 and A2. The rest of the section adapts the theory of Hamilton [1986] to our model using appropriate economic theory and assumptions.

The basis of a Kalman filter is the separation of a model into a set of observation equations and state equations. The system in (1.22) is the representation of the observation equations for a country i at time t. Thus, the observation equations at time t of all m countries of the dataset can be written as,

$$\boldsymbol{y}_t = \Lambda \overline{\boldsymbol{y}}_t + \tilde{\boldsymbol{y}}_t, \tag{1.23}$$

where Λ is a matrix of loadings that allows to represent the specifics of the relationship between each trends and each observables. In this form, we also define $\boldsymbol{y}_t$ as a vector of size $n \times 1$ stacking all the observed data at time t, i.e.,

$$\boldsymbol{y}_t \equiv \begin{bmatrix} \boldsymbol{R}_t \\ \boldsymbol{R^L}_t \\ \boldsymbol{\pi}_t \end{bmatrix}, \tag{1.24}$$

where $\boldsymbol{R_t}$, $\boldsymbol{R^L_t}$ and $\boldsymbol{\pi_t}$ are $m \times 1$ vectors holding the values of all m countries of the data for their respective variables $R_{i,t}$, $R^L_{i,t}$ and $\pi_{i,t}$. Similarly, $\overline{\boldsymbol{y}}_t$ and $\tilde{\boldsymbol{y}}_t$ are stacking vectors for the trends and the cyclical components respectively. Equation (1.23) is the global trend-cycle decomposition of the model. To stay consistent with the notation in Appendix A1, and because we include no exogenous variables and assume the absence measurement error, let us write equation (1.23) in the form,

$$\boldsymbol{y}_t = \boldsymbol{H}'\boldsymbol{\xi}_t. \tag{1.25}$$

Equation (1.25) is the measurement equation of the state-space model we use for the rest of the analysis. The $(\tau + n) \times 1$ vector $\boldsymbol{\xi}_t$ holds all the latent variables in (1.22) in the specific order,

$$\boldsymbol{\xi_t} \equiv \begin{bmatrix} \overline{r}^w_t & \overline{\boldsymbol{r}}_t & \overline{ts}^w_t & \overline{\boldsymbol{ts}}_t & \overline{\pi}^w_t & \overline{\boldsymbol{\pi}}_t & \tilde{\boldsymbol{R}}_t & \tilde{\boldsymbol{R}}^L_t & \tilde{\boldsymbol{\pi}}_t \end{bmatrix}', \tag{1.26}$$

where $\overline{r}^w_t$, $\overline{ts}^w_t$ and $\overline{\pi}^w_t$ are scalars for the global components at time t, and $\overline{\boldsymbol{r}}_t$, $\overline{\boldsymbol{ts}}_t$, $\overline{\boldsymbol{\pi}}_t$, $\tilde{\boldsymbol{R}}_{\boldsymbol{t}}$, $\tilde{\boldsymbol{R}}^{\boldsymbol{L}}_{\boldsymbol{t}}$, $\tilde{\boldsymbol{\pi}}_{\boldsymbol{t}}$ are the trend and cyclical $m \times 1$ vectors holding the values of $\overline{r}^i_t$, $\overline{ts}^i_t$, $\overline{\pi}^i_t$ of all m countries (see (A3.1) for detailed forms). The output matrix $\boldsymbol{H}'$ is the unknown matrix of coefficients that imply the complete relationship between the observed data and the unobserved variables that we aim to estimate. It will be partially restricted using economic theory, that is to respect the assumed system in (1.22), and partially estimated in the process. The specific form of the output matrix $\boldsymbol{H}'$ is presented in equation (A3.2).

We also need to give additional information about the behavior of the latent variables to the algorithm, which will form the state equation of the model. We assume that the $\tau \times 1$ vector of trends $\overline{\boldsymbol{y}}_t$ follows a random walk,

$$\overline{\boldsymbol{y}}_t = \overline{\boldsymbol{y}}_{t-1} + \boldsymbol{\epsilon}_t, \tag{1.27}$$

where the vector of shocks $\boldsymbol{\epsilon}_t$ is independently and identically distributed,

$$\boldsymbol{\epsilon}_t \sim \mathcal{N}\Big(\mathbf{0}_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}\Big). \tag{1.28}$$

Additionally, we assume that the vectors $\tilde{\boldsymbol{R}}_t, \tilde{\boldsymbol{R}}^{\boldsymbol{L}}{}_t, \tilde{\boldsymbol{\pi}}_t$ are stationary components which together evolve according to a VAR process with p lags. We write the behavior of the $n \times 1$ vector $\tilde{y}_t$ as,

$$\left(\boldsymbol{I} - \sum_{l=1}^{p} \boldsymbol{\Phi}_l L^l\right) \tilde{\boldsymbol{y}}_t = \boldsymbol{\varepsilon}_t, \tag{1.29}$$

where the vector of shocks $\boldsymbol{\varepsilon}_t$ is distributed according to,

$$\boldsymbol{\varepsilon}_t \overset{i.i.d}{\sim} \Big(\mathbf{0}_{\boldsymbol{n}}, \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}\Big). \tag{1.30}$$

For computability purposes, we restrict the process to a VAR(1) ($p = 1$ lag). We can thus write the distribution of $\tilde{\boldsymbol{y}}_t$ as,

$$\tilde{\boldsymbol{y}}_t = \boldsymbol{\Phi} \tilde{\boldsymbol{y}}_{t-1} + \boldsymbol{\varepsilon}_t. \tag{1.31}$$

The matrix $\boldsymbol{\Phi}$ of size $n \times n$ is restricted so that the cyclical components of a country i at time t is contigent on the country's cyclical components at time $t-1$ but not on any value of another country $j \neq i$. The form of the matrix $\boldsymbol{\Phi}$ is presented in equation (A3.4). As such, we can write that the vector of latent variables $\boldsymbol{\xi_t}$ evolves according to,

$$\boldsymbol{\xi}_t = \boldsymbol{F} \boldsymbol{\xi}_{t-1} + \boldsymbol{v}_t \tag{1.32}$$

$$\mathbb{E}_{t-1}\left[\boldsymbol{v}_t \boldsymbol{v}_t'\right] = \boldsymbol{Q}. \tag{1.33}$$

where the vector of shocks $\boldsymbol{v}_t \equiv \begin{bmatrix} \boldsymbol{\epsilon}_t & \boldsymbol{\varepsilon}_t \end{bmatrix}'$, the state matrix $\boldsymbol{F}$ and the covariance matrix $\boldsymbol{Q}$ represent the movements of $\overline{\boldsymbol{y}}_t$ and $\tilde{\boldsymbol{y}}_t$ in (1.27) and (1.31). Both matrices $\boldsymbol{F}$ and $\boldsymbol{Q}$ are constructed on economic assumptions and partially estimated in the process. Their specific forms are presented respectively in equations (A3.3) and (A3.5). Equation (1.32) is the state equation of the model.

From a determined set of initial values $\hat{\boldsymbol{\xi}}_{1|0}, \boldsymbol{P}_{1|0}$ and joint probability distribution theory, the estimated latent variables of each time t are extracted from the prediction step,

$$\begin{aligned} \hat{\boldsymbol{\xi}}_{t|t-1} &= \boldsymbol{F}\hat{\boldsymbol{\xi}}_{t-1|t-1} \\ \boldsymbol{P}_{t|t-1} &= \boldsymbol{F}\boldsymbol{P}_{t-1|t-1}\boldsymbol{F}' + \boldsymbol{Q}, \end{aligned} \tag{1.34}$$

followed by the update step,

$$\begin{aligned} \hat{\boldsymbol{\xi}}_{t|t} &= \hat{\boldsymbol{\xi}}_{t|t-1} + \boldsymbol{K}_t(\boldsymbol{y}_t - \boldsymbol{H}'\hat{\boldsymbol{\xi}}_{t|t-1}) \\ \boldsymbol{P}_{t|t} &= \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_t(\boldsymbol{H}'\boldsymbol{P}_{t|t-1}\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{P}_{t|t-1}, \end{aligned} \tag{1.35}$$

from time 1 until time T, the end of the sample. $\boldsymbol{K}_t \equiv \boldsymbol{P}_{t|t-1}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{P}_{t|t-1}\boldsymbol{H})^{-1}$ represents the Kalman gain updating optimally the prediction $\hat{\boldsymbol{\xi}}_{t|t-1}$ using the forecast error $\boldsymbol{y}_t - \boldsymbol{H}'\hat{\boldsymbol{\xi}}_{t|t-1}$. The detailed process is described in Appendix A1. After the preliminary results by the Kalman filter, we get estimates of the latent variables $\hat{\boldsymbol{\xi}}_{t|t-1}$,$\hat{\boldsymbol{\xi}}_{t|t}$ and their respective mean squared errors $\boldsymbol{P}_{t|t}$,$\boldsymbol{P}_{t|t-1}$ for all T times of the sample.

The log likelihood associated with these estimations and the observations $\boldsymbol{y}_t$, considering a set of parameters $\boldsymbol{\theta}$ is,

$$\begin{aligned} \sum_{t=1}^{T} \log f(\boldsymbol{y}_t|\boldsymbol{\zeta}_{t-1};\boldsymbol{\theta}) = &- \frac{Tn}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log \mid \boldsymbol{\Sigma}_t \mid \\ &- \frac{1}{2}\sum_{t=1}^{T}\left[\boldsymbol{y}_t - \boldsymbol{\mu}_t\right]' \left[\boldsymbol{\Sigma}_t\right]^{-1} \left[\boldsymbol{y}_t - \boldsymbol{\mu}_t\right]', \end{aligned} \tag{1.36}$$

where,

$$\begin{aligned} \boldsymbol{\mu}_t &= \boldsymbol{H}'\hat{\boldsymbol{\xi}}_{t|t-1}, \\ \boldsymbol{\Sigma}_t &= \boldsymbol{H}'\boldsymbol{P}_{t|t-1}\boldsymbol{H}. \end{aligned}$$

From an initial set of parameter $\boldsymbol{\theta^0}$, we use computational maximization of the log likelihood in (1.36) with respect to $\boldsymbol{\theta}$ and obtain estimations of $\hat{\boldsymbol{\xi}}_{t|t}$ and $\boldsymbol{P}_{t|t}$ based on optimal parameters. Finally, the Rauch-Tung-Striebel smoother goes recursively over the T estimations and updates each of them using the information set $\boldsymbol{\zeta}_T$. We denote those smoothed inferences as $\hat{\boldsymbol{\xi}}_{t|T}, \boldsymbol{P}_{t|T}$. Appendix A2 describes the smoothing process. From the set of estimations $\hat{\boldsymbol{\xi}}_{t|T}$ and their corresponding mean squared errors $\boldsymbol{P}_{t|T}$ we are able to extract the desired variables. It should be noted that the Kalman filter can directly accommodate missing values. In the presence of missing observations at a time t, the update step in (1.35) applies an alternative correction to the predictions $\hat{\boldsymbol{\xi}}_{t|t-1}$ using the Kalman gain and the observed measurement errors.

2 Trend-Cycle Decompositions of Interest Rates

We present in this part of the paper the analysis based on the model described in Section 1. First, we present the data that we used to perform our calculations, the effective restrictions we have imposed on the parameters we must estimate discussed in 1.3 and the values we have chosen as initialization of the Kalman filter. Then, we present the estimations of the trends discussed in 1.2 and discuss their suitability with historical evidence as well as with the results of other researchers.

2.1 Data

Three measures are necessary to extract all variables of interest in the system (1.22) : short-term nominal interest rate, long-term nominal interest rate and inflation rate. All the data we use in this section is retrieved from Jordà et al. [2017], who have made their *Macrohistory Database* publicly accessible. The nominal interest rates are used in their original form. The inflation rate is constructed from data on consumer price indices, as log-differences. The base year of all the consumer prices is 1990, expressed as 100. The database itself is built on various sources between countries and periods of time. The short-term nominal interest rates are mainly annual averages of 3-month treasury bill yields when available, or closest equivalent. The long-term nominal interest rates are mostly annual averages of 10-year, 15-year or 20-year maturity government bond rates, or closest equivalent. All the rates are expressed as percentage point. Each set of three variables is retrieved for 8 OECD countries : Canada, France, Germany, Italy, Japan, Switzerland, the United Kingdom and the United States, from 1870 to 2016. For computability purposes, we clean extreme values of periods of hyperinflation. Every computation of log-differences over the value of 20 is replaced by a missing value. Over this value, the first order approximation $\ln(1+x) \approx x$ is too loose and denotes of higher values of real inflation, non-computable by the econometric model described in 1.3. As such, the model estimated with this collected data on $m = 8$ countries results in $\tau = 27$ trends and $n = 24$ cyclical components over $T = 147$ periods of time.

2.2 Restrictions

We present here all the restrictions that we imposed on the estimated parameters of the model, so that estimations stay consistent with economic theory and are more likely to appear as envisioned results. This translates first into restricting long-term movements of all the trends variables in $\overline{\boldsymbol{y}}_t$ through the set of standard deviations $\left[\sigma_{\overline{r}^w}, \sigma_{\overline{r}^i}, \sigma_{\overline{ts}^w}, \sigma_{\overline{ts}^i}, \sigma_{\overline{\pi}^w}, \sigma_{\overline{\pi}^i}\right]$. To make sure that trends reflect only low-frequency fluctuations and that high-frequency movements are captured by cyclical variables, we impose range restrictions on the standard deviation of any trend variable and no restriction on the standard deviation of cyclical components (denoted as $\sigma_{\tilde{R}_i}, \sigma_{\tilde{R}_i^L}, \sigma_{\tilde{\pi}_i}$). We set the maximum of variances to be 1/50 for world trends in real interest rates and term spreads and at 1/10 for the world trend in inflation. This means that the model cannot estimate the expected change of each variable to have a standard deviation higher than one percent over respectively half a century ($\overline{r}_t^w$,$\overline{ts}_t^w$) and a decade ($\overline{\pi}_t^w$). Those values also express our belief that nominal trends might be substantially more volatile than real trends. Additionally, we let the idiosyncratic trends to be twice as volatile as their global counterparts. Those restrictions are loosely based on the prior and posterior medians in Del Negro et al. [2018]. If those restrictions proved to be too permissive, it would only result in trends capturing slightly higher-frequency fluctuations, with no consequence of importance to the analysis.

The second set of restriction is regarding the matrix of loading parameters $\boldsymbol{\lambda_\pi}$. Because each coefficient λ_i^π is the loading of the trend in global inflation of a country i and it appears in all terms of the observation equation (1.22), we impose an arbitrary maximum on all of them so that it does not absorb fluctuations of other variables. We also impose a lower-bound of 0, since any negative deviation from the world trend should be captured by the idiosyncratic trend $\overline{\pi}_t^i$ or the cyclical component $\tilde{\pi}_{i,t}$. The upper limit value is determined from an unrestricted pre-estimation, of which the results are presented in Appendix B3.1. We round up the maximum value of the estimated loadings to the value of 1.6, which we use as upper limit in our baseline (restricted) model. Additionally, for identification purposes we set the loading coefficient of the United States to the value of $\lambda^{\pi}_{US} = 1$, under the hypothesis that the U.S. economy is the closest to representing the global component in inflation out of the sampled countries. If that assumption proved wrong, it would only reflect in the U.S. trend in deviation from the world inflation $\overline{\pi}^{US}_t$ without incidence on other estimation. Appendix B3.2 shows the estimated loadings when the only restricted parameter is $\lambda^{\pi}_{CA} = 1$. We can see then that the estimation of the loading coefficient of the U.S. is, as assumed, very close to 1.

The final restriction of the model is the constraint of the vector autoregressive parameters ϕ_{XY} of the matrix $\mathbf{\Phi}$ (see equation (A3.4)). We set each of them to be in the range $[0; 1)$ so that all the cyclical components be stationary. Moreover, for computability purposes, we set that the matrix of coefficient linking a country i's cyclical variables $\tilde{\boldsymbol{y}}_{i,t}$ to their one-lag value $\tilde{\boldsymbol{y}}_{i,t-1}$ is similar across all countries of the sample. This means that only 9 parameters will be estimated as part of the VAR matrix $\mathbf{\Phi}$.

The constrained parameters and their respective restrictions are summarized in table 2.1.

2.3 Initialization

As discussed in 1.3, the Kalman filter estimation depends on a set of initial values which we denote as $\hat{\boldsymbol{\xi}}_{1|0}, \boldsymbol{P}_{1|0}$. The values in $\hat{\boldsymbol{\xi}}_{1|0}$ about trends represent an educated guess as to their respective pre-sample value. The corresponding values of $\boldsymbol{P}_{1|0}$ represent the uncertainty surrounding this guess. The values in $\hat{\boldsymbol{\xi}}_{1|0}$ about stationary cyclical variables are taken from the unconditional mean of their respective process. We have based our guesses for initial expected trends on loose estimations of posterior medians in Del Negro et al. [2018]. These values are 2 for the trend in world real interest rate, 0 for the trend in world term spread and 2.5 for the trend in world inflation.[6] All the country-specific trends are initialized at expected values of 0.[7] The standard deviation corresponding each of these initial guesses are 1 for the world real trends, 0.5 for the country-specific real trends, 2

[6]We denote those values as $\overline{r}^{w}_{1|0}, \overline{ts}^{w}_{1|0}$ and $\overline{\pi}^{w}_{1|0}$.

[7]We denote those values as $\overline{r}^{i}_{1|0}, \overline{ts}^{i}_{1|0}$ and $\overline{\pi}^{i}_{1|0}$.

Table 2.1: Restrictions on parameters

Parameter	Lower Limit	Upper Limit
λ_i^π	0.00	1.60
λ_{US}^π	1.00	1.00
$\sigma_{\overline{r}^w}$	0.00	$\sqrt{1/50}$
$\sigma_{\overline{r}^i}$	0.00	$\sqrt{1/25}$
$\sigma_{\overline{ts}^w}$	0.00	$\sqrt{1/50}$
$\sigma_{\overline{ts}^i}$	0.00	$\sqrt{1/25}$
$\sigma_{\overline{\pi}^w}$	0.00	$\sqrt{1/10}$
$\sigma_{\overline{\pi}^i}$	0.00	$\sqrt{1/4}$
ϕ_{XY}	0.00	0.99

Note: Table 2.1 presents the restrictions that are imposed on some of the parameters which are to be estimated in the process described in 1.3. These restrictions are discussed in 2.2. The left column denotes the restricted variable while the middle and right columns denote respectively of the lower and upper limits of the constraint. Each parameter denoted with a subscript or a superscript i is applicable for all 8 countries of the sample, with the exception of λ_i^π which exclude λ_{US}^π. The parameter ϕ_{XY} denotes all the autoregressive coefficients of the matrix $\mathbf{\Phi}$ (see equation (A3.4) for details).

for the trend in global inflation and 1 for the country-specific trens in inflation.[8] The whole set of initial expected standard deviations are based on the priors in Del Negro et al. [2018] and Negro et al. [2017]. The cyclical variables are all initialized at their unconditional mean of 0.[9] The corresponding set of initial variances is the unconditional variances of their respective process (see (A1.11) for details on the unconditional variance computation). These values of $\boldsymbol{P}_{1|0}$ depend on the matrix $\boldsymbol{Q}$ and as such are computed post-estimation of the parameters in $\boldsymbol{Q}$.[10]

A brief summary of all the discussed initialization are listed in tables 2.2 and 2.3.

[8]By abuse of notation, we write those values as $P_{1|0}^{\overline{r}^w}$, $P_{1|0}^{\overline{ts}^w}$, $P_{1|0}^{\overline{\pi}^w}$, $P_{1|0}^{\overline{r}^i}$, $P_{1|0}^{\overline{ts}^i}$ and $P_{1|0}^{\overline{\pi}^i}$. These correspond in reality to the square root of the corresponding diagonal element of the variance covariance matrix $P_{1|0}$.

[9]We denote those values as $\tilde{R}_{i,1|0}$, $\tilde{R}_{i,1|0}^L$ and $\tilde{\pi}_{i,1|0}$.

[10]By abuse of notation, we denote those values as $P_{1|0}^{\tilde{R}_i}$, $P_{1|0}^{\tilde{R}_i^L}$ and $P_{1|0}^{\tilde{\pi}_i}$ (see note above for details).

Table 2.2: Initial Expected Values, Baseline Model

$\overline{r}^w_{1\|0}$	2.00	$\overline{r}^i_{1\|0}$	0.00
$\overline{ts}^w_{1\|0}$	0.00	$\overline{ts}^i_{1\|0}$	0.00
$\overline{\pi}^w_{1\|0}$	2.50	$\overline{\pi}^i_{1\|0}$	0.00

Note: Table 2.2 presents the initial expected values of the vector of latent variables $\hat{\boldsymbol{\xi}}_{1|0}$ discussed in 2.3. The vector $\hat{\boldsymbol{\xi}}_{1|0}$ serves as initialization of the Kalman filter as discussed in 1.3.

Table 2.3: Initial Standard Deviations, Baseline Model

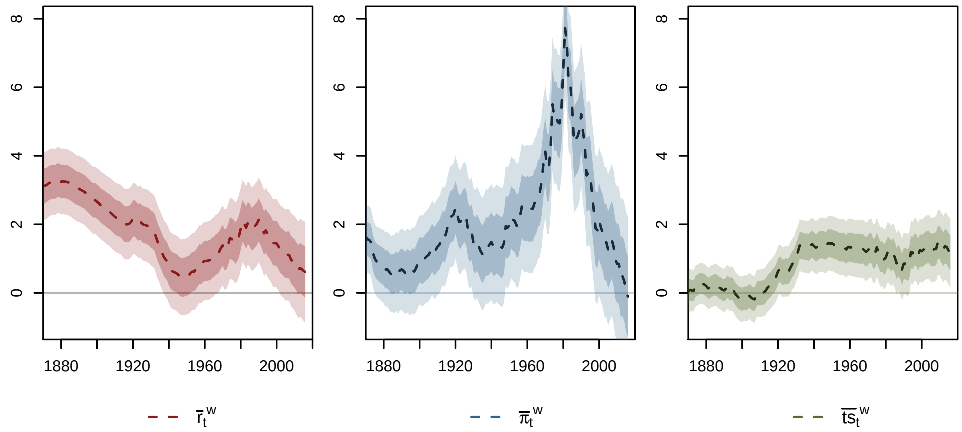
$P^{\overline{r}^w}_{1\|0}$	1.00	$P^{\overline{r}^i}_{1\|0}$	0.50
$P^{\overline{ts}^w}_{1\|0}$	1.00	$P^{\overline{ts}^i}_{1\|0}$	0.50
$P^{\overline{\pi}^w}_{1\|0}$	2.00	$P^{\overline{\pi}^i}_{1\|0}$	1.00

Note: Table 2.3 presents the standard deviations associated with the expected initial values $\hat{\boldsymbol{\xi}}_{1|0}$. These values are discussed in 2.3. By abuse of notation, we write the initial standard deviation of a variable y as $P^y_{1|0}$. These values correspond in reality to the square root of the corresponding diagonal element of the variance covariance matrix $\boldsymbol{P}_{1|0}$.

2.4 Results

We present in this final part of the section the results based on the initialization presented in 2.3 and under the constraints discussed in 2.2. Figure 2.1 shows the smoothed estimations of the world trends in real interest rate, inflation and term spread alongside their respective confidence intervals of 95 and 68 coverage probabilities. As the three panels use the same scale, we notice easily that the trends of real variables are, indeed, less volatile than the trend in inflation. The trend in the term spread has been the most stable over

Figure 2.1: Estimated world trends in short-term real interest rate, inflation and term spread

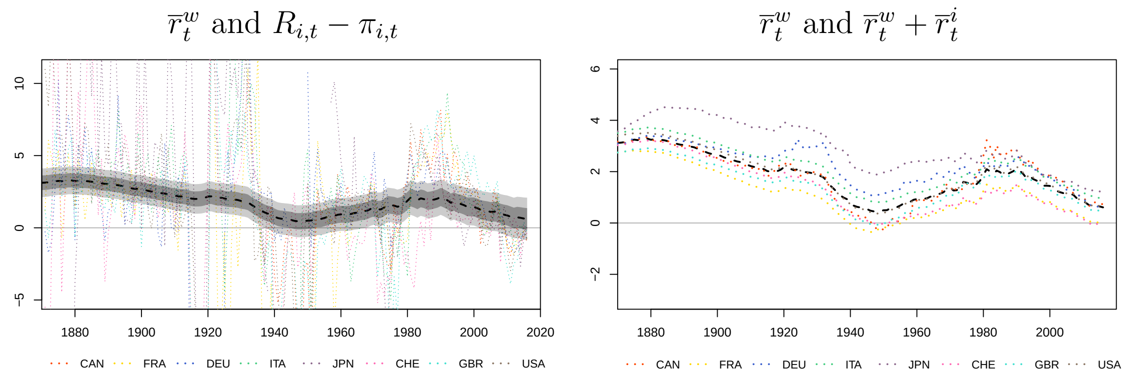


Note: From left to right, the panels show respectively the worlds trends $\overline{r}_t^w$ (dashed red line; see legend), $\overline{\pi}_t^w$ (dashed blue line; see legend) and $\overline{ts}_t^w$ (dashed green line; see legend) together with their 68 and 95 confidence intervals (represented with the shaded areas).

the whole sample. The coverage intervals show that the uncertainty surrounding all the trends is quite wide. An illustration of this are the final values of the world real interest rate, which according to the 68% confidence interval could be as high as 1.4 percent or as low as -0.15 percent. However, the movements of the world inflation and real interest rate seem to be in par with empirical fluctuations and other researches.

When comparing our estimation of the world trend in real interest rate to Negro et al. [2017], Jordà et al. [2017] and Desroches and Francis [2006], we seem to have found a middle-ground. Our results are not as volatile as the decadal moving averages of Jordà et al. [2017] and not as stable over the first half of the sample as Negro et al. [2017]. We also find as Desroches and Francis [2006] that the real interest rate had two peaks in the 10-year interval between 1980 and 1990, meaning the structural changes behind the fall in real interest rates might have began only after 1990. We also find values distinguishably higher in the first 50 years of the sample than estimated by Negro et al. [2017] : whereas they estimate that the world trend has been stable for a century around 1.5 percent, we find that from 3 percent at the beginning of the sample, it has steadily decreased until the end of World War I. This could indicate that the decades-long fall in real interest rate since 1990 is not unprecedented. Our results are in fact more similar to their analysis with looser priors on the fluctuations of trends (see Figure A3 in Negro et al. [2017]), enforcing

Figure 2.2: Estimated trends and observed data for short-term real interest rates



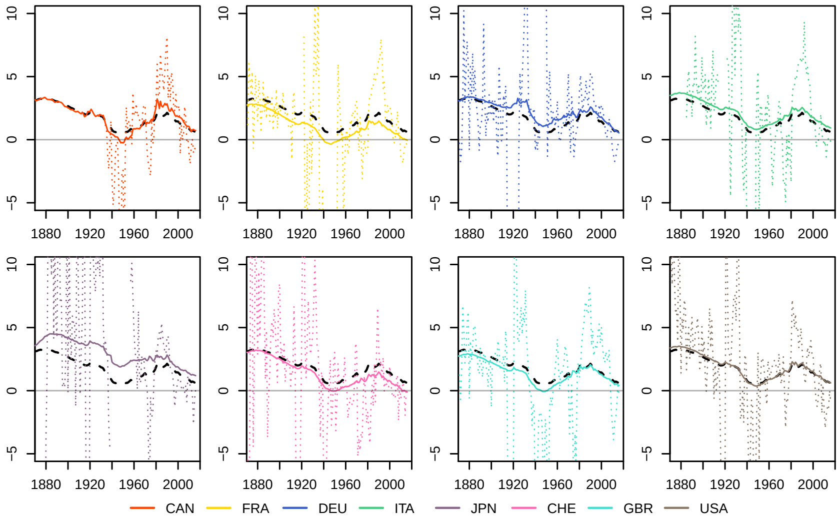
Note: The left panel presents the observed ex-post real interest rate $R_{i,t} - \pi_{i,t}$ for each country i of the sample (dotted lines), together with the estimated world trend $\overline{r}_t^w$ (dashed black line) and its corresponding 68 and 95 confidence intervals (grey shaded areas). The right panel shows the estimated overall trend in real interest rate $\overline{r}_{i,t} = \overline{r}_t^w + \overline{r}_t^i$ for each country i of the sample (dotted lines), together with the estimated world trend $\overline{r}_t^w$ (dashed black line).

our assumption that our looser restrictions caused these differences. The movements in the trend reflect the low interest rates during World War I, as well as, after a brief recovery period, the subsequent fall in the late 1920s, during the Great Depression and until the end of World War II. The low-interest period after the Great Depression and until the war is precisely when Hansen [1939] publicly shared his concerns of secular stagnation. Contrary to his belief, the world real interest rates picked up again after 1945 during the postwar economic boom, until reaching a new high in 1980. This high trend in the world interest rate, hovering above 2 percent, lasted a decade. It was the first time real interest rates were as high since the 1920s. Then, from the second peak in 1990, the trend has decreased by over 1.5 percent point, of which less than 0.4 occurred since the 2007 economic crisis. The middle panel shows interesting movements of inflation throughout the last century. The period of global deflation during the 1930s is reflected in the fast decrease of the trend in world inflation. The world trend in inflation then took off exponentially until 1980 before showing a disinflation period just as fast. The last value shows that the estimation of inflation in 2016 has reached 0, which is in par with the results in Negro et al. [2017]. The right panel on term spreads shows that the trend has essentially been stable around 1.5 percent for the last century.

The left panel in figure 2.2 shows the estimated trend in the world real interest rate alongside the observed ex-post real rate, i.e. calculated from the data as the difference between short term interest rate and inflation at each time. The extreme high-frequency fluctuations of the real interest rates across all countries in the first half of the sample emphasizes the necessity of the constraints on standard deviations discussed in 2.2. Figure B3.1 in Appendix shows the same analysis but without any restriction on the standard deviations. It is easy to see that some high frequency fluctuations are captured in the trends by the algorithm, and that the estimated trend in the Unrestricted Model is closer the decadal moving average (solid red line). The downside of imposing these volatility constraints is that the very clear and sharp fall of the real interest rates over the world is not as reflected in the movement of the trend. The peak during the 1980s is not as high and the most recents points are not as low. In fact the data on ex-post real interest rates $R_{i,t} - \pi_{i,t}$ shows that it has been mostly at or below zero for the last ten years across all 8 countries. The right panel shows the estimated trend in the world real interest rate alongside the overall trend in real interest rates of all the sampled countries. Almost all the economies had reached a peak in the 1980s and are today at one of their lowest point of the sample. Interestingly, both panels show that the co-movement of all the countries' real interest rate has gradually become stronger from the 1940s but is especially evident over the last 40 years. This is coherent with the idea that these past decades the countries' rates have been driven more by global drivers than their own-countries. They have also been slowly converging to the global rate, meaning that the idiosyncratic components have been decreasing, bringing the global economy closer to RIRP due to globalization and liberalization of capital markets.

Figure 2.3 shows the overall trend in real interest of all eight countries, calculated as the sum of the global and the idiosyncratic components (see equation 1.18). Each panel presents additionally the observed ex-post real interest rate of a country and the world trend, and allows for an interesting reflection of the theory on convenience yields discussed in 1.2. We assumed in the model that real interest rates between countries differed in the long term due to a return on the liquidity and safety properties of their respective governments bills. As such, looking at their different deviations from the world trend might give an idea of such returns. If the overall trend of a country is noticeably below the global trend, it could indicate that investors are willing to pay for their convenience properties. France and Switzerland would be two examples of high convenience yields, while interest rates in Japan and Italy would indicate that investors earn a higher return

Figure 2.3: Estimated trends and observed data for short-term real interest rates by country

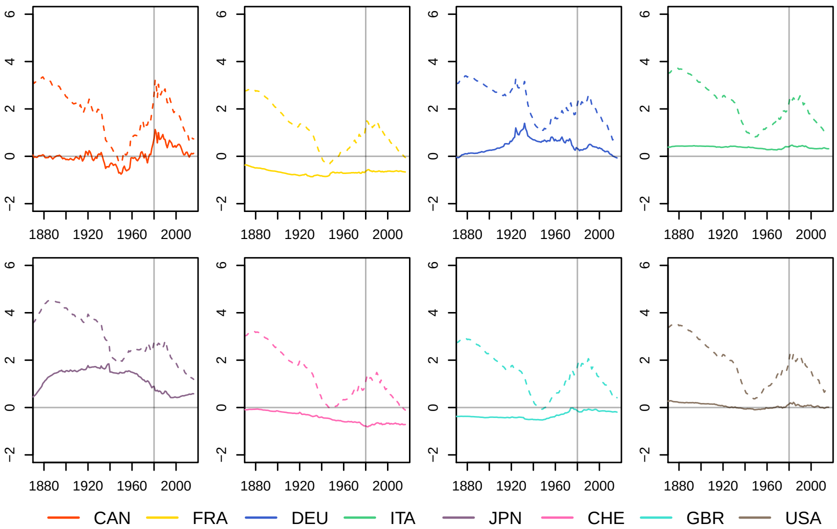


Note: From left to right, each panel presents the observed ex-post real interest rate $R_{i,t} - \pi_{i,t}$ for each country i of the sample (dotted line), the overall trend in real interest rate of the country (straight line) together with the estimated world trend $\bar{r}_t^w$ (dashed black line) in the order : Canada, France, Germany, Italy, Japan, Switzerland, United Kingdom, United States (see legend for colors).

due to uncertainty surrounding the safety and liquidity properties of both bonds. The Canadian trend has been oscillating above and below the world trend but seems to have converged to the world trend over the last 20 years, as did Germany. The U.K and the U.S. have been at level with the world trend for the past fifty years. More interestingly, the panel for the United States could point to the U.S. trend in real interest as an important driver of the world real interest rate, since their levels have been equal for a century. Switzerland has show the lowest trend in real interest rate since the late 1970s and the trend went negative as early as 2012, indicating it to be the country with the safest and most convenient government bonds.

Figure 2.4 presents the overall trend in real interest rate for each country, as well as its idiosyncratic part. All panels show that the country specific trends are either stable since the 1980s or capture only a small part of a country's fall in real interest rate, meaning that for the most part it is indeed attributed to the world component. France, Switzerland, the United Kingdom and the United States show a change in idiosyncratic trend close to zero. While Germany and Japan experienced some fluctuations, the level in 2016 is very close to the level in 1980, which is obviously not the case for their respective overall trend.

Figure 2.4: Estimated trends for short-term real interest rates by countryl

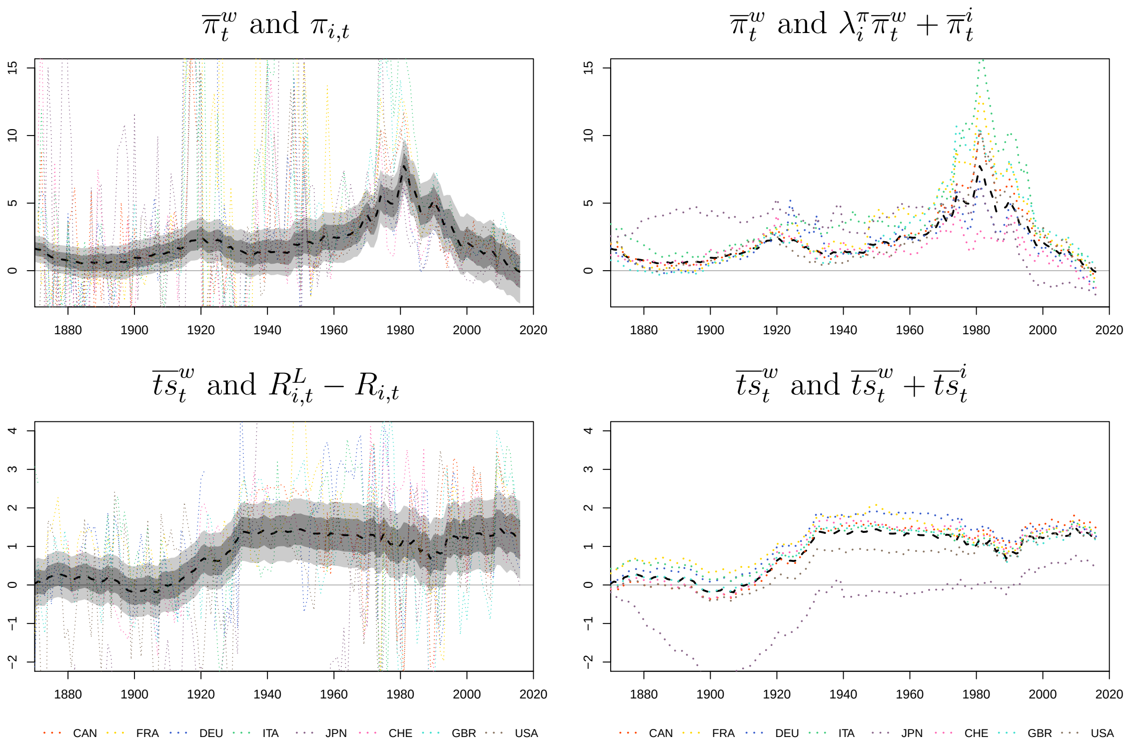


Note: From left to right, each panel presents both the overall trend in real interest rate $\overline{r}_{i,t} = \overline{r}_t^w + \overline{r}_t^i$ (dashed line) and its idiosyncratic component $\overline{r}_t^i$ separately (straight line) for a country i in the order : Canada, France, Germany, Italy, Japan, Switzerland, United Kingdom, United States (see legend for colors).

Only Canada experience a sensible decrease in its deviation from the world real interest rate.

Figure 2.5 shows similar representations as figure 2.2, for estimated trends in inflation and term spreads. The left panels show the world components $\overline{\pi}_t^w$ and $\overline{ts}_t^w$ as well as their confidence intervals and their empirical equivalents $\pi_{i,t}$ and $R_{i,t}^L - R_{i,t}$. The right panels show both the estimated world and overall trends, and they corroborate our previously discussed observation : the trends of all countries denote of closer co-movements in the last 40 years and convergence towards the world trends. In both cases, all countries except for Japan deviate today extremely little from the world trend. The upper left panel of figure 2.5 shows interesting movements of world trend in inflation over the whole horizon. We can see that for the first part of the sample, the world inflation stayed in a reasonable range of 1 to 3 percent. It should be noted however that a number of data in the 1920s were removed due to their extreme values. It could result in the estimated trend at that time to be uncertain. The world trend denotes of the quick rise in inflation from 1970, and specially following both oil crisis in 1973 and 1979. In 1981, it reached a peak of over 8 percent, which was more than 4 times its century average. The global inflation of the early 1980s reflected in all G7 countries' monetary policy which increased interest rates in

Figure 2.5: Estimated trends and observed data for inflation and term spread

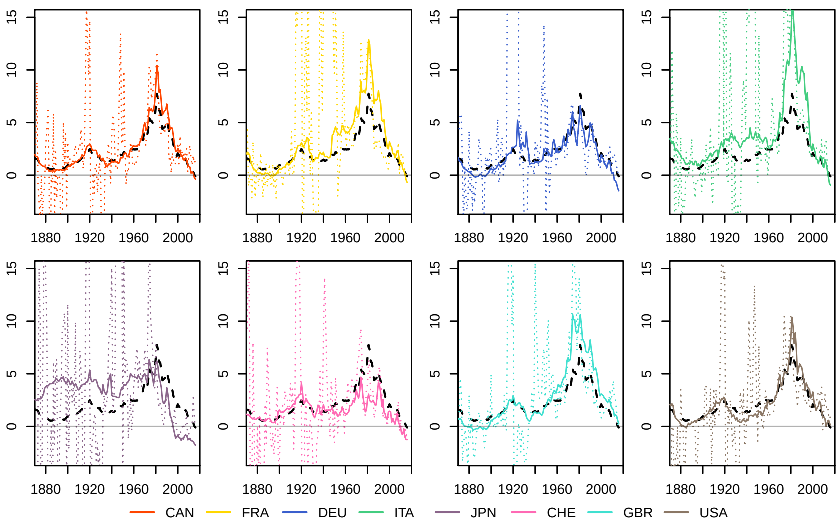


Note: The upper left panel presents the observed inflation $\pi_{i,t}$ for each country i of the sample (dotted lines), together with the estimated world trend in inflation $\overline{\pi}_t^w$ (dashed black line) and its corresponding 68 and 95 confidence intervals (grey shaded areas). The upper right panel shows the estimation of overall trends in inflation $\overline{\pi}_{i,t} = \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i$ for each country i (dotted lines), together with the estimated world trend $\overline{\pi}_t^w$ (dashed black line).The lower left panel presents the observed term spread $R_{i,t}^L - R_{i,t}$ for each country i of the sample (dotted lines), together with the estimated world trend in term spread $\overline{ts}_t^w$ (dashed black line) and its corresponding 68 and 95 confidence intervals (grey shaded areas). The lower right panel shows the estimation of overall trends in term spread $\overline{ts}_{i,t} = \overline{ts}_t^w + \overline{ts}_t^i$ for each country i (dotted lines), together with the estimated world trend $\overline{ts}_t^w$ (dashed black line).

order to restrain the exploding inflation. From then on, inflation has been continuously decreasing, to an all-time low at the very end of the time horizon.

Figure 2.6 shows for each country of the sample a panel superposing the trend in the global inflation, the country's overall trend in inflation as well as the observed inflation. We can see in each panel interesting country-specific movements. The panels for Canada, Germany and the United States show that these countries have deviated the least from the world trend. The panels for France, Italy and the United Kingdom show that these countries have suffered from even higher inflation following the oil crisis, however they quickly reduced their inflation levels and converged to the world trend. This is especially the case for Italy, which showed an inflation rate of almost twice the world level in 1980, but the disinflation allowed the economy to be back at the world trend level as early as the late 1990s. The panel for Japan shows that the trend in inflation of the economy has constantly been high and relatively stable for the first century of the sample. The economy was also fast to contain inflation during and after the 1970s shocks. Japanese inflation from 1980s decreased along with every other countries, except at a quicker pace. Especially, the trend in deflation from the early 1990s denotes of the Japanese economy stuck in a liquidity trap during the Lost Decade. The panel for Switzerland shows that the country has been the most stable out of the whole sample, even managing to avoid the high inflation following the oil crisis. The trend also captures the deflation in the Swiss economy for the past decade.

Figure 2.6: Estimated trends in inflation by country



Note: From left to right, each panel presents the observed inflation $\pi_{i,t}$ for each country i of the sample (dotted line), the overall trend in inflation of the country $\overline{\pi}_{i,t} = \lambda_i^{\pi}\overline{\pi}_t^w + \overline{\pi}_t^i$ (straight line) together with the estimated world trend $\overline{\pi}_t^w$ (dashed black line) in the order : Canada, France, Germany, Italy, Japan, Switzerland, United Kingdom, United States (see legend for colors)

Appendix B2 presents additional results of the baseline model, including the parameters estimated by maximum likelihood in the process.

3 Determinants of World and Country-specific Trends

This final section pursues further analysis on the results presented in 2.4. First, using economic theory, we make assumptions about the relationship between a selection of potential secular drivers and real interest rates. Then, we present the data which we have collected as proxies of the discussed drivers. Finally, we discuss the relationship between the chosen variables and the estimated trends in real interest rates using different forms of linear regression.

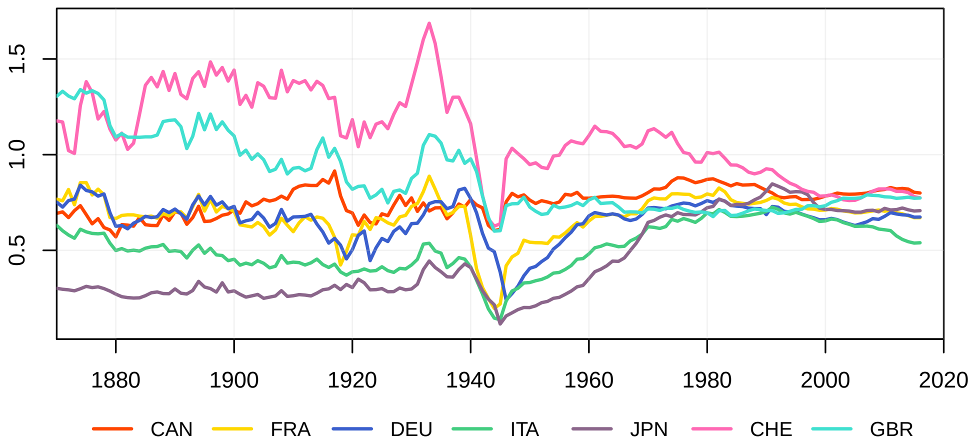
3.1 Rationale behind the determinants

While literature about secular stagnation discusses an eminently wide range of determinants of real interest rates, three particular sources are most commonly accepted : changes in global growth, shifts in investment levels and changes in desired savings. We focus our analysis on all three of them.

3.1.1 Global growth

In neoclassical theory, changes in global growth are assumed to put pressure on households' expected earnings, changing their incentive to save. A simplified model as the Ramsey growth model shows that the steady state real interest rate is partly determined by the pace of technological progress : positive changes in the technological level will give households a positive signal about their future income and decrease their need for savings. This decrease in savings translates into lower investment in capital and thus, lower capital accumulation. Because real interest rates equal marginal product of capital in the model and because capital accumulation has decreased, both the marginal product of capital and the real rates increase. The amplitude of the households' reaction to higher expected income depends on their preference for smoothed consumption. Similarly, in the Solow model, the marginal product of capital is determined by both productivity growth and population growth. If labour and capital are complements in the production function, then more workers means an increase in both the marginal return of capital and the real interest rate. Hansen [1934] agreed with this relationship, and feared that the population stabilization would be determinant in the future of real interest rates and a crucial point

Figure 3.1: GDP (PPP) per capita in U.S. dollars to U.S. GDP (PPP) per capita



Note: Figure 3.1 presents the measure of catch-up growth discussed in 3.1.1. This variable is constructed as the ratio of a country's GDP per capita based on purchasing power parity to the U.S. equivalent, such that a ratio of 1 expresses the same GDP (PPP) per capita as the U.S. at a specific time. Source : Jordà et al. [2017].

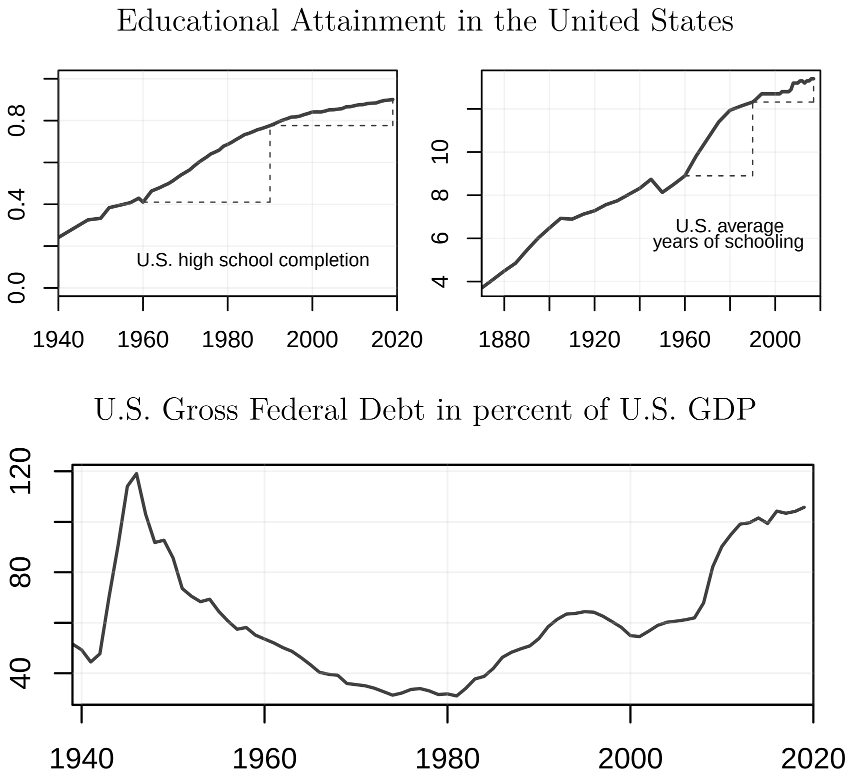
in his theory of secular stagnation. Consequently, we will focus on technological progress and change in labour supply as proxies of global growth. Additionally, we include a direct measure of expected growth. As for the change in technological level, it proves interesting to separate it into two measures to study their effect independently as explored in Rachel and Smith [2015]. Thus, instead of technological growth as a whole, we include measures of growth of productivity at the technological frontier and catch-up growth of the countries behind said technological level. The former is proxied by U.S. productivity growth and the latter by the ratio of a country's GDP per capita to the U.S. equivalent. Figure 3.1 presents the measure of catch-up growth and shows that it had been quite stable over time until the beginning of World War II. At the beginning of 1940, the U.S. economy boomed with the involvement of the U.S. army in the conflict, translating into all economies lagging far behind until the end of the war. It then took up to 40 years for the economies to catch-up to their previous levels. The decade between 1980 and 1990 was the point where all economies were jointly their closest to the U.S. level. In the past 30 years however, catch-up growth has stayed largely stable across all countries.

We turn now to measuring U.S. productivity growth. We focus on U.S. educational attainment and government debt as proxy of growth at the frontier, inspired by Gordon [2012] and Gordon [2014], in which he measured perspective frontier growth and the structural headwinds holding it back.[11] The effect of education level can be theoretically explained by the models in Romer [1986], Romer [1990] and Lucas [1988], in the form of human capital accumulation. Endogenous productivity growth comes from accumulation of knowledge in Romer [1990] and schooling in Lucas [1988], making for more efficient workers (i.e. more human capital). Fernald and Jones [2014] agree about the determinant role of education on the past U.S. economic growth. We present in Figure 3.2 the development of U.S. educational attainment through two measures. It can be seen on both panels that the change in 30 years is substantially lower from 1990 to 2020 than from 1960 to 1990, meaning that human capital accumulation has slowed down and the U.S. might have reached an educational plateau. This would mean that the upward pressure of change in U.S. education level on real interest rates seen in the past is unlikely to happen again.

As for the relationship between level of debt and real interest rate, it is not as apparent and still up for debate. A simple macroeconomic model where capital is crowded out by government debt shows that an increase in debt level results in an increase in marginal return of capital. Such a model is the basic theory behind Engen and Hubbard [2004]. Their results however are conflicted : while some specifications account for a positive effect of government debt on real interest rate, others show no significant relationship. Reinhart and Rogoff [2010] find that the effect of debt-to-GDP ratio on growth is insignificant for levels below 90 percent, and negative above that threshold. Kumar and Woo [2010] agree with a non-linear relationship, with negative effects at high levels of initial debt. Checherita-Westphal and Rother [2012] show that the negative effect could begin at a threshold as low as 70 percent. Rachel and Smith [2015], however, believe that fiscal support in the U.S. could have artificially boosted its growth in the last 40 years. This increase in government debt from 1980 can be seen in the lower panel of Figure 3.2, which shows the evolution of the level of the U.S. debt-to-GDP ratio.

[11] Gordon [2012] came to the conclusion that six structural headwinds were holding the U.S. economy back. In Gordon [2014] he pursued his research on four of them he deemed "widely recognized and uncontroversial" : demographic shifts, rising inequality, public indebtedness and educational attainment. Because we are working with inequality and demographics in the next part, we choose to focus here on education and government debt.

Figure 3.2: Proxies of growth at the technological frontier



Note: Figure 3.2 presents the variables used as proxy measures of growth at the technological frontier. The upper panels shows the level of educational attainment in the United States in two different forms. The upper left panel shows the proportion of U.S. citizens aged 25+ who have completed at least high school, from 1940 to 2019. The data is linearly approximated between 2 to 5 years from 1940 to 1964. The upper right panel shows the average number of years of schooling of U.S. citizens aged 25+. The data is linearly approximated over 5 years from 1870 to 1990. The lower panel presents the ratio of the United States' Gross Federal Debt to the U.S. Gross Domestic Product, from 1939 to 2019. Source : U.S. Census Bureau (upper left panel), Our World in Data (upper right panel), Federal Reserve Bank of St. Louis (lower panel)

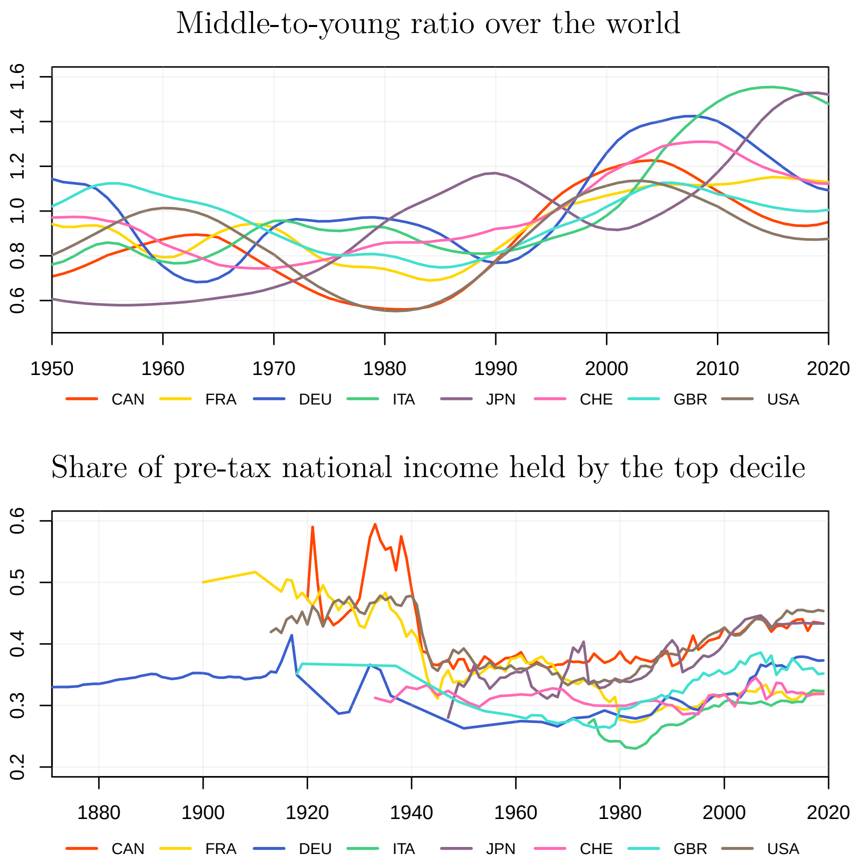
3.1.2 Saving rates

We discuss here shifts in the desired savings schedule as a potential force behind the fall of the world real interest rates. A simple investment-savings (I-S) theoretical framework can describe forwardly the relationship between the schedules in levels of saving and investment and real interest rates, where the equilibrium ratio is the one equating both levels as presented in Rachel and Smith [2015]. A shift towards higher propensity to save for a similar investment schedule would result in an excess in savings, dragging the real interest rate down to a new, higher saving and investment equilibrium rate. We discuss here two variables as proxy for a preference shift towards higher saving rates : changes in the age structure of the population and inequality level in the distribution of income. Consumption smoothing and the life distribution of earnings shows in a simple way how age structure might affect the saving schedule in an economy. As individuals try to even out their consumption level over time and given the bell-shaped distribution of the lifetime income, there are 3 cycles of desired saving. People borrow in the first part of their life, save money when their income level is over their desired consumption and squander their savings in the final part of their life to maintain the consumption level. A shift in the age demographics of a country could then move the savings rate, depending if more people are in the saving or in the dissaving part of their lifecycle. The literature on age structure and movements in the real interest rate is quite wide, and so are the measures of demographics shift. Aksoy et al. [2019] researched the relationship between age profiles of a population and low-frequency movements in macroeconomic variables. Using different baskets of age structure, they find a statistically and economically significant effect of age demographics on real interest rate. Carvalho et al. [2016] have estimated that demographic developments in advanced economies, specifically changes in longevity, can account for up to 150 basis point out of the 450 bps decline in real interest rates since 1990. A higher life expectancy prompts all age profiles to save more to ensure smooth consumption over the longer life span. Many others find supportive evidence as well as projecting that continuous change in age structure will keep putting downwards pressure on real interest rates in the foreseeable future (see Ferrero et al. [2017]; Rachel and Smith [2015]; Gagnon et al. [2016]). We use the ratio of middle-aged to young adults (MY variable) as representative of age structure, based on results by Favero et al. [2016].[12] The upper panel in Figure 3.3 shows the evolution of the MY ratio, constructed as the ratio of individuals aged 40 to 49 to individuals aged 20 to 29 of a population. The overall increase in the ratio from below 1 in 1980 to almost twice that number in the 2000s, i.e. a predominance of young over middle-aged people to the reverse, accounts for a shifts in saving schedule toward higher desired rates. However, it should be noted that the MY variable decreased in all countries, Italy and Japan excluded, in the late 2000s. This means that demographic developments could only explain the pre-crisis fall in real interest rates.

We turn now to discussing changes in income as a driver behind shifts in desired saving level. The relationship between income and savings has been debated for decades. While Keynesian economic theories assume that an increase in present income necessarily translates into an increase in consumption, Friedman [1957] believed that only the expected lifetime income has an effect on consumption and saving levels (the Permanent Income Hypothesis). As such, we choose to focus on changes in income distribution and not on income levels, based on evidence in recent literature supporting that the rich have a higher propensity to save. Dynan et al. [2004] have found that saving rates go up with every quantile of income distribution, with numbers ranging from less than 5 percent of income for the bottom quintile, up to more than 40 percent for the top 5 percent. Alvarez-Cuadrado and El-Attar Vilalta [2012] find supporting evidence of the higher propensity to save of the rich, although they estimate that an increase in inequality would trigger a decrease in aggregate savings. On the contrary, Lancastre [2016] found that rises in income inequality resulted in an expansion of saving rates. These results are based on an overlapping generations New Keynesian model with which he estimated higher marginal saving rates in the upper quantiles of income distribution. The lower panel of Figure 3.3 shows the evolution of the share of pre-tax national income held by the top decile of a country's population. We can see an overall upwards movement of inequality levels from the early 1980s. Changes range from 6 percentage point in France and Switzerland up to 11 in the United States. To illustrate these numbers, this last measure of change means that the 10% of the U.S. population with the highest pre-tax income held 34% of total national income in 1980 and holds 45% in 2019. However, as for demographic

[12]The MY variable was initially proposed by Geanakoplos et al. [2004] in their research about real rates of return on equity and bonds. Favero et al. [2016] have expressed their belief that out of many demographic variables, the MY ratio captures the best the part of active savers in an economy.

Figure 3.3: Proxies for shifts in desired levels of saving



Note: Figure 3.3 presents the two variables used as proxies for shifts in desired saving schedules. The upper panel presents the *middle-to-young* (MY) variable as a measure of demographics development. The MY variable is constructed as the ratio of all individuals aged 40 to 49 to all individuals aged 20 to 29, from 1950 to 2020. The lower panel presents the share of pre-tax national income held by the top decile of a country's population, as a measure of inequality level. Both panels show the measures for all 8 countries of the sample (Canada, France, Germany, Italy, Japan, Switzerland, United Kingdom, United States; see legend for colors). Source : United Nations (upper panel), World Inequality Database (lower panel)

developments, this increase in inequality seems to stop in the late 2000s, giving it low explanatory power in the post-crisis fall in real interest rates.

3.1.3 Investment rates

We discuss in this final part the potential effect of changes in the desired investment rate on real interest rate. As discussed in 3.1.2, real interest rates stabilize at the equilibrium between desired saving and investment schedules and we have thus explored potential sources of shifts in the saving schedule. We expressed that such shifts would result in higher global saving and investment ratios in addition to lower real interest rates, based on the I-S framework. However, empirical evidence shows that they have in fact stayed stable since 1980, which implies that the investment schedule might have shifted as well, towards lower desired levels (see Rachel and Smith [2015]; Eichengreen [2015]). The combined shifts in saving and investment schedule would result in even lower rates and no apparent change in the investment and saving to GDP ratio. As such, we include in our set of determinant variables the ratio of public investment to GDP to account for shifts in investment levels. The IMF [2014] presents evidence of declining rates of public investment and the subsequent effect on real interest rates. Rachel and Smith [2015] agree with this positive influence of public indebtedness, however they measure a relatively small effect of the change in global investment level (20 basis point out of 450).

3.2 Data

We present in Table 3.1 the data that was used to perform further analysis on the determinants of the world real interest rates, based on the discussion in 3.1.

3.3 Results

This final part presents the results of the empirical-based analysis of the structural relationship between the estimated trends in real interest rates in 2.4 and the potential drivers discussed in 3.1. There are two parts to the analysis : the first uses transformation of the panel data into weighted averages as well as U.S. specific variables to quantify the change in the world real interest rate, $\bar{r}_t^w$. The second part is a cross-country panel-based regression, which aims to understand the fluctuations of the country-specific component of the trend in real interest rates, $\bar{r}_t^i$.

3.3.1 Regressions on the world trend in real interest rate

Table 3.2 presents the regressions of the world trend in real interest rate $\bar{r}_t^w$ on different sets of variables. Columns (1) to (13) show the result of univariate regressions on respectively : education, indebtedness, inequality, investment, labour supply, productivity growth, age structure, technological progress, U.S. educational attainment, catch up growth, U.S.

Table 3.1: Collected data for secular drivers

Variable	Data description	Period	Source
Middle-to-young ratio	Ratio of people aged 40-49 to people aged 20-29	1950-2020	United Nations' Department of Economic and Social Affairs
Average years of schooling	Annual growth rate of the average number of years of total schooling of people aged 25+.	1870-2017	Our World In Data
Catch-up growth	Ratio of a country's GDP PPP per capita to the U.S. equivalent	1870-2016	Jordà-Schularick-Taylor Macrohistory Database
Educational attainment	Share of population aged 25-64 of education below upper secondary level	1981-2019	OECD's Education at a glance
Expected growth	Expected annual growth rate of the real GDP	1985-2019	OECD Economic Outlook
Government debt level	General government debt-to-GDP ratio	1995-2019	OECD National Accounts
Inequality	Pre-tax income share held by a the top decile of a population	1871-2018	World Inequality Database
Labour supply growth	Annual growth rate of the labour force	1990-2020	International Labour Organization
MFP growth	Annual growth rate of the multifactor productivity (MFP)	1985-2019	OECD Compendium of Productivity Indicators
Public investment	General government investment as percentage of GDP	1960-2017	International Monetary Fund (IMF)
R&D spending growth	Gross domestic spending on R&D as a percentage of GDP	1981-2019	OECD Main Science and Technology Indicators
US debt	U.S. Gross Federal Debt as Percent of U.S. GDP	1939-2019	Federal Reserve Bank of St. Louis
US educational attainment	Rate of U.S. citizens age 25+ who completed high school at minimum	1940-2019	U.S. Census Bureau
US spread	Spread between annual averages of long-term BAA rated U.S. corporate bond yields and the U.S. government bond long-term interest rate	1919-2020	Moody's Investor Service, Jordà-Schularick-Taylor Macrohistory Database

Note: Table 3.1 presents the data collected based on the discussion in 3.1. All the data is collected for all 8 countries of the sample (Canada, France, Germany, Italy, Japan, Switzerland, U.K., U.S.). Exceptions to this include ; US debt, US educational attainment, US spread (U.S. only data), Educational attainment (no data for Japan), R&D spending (no data for Switzerland).

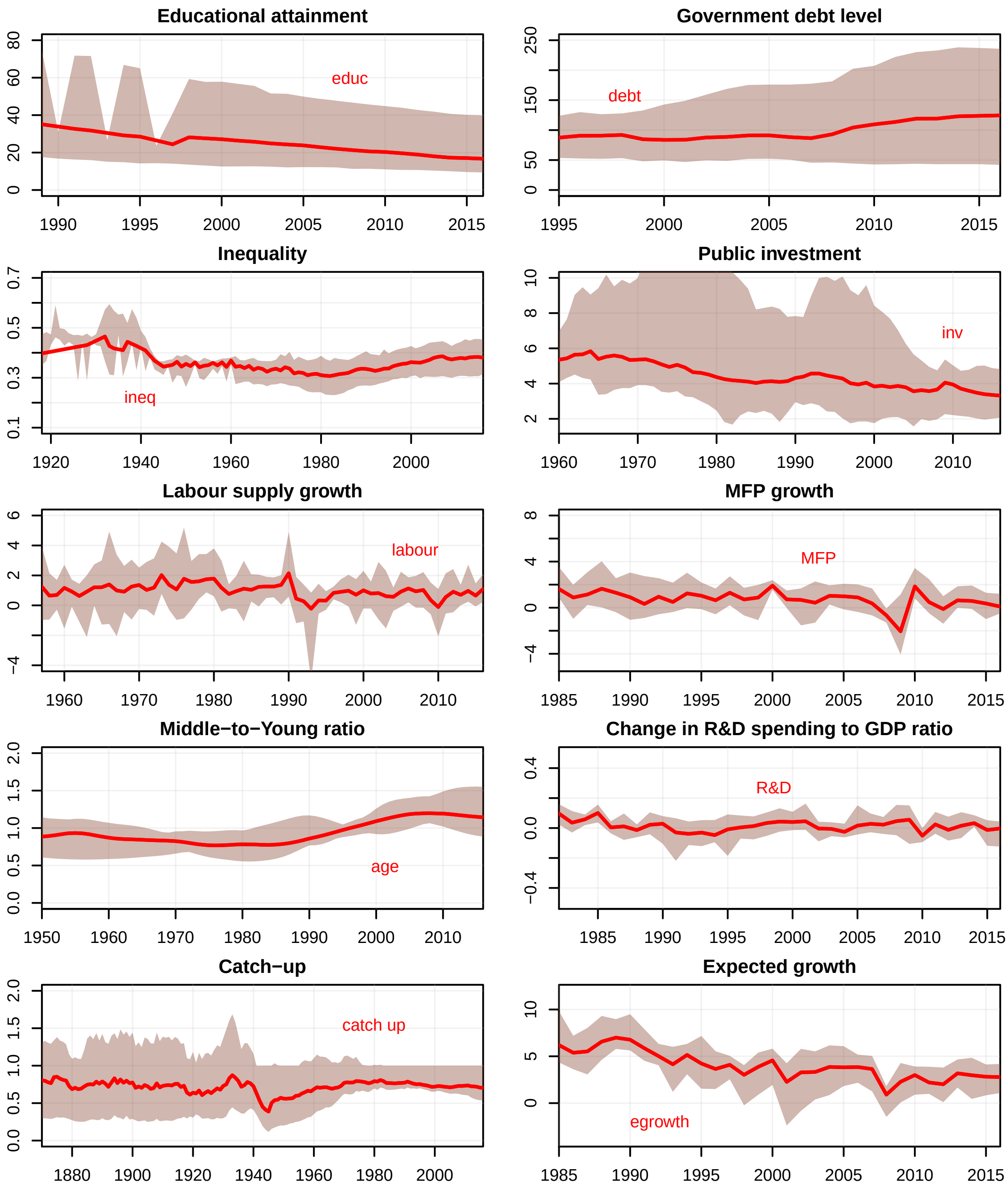
Table 3.2: Determinants of the world trend in real interest rate

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
educ	.08***													-.01***
debt		-.02***												-.00
ineq			-3.77											-.39
inv				.02										.20***
labour					.21*									.01
MFP						.29**								.02***
age							-1.48***							-1.82***
R&D								1.70						.54**
US educ									.27*					.00
catch up										5.40***				2.55***
US debt											-.01***			-.01***
US spread												.21*		-.02
egrowth													.28***	.02***
Obs	25	22	82	57	60	32	67	35	53	147	78	98	32	21
R^2	.95	.74	.07	.00	.05	.20	.22	.02	.14	.26	.41	.15	.72	1.00[(1)]

Note: Table 3.2 presents the results of the regressions on the potential secular drivers discussed in 3.1. The dependent variable is the estimation of the trend in world real interest rate $\overline{r}_t^w$ presented in 2.4. Each column shows the result of a univariate regression, including, but omitting the resulting estimation of, a constant. The explanatory variables are calculated as the weighted average all 8 countries of the sample. We denote them as : *educ* = Educational attainment, *debt* = Government debt level, *ineq* = Inequality, *inv* = Public investment, *labour* = Labour supply growth, *MFP* = MFP growth, *age* = Middle-to-young ratio, *R&D* = R&D spending, *US educ* = U.S. educational attainment, *catchup* = Catch up growth, *US debt* = U.S. level of federal debt, *US spread* = Spread between long term BAA rated U.S. corporate bond yields and the U.S. long term interest rate, *egrowth* = Expected growth (see Table 3.1 for details). Heteroskedasticity and Autocorrelation-Consistent (HAC) standard errors are used to compute the p-values but are omitted. The significance levels of the estimated coefficients are 1%, 5% and 10%, represented respectively with ***, ** and * symbols. The last column presents the results of the multivariate regression on all the variables of the table.
[(1)] Adjusted R^2

debt, U.S. spread and expected GDP growth rate. The U.S. spread between long-term BAA rated corporate bond yields and long-term interest rates on government bonds is used as a proxy for the world convenience yield discussed in the theoretical framework in 1.1. Column (14) presents the results of the multivariate regression on all listed variables. All global variables are calculated as weighted averages of the eight countries of the sample. The weights are calculated as the ratio of a country's real GDP per capita based on purchasing power parity to the U.S. GDP per capita (PPP). Figure 3.4 presents the constructed global variables, as well as the range from minimal to maximal observed value across all 8 countries at each time. The estimated coefficients in the multivariate regression give an interesting insight related to the discussion in 3.1. The results show positive and significant effects of educational attainment, public investment, technological progress, catch-up growth and expectations of growth, as well as a negative and significant effect of the age structure as measured by the MY variable. Growth at the technological

Figure 3.4: Constructed global variables



Note: Figure 3.4 presents the global variables selected as potential drivers of the world real interest rate $\bar{r}_t^w$. These global variables are constructed as the weighted averages of the panel data collected and presented in Table 3.1. The weights are calculated as the ratio of a country's real GDP (PPP) per capita to the U.S. GDP (PPP) per capita. The red straight line represents that weighted average, while the shaded area covers the interval containing the value of all countries at each time. The global variable for Catch-up growth is constructed as the weighted average excluding the United States.

frontier as proxied by U.S. educational attainment shows no effect, while the measure of U.S. level of public indebtedness could have put downwards pressure on the world real interest rate. The overall levels of public debt, income inequality, labour supply growth and U.S. convenience yield cannot account for the decrease fall in the world real interest rate. Using these results, we are able to estimate the effect of each of these variables on the trend in global real interest rate. Table 3.3 presents these estimations, for each statistically significant variable in the multivariate regression in Table 3.2. These effects are separated into pre-crisis and post-crisis decrease in the real interest rate. As such, we can see that our model estimates that demographic developments have put the most prominent downwards pressure on the world real interest rate, although they only account for the pre-crisis fall. The same can be said about expected growth, the fall of which accounts for 15 basis point of the 1980-2008 fall of the interest rate. Post-crisis however, expectations of growth seem to have stabilized. Global productivity growth, proxied by MPF growth and R&D spending, seem to have had an overall stable and relatively low effect on $\overline{r}^w$ (about 10 bps out of the whole 150). The fall in public investment levels accounts for a large part of the post-crisis fall with 7 basis points out of 37, and 20 out of 150 over the whole period. Similar effects of catch-up growth are estimated. As assumed, educational attainment played a part in pushing real interest rates upwards, but educational plateaus in advanced economies might have diminished that effect in the last decades and could continue to do so in the future. As for growth at the frontier, we rather take no position as to the results. Proxied by the level of U.S. government debt, they would imply that it played the second biggest role in the decrease of the world real interest rate (over 70 bps overall). However, with other measures of the U.S. growth proving insignificant and considering the sign and the risk of mis-specification, we give it reasonable doubt.

Table 3.3: World trend in real interest rate accounting

	1980 to 2008	2008 to 2016
$\overline{r}_t^w$	-112 bps	-37 bps
Education	9 bps	3 bps
Public investment	-12 bps	-7 bps
MFP growth	-4 bps	1 bps
Age structure	-76 bps	10 bps
R&D spending	-3 bps	-3 bps
Catch-up growth	-15 bps	-6 bps
US government debt	-37 bps	-37 bps
Expected growth	-14 bps	4 bps

Note: Table 3.3 presents the accounted effect on the world real interest rate between 1980 and 2016 of the significant drivers in Table 3.2. The row for $\overline{r}_t^w$ represent the estimated change of the variable from the beginning to the end of a time horizon (see column names). The subsequent rows present how much of the change in the world interest rate during the corresponding time horizon can be explained by the change in their corresponding variable.

We present additional results in Table B4.1 using an alternative computation of global measures. The regression specifications are identical as in Table 3.2, however the global variables are constructed as the unweighted average of the eight countries of the sample for each variable. The results show that the coefficient estimations and their significance level is robust to the change in weighting.

3.3.2 Panel

Table 3.4 presents the results of the panel regressions on education, catch-up growth, government indebtedness, inequality, public investment, labour supply, productivity growth, age structure and expected GDP growth as determinants drivers of the idiosyncratic trend in real interest rate. It should be noted that the measure of educational attainment in Table 3.4 is different from the measure used for global regressions in Table 3.2. Due to missing data for Japan, the educational attainment is measured as the change in average number of years of schooling (see Table 3.1 for details). Additionally, technological progress in the form of change in R&D spending is not included in the model due to missing data for Switzerland. As such, columns (1) to (9) show the results of univariate panel-based regressions including country-specific fixed effect of the form,

$$\overline{r}_t^i = \gamma_i + \beta_1 x_{i,t} + u_{i,t}.$$

Column (10) presents the results of the multivariate panel regression on all nine variables of the table including, but omitting the results of, country fixed-effect. We denote the specification of the regression as,

$$\begin{aligned}\overline{r}_t^i = \gamma_i &+ \beta_1 educ_{i,t} + \beta_2 catchup_{i,t} + \beta_3 debt_{i,t} + \beta_4 ineq_{i,t} + \beta_5 inv_{i,t} \\ &+ \beta_6 labour_{i,t} + \beta_7 MFP_{i,t} + \beta_8 age_{i,t} + \beta_9 egrowth_{i,t} + u_{i,t},\end{aligned}$$

where γ_i captures the country fixed-effect. The results are noticeably different from the estimations in Table 3.2, and less consistent with the theorized relationships in 3.1. Columns (3) and (6) show no significant relationship between trends in the country-specific components of real interest rates and government debt levels or growth of labour supply. While column (1) shows a positive and significant effect of change in educational attainment in a univariate regression, this effect is no longer significant in the multivariate regression in column (10). Column (2) shows that the effect of catch-up growth in a univariate regression is negative, which is not in par with our discussion in 3.1 and previous results in Table 3.2. Columns (5), (7) and (8) show significant effects of expected sign in the univariate regressions, but these effects are reversed in the multivariate regression. Only the estimated effect of the inequality level and expectations of growth show a significant relationship of expected sign in both their respective univariate regression (column (4) and column (9)) and the multivariate panel regression in column (10). As such, we conclude that we find no convincing evidence of the determinants of deviations from the world trend in our selected data. Reflecting on our theoretical framework, it would prove particularly interesting to include measures of convenience yields on each government bond in the multivariate panel regression.

Table 3.4: Determinants of the idiosyncratic trends in real interest rate

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
educ	.01***									.01
catchup		-.25**								-.20
debt			.00							.00
ineq				-1.32***						-3.90***
inv					.03**					-.06***
labour						.00				-.00
MFP							.01***			-.01*
age								-.45***		.08*
egrowth									.03***	.02***
Obs	208	1176	172	637	450	446	249	536	256	172
R^2	.04	.02	.00	.06	.03	.00	.02	.14	.22	.56[1]

Note: Table 3.4 presents the results of the cross-country panel regressions on the drivers discussed and presented in subsection 3.1. The dependent variables are the idiosyncratic component of the trend in real interest rate, $\overline{r}_t^i$, estimated in subsection 2.4 and presented in Figure 2.4. All the regressions include a country-specific fixed effect. Heteroskedasticity and Autocorrelation-Consistent (HAC) standard errors are used to compute the p-values but are omitted. The significance levels of the estimated coefficients are 1%, 5% and 10%, represented respectively with ***, ** and * symbols. The last column presents the results of the multivariate panel regression on all the variables excluding the change in R&D spendings, as no data is available for Switzerland.

[1] Adjusted R^2

Conclusion

Let us conclude this paper summarizing our methods and results. Using data on short-term and long-term nominal interest rates as well as inflation of advanced economies, we conducted a trend-cycle decomposition with the use of a Kalman filter. Furthermore, the cross-section of countries allowed us to further separate each trend into a global and a country-specific component. Thus, we were able to find convincing evidence of the existence of a world real interest rate driving the real interest rates of most advanced economies. We found that the estimations of said world real interest rate prove to be efficient in explaining the co-movements of the real interest rates among advanced economies over the whole horizon of the sample. We have found that countries' deviations from the trend in world real interest rate are partly due to high-frequency fluctuations and partly to idiosyncratic trends. However, those deviations have been slowly diminishing as we have observed convergence of all advanced economies towards the world interest rate over the last 40 years of the sample. The results show that the trend in the world real interest rate as well as all the sampled countries' real interest rates have decreased to levels not seen since the 1960s, following a decade-long high in the 1980s. This decrease is estimated to be about 1.5 percentage point. Together, these results suggest that the fall in real interest rates over the world witnessed in the past 30 years are driven by global determinants rather by own-countries.

Pursing analysis on these estimations, we found that shifts in investment and saving schedules, as well as changes in expectations of growth can explain a substantial part of the decrease in the world real interest rate. We found that global demographics development and changes in the level of public investment can account for 85 out of the 150 basis point decrease of the trend in the world real interest rate. However, we find no convincing evidence of the collected variables being significant drivers of the idiosyncratic trends in real interest rates. This particular lack of result leaves room for further analysis on countries' deviations from the world real interest rate.

These results could allow for an interesting discussion on the future of the world real interest rates. Using trends and predictions of those secular drivers would make it possible to forecast changes in real interest rates of advanced economies and make for an interesting questioning of the role of monetary policy on the future of real interest rates. We leave it to a future research to pursue such an analysis on all of the determinants of the world trend in real interest rate.

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Appendices

A Additional material

A1 Kalman Filter

This section briefly introduces the theory behind the application of the Kalman filter that was used to estimate the unknown parameters of the dynamic model as well as to make inference about unobservable variables. We follow here the notations of Hamilton [1986].

The Kalman filter represents linear dynamic systems using a *state-space* form. The *measurement vector* $\boldsymbol{y}_t$ is an $(n \times 1)$ vector of observed variables at time t. The *state vector* $\boldsymbol{\xi}_t$ is an $(r \times 1)$ vector of unobserved state variables. The state vector $\boldsymbol{\xi}_t$ together with the $(k \times 1)$ vector $\boldsymbol{x}_t$ of exogenous variables and the $(n \times 1)$ white noise vector $\boldsymbol{w}_t$ describe the dynamics of the observed variables $\boldsymbol{y}_t$ through the *observation equation*,

$$\boldsymbol{y}_t = \boldsymbol{A}'\boldsymbol{x}_t + \boldsymbol{H}'\boldsymbol{\xi}_t + \boldsymbol{w}_t, \tag{A1.1}$$

where $\boldsymbol{A}'$ and $\boldsymbol{H}'$ are respectively $(n \times k)$ and $(n \times r)$ matrices of coefficients at least partially unkown.

The dynamics of the state vector $\boldsymbol{\xi}_t$ are described through the *state equation*,

$$\boldsymbol{\xi}_t = \boldsymbol{F}\boldsymbol{\xi}_{t-1} + \boldsymbol{v}_t, \tag{A1.2}$$

where the matrix of coefficients $\boldsymbol{F}$ is of size $(r \times r)$. The white noise vectors $\boldsymbol{w}_t$ and $\boldsymbol{v}_t$, are supposed to be independent and respectively *i.i.d.* $\mathcal{N}(\boldsymbol{0}, \boldsymbol{R})$ and *i.i.d.* $\mathcal{N}(\boldsymbol{0}, \boldsymbol{Q})$. Together with equations (A1.1) and (A1.2), they form the state-space model,

$$\begin{aligned}
\underset{(n\times1)}{\boldsymbol{y}_t} &= \underset{(n\times k)(k\times1)}{\boldsymbol{A}'\boldsymbol{x}_t} + \underset{(n\times r)(r\times1)}{\boldsymbol{H}'\boldsymbol{\xi}_t} + \underset{(n\times1)}{\boldsymbol{w}_t} \\
\mathbb{E}(\boldsymbol{w}_t\boldsymbol{w}_t') &= \underset{(n\times n)}{\boldsymbol{R}} \\
\underset{(r\times1)}{\boldsymbol{\xi}_t} &= \underset{(r\times r)(r\times1)}{\boldsymbol{F}\boldsymbol{\xi}_{t-1}} + \underset{(n\times1)}{\boldsymbol{v}_t} \\
\mathbb{E}(\boldsymbol{v}_t\boldsymbol{v}_t') &= \underset{(r\times r)}{\boldsymbol{Q}} .
\end{aligned}$$

The Kalman filter proceeds with two distincts steps : one of *prediction* and one of *update*. The prediction step proceeds as follows. Considering the information set of time $t-1$,

$$\boldsymbol{\zeta}_{t-1} \equiv (\boldsymbol{y}'_{t-1}, \boldsymbol{y}'_{t-2}, \ldots, \boldsymbol{y}'_1, \boldsymbol{x}'_{t-1}, \boldsymbol{x}'_{t-2}, \ldots, \boldsymbol{x}'_1)', \tag{A1.3}$$

the conditional distribution of $\boldsymbol{\xi}_t$ on $\boldsymbol{\zeta}_{t-1}$ is,

$$\boldsymbol{\xi}_t | \boldsymbol{\zeta}_{t-1} \sim \mathcal{N}\Big(\hat{\boldsymbol{\xi}}_{t|t-1}, \boldsymbol{P}_{t|t-1}\Big), \tag{A1.4}$$

where,

$$\hat{\boldsymbol{\xi}}_{t|t-1} = \boldsymbol{F}\hat{\boldsymbol{\xi}}_{t-1|t-1}\boldsymbol{P}_{t|t-1} \quad = \boldsymbol{F}\boldsymbol{P}_{t-1|t-1}\boldsymbol{F}' + \boldsymbol{Q}. \tag{A1.5}$$

The prediction step calculates the best estimation at time $t-1$ of the state vector at time t, $\mathbb{E}(\boldsymbol{\xi}_t|\boldsymbol{\zeta}_{t-1})$, as well as the covariance matrix $\mathbb{E}(\boldsymbol{\xi}_t\boldsymbol{\xi}'_t|\boldsymbol{\zeta}_{t-1})$. Those predictions are written as $\hat{\boldsymbol{\xi}}_{t|t-1}$ and $\boldsymbol{P}_{t|t-1}$.

Using the additional information of time t, the update step upgrades the estimation of $\boldsymbol{\xi}_{t|t-1}$ combining the prediction $\hat{\boldsymbol{\xi}}_{t|t-1}$ and the forecast error,

$$\boldsymbol{y}_t - \mathbb{E}(\boldsymbol{y}_t|\boldsymbol{x}_t, \boldsymbol{\zeta}_{t-1}) = \boldsymbol{y}_t - \boldsymbol{A}'\boldsymbol{x}_t - \boldsymbol{H}'\hat{\boldsymbol{\xi}}_{t|t-1}. \tag{A1.6}$$

More precisely, we know that the joint distribution of $\boldsymbol{y}_t$ $\boldsymbol{\xi}_t$ conditional on $\boldsymbol{x}_t, \boldsymbol{\zeta}_{t-1}$ is,

$$\begin{bmatrix} \boldsymbol{y}_t|\boldsymbol{x}_t, \boldsymbol{\zeta}_{t-1} \\ \boldsymbol{\xi}_t|\boldsymbol{x}_t, \boldsymbol{\zeta}_{t-1} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \boldsymbol{A}'\boldsymbol{x}_t + \boldsymbol{H}'\hat{\boldsymbol{\xi}}_{t|t-1} \\ \hat{\boldsymbol{\xi}}_{t|t-1} \end{bmatrix}, \begin{bmatrix} \boldsymbol{H}'\boldsymbol{P}_{t|t-1}\boldsymbol{H} + \boldsymbol{R} & \boldsymbol{H}'\boldsymbol{P}_{t|t-1} \\ \boldsymbol{P}_{t|t-1}\boldsymbol{H} & \boldsymbol{P}_{t|t-1} \end{bmatrix}\right). \tag{A1.7}$$

Furthermore, we know from equation (A1.7) and joint distribution theory that the distribution of $\boldsymbol{\xi}_t|\boldsymbol{y}_t, \boldsymbol{x}_t, \boldsymbol{\zeta}_{t-1} \equiv \boldsymbol{\xi}_t|\boldsymbol{\zeta}_t$ is,

$$\boldsymbol{\xi}_t|\boldsymbol{\zeta}_t \sim \mathcal{N}\Big(\hat{\boldsymbol{\xi}}_{t|t}, \boldsymbol{P}_{t|t}\Big), \tag{A1.8}$$

where,

$$\begin{aligned} \hat{\boldsymbol{\xi}}_{t|t} &= \hat{\boldsymbol{\xi}}_{t|t-1} + \boldsymbol{P}_{t|t-1}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{P}_{t|t-1}\boldsymbol{H} + \boldsymbol{R})^{-1}(\boldsymbol{y}_t - \boldsymbol{A}'\boldsymbol{x}_t - \boldsymbol{H}'\hat{\boldsymbol{\xi}}_{t|t-1}) \\ \boldsymbol{P}_{t|t} &= \boldsymbol{P}_{t|t-1} - \boldsymbol{P}_{t|t-1}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{P}_{t|t-1}\boldsymbol{H} + \boldsymbol{R})^{-1}\boldsymbol{H}'\boldsymbol{P}_{t|t-1}, \end{aligned} \tag{A1.9}$$

$\hat{\boldsymbol{\xi}}_{t|t}$ is updated estimation is written as and depends on the Kalman gain to weight the adjustment by the forecast error. The optimal Kalman gain is defined as,

$$\boldsymbol{K}_t = \boldsymbol{P}_{t|t-1}\boldsymbol{H}'(\boldsymbol{H}'\boldsymbol{P}_{t|t-1}\boldsymbol{H} + \boldsymbol{R})^{-1}. \tag{A1.10}$$

To operate the recursion, the filter must be initialized. We denote the initial values as $\hat{\boldsymbol{\xi}}_{1|0}$ and $\boldsymbol{P}_{1|0}$. When considering a stationary process, $\hat{\boldsymbol{\xi}}_{1|0}$ is set as the process' unconditional mean and $\boldsymbol{P}_{1|0}$ as the unconditional variance which solves,

$$\text{vec}(\boldsymbol{P}_{1|0}) = [\boldsymbol{I}_{r^2} - (\boldsymbol{F} \otimes \boldsymbol{F})]^{-1} \cdot \text{vec}(\boldsymbol{Q}) \tag{A1.11}$$

A non-stationary process can be initialized with a guess using outside prior knowledge, such as economic theory. The filter then iterates over all the available period of times using at the update step,

$$\begin{aligned} \hat{\boldsymbol{\xi}}_{t|t} &= \hat{\boldsymbol{\xi}}_{t|t-1} + \boldsymbol{K}_t(\boldsymbol{y}_t - \boldsymbol{A}'\boldsymbol{x}_t - \boldsymbol{H}'\hat{\boldsymbol{\xi}}_{t|t-1}) \\ \boldsymbol{P}_{t|t} &= \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_t\boldsymbol{H}'\boldsymbol{P}_{t|t-1}, \end{aligned} \tag{A1.12}$$

and at the prediction step,

$$\begin{aligned} \hat{\boldsymbol{\xi}}_{t+1|t} &= \boldsymbol{F}\hat{\boldsymbol{\xi}}_{t|t} \\ \boldsymbol{P}_{t+1|t} &= \boldsymbol{F}\boldsymbol{P}_{t|t}\boldsymbol{F}' + \boldsymbol{Q}. \end{aligned} \tag{A1.13}$$

A2 Rauch-Tung-Striebel smoother

The Rauch-Tung-Striebel smoother is a fixed-interval smoother which updates each estimation $\hat{\boldsymbol{\xi}}_{t|t}$ and its corresponding mean squared error $\boldsymbol{P}_{t|t}$ using the information set of the end of the sample, $\boldsymbol{\zeta}_T$. Those new inferences are then denoted as $\hat{\boldsymbol{\xi}}_{t|T}$ and $\boldsymbol{P}_{t|T}$. From the last estimation by the Kalman filter $\hat{\boldsymbol{\xi}}_{T|T}, \boldsymbol{P}_{T|T}$, the Rauch-Tung-Striebel smoother goes reversely over the $T-1$ estimations, updating each of them by iterating from $T-1$ to 1 on,

$$\begin{aligned} \hat{\boldsymbol{\xi}}_{t|T} &= \hat{\boldsymbol{\xi}}_{t|t} + \boldsymbol{J}_t(\hat{\boldsymbol{\xi}}_{t+1|T} - \hat{\boldsymbol{\xi}}_{t+1|t}) \\ \boldsymbol{P}_{t|T} &= \boldsymbol{P}_{t|t} + \boldsymbol{J}_t(\boldsymbol{P}_{t+1|T} - \boldsymbol{P}_{t+1|t})\boldsymbol{J}_t', \end{aligned} \tag{A2.1}$$

where,

$$\boldsymbol{J}_t = \boldsymbol{P}_{t|t}\boldsymbol{F}'\boldsymbol{P}^{-1}_{t+1|t}. \tag{A2.2}$$

A3 Detailed Econometric Framework

The observation equation of the state-space representation of the system in 1.22 is :

$$\mathbf{y_t} = \boldsymbol{H'}\boldsymbol{\xi_t},$$

where,

$$\boldsymbol{y}_t = \begin{bmatrix} \boldsymbol{R}_t & \boldsymbol{R^L}_t & \boldsymbol{\pi}_t \end{bmatrix}', \boldsymbol{\xi_t} = \begin{bmatrix} \overline{r}_t^w & \boldsymbol{\overline{r}_t} & \overline{ts}_t^w & \boldsymbol{\overline{ts}_t} & \overline{\pi}_t^w & \boldsymbol{\overline{\pi}_t} & \boldsymbol{\tilde{R}_t} & \boldsymbol{\tilde{R}_t^L} & \boldsymbol{\tilde{\pi}_t} \end{bmatrix}',$$

$$\begin{aligned}
\boldsymbol{R}_t &= \begin{bmatrix} R_{CA,t} & R_{FR,t} & R_{DE,t} & R_{IT,t} & R_{JP,t} & R_{CH,t} & R_{UK,t} & R_{US,t} \end{bmatrix}', \\
\boldsymbol{R}_t^L &= \begin{bmatrix} R^L_{CA,t} & R^L_{FR,t} & R^L_{DE,t} & R^L_{IT,t} & R^L_{JP,t} & R^L_{CH,t} & R^L_{UK,t} & R^L_{US,t} \end{bmatrix}', \\
\boldsymbol{\pi}_t &= \begin{bmatrix} \pi_{CA,t} & \pi_{FR,t} & \pi_{DE,t} & \pi_{IT,t} & \pi_{JP,t} & \pi_{CH,t} & \pi_{UK,t} & \pi_{US,t} \end{bmatrix}', \\
\boldsymbol{\overline{r}}_t &= \begin{bmatrix} \overline{r}_t^{CA} & \overline{r}_t^{FR} & \overline{r}_t^{DE} & \overline{r}_t^{IT} & \overline{r}_t^{JP} & \overline{r}_t^{CH} & \overline{r}_t^{UK} & \overline{r}_t^{US} \end{bmatrix}', \\
\boldsymbol{\overline{ts}}_t &= \begin{bmatrix} \overline{ts}_t^{CA} & \overline{ts}_t^{FR} & \overline{ts}_t^{DE} & \overline{ts}_t^{IT} & \overline{ts}_t^{JP} & \overline{ts}_t^{CH} & \overline{ts}_t^{UK} & \overline{ts}_t^{US} \end{bmatrix}', \\
\boldsymbol{\overline{\pi}}_t &= \begin{bmatrix} \overline{\pi}_t^{CA} & \overline{\pi}_t^{FR} & \overline{\pi}_t^{DE} & \overline{\pi}_t^{IT} & \overline{\pi}_t^{JP} & \overline{\pi}_t^{CH} & \overline{\pi}_t^{UK} & \overline{\pi}_t^{US} \end{bmatrix}', \\
\boldsymbol{\tilde{R}}_t &= \begin{bmatrix} \tilde{R}_{CA,t} & \tilde{R}_{FR,t} & \tilde{R}_{DE,t} & \tilde{R}_{IT,t} & \tilde{R}_{JP,t} & \tilde{R}_{CH,t} & \tilde{R}_{UK,t} & \tilde{R}_{US,t} \end{bmatrix}', \\
\boldsymbol{\tilde{R}}_t^L &= \begin{bmatrix} \tilde{R}^L_{CA,t} & \tilde{R}^L_{FR,t} & \tilde{R}^L_{DE,t} & \tilde{R}^L_{IT,t} & \tilde{R}^L_{JP,t} & \tilde{R}^L_{CH,t} & \tilde{R}^L_{UK,t} & \tilde{R}^L_{US,t} \end{bmatrix}', \\
\boldsymbol{\tilde{\pi}}_t &= \begin{bmatrix} \tilde{\pi}_{CA,t} & \tilde{\pi}_{FR,t} & \tilde{\pi}_{DE,t} & \tilde{\pi}_{IT,t} & \tilde{\pi}_{JP,t} & \tilde{\pi}_{CH,t} & \tilde{\pi}_{UK,t} & \tilde{\pi}_{US,t} \end{bmatrix}'.
\end{aligned} \tag{A3.1}$$

The coefficient matrix $\boldsymbol{H'}$ is defined as :

$$\boldsymbol{H'} = \begin{bmatrix} \mathbf{1} & \boldsymbol{I_m} & \mathbf{0} & \mathbf{0_m} & \boldsymbol{\lambda^\pi} & \boldsymbol{I_m} & \boldsymbol{I_m} & \mathbf{0_m} & \mathbf{0_m} \\ \mathbf{1} & \boldsymbol{I_m} & \mathbf{1} & \boldsymbol{I_m} & \boldsymbol{\lambda^\pi} & \boldsymbol{I_m} & \mathbf{0_m} & \boldsymbol{I_m} & \mathbf{0_m} \\ \mathbf{0} & \mathbf{0_m} & \mathbf{0} & \mathbf{0_m} & \boldsymbol{\lambda^\pi} & \boldsymbol{I_m} & \mathbf{0_m} & \mathbf{0_m} & \boldsymbol{I_m} \end{bmatrix}, \tag{A3.2}$$

where $\mathbf{1}$ and $\mathbf{0}$ are of size $m \times 1$, $\mathbf{0_m} = m \times m$ and $\boldsymbol{I_m}$ is the identity matrix of dimension m. The vector $\boldsymbol{\lambda^\pi}$ holds the inflation loadings in the form,

$$\boldsymbol{\lambda^\pi} = \begin{bmatrix} \lambda^\pi_{CA} & \lambda^\pi_{FR} & \lambda^\pi_{DE} & \lambda^\pi_{IT} & \lambda^\pi_{JP} & \lambda^\pi_{CH} & \lambda^\pi_{UK} & \lambda^\pi_{US} \end{bmatrix}'.$$

The state equation of the state-space model is,

$$\boldsymbol{\xi}_t = \boldsymbol{F}\boldsymbol{\xi}_{t-1} + \boldsymbol{v}_t,$$

where the state matrix $\boldsymbol{F}$ and the vector of shocks $\boldsymbol{v}_t$ are defined as,

$$\boldsymbol{F} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & \ddots & \dots & \vdots \\ \vdots & \vdots & 1 & 0 \\ 0 & \dots & 0 & \boldsymbol{\Phi} \end{bmatrix}, \tag{A3.3}$$

$$\boldsymbol{v}_t = \begin{bmatrix} \epsilon_t \\ \varepsilon_t \end{bmatrix}.$$

The VAR matrix $\boldsymbol{\Phi}$ is thus constructed as,

$$\boldsymbol{\Phi} = \begin{bmatrix} \phi_{RR}\boldsymbol{I_m} & \phi_{RR^L}\boldsymbol{I_m} & \phi_{R\pi}\boldsymbol{I_m} \\ \phi_{R^LR}\boldsymbol{I_m} & \phi_{R^LR^L}\boldsymbol{I}_m & \phi_{R^L\pi}\boldsymbol{I_m} \\ \phi_{\pi R}\boldsymbol{I_m} & \phi_{\pi R^L}\boldsymbol{I_m} & \phi_{\pi\pi}\boldsymbol{I_m} \end{bmatrix}, \tag{A3.4}$$

where the coefficients ϕ_{XY} satisfy,

$$\begin{bmatrix} \tilde{R}_{i,t} \\ \tilde{R}^L_{i,t} \\ \tilde{\pi}_{i,t} \end{bmatrix} = \begin{bmatrix} \phi_{RR} & \phi_{RR^L} & \phi_{R\pi} \\ \phi_{R^LR} & \phi_{R^LR^L} & \phi_{R^L\pi} \\ \phi_{\pi R} & \phi_{\pi R^L} & \phi_{\pi\pi} \end{bmatrix} \begin{bmatrix} \tilde{R}_{i,t-1} \\ \tilde{R}^L_{i,t-1} \\ \tilde{\pi}_{i,t-1} \end{bmatrix}.$$

The covariance matrix of the vector of shock $\mathbb{E}(\boldsymbol{v}_t\boldsymbol{v}_t') = \boldsymbol{Q}$ is a diagonal matrix denoted as,

$$\boldsymbol{Q} = \begin{bmatrix} \sigma^2_{\overline{r}^w} & 0 & \dots & & & & & & 0 \\ 0 & \boldsymbol{\Sigma}_{\overline{r}^i} & 0 & \dots & & & & & 0 \\ 0 & 0 & \sigma^2_{\overline{ts}^w} & 0 & \dots & & & & 0 \\ 0 & \dots & 0 & \boldsymbol{\Sigma}_{\overline{ts}^i} & 0 & \dots & & & 0 \\ 0 & \dots & & 0 & \sigma^2_{\overline{\pi}^w} & 0 & \dots & & 0 \\ 0 & \dots & & & 0 & \boldsymbol{\Sigma}_{\overline{\pi}^i} & 0 & \dots & 0 \\ 0 & \dots & & & & 0 & \boldsymbol{\Sigma}_{\tilde{R}_i} & 0 & 0 \\ 0 & \dots & & & & & 0 & \boldsymbol{\Sigma}_{\tilde{R}_i^L} & 0 \\ 0 & \dots & & & & & & 0 & \boldsymbol{\Sigma}_{\tilde{\pi}_i} \end{bmatrix}, \tag{A3.5}$$

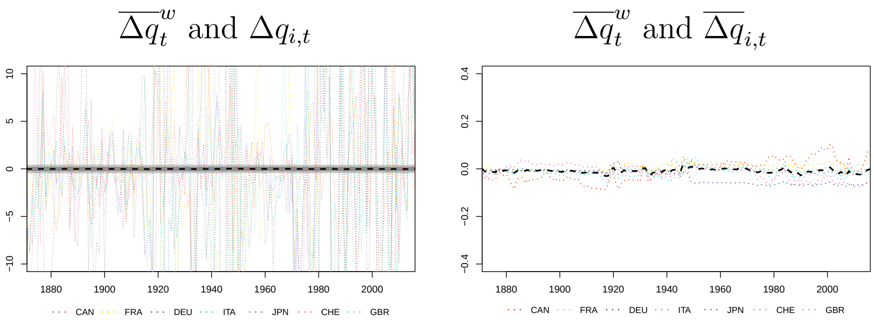
where the matrices $\boldsymbol{\Sigma}_X$ hold the variances :

$$\boldsymbol{\Sigma}_X = \begin{bmatrix} \sigma^2_{X_{CA}} & 0 & & & \dots & & & 0 \\ 0 & \sigma^2_{X_{FR}} & & & & & & \\ & & \sigma^2_{X_{DE}} & & & & & \\ & & & \sigma^2_{X_{IT}} & & \ddots & & \vdots \\ \vdots & & & & \sigma^2_{X_{JP}} & & & \\ & & \ddots & & & \sigma^2_{X_{CH}} & & \\ & & & & & & \sigma^2_{X_{UK}} & 0 \\ 0 & & & \dots & & & 0 & \sigma^2_{X_{US}} \end{bmatrix}$$

B Additional Tables and Figures

B1 Real Exchange Rate Model

Figure B1.1: Estimated trends and observed data for growth of real exchange rates



Note: Figure B1.1 presents the results of a trend-cycle decomposition of the growth of real exchange rates relative to the U.S. dollar $\Delta q_{i,t}$ using a Kalman filter. The left panel shows the observed growth of real exchange rates relative to the U.S. dollar $\Delta q_{i,t}$ for each country i (dotted colored lines; see legend), together with the estimated world trend $\overline{\Delta q}_t^w$ (dashed black line) and its corresponding 68 and 95 confidence intervals (grey shaded areas). The right panel shows the estimation of $\overline{\Delta q}_{i,t} = \overline{\Delta q}_t^w + \overline{\Delta q}_t^i$ for each country i (dotted colored lines; see legend), together with the estimated world trend $\overline{\Delta q}_t^w$ (dashed black line).

B2 Baseline Model, additional results

Table B2.1: Estimated inflation loadings λ_i^π

λ_{CA}^π	1.28
λ_{FR}^π	1.60
λ_{GE}^π	1.21
λ_{IT}^π	1.60
λ_{JP}^π	1.04
λ_{CH}^π	0.84
λ_{UK}^π	1.18
λ_{US}^π	1.00

Note: Table B2.1 presents the estimations of the country-specific inflation loading parameters λ_i^π, under the restrictions discussed in 2.2.

Table B2.2: Estimated autoregressive parameters ϕ_{XY}

ϕ_{RR}	0.56	ϕ_{RR^L}	0.00	$\phi_{R\pi}$	0.00
ϕ_{R^LR}	0.02	$\phi_{R^LR^L}$	0.85	$\phi_{R^L\pi}$	0.00
$\phi_{\pi R}$	0.00	$\phi_{\pi R^L}$	0.48	$\phi_{\pi\pi}$	0.51

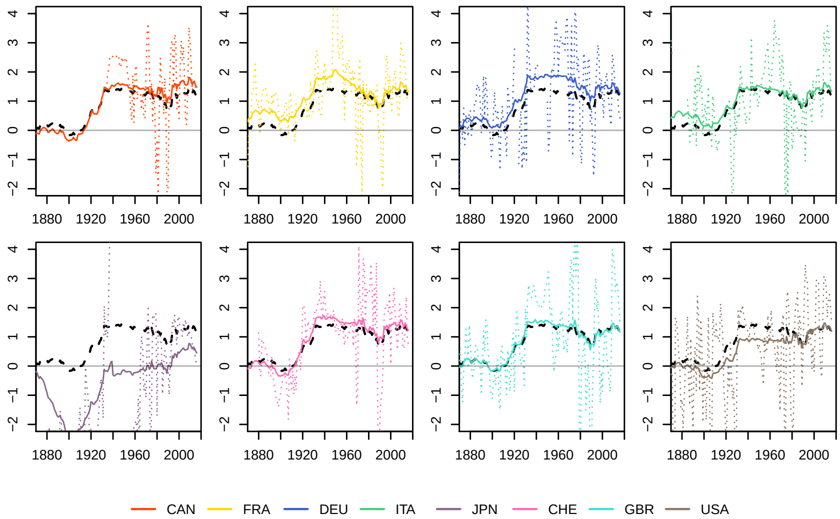
Note: Table B2.2 presents the estimated autoreressive parameters ϕ_{XY} of the VAR matrix $\mathbf{\Phi}$, under the restriction $\phi \in [0, 1)$ discussed in 2.2.

Table B2.3: Estimated standard deviations of the world trends $\sigma_{\overline{x}^w}$

$\sigma_{\overline{r}^w}$	$\sqrt{1/50}$
$\sigma_{\overline{ts}^w}$	$\sqrt{1/50}$
$\sigma_{\overline{\pi}^w}$	$\sqrt{1/10}$

Note: Table B2.3 presents the estimated standard deviations of the world trends in real interest rate $\sigma_{\overline{r}^w}$, term spread $\sigma_{\overline{ts}^w}$ and inflation $\sigma_{\overline{\pi}^i}$ under the restrictions discussed in 2.2.

Figure B2.1: Estimated trends and observed data for term spread by country



Note: From left to right, each panel presents the observed term spread $R^L_{i,t} - R_{i,t}$ for each country i of the sample (dotted line), the overall trend in term spread of the country $\overline{ts}_{i,t} = \overline{ts}^w_t + \overline{ts}^i_t$ (straight line) together with the estimated world trend $\overline{ts}^w_t$ (dashed black line) in the order : Canada, France, Germany, Italy, Japan, Switzerland, United Kingdom, United States (see legend for colors)

B3 Unrestricted Model

We present here the results of a modified version of the baseline model described in 2.2. We call the Unrestricted Model a version of the baseline model where we impose no restriction on the standard deviations $\left[\sigma_{\bar{r}^w}, \sigma_{\bar{r}^i}, \sigma_{\overline{ts}^w}, \sigma_{\overline{ts}^i}, \sigma_{\bar{\pi}^w}, \sigma_{\bar{\pi}^i}\right]$ or on the inflation loadings λ^π_i. Only the autoregressive parameters of the VAR matrix $\mathbf{\Phi}$ and the U.S. inflation loading λ^π_{US} are constrained as they are in the baseline model. Table B3.2 presents estimated parameters in an alternative version of the Unrestricted Model where the condition $\lambda^\pi_{US} = 1$ is replaced with $\lambda^\pi_{CA} = 1$.

Table B3.1: Estimated inflation loadings λ_i^π, Unrestricted Model

λ_{CA}^π	0.99
λ_{FR}^π	1.41
λ_{GE}^π	0.92
λ_{IT}^π	1.50
λ_{JP}^π	0.81
λ_{CH}^π	0.89
λ_{UK}^π	1.02
λ_{US}^π	1.00

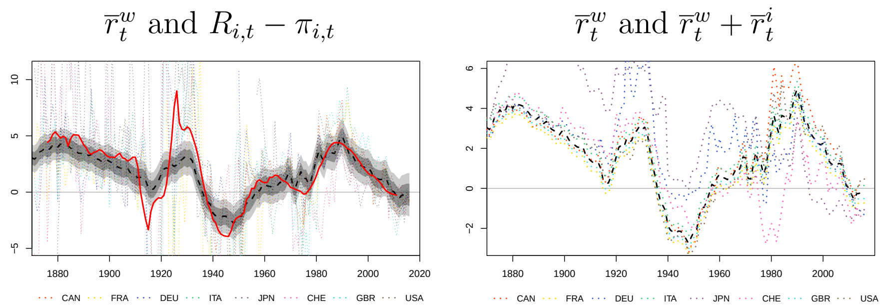
Note: Table B3.1 presents the estimations of the country-specific inflation loadings λ_i^π of the Unrestricted Model, considering the condition $\lambda_\pi^{US} = 1$.

Table B3.2: Estimated inflation loadings λ_i^π, Unrestricted Model Variant

λ_{CA}^π	1.00
λ_{FR}^π	1.43
λ_{GE}^π	0.93
λ_{IT}^π	1.52
λ_{JP}^π	0.82
λ_{CH}^π	0.90
λ_{UK}^π	1.03
λ_{US}^π	1.01

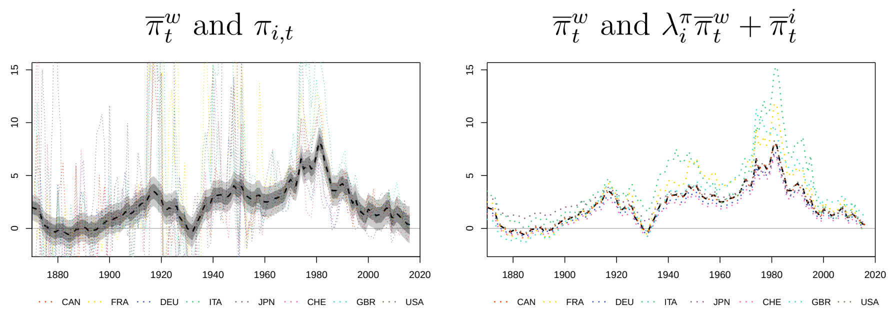
Note: Table B3.2 presents the estimations of the country-specific inflation loadings λ_i^π of the Unrestricted Model, considering the condition $\lambda_\pi^{CA} = 1$.

Figure B3.1: Estimated trends and observed data for short-term real interest rates, Unrestricted Model



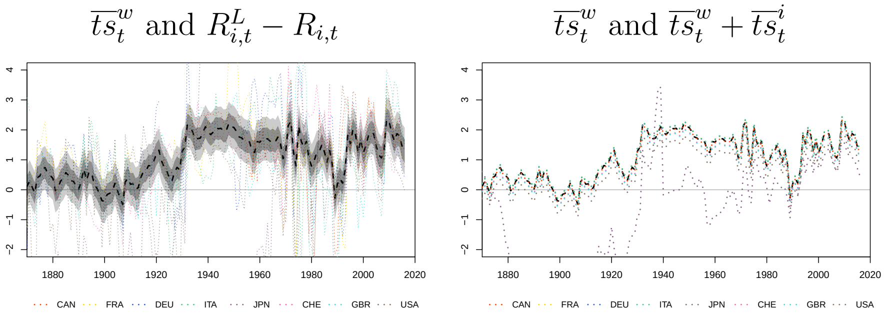
Note: The left panel shows the observed ex-post real interest rate $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines; see legend), together with the estimated world trend $\overline{r}_t^w$ (dashed black line) and its corresponding 68 and 95 confidence intervals (grey shaded areas). The straight red line represents the decadal moving average of the observed ex-post real interest rates of all the sampled countries. The right panel shows the estimation of the overall trend in real interest rate $\overline{r}_{i,t} = \overline{r}_t^w + \overline{r}_t^i$ for each country i (dotted lines; see legend), together with the estimated world trend $\overline{r}_t^w$ (dashed black line).

Figure B3.2: Estimated trends and observed data for inflation, Unrestricted Model



Note: The left panel presents the observed inflation $\pi_{i,t}$ for each country i of the sample (dotted lines), together with the estimated world trend in inflation $\overline{\pi}_t^w$ (dashed black line) and its corresponding 68 and 95 confidence intervals (grey shaded areas). The right panel shows the estimation of overall trends in inflation $\overline{\pi}_{i,t} = \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i$ for each country i (dotted lines), together with the estimated world trend $\overline{\pi}_t^w$ (dashed black line).

Figure B3.3: Estimated trends and observed data for term spreads, Unrestricted Model



Note: The left panel presents the observed term spread $R_{i,t}^L - R_{i,t}$ for each country i of the sample (dotted lines), together with the estimated world trend in term spread $\overline{ts}_t^w$ (dashed black line) and its corresponding 68 and 95 confidence intervals (grey shaded areas). The right panel shows the estimation of overall trends in term spread $\overline{ts}_{i,t} = \overline{ts}_t^w + \overline{ts}_t^i$ for each country i (dotted lines), together with the estimated world trend $\overline{ts}_t^w$ (dashed black line).

B4 Global Regressions, additional results

Table B4.1: Determinants of the world trend in real interest rate, alternative results

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
educ	.08***													-.01***
debt		-.02***												-.00
ineq			-3.06											-.34
inv				.01										.17***
labour					.22*									-.01
MFP						.29**								.01***
age							-1.29***							-1.83***
R&D								1.43						.53*
US educ									.27*					.01
catch up										5.40***				2.56***
US debt											-.01***			-.01***
US spread												.21*		-.02
egrowth													.27***	.03**
Obs	25	22	82	57	60	32	67	35	53	147	78	98	32	21
R^2	.94	.77	.05	.00	.05	.21	.18	.01	.14	.26	.41	.15	.75	1.00$^{(1)}$

Note: This table presents the results of the regressions on the potential secular drivers discussed in 3.1. The dependent variable is the estimation of the trend in world real interest rate $\overline{r}_t^w$ presented in 2.4. Each column shows the result of a univariate regression, including, but omitting the resulting estimation of, a constant. The explanatory variables are calculated as the unweighted average all 8 countries of the sample. We denote them as : *educ* = Educational attainment, *debt* = Government debt level, *ineq* = Inequality, *inv* = Public investment, *labour* = Labour supply growth, *MFP* = MFP growth, *age* = Middle-to-young ratio, *R&D* = R&D spending, *US educ* = U.S. educational attainment, *catchup* = Catch up growth, *US debt* = U.S. level of federal debt, *US spread* = Spread between long term BAA rated U.S. corporate bond yields and the U.S. long term interest rate, *egrowth* = Expected growth (see table 3.1 for details). Heteroskedasticity and Autocorrelation-Consistent (HAC) standard errors are used to compute the p-values but are omitted. The significance levels of the estimated coefficients are 1%, 5% and 10%, represented respectively with ***, ** and * symbols. The last column presents the results of the multivariate regression on all the variables of the table.

(1) Adujsted R^2