PAG Static & Dynamic Analysis of Spring Constant

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```r
library(ggplot2)

U_x = 0.002
pc_U_m = 5
U_20T = 0.2
U_T = U_20T / 20

g = 9.80665

static_df <- read.table("static.csv", sep=",", header=TRUE)

static_df$x <- rowMeans(subset(static_df, select = c(x1, x2, x3, x4, x5)))
static_df$F <- g * static_df$m

static_df

static_model <- lm(F ~ x, data=static_df)
summary(static_model)

qplot(x, F, data=static_df,
      main="Force against extension in a spring (Hooke's Law)",
      xlab="x / m", ylab="F / N") +
      geom_errorbar(mapping=aes(ymin=g * (m * (100 + pc_U_m) / 100),
                                ymax=g * (m * (100 - pc_U_m) / 100),
                                width=0.005)) +
      geom_errorbarh(mapping=aes(xmin=x - U_x,
                                 xmax=x + U_x,
                                 height=0.1)) +
      theme(panel.grid.minor = element_line(colour="gray", size=0.4),
            panel.grid.major = element_line(colour="gray", size=1),
            panel.background = element_blank()) +
      scale_y_continuous(minor_breaks = seq(0, 7, 0.2),
                         breaks = seq(0, 7, 1)) +
      scale_x_continuous(minor_breaks = seq(0.5, 1, 0.01),
                         breaks = seq(0.5, 1, 0.05)) +
      geom_smooth(method = "lm", color = "red", size=.1)

dynamic_df <- read.table("dynamic.csv", sep=",", header=TRUE)
dynamic_df$T <- rowMeans(subset(dynamic_df, select=c(T1, T2, T3, T4, T5))) / 20
dynamic_df$plot_y <- (2 * pi / dynamic_df$T) ^ 2
```

```r
dynamic_df$plot_x <- 1 / dynamic_df$m

dynamic_df

dynamic_model <- lm(plot_y ~ plot_x, data=dynamic_df)
summary(dynamic_model)

qplot(plot_x, plot_y, data=dynamic_df,
      main="Dynamic analysis of spring constant: Period against mass",
      xlab="kg / m", ylab="(Angular Frequency)^2/Hz^2") +
      geom_errorbar(mapping=aes(ymin=(2 * pi / (T + U_T))^2,
                                ymax=(2 * pi / (T - U_T))^2),
                                width=0.2) +
      geom_errorbarh(mapping=aes(xmin=1 / (m * (100 + pc_U_m) / 100),
                                 xmax=1 / (m * (100 - pc_U_m) / 100))) +
      theme(panel.grid.minor = element_line(colour="gray", size=0.4),
            panel.grid.major = element_line(colour="gray", size=1),
            panel.background = element_blank()) +
      scale_y_continuous(minor_breaks = seq(0, 250, 10),
                         breaks = seq(0, 250, 50)) +
      scale_x_continuous(minor_breaks = seq(0, 11, 0.4),
                         breaks = seq(0, 11, 2)) +
      geom_smooth(method = "lm", color = "steelblue", size=.1)
```

Listing 1: R source

```
    m    x1    x2    x3    x4    x5      x        F
1 0.1 0.585 0.586 0.587 0.587 0.588 0.5866 0.980665
2 0.2 0.625 0.626 0.625 0.628 0.627 0.6262 1.961330
3 0.3 0.663 0.665 0.664 0.665 0.664 0.6642 2.941995
4 0.4 0.702 0.704 0.701 0.704 0.704 0.7030 3.922660
5 0.5 0.741 0.742 0.739 0.742 0.739 0.7406 4.903325
6 0.6 0.776 0.777 0.775 0.776 0.780 0.7768 5.883990

Call:
lm(formula = F ~ x, data = static_df)

Residuals:
        1         2         3         4         5         6
 0.027502 -0.011303 -0.008916 -0.027125 -0.014441  0.034284

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -14.1484     0.1195  -118.4 3.06e-08 ***
x            25.7442     0.1742   147.7 1.26e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.02776 on 4 degrees of freedom
Multiple R-squared:  0.9998,	Adjusted R-squared:  0.9998
F-statistic: 2.183e+04 on 1 and 4 DF,  p-value: 1.259e-08
```

```

    m    T1    T2    T3    T4    T5      T    plot_y    plot_x
1 0.1  7.61  7.90  8.15  8.22  8.23 0.4011 245.38862 10.000000
2 0.2 11.54 11.24 11.16 11.31 11.23 0.5648 123.75717  5.000000
3 0.3 13.42 13.64 13.68 13.40 13.53 0.6767  86.21193  3.333333
4 0.4 15.71 15.69 15.50 15.55 15.52 0.7797  64.93886  2.500000
5 0.5 17.33 17.42 17.31 17.19 17.26 0.8651  52.75056  2.000000
6 0.6 18.63 18.64 18.60 18.69 18.88 0.9344  45.21620  1.666667

Call:
lm(formula = plot_y ~ plot_x, data = dynamic_df)

Residuals:
      1       2       3       4       5       6
 0.3031 -1.2932  1.1733 -0.0939 -0.2787  0.1893

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   5.0152     0.6427   7.804  0.00145 **
plot_x       24.0070     0.1289 186.242 4.99e-09 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.9032 on 4 degrees of freedom
Multiple R-squared:  0.9999,	Adjusted R-squared:  0.9999
F-statistic: 3.469e+04 on 1 and 4 DF,  p-value: 4.986e-09
```

Listing 2: Model results

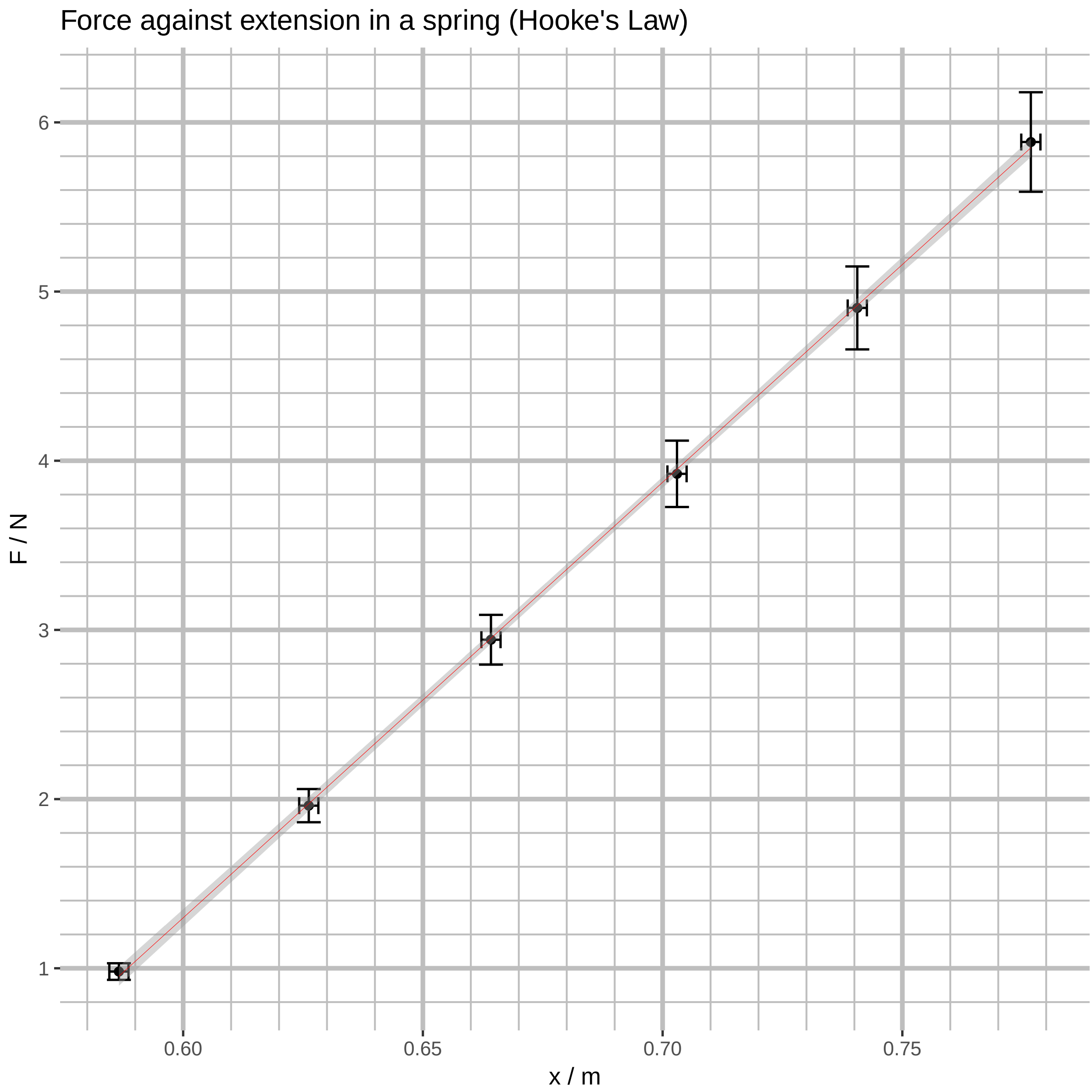

Figure 1: Static analysis: Graph of F/N against x/m

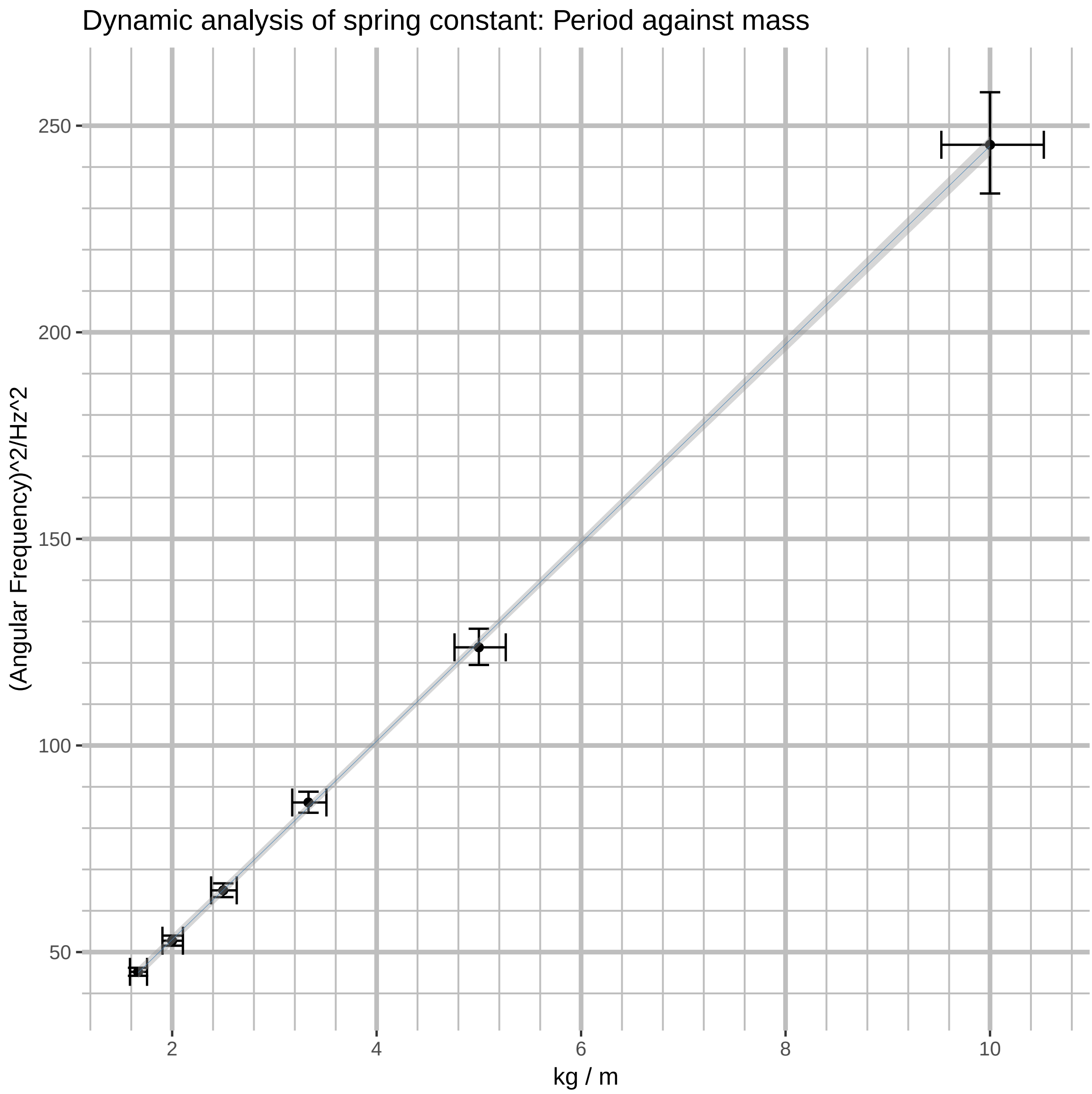

Figure 2: Dynamic analysis: Graph of ω^2/Hz^2 against kg$/m$

This uses the linearisation from $\omega^2 = \frac{k}{m} \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \left(\frac{1}{m}\right) k$

The linear regression in figure 1 calculated $k \pm \sigma_{est}$ as $(25.7442 \pm 0.1742)\,\mathrm{N\,m^{-1}}$.

The linear regression in figure 2 calculated $k \pm \sigma_{est}$ as $(24.0070 \pm 0.1289)\,\mathrm{N\,m^{-1}}$.

This discrepancy can most likely be explained by the fact that when the spring was oscillating, the stand was also oscillating, probably causing some kind of damping.

I'm inclined to prefer the static experiment (Graph 1), as the uncertainty remains globally quite low and it produces data that it's easier to fit a line to (even after linearisation), while not introducing any systematic error from air resistance or shaking apparatus.