PAG: Determination of the half-thickness of aluminium

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By linear regression of $\ln(I/\mathrm{Bq})$ (Figure 1) against x/mm, I determined the value of the half thickness of aluminium as $x_{1/2} = 0.000\,379\,6\,\mathrm{m}$, or about $0.38\,\mathrm{mm}$. As an order of magnitude, this is consistent with common sense, as, very roughly speaking, the intensity halved when adding about $0.5\,\mathrm{mm}$ of aluminium.

A less precise calculation from the fitted exponential curve in Figure 2, and particularly the consideration of the points $(0.515\,\mathrm{mm}, 18.00\,\mathrm{Bq})$ and $(0.894\,\mathrm{mm}, 9.01\,\mathrm{Bq})$ suggests a half-thickness of about $(0.894 - 0.515)\mathrm{mm} = 0.379\,\mathrm{mm}$, which is consistent with the actual analysis.

```
1  library(ggplot2)
2
3  dt = 10
4  C_bg = (3 + 0 + 2) / 3
5
6  ratio_U_C = 1.1
7  U_x = 0.5e-3 / 100
8
9  N = 500
10
11 beta_df <- read.table("data.csv", sep=",", header=TRUE)
12 beta_df$x <- beta_df$x * 1e-3
13 beta_df$I <- (rowMeans(beta_df[c("C1", "C2", "C3")]) - C_bg) / dt
14
15 show(beta_df)
16 show(log(beta_df$I))
17
18 model <- lm(log(I) ~ x, data=beta_df)
19 summary(model)
20 mu <- coef(summary(model))["x", "Estimate"]
21 x_half = log(2) / mu
22 print(paste("Half thickness: ", x_half))
23
24 qplot(x, log(I), data=beta_df,
25       main="Linearised intensity vs thickness of absorbent aluminium",
26       xlab="x/m", ylab="ln(I/Bq)") +
27       geom_errorbarh(data=beta_df,
28                      mapping=aes(xmin=x-U_x,
29                                  xmax=x+U_x)) +
30       geom_errorbar(data=beta_df,
31                     mapping=aes(ymin=log(I / ratio_U_C),
32                                 ymax=log(I * ratio_U_C))) +
```

```
      theme(panel.grid.minor = element_line(colour="gray", size=0.4),
            panel.grid.major = element_line(colour="gray", size=1),
            panel.background = element_blank()) +
      geom_smooth(method="lm")

prediction <- data.frame(
    x = c(beta_df$x, seq(min(beta_df$x), max(beta_df$x), length.out=N)),
    I = c(beta_df$I, rep(NA, N))
)
prediction$pred <- exp(predict(model, prediction))

#show(prediction)

qplot(x, I, data=beta_df,
      main="Intensity vs thickness of absorbent aluminium",
      xlab="x/m", ylab="I/Bq") +
      geom_errorbarh(data=beta_df,
                     mapping=aes(xmin=x-U_x,
                                 xmax=x+U_x)) +
      geom_errorbar(data=beta_df,
                    mapping=aes(ymin=(I / ratio_U_C),
                                ymax=(I * ratio_U_C))) +
      theme(panel.grid.minor = element_line(colour="gray", size=0.4),
            panel.grid.major = element_line(colour="gray", size=1),
            panel.background = element_blank()) +
      geom_line(size=1, color="steelblue", data = prediction, aes(y = pred))
```

Listing 1: Source code of the program `analyse.r`.

```
        x  C1  C2  C3         I
1 0.0005 178 155 173 16.700000
2 0.0007 126 136 119 12.533333
3 0.0010  83  64  90  7.733333
4 0.0012  55  56  58  5.466667
5 0.0014  37  37  41  3.666667
6 0.0015  39  39  37  3.666667
7 0.0017  23  15  29  2.066667
8 0.0019  18  14   8  1.166667
9 0.0020  15  13  13  1.200000
[1] 2.8154087 2.5283918 2.0455400 1.6986690 1.2992830 1.2992830 0.7259370
[8] 0.1541507 0.1823216

Call:
lm(formula = log(I) ~ x, data = beta_df)

Residuals:
      Min        1Q    Median        3Q       Max
-0.207413 -0.024306  0.003353  0.040626  0.207342

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)     3.8309     0.1164   32.91 6.19e-09 ***
x           -1825.9453    82.5616  -22.12 9.76e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1212 on 7 degrees of freedom
Multiple R-squared:  0.9859,        Adjusted R-squared:  0.9839
F-statistic: 489.1 on 1 and 7 DF,  p-value: 9.764e-08

[1] "Half thickness:  -0.000379610048983322"
```

Listing 2: Output of `analyse.r` (1) when run.

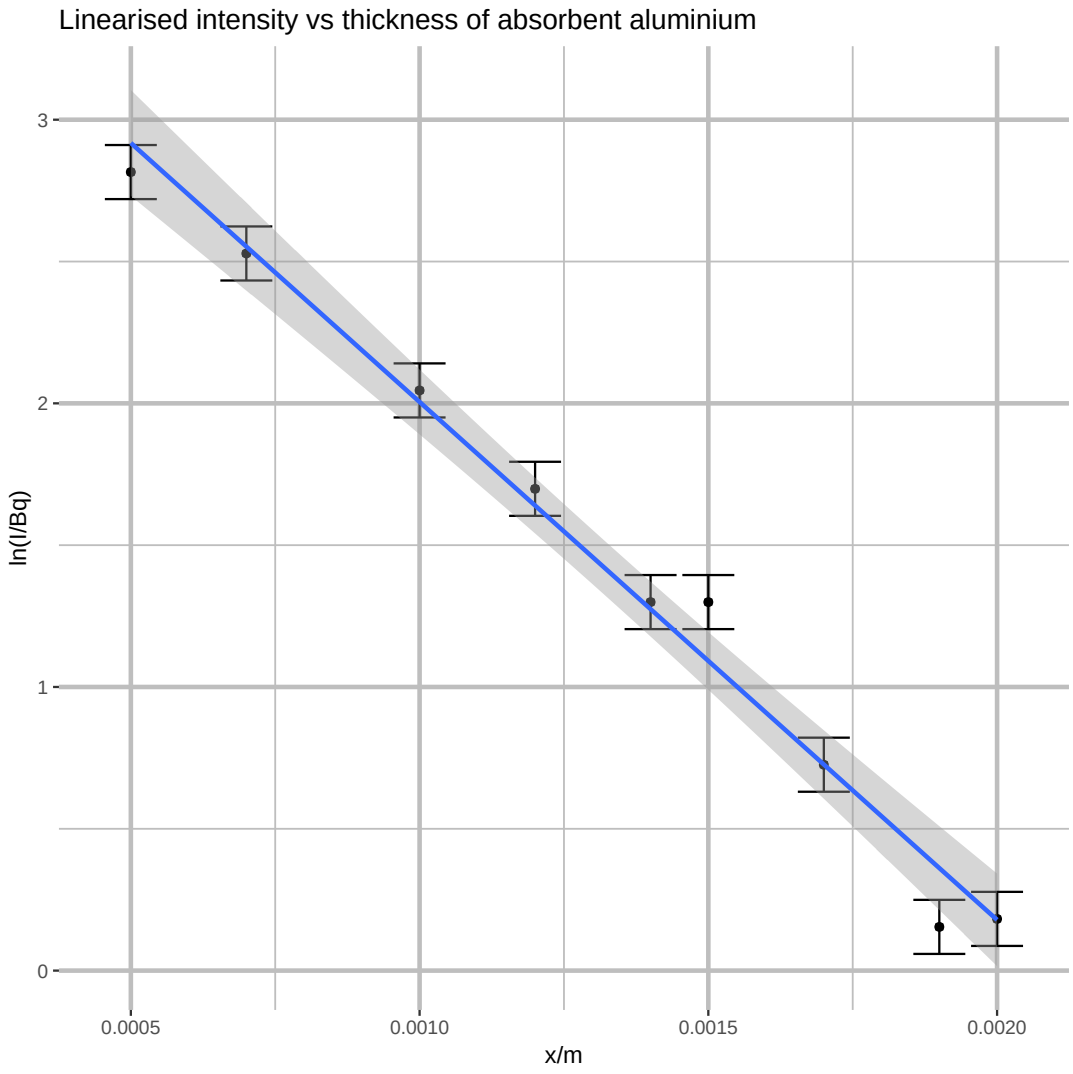

Figure 1: Plot of $\ln(I/\mathrm{Bq})$ against x/mm

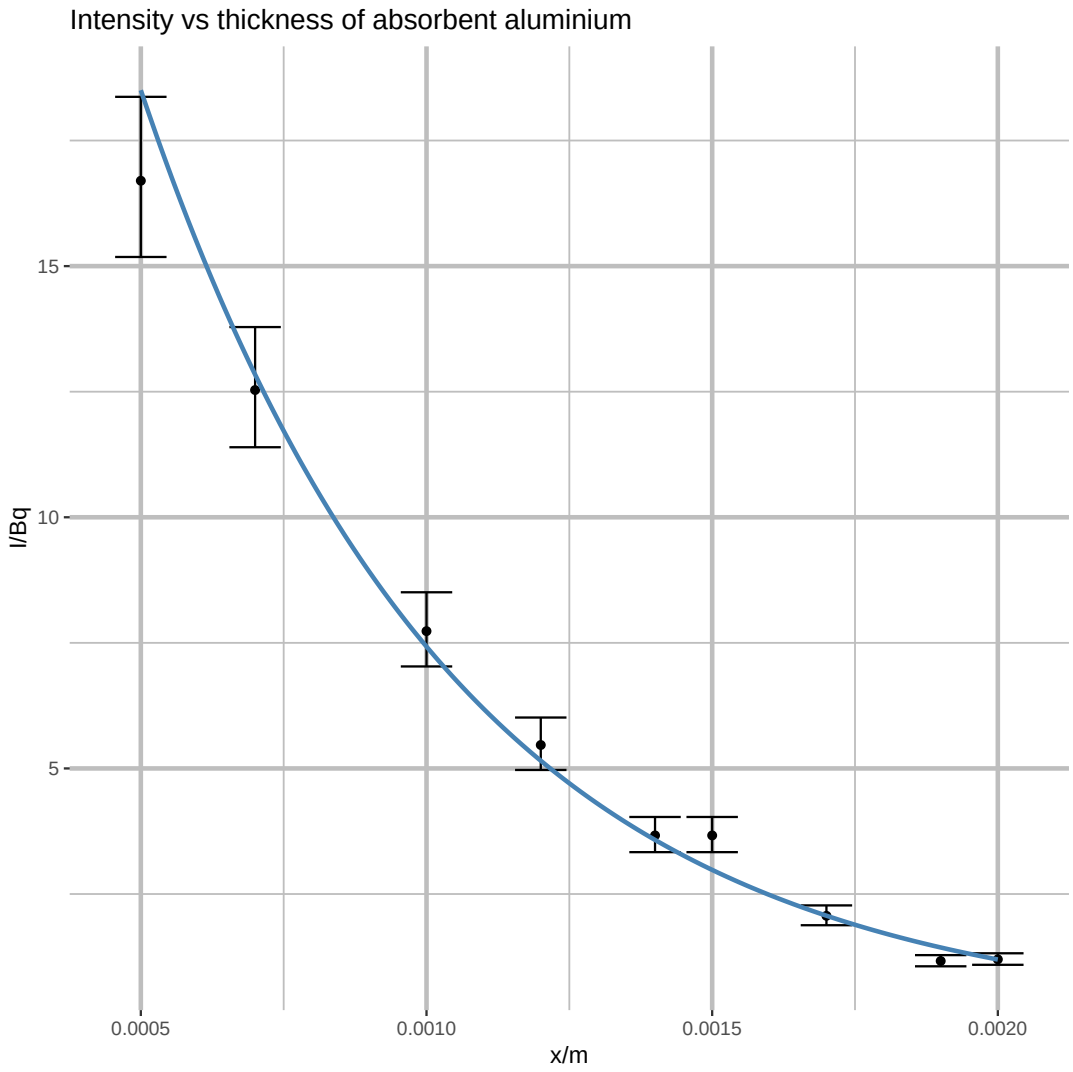

Figure 2: Plot of maximum displacements in experiment 2.