# Circular motion PAG

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Initially, the linear regression shown in Figure 1 gives a value of $g = (8.3 \pm 0.6)\,\mathrm{ms}^{-2}$ (Listing 1). However, in doing the experiment I noticed the large impact of static frictional effects in the larger radii.

Here, a lower velocity can be sustained without the string slipping, because it becomes harder to sustain the motion of the bung in the plane perpendicular to the ground, and there is an increased normal interaction between the tip of the tube and the string to counteract the downwards component of its tension. This results in a greater frictional force, so the string won't slip.

I especially felt this effect in the longest radius we tested, of 680 mm, and we had some trouble taking measurements for this. It can also be seen in Figure 1 that the most extreme point seems to lie below the regression line.

Excluding this point yields the plot in Figure 2. This gives a value of $g = (9.5 \pm 0.5)\,\mathrm{ms}^{-2}$ (Listing 2), which is obviously a much more desirable result.

Particularly, in excluding this point, the RSE is reduced from 0.03487 to 0.02276 (Listings 1, 2). This reduction of 34.7% is, in my mind, ample justification of this exclusion.

```
Call:
lm(formula = y ~ x, data = time_df)

Residuals:
      Min        1Q    Median        3Q       Max
-0.043362 -0.016395  0.002467  0.013296  0.058110

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.07442    0.02332   3.191   0.0188 *
x            8.33833    0.56323  14.804 5.97e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.03487 on 6 degrees of freedom
Multiple R-squared:  0.9734,	Adjusted R-squared:  0.9689
F-statistic: 219.2 on 1 and 6 DF,  p-value: 5.973e-06
```

Listing 1: Model results of (1)

```
Call:
```

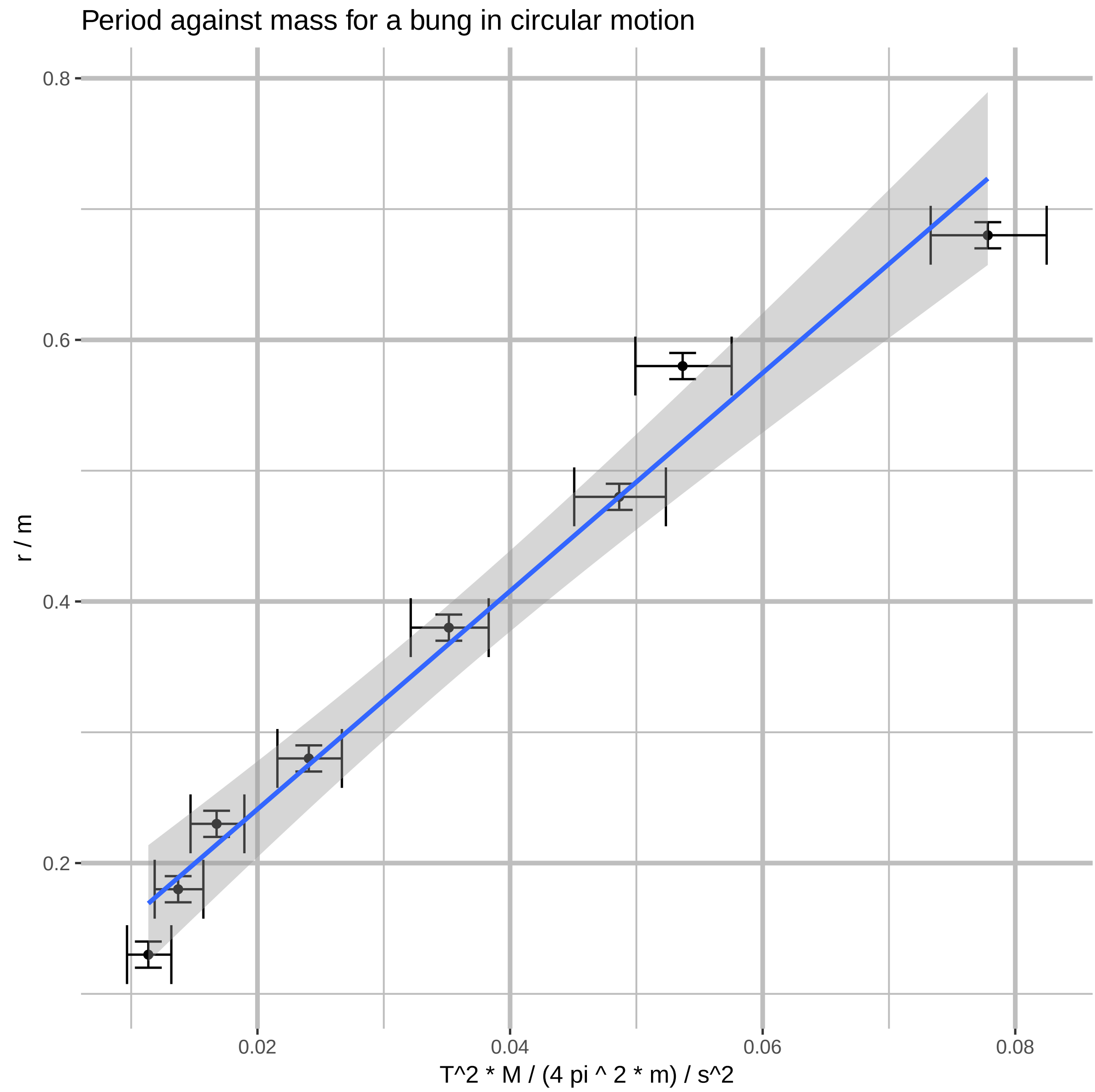


Figure 1: Plot of $r$/m against $\dfrac{T^2 M}{4\pi^2 m}/\mathrm{s}^2$

```
lm(formula = y ~ x, data = time_df)

Residuals:
        1         2         3         4         5         6         7 
-0.029859 -0.001103  0.004765  0.024447  0.003507 -0.023940  0.022183 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.04552    0.01799    2.53   0.0525 .  
x            9.54637    0.54401   17.55  1.1e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.02276 on 5 degrees of freedom
```

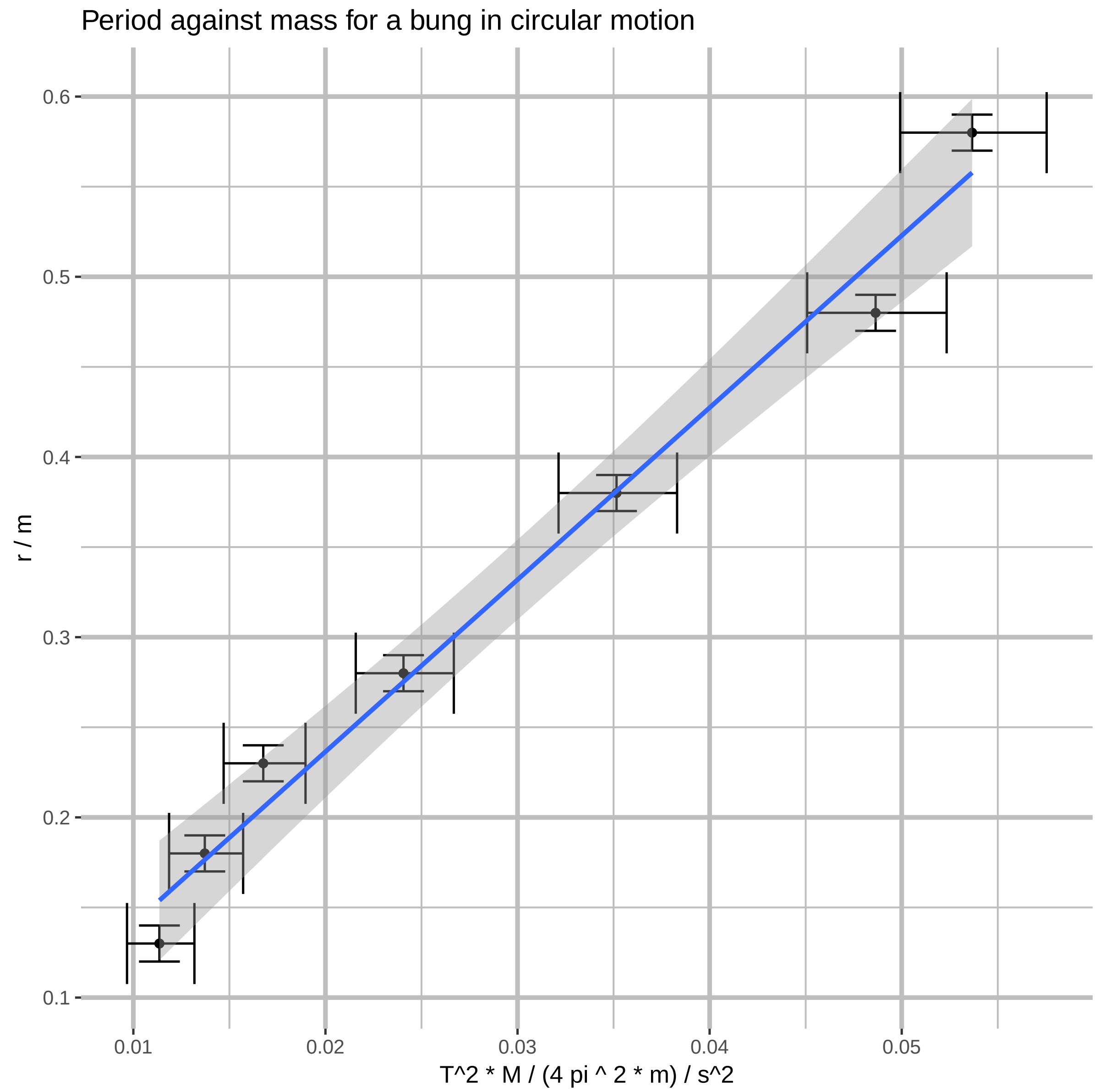


Figure 2: Same plot as (1), but removing the largest datapoint

```
Multiple R-squared:  0.984,	Adjusted R-squared:  0.9808
F-statistic: 307.9 on 1 and 5 DF,  p-value: 1.102e-05
```

Listing 2: Model results of (2)

```r
library(ggplot2)

U_T = 0.02
U_r = 0.01

M = 1e-3 * (59.69 + 59.70 + 59.67) / 3
m = 1e-3 * (8.99 + 8.9 + 8.90) / 3

time_df <- read.table("data.csv", sep=",")
colnames(time_df) <- c("r_mm", "10T1", "10T2", "10T3")
```

```r
time_df$T_avg <- rowMeans(time_df[c("10T1", "10T2", "10T3")]) / 10
time_df$r <- time_df$r / 1000

time_df$x = time_df$T_avg^2 / (4 * pi ^2 * m / M)
time_df$y = time_df$r

model <- lm(y ~ x, data=time_df)
summary(model)

qplot(x, y, data=time_df,
      main="Period against mass for a bung in circular motion",
      xlab="T^2 * M / (4 pi ^ 2 * m) / s^2", ylab="r / m") +
      geom_errorbarh(data=time_df,
                     mapping=aes(xmin=(T_avg - U_T)^2 / (4 * pi ^ 2 * m / M),
                                 xmax=(T_avg + U_T)^2 / (4 * pi ^ 2 * m / M)))
                                 ↪ +
      geom_errorbar(data=time_df, mapping=aes(ymin=r-U_r, ymax=r+U_r)) +
      theme(panel.grid.minor = element_line(colour="gray", size=0.4),
            panel.grid.major = element_line(colour="gray", size=1),
            panel.background = element_blank()) +
      geom_smooth(method="lm")
```