PAG Ionisation energy of a thermistor

Izaak van Dongen

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```r
library(ggplot2)

k = 1.3806488e-23

U_I = 0.01e-3
U_T = 0.2

V_values <- c(5.11, 5.08, 5.15, 5.27)
V <- mean(V_values)
V

current_df <- read.table("data.csv", sep=",")
colnames(current_df) <- c("T_celsius", "I_ma")

current_df$T <- current_df$T_celsius + 273.15
current_df$I <- current_df$I_ma / 1000

current_df$T_recip = 1 / current_df$T
current_df$ln_I = log(current_df$I)

pc_U_I = U_I / median(current_df$I)
pc_U_T = U_T / median(current_df$T)

pc_U_I
pc_U_T

current_df

nl_model <- nls(I~A*exp(-E/(k * T)),
                start=list(A=4, E=0.3 * 1.6e-19), data=current_df)
nl_model

qplot(T, I, data=current_df,
      main="Current through thermistor against temperature",
      xlab="T / K", ylab="I / A") +
      geom_errorbar(data=current_df, mapping=aes(ymin=I-U_I, ymax=I+U_I)) +
      geom_errorbarh(data=current_df, mapping=aes(xmin=T-U_T, xmax=T+U_T)) +
      theme(panel.grid.minor = element_line(colour="gray", size=0.4),
            panel.grid.major = element_line(colour="gray", size=1),
            panel.background = element_blank()) +
      scale_y_continuous(minor_breaks = seq(0.1e-3, 1.1e-3, 0.2e-4),
```

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                              breaks = seq(0.1e-3, 1.1e-3, 0.1e-3)) +
       scale_x_continuous(minor_breaks = seq(280, 320, 1),
                              breaks = seq(280, 320, 5)) +
       geom_line(aes(y = predict(nl_model)), color="steelblue")

model <- lm(ln_I ~ T_recip, data=current_df)
model

qplot(T_recip, ln_I, data=current_df,
       main="ln(I) against 1/T",
       xlab="K / T", ylab="ln(I) - ln(A)") +
       geom_errorbar(data=current_df,
                        mapping=aes(ymin=ln_I - pc_U_I,
                                    ymax=ln_I + pc_U_I)) +
       geom_errorbarh(data=current_df,
                        mapping=aes(xmin=(1 - pc_U_T) * T_recip,
                                    xmax=(1 + pc_U_T) * T_recip)) +
       theme(panel.grid.minor = element_line(colour="gray", size=0.4),
             panel.grid.major = element_line(colour="gray", size=1),
             panel.background = element_blank()) +
       scale_y_continuous(minor_breaks = seq(-10, -5, 0.05),
                              breaks = seq(-10, -5, 0.2)) +
       scale_x_continuous(minor_breaks = seq(0.003, 0.004, 0.00002),
                              breaks = seq(0.003, 0.004, 0.0001)) +
       geom_line(aes(y = predict(model)), color="steelblue")
```

Listing 1: R source

```
[1] 5.1525
[1] 0.01818182
[1] 0.0006685609
   T_celsius I_ma      T       I     T_recip      ln_I
1          9 0.28 282.15 0.00028 0.003544214 -8.180721
2         10 0.29 283.15 0.00029 0.003531697 -8.145630
3         11 0.30 284.15 0.00030 0.003519268 -8.111728
4         12 0.31 285.15 0.00031 0.003506926 -8.078938
5         13 0.33 286.15 0.00033 0.003494671 -8.016418
6         14 0.34 287.15 0.00034 0.003482500 -7.986565
7         15 0.35 288.15 0.00035 0.003470415 -7.957577
8         16 0.37 289.15 0.00037 0.003458413 -7.902008
9         17 0.39 290.15 0.00039 0.003446493 -7.849364
10        18 0.40 291.15 0.00040 0.003434656 -7.824046
11        19 0.42 292.15 0.00042 0.003422899 -7.775256
12        20 0.43 293.15 0.00043 0.003411223 -7.751725
13        21 0.45 294.15 0.00045 0.003399626 -7.706263
14        22 0.47 295.15 0.00047 0.003388108 -7.662778
15        23 0.48 296.15 0.00048 0.003376667 -7.641724
16        24 0.51 297.15 0.00051 0.003365304 -7.581100
17        25 0.53 298.15 0.00053 0.003354016 -7.542634
18        26 0.55 299.15 0.00055 0.003342805 -7.505592
19        27 0.57 300.15 0.00057 0.003331667 -7.469874
```

```
20         28 0.59 301.15 0.00059 0.003320604 -7.435388
21         29 0.61 302.15 0.00061 0.003309614 -7.402052
22         30 0.63 303.15 0.00063 0.003298697 -7.369791
23         31 0.66 304.15 0.00066 0.003287851 -7.323271
24         32 0.68 305.15 0.00068 0.003277077 -7.293418
25         33 0.70 306.15 0.00070 0.003266373 -7.264430
26         34 0.73 307.15 0.00073 0.003255738 -7.222466
27         35 0.75 308.15 0.00075 0.003245173 -7.195437
28         36 0.78 309.15 0.00078 0.003234676 -7.156217
29         37 0.81 310.15 0.00081 0.003224246 -7.118476
30         38 0.84 311.15 0.00084 0.003213884 -7.082109
31         39 0.87 312.15 0.00087 0.003203588 -7.047017
32         40 0.89 313.15 0.00089 0.003193358 -7.024289
33         41 0.93 314.15 0.00093 0.003183193 -6.980326
34         42 0.96 315.15 0.00096 0.003173092 -6.948577
35         43 1.01 316.15 0.00101 0.003163056 -6.897805
Nonlinear regression model
  model: I ~ A * exp(-E/(k * T))
   data: current_df
        A         E
4.003e+01 4.627e-20
 residual sum-of-squares: 6.51e-10

Number of iterations to convergence: 11
Achieved convergence tolerance: 2.081e-06

Call:
lm(formula = ln_I ~ T_recip, data = current_df)

Coefficients:
(Intercept)      T_recip
      3.695    -3353.201
```

Listing 2: Model results

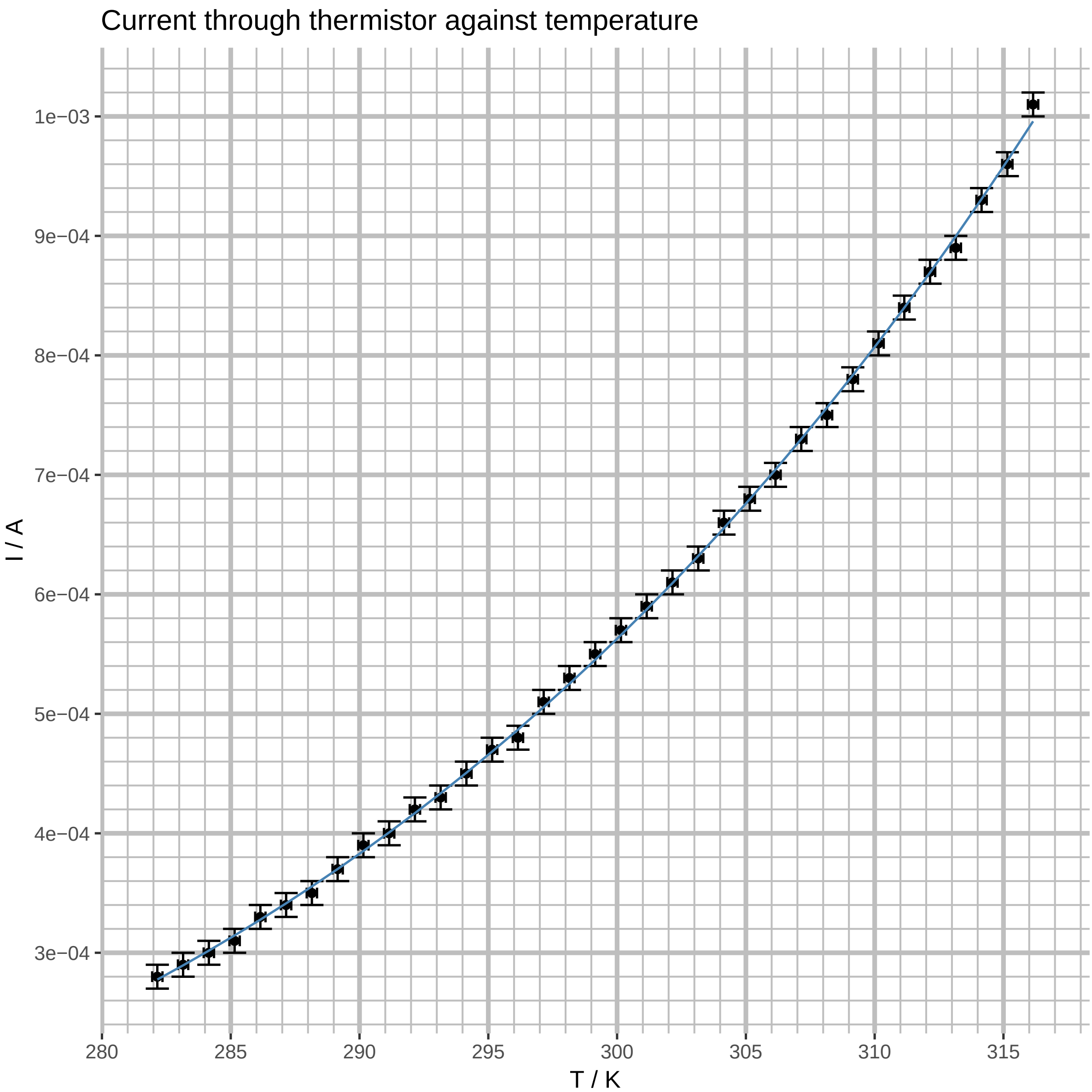

Figure 1: Graph of I/A against T/K

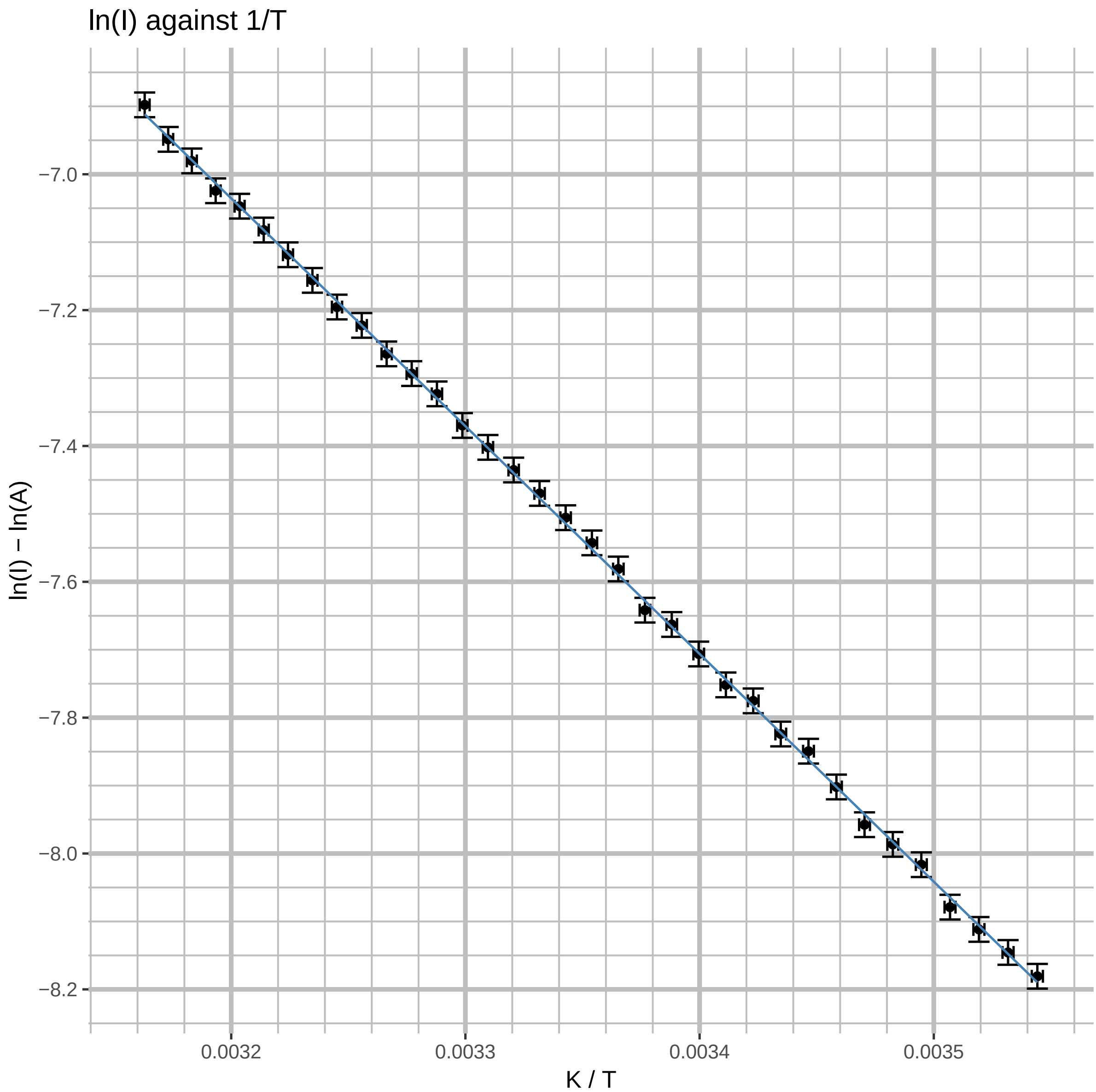

Figure 2: Linearised graph $\ln I - \ln A$ against K/T

The nonlinear regression in figure 1 found a value for ε as 4.627×10^{-20} J.

The linear regression in figure 2 found $-\frac{\varepsilon}{k} = -3353.201\,\mathrm{K} \Rightarrow \varepsilon = 3353.201 \times 1.381 \times 10^{-23}\,\mathrm{J} = 4.629 \times 10^{-20}\,\mathrm{J}$

So $\varepsilon = 4.63 \times 10^{-20}$ J to 3 SF is consistent with both analyses.

Alternatively expressed, $\varepsilon = 0.290\,\mathrm{eV} = 27.9\,\mathrm{kJ\,mol^{-1}} \approx 11kT$, where $T = 300\,\mathrm{K}$.