Implementation and Performance Analyses of a Highly Efficient Algorithm for Pressure-Velocity Coupling

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10.1 Effect of Different Non-Orthogonal Correctors

Section 3.3 introduced different ways to address the degradation of the grid quality due to non-orthogonality of the grid. This section compares the three different correctors, the orthogonal correction, the minimum correction and the overrelaxed approach, which are used in the discretization process of the gradients at cell boundary faces. For this purpose the grid generator program of section **??** was extended to generate skewed grids. For this purpose a random number generator was used to randomly move each inner grid point within a specified neighborhood of the original location. Figure 22 illustrates the concept for a two-dimensional 8 grid. To maintain the grid's integrity after the movement of the grid points the neighborhoods were limited by half the distance Δx of two neighboring grid points.

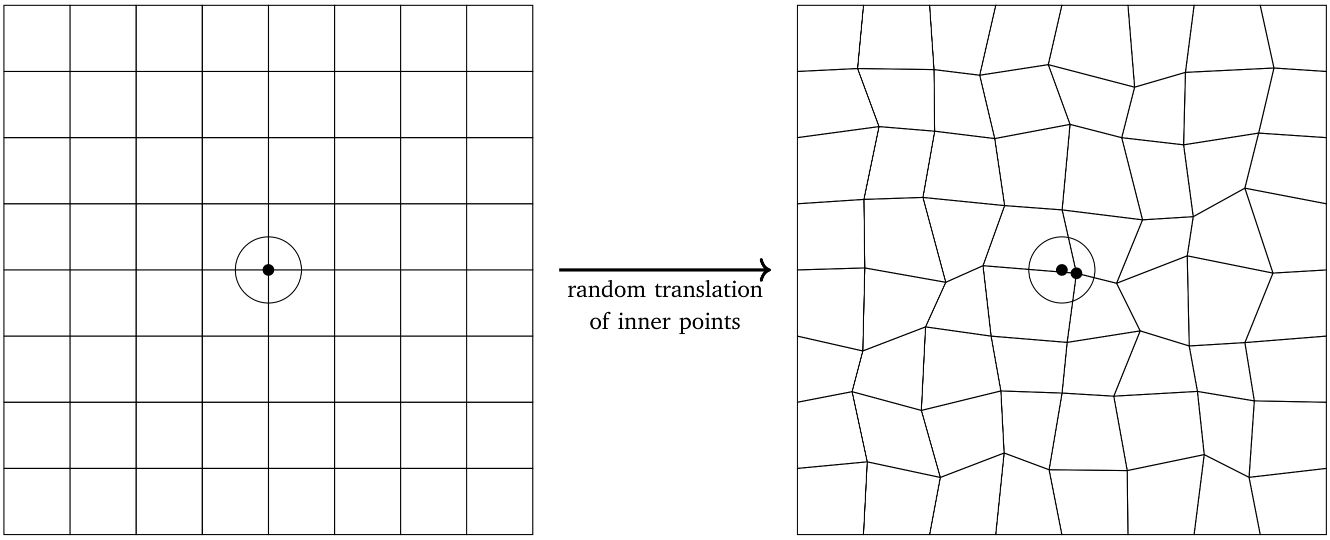

Figure 22: Skewing of an initially equidistant structured orthogonal grid via random movement of inner grid points within a small neighborhood of their original location. The neighborhood with maximal diameter is indicated by a circle around an inner grid point

To measure the effect of different non-orthogonal corrections on the solution process, tests for different skewed grids were performed, measuring the number of needed outer iterations. The grid skewness was parametrized with the relative diameter α of the maximal neighborhood, which constrains the movement of grid points. $\alpha \in [0, 1)$, where $\alpha = 0$ corresponds to no movement of grid points at all and $\alpha = 1$ would move grid points up to $\frac{\Delta x}{2}$ away from their original location. The choice of $\alpha = 1$ is not permitted to maintain the grid's integrity. Such a choice would permit the grid generator to place two grid points at the same location.

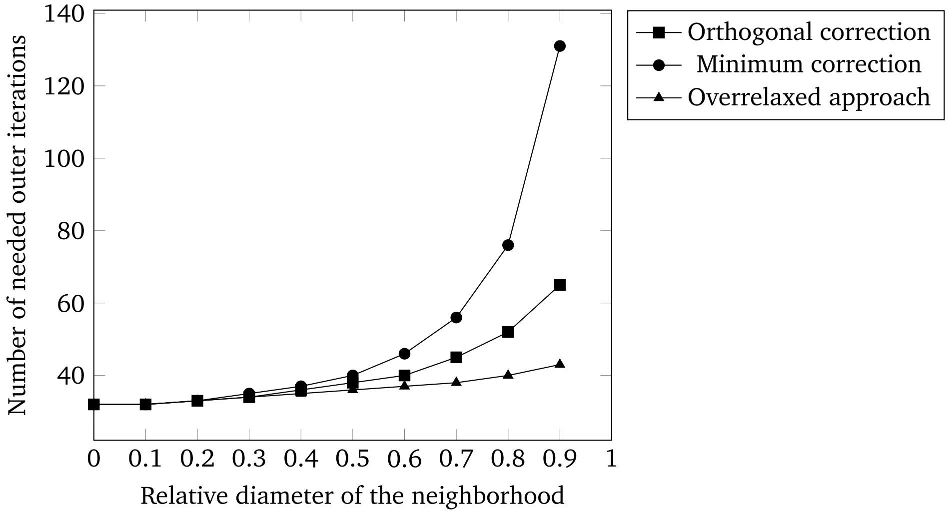

Figure 23: Number of needed outer iterations for different relative diameters of the neighborhood in which the movement of grid points takes place parametrized by the orthogonal corrector

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