(7) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^{n-1}$

$\displaystyle = \lim_{k\to\infty}\sum_{n=1}^{k} 1\cdot\left(\frac{1}{3}\right)^{n-1}$

$\displaystyle = \lim_{k\to\infty}\frac{1\left(1-\left(\frac{1}{3}\right)^{k}\right)}{1-\frac{1}{3}}$

$\displaystyle = \lim_{k\to\infty}\frac{3}{2}\left\{1-\left(\frac{1}{3}\right)^{k}\right\}$

$\displaystyle = \frac{3}{2}$ //

[36] 次の関数の極限を求めよ.

(1) $\displaystyle\lim_{x\to 2}(x^3-3x^2+3x-1)$

$= 8-12+6-1$
$= 1$ //

(3) $\displaystyle\lim_{x\to\pi}\frac{\cos x}{x}$

$\displaystyle = \frac{\cos\pi}{\pi} = -\frac{1}{\pi}$ //

(5) $\displaystyle\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}$

$\displaystyle = \lim_{x\to 2}\frac{(x-2)(x+3)}{(x-2)(x+2)}$

$\displaystyle = \frac{5}{4}$ //

(7) $\displaystyle\lim_{x\to 0}\frac{1}{x}\left(1+\frac{1}{x-1}\right)$

$\displaystyle = \lim_{x\to 0}\frac{1}{x}\cdot\frac{x}{x-1}$

$= -1$ //

(8) $\displaystyle\sum_{n=1}^{\infty}3\left(\frac{1}{2}\right)^{n-3}$

$\displaystyle = \lim_{k\to\infty}\sum_{n=1}^{k}12\left(\frac{1}{2}\right)^{n-1}$

$\displaystyle = \lim_{k\to\infty}\frac{12\left(1-\left(\frac{1}{2}\right)^{k}\right)}{1-\frac{1}{2}}$

$= 24$ //

(2) $\displaystyle\lim_{x\to 1}\frac{x-1}{x^2+1}$

$\displaystyle = \frac{0}{2} = 0$ //

(4) $\displaystyle\lim_{x\to 0}2^{x}(x^2-2x-3)$

$= 1(0-0-3) = -3$ //

(6) $\displaystyle\lim_{x\to 1}\frac{x^2-1}{x^2-3x+2}$

$\displaystyle = \lim_{x\to 1}\frac{(x+1)(x-1)}{(x-2)(x-1)}$

$= -2$ //

(8) $\displaystyle\lim_{x\to -2}\frac{1}{x+2}\left(2+\frac{4}{x}\right)$

$\displaystyle = \lim_{x\to -2}\frac{1}{x+2}\cdot\frac{2(x+2)}{x}$

$\displaystyle = \lim_{x\to -2}\frac{2}{x} = -1$ //

(9) $\displaystyle\lim_{x \to -1} \frac{\sqrt{x^2+3}+2x}{x+1}$

$\displaystyle = \lim_{x \to -1} \frac{x^2+3-4x^2}{(x+1)(\sqrt{x^2+3}-2x)}$

$\displaystyle = \lim_{x \to -1} \frac{-3(x^2-1)}{(x+1)(\sqrt{x^2+3}-2x)}$

$\displaystyle = \lim_{x \to -1} \frac{-3(x-1)(x+1)}{(\sqrt{x^2+3}-2x)(x+1)}$

$\displaystyle = \lim_{x \to -1} \frac{-3(x-1)}{\sqrt{x^2+3}-2x}$

$\displaystyle = \frac{6}{2+2} = \frac{3}{2}$ //

(10) $\displaystyle\lim_{x \to 0} \frac{\sqrt{3x^2+2x+1}-\sqrt{1+3x}}{3x}$

$\displaystyle = \lim_{x \to 0} \frac{3x(x-\frac{1}{3})}{3x(\sqrt{3x^2+2x+1}+\sqrt{1+3x})}$

$\displaystyle = \lim_{x \to 0} \frac{x-\frac{1}{3}}{\sqrt{3x^2+2x+1}+\sqrt{1+3x}}$

$\displaystyle = \frac{-\frac{1}{3}}{2} = -\frac{1}{6}$ //

(11) $\displaystyle\lim_{x \to -\infty} \left(1-\frac{1}{x^2}\right)$

$= 1$ //

(12) $\displaystyle\lim_{x \to -\infty} \frac{x^3+2}{x-1}$

$\displaystyle = \lim_{x \to -\infty} \frac{x^3(1+\frac{2}{x^3})}{x(1-\frac{1}{x})}$

$\displaystyle = \lim_{x \to -\infty} x^2 = \infty$ //